Stable Gait Optimization for Small-Sized Humanoid Robot Using CFO

Tran Thien Huan, Khuu Bach Thy, Nguyen Ho Hieu Trung, Ho Pham Huy Anh

Abstract—This paper proposes a new way to optimize the gait design for human robots that allows stable stepping with preset foot-lifting magnitude. The novel Central Force Optimization (CFO) algorithm is used to optimize the gait parameters to help humanoid robot walk steadily. The efficiency of the proposed method is compared with the GA-Genetic Algorithm, PSO-Particle Swarm Optimization and improved differential evolution algorithm (MDE-Modified Differential Evolution). The simulated and experimental results applied on the small-sized humanoid robot show that the newly proposed algorithm offers an efficient and stable gait for humanoid robots with accurate foot-lifting magnitude.

Keywords—Gait optimization for humanoid robot, Zero Moment Point (ZMP), inverse and forward kinematics, Central Force Optimization (CFO) algorithm, Modified Differential Evolution (MDE) algorithm, Particle Swarm Optimization (PSO), Genetic Algorithm (GA).

I. INTRODUCTION

The walking gesture of human always contains many so sophisticated points that until now the biped robot still can not fully demonstrate. Therefore, the research of humanoid robot walking mechanism is being developed in different domains. Several standards have been applied to human robots to ensure stable and natural gait. Static walking is the first applied principle, in which the vertical projection of center of mass (COM) on the ground is always in the supporting foot. In other words, humanoid robots can stop at any times when walking without falling apart. By its very simple concept, this principle is efficiently applied to slowspeed biped robots whereby dynamic effects can be ignored [1-2]. Then researchers began to focus on developing dynamic walking [3]. This method allows the humanoid robot to speed up the pace. However, during the biped robot walking process, the robot may fall due to environmental interference and biped can not stop abruptly. Therefore, ZMP-based walking is proposed to help control and to efficiently manipulate biped inertia [4-5].

Recently, several studies have focused on improving the performance of biped robot walking gesture. Huang in [6] introduces a stable gait using the gaiter to use the interpolation function. The method developed by D. Huan, through the GA algorithm, optimizes the generator to help robot move steadily with the least amount of energy [7]. Dip et al. [8] exhibit steady gait at constant velocity using the sine wave generator. Intelligent algorithms are applied in this method to optimize the gait generator for humanoid

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robots such as genetic algorithm (GA) [8], algorithm for optimal swarm (PSO) [9], advanced differential algorithm (MDE) [10]. Shaffi in [11] introduces the biped robot which achieves a stable gait by using the Fourier series gait generator. This method uses intelligent algorithms to develop human robot walking movement, such as the bee swarm algorithm [12], the fuzzy TS controller [13], the evolution algorithm [14]. In particular, the very powerful potentiality of the CFO-Central Force Optimization algorithm has not yet to be applied to optimally identify the biped robot's walking gait parameters.

To overcome this gap, this paper proposes the Central Force Optimization (CFO) algorithm used to optimally identify the biped walking gait parameters, based on the dynamic walking method, and use ZMP concept to maintain biped stability. The simulation and empirical results show that the newly proposed CFO algorithm allows optimal identification of the gait parameters for the biped robot to obtain steady gait with accurate foot-lifting magnitude. Small-sized humanoid robot model, namely HUBOT-5, is used to investigate the experimental results.

II. HUMANOID ROBOT MODEL

Small-sized humanoid robot (HUBOT - 5) has the upper torso and two legs as described in figure 1. Each leg consists of 3 parts, which are femoral, legs, and foot with a total of 6 DOF (Degrees Of Freedom). There are 3 DOF at the hip, 1 at the knee, HUBOT – 5 can mimic the walking gesture of human according to the front size interface (YZ – frontal view) and the side view (XZ - Sagittal view). Total weight of the HUBOT is about 1,5 kg (including the dynamic actuator, sensor, controller and amplifier) and has a height of about 50 cm. HUBOT - 5 is designed to ensure full dynamic structure, with each dof is corresponding with 1 independent actuator. The Servo DC engine HD - 1501 is used as the actuator. The significant advantage of HD – 1501 is small, compact and light (60 g) with twisted momentum 17 kg high. The control signal of the servo is obtained via MATLAB Simulink with the RS-485 transmission standard.

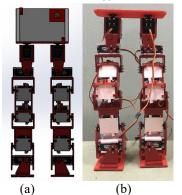


Figure 1. HUBOT-5 humanoid robot configuration with 12 DOF

This research only focus on the straight walking gait of humanoid robot, then the upper torso is fixated with only the 10 engines of 10 DOF being controlled and defined in Fig. 2.

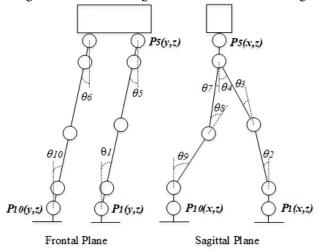


Figure 2. Humanoid HUBOT-5 structural model.

III. GAIT GENERATION FOT BIPED ROBOT

Four important variables of the humanoid robot that play an essential role in Stable gait generation, are S-Walking length [m], H-Leg lifting [m], h-Leg kneeling [m] and n-Hip swinging are described in Figure 3. In which, d0 is the height of the torso, d1 is the distance between the 2 dof at the knee joints, d2 is the length of the leg, d3 is the length of the femoral and d4 represents the distance between 2 hips.

As described in Figure 3, the Hip trajectory $P_5 = [P_{5x}, P_{5y}, P_{5z}]$ and ankle trajectory $P_1 = [P_{1x}, P_{1y}, P_{1z}]$ of the supporting leg, ankle trajectory $P_{10} = [P_{10x}, P_{10y}, P_{10z}]$ of the moving legs will depend on 4 variables (S, H, h, n) in both the frontal $(YZ-Frontal\ View)$ and sagittal $(XZ-Sagittal\ View)$ interface.

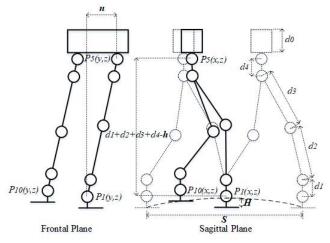


Figure 3. Four Variables influences the human walking gait of Humanoid Robot HUBOT - 5

The trajectories P_1 , P_5 , P_{10} are sine-time dependent, as consulted in [15]. Using the equations described in [10], 10

trajectories of the 2 leg rotary angles of HUBOT-5 in one interval cycle are used to control the walking gait.

Thus the set of four parameters H, h, s and n needs to be carefully selected so that the ZMP parameter ensures that the human robot can walk steadily with the foot-lifting set up in advance. This paper uses the CFO algorithm to satisfactorily address the above problem.

IV. GAIT PARAMETERS OPTIMIZATION USING CFO

A. CFO algorithm

The CFO was introduced in 2007 by R. A. Formato [16]. The algorithm is a somewhat new optimization technique which is nature-based, multi-dimensional, metaheuristic, and based heavily on the gravitational kinematics concept, for example, object fly through space. Unlike other evolutionary algorithm (GA, PSO), CFO does not have randomness characteristics, and that makes the algorithm deterministic.

CFO models inspired from the idea of "probe" flying through spaces, the probes will also be attracted by another probe which has equal or greater mass, and the heavier probe will not be influenced. The probes are model based on known equation emphasis on physical attributes, so their path can be mimical and no random changes will be made there after. The position of the probes will be updated continuously, along with their accelerations. The process will continue until global optimum solution is found.

The CFO algorithm consists of following steps:

- 1. Calculate probe initial position, evaluate their value and their accelerations, computed their mass.
- 2. Based on the evaluated acceleration, find new position of the probes.
- 3. Check if each probe mass and position is located inside the decision space, or whether is it converged.
- 4. Update the fitness value with new position.
- 5. Update acceleration.
- 6. Loop until stopping the criterion is satisfied.

CFO notable parameters:

- Alpha coefficient is the influence of probes to each other, using the difference of mass. The lower alpha is, the less effect that probe with approx. mass will influence each other.
- *Beta*, as extended distance, determines the remaining influence of probes to each other in space. The lower beta is, the more effect one probe can have to others as great distance.
- Gamma, simply determines accelerations, the higher gamma is, the faster the probe accelerate. This can be good when the user wants to quickly find the solution.
 However, this may have a bad effect on the convergence characteristics.

B. Specify the Objective Function

To evaluate human gait parameters of the humanoid robot, one must define the objective function. The goal of the HUBOT-5 human robot is to achieve a stable gait with a preset lifting amplitude. For this purpose, the ZMP point projection on the foot area according to the ZMP point

principle will be used. When the feet touch the ground, the area of the supporting foot is the area between the two feet of the human robot, and when one foot touches the ground, the foot area is the area of the foot touching the ground.

The sum of the squared distance from the ZMP to the center of the supporting foot in 1 step walking of HUBOT -5 (Equation 1) is the first objective function.

$$f_1 = \int_{0}^{T} \sqrt{x_{ZMP}^2 + y_{ZMP}^2} . dt$$
 (1)

in which T is the stepping cycle and (x_{ZMP}, y_{ZMP}) is the coordination of ZMP in the process of the robot to perform the step. The smaller f_1 is the more stable the gait will become.

Additionally, for the humanoid robot to follow the pre-set lighting height value $-H_{\it ref}$, the difference between the magnitude of the foot lift parameter - and the foot-lift set $-H_{\it ref}$ (Equation 2) is the second objective function. The smaller f_2 is, the more strictly the lifting magnitude will follow $H_{\it ref}$.

$$f_{\gamma} = \left| H_{vef} - H \right| \tag{2}$$

Thus, in order for HUBOT-5 to have a steady gait with the foot set up in advance, we find the minimum value of the two objective functions f_1 and f_2 , or to find the minimum of the function f as:

$$f = \lambda \cdot \left(\int_{0}^{T} \sqrt{x_{\text{ZMP}}^{2} + y_{\text{ZMP}}^{2}} dt \right) + \left(1 - \lambda \right) \cdot \left| H_{\text{ref}} - H \right|$$
 (3)

In which, $\lambda (0 < \lambda \le 1)$ is select to prioritize of the (λ increase) and difference with the desired lifting magnitude (λ decreased).

C. ZMP trajectory calculation

For HUBOT-5, the volume of the suture is very small than the mass of the joints, so the center of the joint is at the end of the joint. Thus, the moment of inertia of the equation is zero, the Equation for ZMP is calculated as (4):

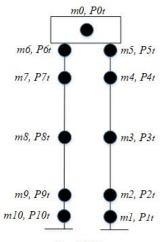
$$\begin{cases} x_{ZMP} = \frac{\sum_{i=1}^{n} m_{i} (\ddot{z}_{i} + g) x_{i} - \sum_{i=1}^{n} m_{i} \ddot{x}_{i} z_{i}}{\sum_{i=1}^{n} m_{i} (\ddot{z}_{i} + g)} \\ y_{ZMP} = \frac{\sum_{i=1}^{n} m_{i} (\ddot{z}_{i} + g) y_{i} - \sum_{i=1}^{n} m_{i} \ddot{y}_{i} z_{i}}{\sum_{i=1}^{n} m_{i} (\ddot{z}_{i} + g)} \end{cases}$$

$$(4)$$

In equation 4, the mass distribution m_i and coordinates (x_i, y_i, z_i) of the stages are defined in Figure 4.

The coordinates $P_{it}(x, y, z)$ of the joints are determined from 10 angles of rotation at one time in one step with the origin coordinatation at the center of the supporting foot by

means of the geometric relation. In which, d_0 , d_1 , d_2 , d_3 and d_4 are illustrated in Figure 3.



Frontal Plane

Figure 4: The mass distribution and coordinates of the stages Finally, the flow chart to calculate the ZMP trajectory from the set of four gait parameters of the human robot is illustrated in Figure 5.

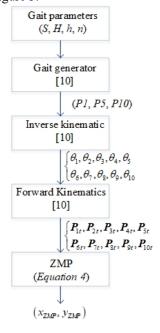


Figure 5. ZMP calculation flow chart

V. SIMULATION AND EXPERIMENT RESULTS

The simulated and experimental results were tested on the HUBOT-5 robot. The physical parameters of the HUBOT-5 robot are presented in Table 1.

TABLE 1. PHYSICAL PARAMTERS OF HUBOT-5

Parameters	Value
d_0	6.000 cm
d_1	4.254 cm
d_2	9.109 cm

d_3	8.063 cm
d_4	9.345 cm
W	8.640 cm
$\begin{cases} m_i \\ i = 010 \end{cases}$	60 gam

In order to find the appropriate value for the coefficients λ of the objective function in Equation (3), by selecting $\lambda = 0.4$ the HUBOT-5 robot has a steady gait with an adjustable foot lift, and this value will be used thorough the testing process of GA, PSO, MDE and proposed CFO. For optimal GA, PSO, MDE and CFO optimization algorithms, gait parameters are limited as given in Table 2.

TABLE 2. BOUNDARIES FOR 4 WALKING GAIT PARAMTERES OF HUMANOID ROBOT HUBOT-5

Parameters	Lower Bound	Upper Bound
S-Step length	1.5 cm	25 cm
H-foot lifting	0.1 cm	10 cm
<i>h</i> -kneeling	0.1 cm	1.5 cm
<i>n</i> -hip swinging	0.1 cm	10 cm

The mathematical properties of GA, PSO, MDE, and CFO optimization algorithms are probabilistic algorithms, so each algorithm performs 10 different training times, each training will repeat 500 times (N=500) with the same population size (NP=32) and the same number of variables (n=4). Table 3 presents GA, PSO, MDE and CFO parameters parameter values.

TABLE 3: PARAMETERS OF GA, PSO, MDE AND CFO ALGORITHM

Method	Paramters	Value	
GA	Mutation Probability	0.4	
	(MP)	0.9	
	Crossover Probability		
	(CP)		
PSO	Accelaration factor (C ₁)	2.0	
	Accelaration factor (C ₂)	2.0	
	Inertia Weight (w)	[0.4; 0.9]	
MDE	Mutation value (F)	Random [0.4;	
	Crossover Probability	1.0]	
	(CR)	Random [0.7;	
		1.0]	
CFO	Alpha	0.25	
	Beta	0.35	
	Gamma	0.95	
	Frep	0.5	
	deltaFrep	0.05	

Specify the lifting height of HUBOT-5 is $H_{ref} = 20mm$.

Table 4 shows the optimum gait value and best value for the target function of 10 runs corresponding to the GA, PSO, MDE and CFO algorithms. Figure 6 illustrates the mean value of the target function after 10 runs of each algorithm (GA: green, PSO: blue, MDE: red, CFO: reddish purple).

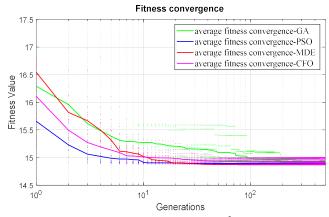


Figure 6: Mean value of f

TABLE 4: OPTIMIZED GAIT VALUE AND BEST TARGET VALUE OF 10 RUNS

Run	OF 10 RUNS. Run Agorithms Walking Gait Parameters value Best					
	11901111111	S	H	h (cm)	n (cm)	firness
		(cm)	(cm)	11 (6111)	1. (e)	value
		(5117)	()			f(cm)
1	GA	15	1.99	0.80	6.86	14.88
	PSO	15	1.99	1.50	6.97	14.89
	MDE	15	1.99	0.80	6.89	14.87
	CFO	15	2.00	0.76	6.53	14.93
2	GA	15	1.99	0.75	6.68	14.88
	PSO	15	2.00	1.50	6.97	14.89
	MDE	15	1.99	0.80	6.89	14.87
	CFO	15	2.00	0.74	6.52	14.97
3	GA	15	2.01	0.78	6.81	14.93
	PSO	15	2.00	0.80	6.89	14.87
	MDE	15	2.00	0.80	6.89	14.87
	CFO	15	1.99	0.50	6.63	14.98
4	GA	15	1.99	0.82	6.97	14.90
	PSO	15	2.00	1.50	6.97	14.89
	MDE	15	2.00	0.80	6.89	14.87
	CFO	15	2.00	1.06	6.93	14.92
5	GA	15	1.99	0.82	6.99	14.87
	PSO	15	2.00	0.80	6.89	14.87
	MDE	15	1.99	0.80	6.89	14.87
	CFO	15	2.00	0.89	6.99	14.87
6	GA	15	2.00	0.80	6.85	14.89
	PSO	15	2.00	0.80	6.89	14.87
	MDE	15	2.00	0.80	6.89	14.87
	CFO	15	2.00	0.78	6.69	14.87
7	GA	15	2.01	0.81	6.93	14.94
	PSO	15	2.00	0.80	6.89	14.87
	MDE	15	2.00	0.80	6.89	14.87
	CFO	15	2.00	0.77	6.67	14.96
8	GA	15	1.99	0.75	6.67	14.88
	PSO	15	1.99	0.80	6.89	14.87
	MDE	15	2.00	0.80	6.89	14.87
	CFO	15	2.00	0.92	6.53	14.90
9	GA	15	1.97	0.81	6.92	14.92
	PSO	15	1.99	0.80	6.89	14.87
	MDE	15	2.00	0.80	6.89	14.87
	CFO	15	2.00	1.04	6.89	14.88
10	GA	15	1.99	0.79	6.84	14.90
	PSO	15	2.00	1.50	6.97	14.89
	MDE	15	1.99	0.80	6.89	14.87
	CFO	15	2.02	0.74	6.64	14.99

Based on Table 4, the optimum set of parameters for the HUBOT-5 conformation to the objective of 10 runs per GA, PSO, MDE and CFO algorithms is shown in Table 5.

TABLE 5. PARAMETRIC SET APPLIED WITH EACH ALGORITHM.

$H_{ref} = 2 cm$					
Agorithms	Walking Gait Parameters value				Best firness
]	S (cm)	H (cm)	h (cm)	n (cm)	value f(cm)
GA	15	1.99	0.82	6.99	14.87
PSO	15	2.00	0.8	6.89	14.87
MDE	15	2.00	0.8	6.89	14.87
CFO	15	2.00	0.89	6.99	14.87

Figure 7 shows resulted ZMP and COM trajectories when HUBOT-5 steps in a stepping cycle (T = 2s) corresponding to the configurations using GA, PSO, MDE and CFO algorithms. The optimal set of parameters for each of the algorithms in Table 5 shows that the target is reached according to the set foot lift. The ZMP and COM trajectories corresponding to each of the algorithms in Figure 7 show that they are always within the footprint and ensuring steady-state goals.

Based on the results described in Figure 6, the CFO algorithm finds an optimal value of 14.93174983 after an average of 89 generations. The CFO algorithm outperforms GA, PSO, and MDE algorithms in terms of convergence statistic.

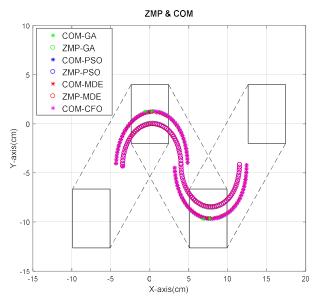


Figure 7: ZMP and COM survey

Table 6 demonstrates the optimized value of the walking gait parameters to make HUBOT-5 walk steadily with 2 cases of differnet foot lift magnitude. ($H_{ref} = 2cm \, \text{và} \, H_{ref} = 4cm$) using CFO.

TABLE 6. RESULTED CFO-BASED OPTIMAL SET OF 4-PARAMETERS

	H_{ref}	CFO optimization Results				
Ì	(cm)	S (cm)	H (cm)	h (cm)	n (cm)	
	2.0	15	2.0	0.89	6.99	
	4.0	15	4.0	1.09	7.12	

Figure 8 show that the HUBOT-5 obtains a pick-up foot-lift with respect to preset value. Figure 9 illustrates the ZMP point and the projection of COM for two different feet lifting amplitude. This shows that the ZMP point is always in the

supporting foot area and then ensuring that the HUBOT-5 always in stable walking.

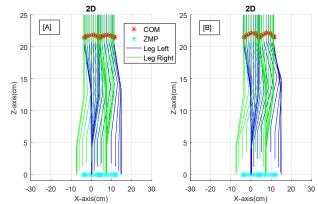


Figure 8: Simulated 2D gait result of biped HUBOT-5 with different footlifting amplitudes

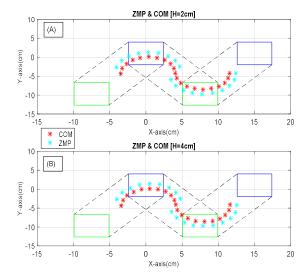


Figure 9. Resulted ZMP và COM trajectories

Figure 10 demonstrates 10 rotary angle trajectories in one stepping cycle of the 2 legs of HUBOT–5 $(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8, \theta_9, \theta_{10})$ when walking with two sets of gait parameters in term to different foot lifts (see Table 9). Using 10 rotary angles to control HUBOT-5 performing two corresponding steps with different foot lifts. Figures 11 illustrates the image of the HUBOT-5 performing a stable step with the foot lift $H_{ref} = 2cm$.

Based on the results of the optimization and simulation run shown in Table 6, Figure 8 and Figure 9, as well as the experimental result presented in Figure 11, show that the work of the preset foot-lifting parameters - $H_{\rm ref}$ and the four key parameters (S-step length, H-foot lifting, h– kneeling, and n–hip swinging) optimally ensuring the HUBOT-5 biped robot to steadily walking without fallen and keeping pace with preset foot-lift amplitude - using CFO algorithm is feasible.

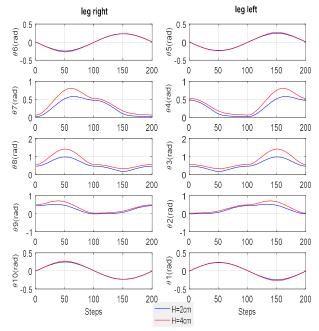


Figure 10: Trajectories of 10 rotary angles of 2 legs of the HUBOT-5 biped

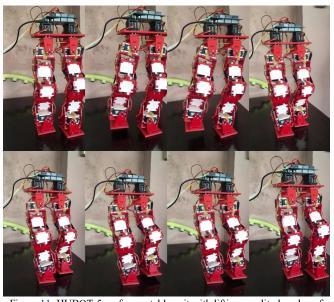


Figure 11: HUBOT-5 performs stable gait with lifting amplitude value of

 $H_{ref} = 2cm$

VI. CONCLUSION

This paper introduces a new algorithm that enables dynamic equilibrium shapes for humanoid robots based on CFO algorithms. First, inverse kinematics is used to estimate the position of the motors located in the joints at the two legs of biped. Then, the CFO optimization algorithm is used to optimally identify the best value for humanoid robot gait parameters so that the distance from ZMP to the center of the foot is the smallest with the preset foot-lifting value. The simulated and experimental results of the small-sized

HUBOT-5 biped robot demonstrate that the use of CFO algorithm with an effective target function that allows the biped robot to walk efficiently and steadily with reduced training time.

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