

Optimal Trajectory Generation for Bipedal Robots

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Abstract—This paper proposes a new method of trajectory planning for biped robots walking on level ground. In this approach, the hip and foot trajectories are designed in Cartesian space using polynomial interpolation. The key parameters which define the trajectories are searched by genetic algorithm. The objective is to obtain the best trajectory that has large stability margin and low energy consumption. ZMP is used as the criterion to ensure physically realizable walking motion. The effectiveness of our method is verified by simulations of a humanoid robot named NUSBIP-II.

I. INTRODUCTION

In order for the humanoid robots to coexist with human beings in a complex human environment, they must have the agility and ability comparable to humans such as operating on uneven terrains, walking on stairs, slopes, etc. Many works have been contributed to the area of walking gait synthesis for bipedal walking [1-15]. Hasegawa *et al.* [15], Roussel *et al.* [12], and Channon *et al.* [13] have proposed methods of walking pattern generation by minimizing the cost function of energy consumption.

Stability is a critical issue in bipedal walking. The zero-moment-point (ZMP) [16] has been widely used as a criterion to ensure stability of bipedal walking in many studies. Takanishi *et al.* [6], Hirai *et al.* [7], Kajita *et al.* [1], Shih *et al.* [11], and Erbatur *et al.* [8] have used the ZMP as the stability index to design the walking pattern for bipedal robots. Basically, in these methods, the desired ZMP trajectory is first prescribed and then the hip or trunk motion is derived to obtain the ZMP trajectory. However, it is impossible to determine which ZMP trajectory is good to generate a smooth and energy efficient motion. In addition, the hip acceleration may need to be very large to achieve a desired ZMP trajectory and this will lead to the increase of energy consumption which is not desirable. To solve this problem, some researchers have proposed methods of gait synthesis without first prescribing the desired ZMP trajectory. Huang *et al.* [10] proposed a method in which the foot and hip trajectories are planned beforehand in Cartesian space using cubic spline interpolation. The hip trajectory is affected by two parameters, these parameters are determined by iterative computation to obtain largest stability margin for the biped. The advantage of this method is that the motion of the hip or trunk is very smooth which make the control of the upper limbs easier. However, since the characteristic of the hip trajectory is only determined by two parameters and the range of these parameters are quite limited therefore the

diversity of the hip trajectory is limited. Hence, the optimal trajectory obtained may not be the best trajectory.

In this study, we propose a method of trajectory generation in which the foot and hip trajectories are planned in the Cartesian space using polynomial interpolation. No desired ZMP trajectory is prescribed. The key parameters of the hip trajectory and foot trajectory will be searched using genetic algorithm (GA) to find out the the optimal trajectory which has large stability margin and low energy consumption.

This paper is organized as follows. Section II describes the life-size humanoid robot NUSBIP-II whose specifications are used in the simulation. Section III presents the generation of foot and hip trajectories and the choice of key parameters. Genetic algorithm implementation is described in Section IV. Section V shows the simulation results and conclusion is made in Section VI.

II. THE BIPED ROBOT

The robot considered in this study is a 7-link biped robot named NUSBIP-II, which was built and developed in our lab (see Fig. 1). The total weight is about 22.2 kg. The biped has 12 DOFs. Each leg has six active DOFs of which three DOFs are at the hip (pitch, yaw, roll), one at the knee (pitch) and two at the ankle joint (pitch, roll). Each DOF is driven by a DC motor and integrated with an angular position sensor to measure the relative angle between two consecutive links. Each of the feet is equipped with four force sensors (two at the toe and two at the heel) to sense the contact forces between the feet and the ground.

The model of the robot is depicted in Fig. 2. The mass of link i th is m_i and the moment of inertia around its COG is I_i .

III. TRAJECTORY GENERATION

A. Foot Trajectory

The walking parameters that define the foot trajectory is shown in Fig. 3 where S is the half walking step length, S_p is the horizontal distance from the start of the step (foot lift-off) to the place where the swing-foot at its highest position, H_p is the highest position of the foot in vertical axis. In this work, the foot is constrained to move such that it is always parallel to the ground. Let T be the period of one walking step, the interval of the k^{th} step is from kT to $(k+1)T$, $k = 1, 2, \dots, N$, N is the number of steps. The corresponding time instant when the foot is at its highest position is $kT + t_p$. Since the walking motion

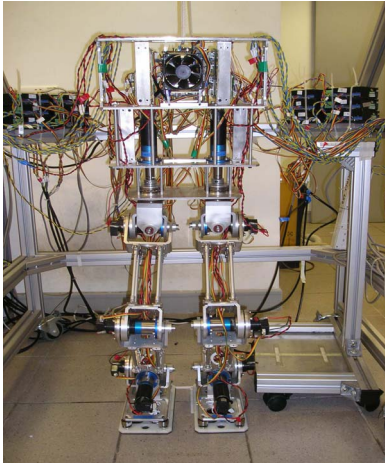


Fig. 1. Picture of NUSBIP-II.

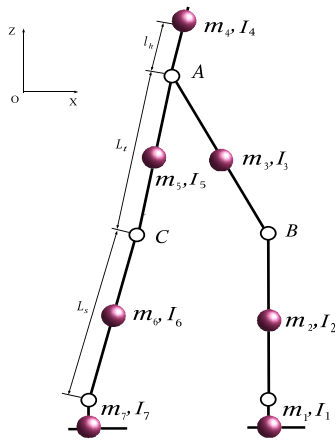


Fig. 2. Model of the robot.

is repeated periodically, we only need to plan the motion in one period. The horizontal motion of the foot trajectory must satisfy the position constraints as in (1), with the coordinate system placed at the stance ankle:

$$x_f(t) = \begin{cases} -S, & t = kT \\ -S + S_p, & t = kT + t_p \\ S, & t = (k+1)T \end{cases} \quad (1)$$

Since the feet fully contact with the ground at $t = kT$ and $t = kT + T$, the following velocity constraints must be included:

$$\begin{cases} \dot{x}_f(kT) = 0 \\ \dot{x}_f(kT + T) = 0 \end{cases} \quad (2)$$

Totally, the horizontal motion of the swing foot must satisfy 5 constraints (position and velocity constraints), a fourth-order polynomial is enough to describe the motion:

$$x_f(t) = a_4 t^4 + a_3 t^3 + a_2 t^2 + a_1 t + a_0 \quad (3)$$

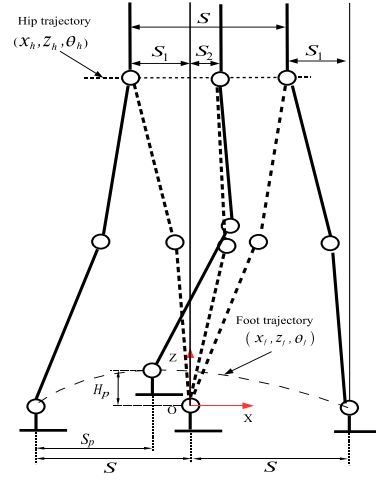


Fig. 3. Parameters defining the walking trajectory of the robot. The stance-leg and swing-leg are represented by dashed lines and solid lines, respectively.

Similarly, the position and velocity constraints of the vertical motion of the swing foot are defined in (4) and (5), respectively.

$$z_f(t) = \begin{cases} 0, & t = kT \\ H_p, & t = kT + t_p \\ 0, & t = (k+1)T \end{cases} \quad (4)$$

$$\begin{cases} \dot{z}_f(kT) = 0 \\ \dot{z}_f(kT + T) = 0 \end{cases} \quad (5)$$

Again, a fourth-order polynomial is used to describe the vertical motion:

$$z_f(t) = b_4 t^4 + b_3 t^3 + b_2 t^2 + b_1 t + b_0 \quad (6)$$

In this study, the half walking step length S and the step period T are given. The maximum foot height H_p is an important parameter that has significant influence on the swing leg dynamics. Therefore, we leave H_p as a free variable and it will be searched by GA to find the optimal value minimizing our proposed cost function.

As an example, we choose the walking parameters as followings: $T = 0.8s$, $t_p = 0.4s$, $S = 0.3m$, $S_p = S$, $H_p = 0.07m$. The foot trajectory corresponding to these parameters is shown in Fig. 4.

B. Hip Trajectory

Hip motion is very critical to stability of the biped because a little change in hip motion may cause the whole dynamics of the biped to change dramatically and thus affect the biped stability the most. Therefore, in this work the hip trajectory will be carefully treated such that stable walking could be achieved. Since the vertical motion of the hip is quite limited, it doesn't affect much on the stability of the biped. For simple analysis, the hip height is constrained to be constant and the pitch angle of the trunk is kept upright (pitch angle $\theta_h = 0$).

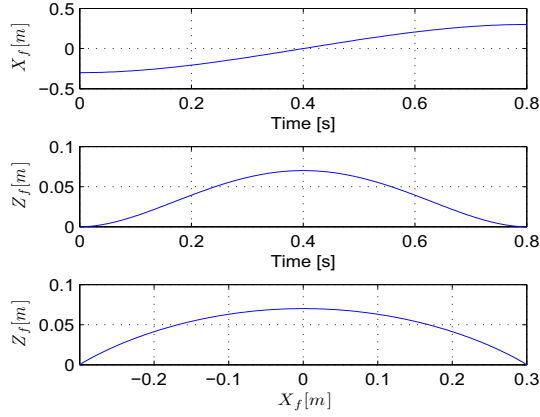


Fig. 4. Sample foot trajectory.

The horizontal motion $x_h(t)$ of the hip is the main factor that affects the stability of the biped in sagittal plane. In this method, instead of using the cubic spline interpolation as in [10] we use the high-order polynomial interpolation to generate the smooth hip trajectory. The order of the polynomial is chosen to be larger than the number of constraints such that we can have more flexibility for the hip trajectory. The higher the order of the polynomial, the more flexible the set of hip trajectories could be obtained. Let's call the difference between the order of the polynomial and the number of constraints the number of redundant-coefficients. Diverse, flexible hip trajectories can be achieved by using different set of redundant-coefficients.

As stated earlier, the reference coordinate is placed at the stance ankle. The following condition for hip motion is obtained:

$$x_h(t) = \begin{cases} -S_1, & t = kT \\ S_2, & t = kT + T/2 \\ S - S_1, & t = (k+1)T \end{cases} \quad (7)$$

In order for the periodic trajectory $x_h(t)$ to be smooth, the following continuous condition must be satisfied:

$$\dot{x}_h(kT) = \dot{x}_h(kT + T) \quad (8)$$

In the process of walking, according to biomechanic studies, the body must slow down and then speed up again during each step. A supporting foot starts out ahead of the body where it tends to slow the body down, and then it passes under the body and to the rear, where it tends to speed up the body again [18]. Therefore, the body acceleration right before the touch down of the swinging foot (end of walking step) is positive and reaches maximum value. Whereas, right after the touch down of the swinging foot (start of walking step) the body acceleration is a positive, minimum value. In addition, the body motion can be reasonably described as an inverted pendulum whose acceleration at the beginning and end of the walking step are equal in magnitude but opposite in sign

[19]. Based on these studies, we can impose the acceleration constraints on the body motion as in (9) to make the walking more human-like.

$$\ddot{x}_h(kT) = -\ddot{x}_h(kT + T) \quad (9)$$

A fourth-order polynomial should be enough to satisfy the five constraints in (7), (8) and (9). However, we wish to have a set of diverse trajectories from which we can find the best trajectory which has the large stability margin and low energy consumption. Therefore, instead of using a fourth-order polynomial for the hip trajectory we use a higher order polynomial which can generate more flexible trajectories by using different set of **redundant-coefficients**. In this work, we found that a seventh-order polynomial is good enough to provide various trajectories by changing the redundant-coefficients, higher order polynomial (8, 9, ...) doesn't make a significant improvement.

$$x_h(t) = a_7 t^7 + a_6 t^6 + a_5 t^5 + a_4 t^4 + a_3 t^3 + a_2 t^2 + a_1 t + a_0 \quad (10)$$

A seventh-order polynomial has 8 coefficients. Since there are five constraints, five coefficients are used to comply with the constraints (let's call these coefficients the constrained-coefficients), the other three coefficients are the redundant-coefficients and are free to choose. We can choose any five coefficients to be the constrained-coefficients provided that they can satisfy the constraint equations in (7), (8) and (9). The five coefficients a_0, a_1, a_2, a_3, a_4 could be the first choice when selecting the constrained-coefficients. However, these coefficients do not comply with the constraints in (7), (8) and (9). As such, another set of five constrained-coefficients are chosen, which are a_0, a_1, a_3, a_4 and a_5 . Accordingly, the other three coefficients a_2, a_6, a_7 are redundant-coefficients. When these redundant-coefficients are given (a_2, a_6, a_7 will be searched by genetic algorithm), the constrained-coefficients are determined as in (11).

$$\begin{aligned} a_5 &= -32(M_2 - M_1/2 + 3M_3T/16 - M_4T^2/32)/5T^5, \\ a_4 &= (M_4 - 2M_3/T - 10T^3a_5)/4T^2, \\ a_3 &= (M_3 - 5T^4a_5 - 4T^3a_4)/3T^2, \\ a_1 &= (M_1 - T^5a_5 - T^4a_4 - T^3a_3)/T, \quad a_0 = -S_1 \end{aligned} \quad (11)$$

where

$$\begin{aligned} M_1 &= S - a_7T^7 - a_6T^6 - a_2T^2 \\ M_2 &= S_2 + S_1 - a_7(T/2)^7 - a_6(T/2)^6 - a_2(T/2)^2 \\ M_3 &= -7a_7T^6 - 6a_6T^5 - 2a_2T \\ M_4 &= -42a_7T^5 - 30a_6T^4 - 4a_2 \end{aligned} \quad (12)$$

Fig. 5 shows a bunch of trajectories generated by choosing **deferent** sets of (a_2, a_6, a_7) . From the figure it can be seen that by changing the higher-order coefficients we can achieve a set of diverse trajectories.

In order to give more freedom to the hip trajectory, the parameters S_1 and S_2 are decided to be free variables. These parameters, along with the coefficients a_2, a_6, a_7 and H_p will

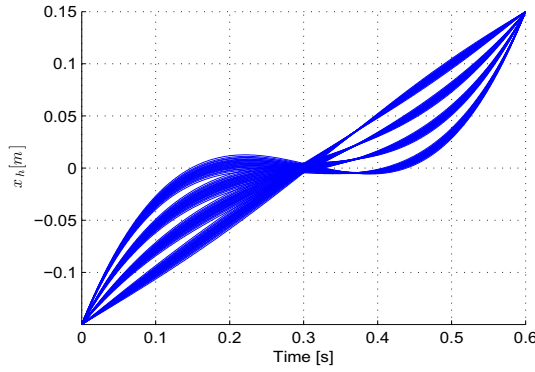


Fig. 5. Diverse hip trajectories generated when $a_2 = \{-6; -4; -2; 0\}$, $a_6 = a_7 = \{-2; -1; 0; 1; 2\}$ and $S_1 = 0.15$, $S_2 = 0$.

be searched by GA to obtain the optimal values resulting in large stability margin and low energy consumption trajectory. Compared to [10], our formulation of hip trajectory can produce a much bigger set of hip trajectories to be selected by GA.

IV. GENETIC ALGORITHM IMPLEMENTATION

Genetic Algorithm (GA) [17] is a famous search algorithm developed by John Holland and his colleagues at the University of Michigan. The basic idea of these algorithms is borrowed from the natural selection process in which the fittest individuals have highest chance to survive in the next generation and vice versa. GA have proved to be very robust, efficient search algorithms and been being applied successfully in a wide range of fields from biology, medicine, computer science, engineering, etc.

A. GA's parameters

As mentioned in the earlier sections, the interested parameters in this study are the maximum height H_p of the swinging foot, the coefficients a_2 , a_6 , a_7 of the polynomial describing the horizontal hip trajectory and the parameters S_1 , S_2 . These parameters will be searched by GA to find the best trajectory that has large stability margin and low energy consumption. All the genetic operations (reproduction, crossover and mutation) will be performed on a string of 60 bits binary number. The real value of the parameters a_2 , a_6 , a_7 and H_p , S_1 , S_2 are extracted from this string.

B. The Fitness Function

In genetic algorithm, fitness function is the core of the searching mechanism. It has the role of guiding the search such that desired effects could be achieved. The objective that we want to achieve is reflected through the fitness value. Therefore, choosing the correct fitness function is necessary otherwise GA may never converge or even if GA converges, desired performance may not be obtained. In this work, our objective is to search for the interested parameters a_2 , a_6 , a_7 and H_p , S_1 , S_2 such that the zero-moment-point (ZMP) is as close to the mid-point of the stance foot as possible (the

TABLE I
SPECIFICATIONS OF NUSBIP-II.

Length (m)	Thigh		Shank		Foot Length		
	0.32		0.32		0.21		
Weight (kg)	m_1	m_2	m_3	m_4	m_5	m_6	m_7
	1.52	1.68	2.68	10.45	2.68	1.68	1.52
Inertia (kgm ²)	I_{1y}	I_{2y}	I_{3y}	I_{4y}	I_{5y}	I_{6y}	I_{7y}
	0.1040	0.2739	0.3458	0.2426	0.3458	0.2739	0.1040

nearer the ZMP to the mid-point of the foot, the higher the stability margin) and the total energy of the walking biped is minimized. For such, the cost function can be expressed as follows:

$$P = w_1(|X_{zmp} - d|_{max} + \frac{1}{n} \sum^n |X_{zmp} - d|) + w_2 \frac{1}{n} \sum^n \sum_{i=1}^J |\tau_i \dot{q}_i| dt \longrightarrow Min \quad (13)$$

where X_{zmp} is the current ZMP location with respect to the reference coordinate located at the stance ankle; d is the offset distance from the reference coordinate to the mid-point of the foot; n is the number of integration steps; J is the number of joints of the biped; τ_i , q_i are the torque and the joint angle of the joint i , respectively; and w_1 , w_2 are the weighting factors.

The terms in the bracket on the righthand side (right after the weighting factor w_1) of (13) are a measure of the closeness of the ZMP trajectory to the mid-point of the foot. The other term (following the weighting factor w_2) is the total energy consumption of all the actuators.

To achieve the above mentioned objective, the cost function must be minimized. As a convention, in GA the fitness function is always being maximized. Therefore, the fitness function can be chosen as the inverse of the cost function:

$$F = \begin{cases} \frac{1}{P} & \text{if ZMP stays inside the stable region} \\ 0 & \text{Otherwise} \end{cases} \quad (14)$$

The flowchart of the proposed method is shown in Fig. 6 where GN is the generation number and GN_{max} is the maximum number of generation.

V. SIMULATION RESULTS

The specifications of the simulated biped are taken from a real biped, which was named NUSBIP-II and developed in our lab (Fig. 1). Table I summarizes the specifications of the biped robot NUSBIP-II. The simulation is done in Yobotics,¹ a dynamic simulation software which allows the running of batches of simulation.

Table II shows the initial conditions for GA where IndivNo is the number of individuals and GenNo is the number of generations that GA will do the search. In this simulation, the step length is chosen to be 0.6m (or $S = 0.3m$), and the walking step period is $T = 0.8s$. As mentioned in the previous

¹<http://www.yobotics.com/simulation/simulation.htm>

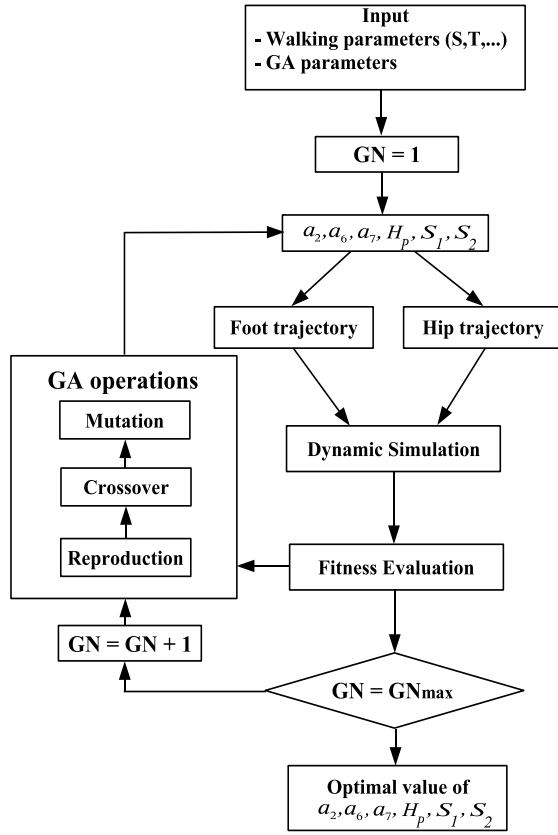


Fig. 6. Flowchart of the proposed method.

TABLE II
INITIAL CONDITIONS FOR GA.

GA parameters			
IndivNo	GenNo	Crossover rate	Mutation rate
200	100	80%	2%

sections, the interested parameters that are searched by GA are the maximum height H_p of the swinging foot during walking and the coefficients a_2, a_6, a_7 of the polynomial describing the hip trajectory and the parameters S_1, S_2 . These parameters will be determined when the GA converges.

In this study, GA converges after 80 generations. The converged values of the interested parameters are $a_2 = -0.9609$, $a_6 = -0.9453$, $a_7 = -1.002$ and $H_p = 0.050$, $S_1 = 0.1362$, $S_2 = 0.013$. Since these parameters are known, the foot trajectory and the hip trajectory are determined. Fig. 7 depicts the optimal hip trajectory of the bipedal robot.

Fig. 8 shows the resulting ZMP trajectory of the biped. It can be seen that, the ZMP is always inside the stable region which means the bipedal walking is physically feasible. In addition, the ZMP stays very close to the middle of the foot which is desirable because the closer the ZMP to the middle of the foot, the more stable the bipedal walking is. The stable region is defined by the upper-bound and the lower-bound. When only one foot is supporting, the upper-bound

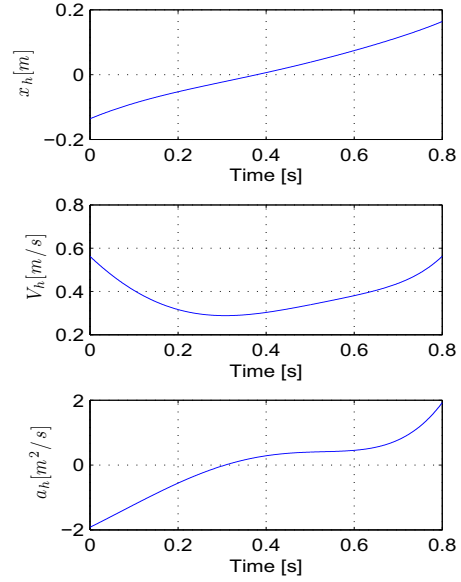


Fig. 7. From top to bottom are the resulting position, velocity and acceleration hip trajectories of the robot.

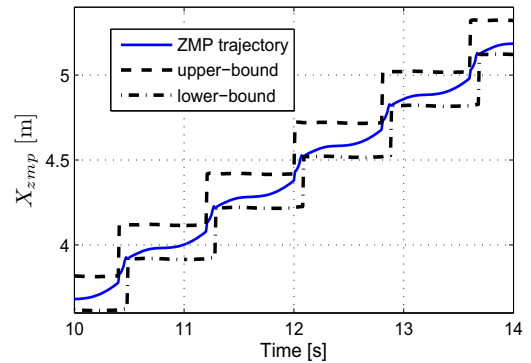


Fig. 8. The resulting ZMP trajectory.

and the lower-bound are the toe and the heel locations of the supporting foot, respectively. When two legs are touching the ground, the upper-bound is the toe of the front foot and the lower-bound is the heel of the behind foot.

The total power of all the actuators is shown in Fig. 9. The minimum power is about 6W, the maximum power is about 60W and the average power is only about 22W. The velocity profile of the trunk of the biped is shown in Fig. 10. From this figure, we can see that the velocity profile is very smooth. The hip, knee and ankle joint angle trajectories are shown in Fig. 11. Fig. The walking motion images (in stick diagram) of the biped captured at 0.05s apart is shown in Fig. 12.

VI. CONCLUSION

In this study, we proposed a method of optimal trajectory generation for bipedal robot. The foot and hip trajectories in sagittal plane are planned in the Cartesian space using

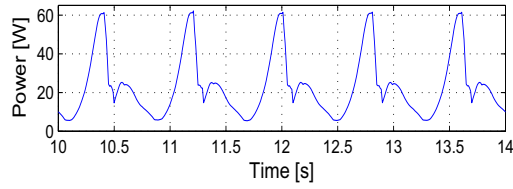


Fig. 9. Total power of all actuators of the biped.

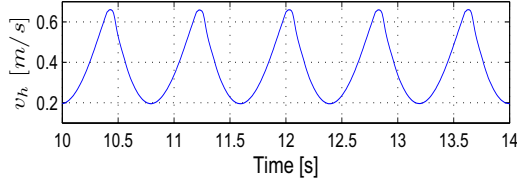


Fig. 10. Velocity profile of the trunk.

polynomial interpolation. The contribution of our work is the proposal of the new way of designing the hip trajectory such that a big set of smooth hip trajectories can be generated by choosing different sets of the 6 key parameters ($a_5, a_6, a_7, H_p, S_1, S_2$). The best trajectory with large stability margin and low energy consumption is then searched by GA. It is obvious that the bigger the set of hip trajectories is generated, the better the resulting optimal trajectory can be obtained. In this study, by using the 6 key parameters as free variables we can create a much bigger set of hip trajectories (compared to other related works) for GA to search. According to our knowledge, this is the first work to consider finding the optimal maximum foot height of the swinging foot (H_p). The simulation results show that the achieved walking motion is very stable with very large stability margin and low energy consumption.

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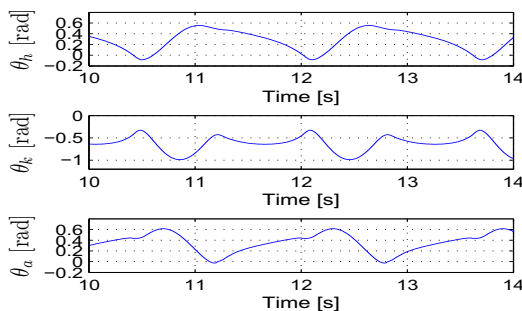


Fig. 11. The hip, knee and ankle joint angle trajectories.

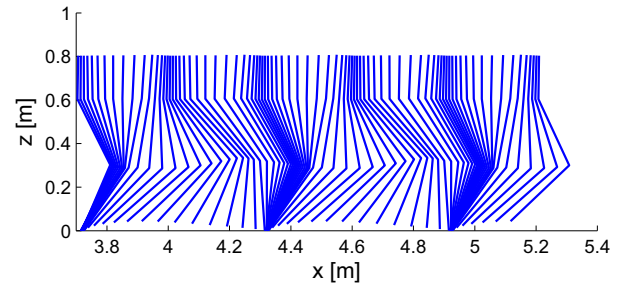


Fig. 12. The stick diagram of the walking robot (showing the right leg only). Images are captured at 0.05s apart starting from time $t_1 = 10s$ to $t_2 = 14s$.

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