Example of NP Classifier.

→ $x \in \mathbb{R}$

→ class Conditional densities are normal with equal Variance.

→ $\mu_0 < \mu_1$

→ Now One can Show that, the NP classifier is

if $x > \tau$ then class 1

where τ is determined on Type-I error bound.

$$\int_{\tau}^{\infty} p_{X|Y=0}(x)\, dx = \alpha$$

Bayes classifier minimizes risk which is a kind of weighted error.

NP classifier is another way of deciding how to trade one type of error with another.

For a 2-Class problem, One Can always trade False +ve rate against False -ve rate.

We would always have a Threshold which can be tuned to effect this trade-off

→ ROC : Receiver operating characteristic is a convenient way of looking at this.

Consider a $x \in \mathbb{R}$, 2 class problem

$h(x) = 0$ if $x < \tau$

→ Consider equal priors

Gaussian class conditionals densities with equal variance

0-1 Loss.

$$\mathbb{P}[\text{error}] = 0.5 \int_{-\infty}^{\tau} p_{X|Y=1}(x)\, dx + 0.5 \int_{\tau}^{\infty} p_{X|Y=0}(x)\, dx$$

$$= 0.5 \left[\phi\left(\frac{\tau - \mu_1}{\sigma}\right) + \left(1 - \phi\left(\frac{\tau - \mu_0}{\sigma}\right)\right) \right]$$

* As we vary τ
  we trade one kind of error with another

* In a 2-class classification there will be 4 possible outcomes.

c_i : denote the probability of wrongly assigning class-i

C_1 - +ve class

C_0 - -ve class

we decide C_1 if $x > \tau$

$c_0 = \mathbb{P}[x \le \tau \mid x \in C_1]$: False -ve rate

$c_1 = \mathbb{P}[x > \tau \mid x \in C_0]$: False +ve rate

$1 - c_0 = \mathbb{P}[x > \tau \mid x \in C_1]$ True +ve rate

$1 - c_1 = \mathbb{P}[x \le \tau \mid x \in C_0]$ True -ve rate

We have 2 quantities of interest

$$\text{precision} = \frac{TP}{TP+FP}$$

$$\text{recall} = \frac{TP}{TP+FN}$$

→ For a Fixed Class Conditional densities.

if we vary τ, the point $(e_1, 1-e_0)$ moves on a Smooth Curve in $\mathbb{R}^2$

* Hence varying τ, we can find ROC & decide which may be the best operating point

* This can be done for any threshold based classifier irrespective of class conditional densities.

when class conditionals densities are normal with equal variance.

$$e_1 = \int_{\tau}^{\infty} p_{X|Y=0}(x) \, dx = 1 - \phi\left(\frac{\tau - \mu_0}{\sigma}\right)$$

$$e_0 = \int_{-\infty}^{\tau} p_{X|Y=1}(x) \, dx = \phi\left(\frac{\tau - \mu_1}{\sigma}\right)$$

From these integrals we get

$$\frac{\tau - \mu_0}{\sigma} = \phi^{-1}(1 - c_1) = a$$

$$\frac{\tau - \mu_1}{\sigma} = \phi^{-1}(1 - (1 - c_0)) = b$$

then $|a - b| = \left|\frac{\mu_1 - \mu_0}{\sigma}\right| = d.$

This is called the discriminator which is not dependent of τ.