Lets consider the 2-class case

Risk Integral is

$$R(h) = \int_{S_1(h)} L(1,0)\, p_{y=0}\, p_{x|y=0}\, dx \;+$$

$$\int_{S_0(h)} L(0,1)\, p_{y=1}\, p_{x|y=1}\, dx$$

we know that

$$p_{y=0} = 1 - p_{y=1}$$

$$R(h) = L(1,0)\, p_{y=0} \int_{S_1(h)} p_{x|y=0}\, dx \;+$$

$$L(0,1)\,(1 - p_{y=0}) \int_{S_0(h)} p_{x|y=1}\, dx$$

$$= L(0,1) \int_{S_0(h)} p_{x|y=1}\, dx \;+$$

$$p_{y=0} \left[L(1,0) \int_{S_1(h)} p_{x|y=0}\, dx - L(0,1) \int_{S_0(h)} p_{x|y=1}\, dx \right]$$

This indicates for a fixed classifier risk varies linearly with prior probability.

Minmax Classifier:

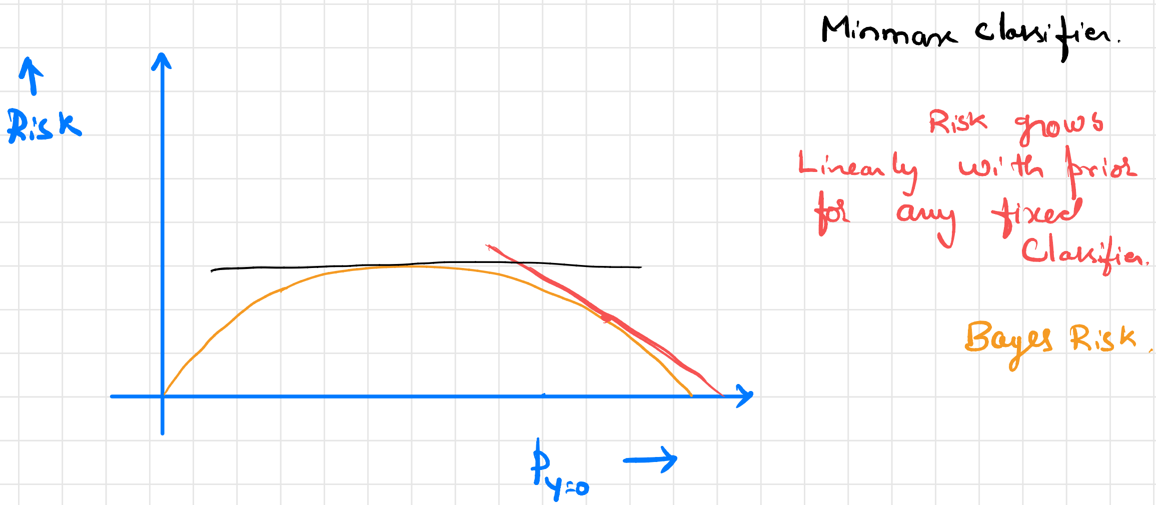
Consider a classifier s.t

$$L(1,0) \int_{S_1(h)} p_{X|Y=0} \, dx = L(0,1) \int_{S_0(h)} p_{X|Y=1} \, dx$$

* For this classifier the risk would be

$$R(h_M) = L(0,1) \int_{S_0(h)} p_{X|Y=1} \, dx.$$

Risk would be independent of priors.



$$L(1,0) \int_{S_1(h)} p_{X|Y=0} \, dx = L(0,1) \int_{S_0(h)} p_{X|Y=1} \, dx$$

Suppose in the above condition

$$L(1,0) = L(0,1)$$

$$\int_{S_1(h)} p_{X|Y=0} \, dx = \int_{S_0(h)} p_{X|Y=1} \, dx.$$

Now, for a simple case of one dimensional feature Normal class conditional densities.

$p_{X|Y=i}$ is normal with μ_i, σ_i^2

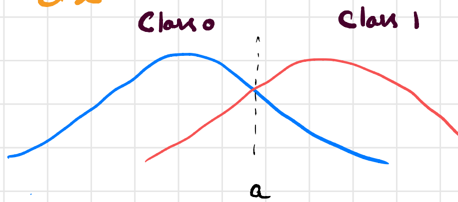
assume $\mu_0 < \mu_1$

$h(x) = 0$ iff

$x < a$

now we will fix the threshold "a", to satisfy the minmax criterion.

$$\int_a^{\infty} p_{X|Y=0} \, dx = \int_{-\infty}^{a} p_{X|Y=1} \, dx$$



These will become the integrals of std normal

$z = \frac{x-\mu_0}{\sigma_0}$ for 1st integral

$z = \frac{x-\mu_1}{\sigma_1}$ for 2nd integral

The threshold should satisfy

$$1-\phi\left(\frac{a-\mu_0}{\sigma_0}\right) = \phi\left(\frac{a-\mu_1}{\sigma_1}\right)$$

WKT

$$1-\phi(z) = \phi(-z)$$

$$\phi\left(\frac{\mu_0-a}{\sigma_0}\right) = \phi\left(\frac{a-\mu_1}{\sigma_1}\right)$$

$$\frac{\mu_0-a}{\sigma_0} = \frac{a-\mu_1}{\sigma_1}$$

$$a = \frac{\mu_0\sigma_1 + \mu_1\sigma_0}{\sigma_0 + \sigma_1}$$

now here we can observe that

1) Min max is a Linear while Bayes classifier for the same case is quadratic.

2) When $\sigma_0 = \sigma_1 = \sigma$

Min max is same as Bayes in this special case.