॥ भगवान् श्री रामकृष्ण शरणम् ॥

PRNN - Course by

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Lecture : 01 5^{th} Jan 2025

* we will look @ ML from probabilistic view point.

* Function approx

$\mathcal{X}$: domain set (Input)

$\mathcal{Y}$: Range set (output)

Given pairs $D = \{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$

where (x_i, y_i) is a pair of observations,

Assume that $\exists$ a fun^c $f : \mathcal{X} \to \mathcal{Y}$

$$f(x_i) = y_i$$

The underlying fun^c "f" is Unknown

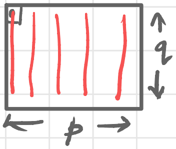
Given D, find f

Start with some initial guess on f.

then refine the guess using "D" (data)

Example:

Consider x_i to be x-ray image



• : value b/w 0-255

$p \times q$: values b/w 0-255

$\begin{bmatrix} | \\ | \\ | \\ \vdots \\ | \end{bmatrix}_{pq \times 1}$ pq sized Vector.

$\in \mathbb{R}^{pq}$

$x_i \in X \subset \mathbb{R}^d$

$y_i \in \{0, 1\}$

$0 \rightarrow$ diseased

$1 \rightarrow$ benign / non- diseased.

$$f : X \rightarrow Y$$

* "f" is complex to be estimated from the physics of the problem

* $\therefore$ Resort to statistical methods

Idea is to make repeated observation & estimate f.

* X, Y in a probabilistic framework are Random Variables.

Recap: Sample Space

Random Variable

Distribution func & Density func

Lecture 2 : 7^{th} Jan 2026 :

* Mathematical framework to work under uncertainties but the decision is deterministic.

* Random experiment

* outcomes of Random experiment : Ω

Sample space : set of all possible outcomes of a Random experiment

$A = [2, 3]$
$B = [6, 14]$
} under the measure of length we can compare.

if working under $\mathbb{R}^2$ we use area measure.

* Sample space may not be numeric

* Consider the subsets of sample space.

* Let $\mathcal{F}$ denote the collection of all possible subset.

Objective : Assign a measure on $\mathcal{F}$

* Probability Measure is one such measure : $\mathbb{P}$

$\mathbb{P} : \mathcal{F} \to [0, 1]$

Properties of $\mathbb{P}$

if $A, B \in \mathcal{F}$

1) $\mathbb{P}(A) \geq 0 \quad \forall A \in \mathcal{F}$

2) $\mathbb{P}(\Omega) = 1 \quad , \quad \mathbb{P}(\phi) = 0$

3) $A \cap B = \phi \quad , \quad \mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$

we can interpret it as a likelihood of events.

* Now we have a probability triplet

$$(\Omega, \mathcal{F}, \mathbb{P})$$

* Ω is abstract, we need numbers to operate on.

Define a fun^c X : Random Variables.

from Ω to $\mathbb{R}$

$$X : \Omega \to \mathbb{R}$$

$\mathcal{F} \to \mathcal{B}$- sigma algebra $\quad (-\infty, x]$

$\mathbb{P} \to \mathbb{P}_X$: distribution fun^c

$$\mathbb{P}_X(x) \triangleq \mathbb{P}[A : X^{-1}(-\infty, x]]$$

$$x \in \mathbb{R}$$

$$(\Omega, \mathcal{F}, \mathbb{P}) \xrightarrow{X} \underbrace{(\mathbb{R}, \mathcal{B}, \mathbb{P}_X)}$$

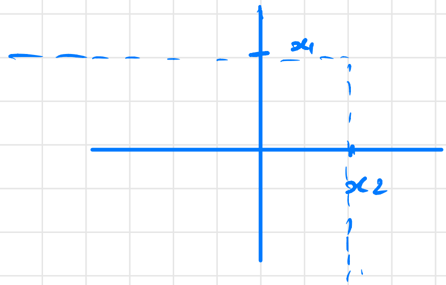
In ML we work with this triplet

* The Random Variables with Vector Valued Range spaces.

In general, RV have $\mathbb{R}^d$ as there range spaces

$$X : \Omega \to \mathbb{R}^d \quad \text{(Vector valued RV)}$$

if $d = 2$, $\mathcal{B}$ is $(-\infty, x_1] \times (-\infty, x_2]$



$\mathbb{P}_X(x)$ = Probability of the inv. image of Cartisian product under X.
$x \in \mathbb{R}^d$