References with abstracts for estimation, modeling, and analysis applied to covariances, correlations and volatilities

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Adams, Z., Fuss, R., and Gluck, T. (2017). "Are correlations constant? Empirical and theoretical results on popular correlation models in finance." In: *Journal of Banking and Finance* 84, pp. 9–24.

Multivariate GARCH models have been designed as an extension of their univariate counterparts. Such a view is appealing from a modeling perspective but imposes correlation dynamics that are similar to time-varying volatility. In this paper, we argue that correlations are quite different in nature. We demonstrate that the highly unstable and erratic behavior that is typically observed for the correlation among financial assets is to a large extent a statistical artifact. We provide evidence that spurious correlation dynamics occur in response to financial events that are sufficiently large to cause a structural break in the time-series of correlations. A measure for the autocovariance structure of conditional correlations allows us to formally demonstrate that the volatility and the persistence of daily correlations are not primarily driven by financial news but by the level of the underlying true correlation. Our results indicate that a rolling-window sample correlation is often a better choice for empirical applications in finance.

Agrawal, R., Roy, U., and Uhler, C. (2021). "Covariance Matrix Estimation under Total Positivity for Portfolio Selection." In: *Journal of Financial Econometrics*.

Selecting the optimal Markowitz porfolio depends on estimating the covariance matrix of the returns of N assets fromT periods of historical data. Problematically, N is typically of the same order as T, which makes the sample covariance matrix estimator perform poorly, both empirically and theoretically. While various other general purpose covariance matrix estimators have been introduced in the financial economics and statistics literature for dealing with the high dimensionality of this problem, we here propose an estimator that exploits the fact that assets are typically positively dependent. This is achieved by imposing that the joint distribution of returns be multivariate totally positive of order 2, MTP2. This constraint on the covariance matrix not only enforces positive dependence among the assets, but also regularizes the covariance matrix, leading to desirable statistical properties such as sparsity. Based on stock-market data spanning over thirty years, we show that estimating the covariance matrix under MTP2 outperforms previous state-of-the-art methods including shrinkage estimators and factor models.

Albanese, C., Li, D., Lobachevskiy, E., and Meissner, G. (2013). "A Comparative Analysis of Correlation Approaches in Finance." In: *The Journal of Derivatives* 21(2), pp. 42–66.

Although volatility is the key parameter for plain vanilla option pricing, many kinds of credit derivatives and exotic options involve multiple risk factors, so correlations must also be modeled. Different types of derivatives entail different types of correlation, from the basic Pearson correlation used in ordinary futures hedging and equity portfolio calculations, to copula methods that allow a wide range of tail dependence properties, to the ubiquitous Gaussian copula of credit risk modeling. Models of stochastically time-varying correlations have been developed, and for the correlation structure within a credit portfolio that may contain thousands of individual loans, top-down methods like Vasicek's large homogeneous portfolio approximation may be required. This article provides a comprehensive review of the many correlation concepts and models that are increasingly necessary for modern derivatives researchers.

Aldridge, I. (2019). "Big data in portfolio allocation: A new approach to successful portfolio optimization." In: *The Journal of Financial Data Science* 1(1), pp. 45–63.

In the classic mean-variance portfolio theory as proposed by Harry Markowitz, the weights of the optimized portfolios are directly proportional to the inverse of the asset correlation matrix. However, most contemporary portfolio optimization research focuses on optimizing the correlation matrix itself, and not its inverse. In this article, the author demonstrates that this is a mistake. Specifically, from the Big Data perspective, she proves that the inverse of the correlation matrix is much more unstable and sensitive to random perturbations than is the correlation matrix itself. As such, optimization of the inverse of the correlation matrix adds more value to optimal portfolio selection than does optimization of the correlation matrix. The author further shows the empirical results of portfolio reallocation under different common portfolio composition scenarios. The technique outperforms traditional portfolio allocation techniques out of sample, delivering nearly 400% improvement over the equally weighted allocation over a 20-year investment period on the SandP 500 portfolio with monthly reallocation. In general, the author demonstrates that the correlation inverse optimization proposed in this article significantly outperforms the other core portfolio allocation strategies, such as equally weighted portfolios, vanilla mean-variance optimization, and techniques based on the spectral decomposition of the correlation matrix. The results presented in this article are novel in the data science space, extend far beyond financial data, and are applicable to any data correlation matrixes and their inverses, whether in advertising, healthcare, or genomics.

Avella-Medina, M. (2020). "Robust Methods for High-Dimensional Regression and Covariance Matrix Estimation." In: *Macroeconomic Forecasting in the Era of Big Data*. Springer International Publishing, pp. 625–653.
We review some basic ideas of the robust statistics literature and define tools that allows us to construct robust statistical procedures. We show how these ideas, originally developed for fixed dimensional settings, can also be applied to high-dimensional problems where the number of unknown parameters can be larger than the sample size. In particular, we build on the theory of M-estimators and adapt it to handle the problems of high-dimensional regression and covariance matrix estimation via regularization. For the former problem we show that penalized M-estimators for high-dimensional generalized linear models can lead to estimators that are consistent when the data is nice and contains no contaminated observations, while importantly remaining stable in the presence of a small fraction of outliers. For the problem of covariance estimation we show that M-estimators be used to significantly weaken the typical requirement of having sub-Gaussian distributions to assuming only a few finite moments. This relaxation cannot be achieved by regularizing the sample covariance as in classical fixed dimensional regimes.

Barroso, P. and Saxena, K. (2021). "Lest we forget: Using Out-Of-Sample Errors in Portfolio Optimization." In: *The Review of Financial Studies*.
Portfolio optimization often struggles in realistic out-of-sample contexts. We deconstruct this stylized fact by comparing historical forecasts of portfolio optimization inputs with subsequent out-of-sample values. We confirm that historical forecasts are imprecise guides of subsequent values, but we discover the resultant forecast errors are not entirely random. They have predictable patterns and can be partially reduced using their own history. Learning from past forecast errors to calibrate inputs (akin to empirical Bayesian learning) generates portfolio performance that reinforces the case for optimization. Furthermore, the portfolios achieve performance that meets expectations, a desirable yet elusive feature of optimization methods.

Bartz, D. (2016). "Advances in high-dimensional covariance matrix estimation." PhD thesis. Technical University Berlin.
Many applications require precise estimates of high-dimensional covariance matrices. The standard estimator is the sample covariance matrix, All which is conceptually simple, fast to compute and has favorable properties in the limit of infinitely many observations. The picture changes When the dimensionality is of the same order: as the number of observations. In examined cases, the eigenvalues of the sample covariance matrix are highly biased, the condition number Becomes large and the inversion of the matrix gets numerically unstable. A number of alternative estimators are superior in the high-dimensional setting, Which include as subcategories structured estimators, regularized estimators and spectrum correction methods. In this thesis I contribute to all three areas. In the area of structured estimation, I focus on models with low intrinsic dimensionality. I analyze the bias in Factor Analysis, the state-of-the-art factor model and propose Directional Variance Adjustment DVA Factor Analysis, which Reduces bias and yields improved estimates of the covariance matrix. Analytical shrinkage of Ledoit and Wolf LW Shrinkage is the most popular regularized estimator. I contribute in three aspects: first, I Provide A theoretical analysis of the behavior of EV Shrinkage in the presence of pronounced intrinsically directions, a case of great practical relevance. I did show LW Shrinkage does not perform well in this setting and propose AOC Shrinkage Which yields significant improvements. Second, I discuss the effect of autocorrelation on LW Shrinkage and review the Sancetta estimator, to extension of LW Shrinkage to autocorrelated data. I show, dass die Sancetta estimator is biased and propose a Theoretically and empirically superior estimator with reduced bias. Third, I propose to extension of shrinkage to shrinkage multiple targets. Multi-Target Shrinkage is not restricted to covariance estimation and Allows for many interesting applications Which go beyond regularization, Including transfer learning. I Provide a detailed theoretical and empirical analysis. Spectrum correction Approaches the Problem of covariance estimation by Improving the estimates of the eigenvalues of the sample covariance matrix. I discuss the state-of-the-art approach, Nonlinear Shrinkage, and propose a cross-validation based covariance CVC estimator Which yields competitive performance at Increased numerical stability and greatly reduced complexity and computational cost. On all datasets Considered, CVC is on par or superior in comparison to the regularized and structured estimators. In the load-chapter, I conclude with a discussion of the advantages and disadvantages of all covariance estimators presented in this thesis and give situation-specific recommendations. In addition, the appendix contains a systematic analysis of linear discriminant analysis as a model application, Which sheds light on the interdependency between the generative model of the data and various covariance estimators. Many applications require high-dimensional covariance estimates. The standard

estimator is the sample covariance. The sample covariance is conceptually easy to calculate quickly and has good properties in the limit of infinitely many observations. This changes when the dimensionality has the same order of magnitude as the number of observations. Then, the eigenvalues of the sample covariance to a high bias, the condition is great and the inverse numerically unstable. There are numerous alternative estimator that are better in this application. This can usually be in the subgroups structured estimator divided regularized estimators and methods for spectrum correction. In this dissertation, I contribute to all of these areas. In structured estimator I focus on models with low intrinsic dimensionality. I analyzed the systematic error of the factor analysis, the prior art in the field factor models, and propose Directional Variance Adjustment DVA before which corrects the systematic errors and improved covariance estimation supplies. Analytical Shrinkage of Ledoit and Wolf LW Shrinkage is the most widely used regularized estimator. To this end, I provide three contributions: First I run a theoretical analysis of the behavior of EV Shrinkage in presence of strong eigendirections ago, a case of great practical relevance. I show that LW Shrinkage unfavorable behavior in this situation and propose the more powerful AOC Shrinkage before. Second, I discuss the effect of autocorrelation on Shrinkage and adjust the Sancetta estimator front, an extension of LW Shrinkage for autocorrelated data. I show that the Sancetta estimator has a bias and suggest a theoretically and empirically superior estimator which has a lower bias. Third, I propose an extension of Shrinkage Shrinkage for multiple targets.

Begusic, S. and Kostanjcar, Z. (2019). "Cluster-Based Shrinkage of Correlation Matrices for Portfolio Optimization." In: *11th International Symposium on Image and Signal Processing and Analysis (ISPA)*. IEEE, pp. 301–305.
The estimation of correlation and covariance matrices from asset return time series is a critical step in financial portfolio optimization. Although sample estimates are reliable when the length of time series is very large compared to the number of assets, in high-dimensional settings estimation issues arise. To reduce estimation errors and mitigate their propagation to out-of-sample performance of portfolios based on noisy estimates, shrinkage methods are applied. In this paper we consider several shrinkage methods for correlation matrix estimation and define a cluster-based shrinkage procedure which introduces information about the structures of communities identified in asset dependence graphs. To test the considered shrinkage methods we apply them in a portfolio optimization scenario using the global minimum variance portfolio, and perform backtests on a large sample of NYSE daily stock return data. We find that shrinkage methods generally improve out-of-sample portfolio performance, and the proposed cluster-based method yields improved results and portfolios which outperform other considered methods.

Benaych-Georges, F., Bouchaud, J.-P., and Potters, M. (2021). "Optimal cleaning for singular values of cross-covariance matrices." In: *arXiv e-Print*.
We give a new algorithm for the estimation of the cross-covariance matrix EXY' of two large dimensional signals $X \in R^n$, $Y \in R^p$ in the context where the number T of observations of the pair (X, Y) is large but n/T and p/T are not supposed to be small. In the asymptotic regime where n, p, T are large, with high probability, this algorithm is optimal for the Frobenius norm among rotationally invariant estimators, i.e. estimators derived from the empirical estimator by cleaning the singular values, while letting singular vectors unchanged.

Berger, T. and Gencay, R. (2020). "Short-run wavelet-based covariance regimes for applied portfolio management." In: *Journal of Forecasting* 39(4), pp. 642–660.
Decisions on asset allocations are often determined by covariance estimates from historical market data. In this paper, we introduce a wavelet-based portfolio algorithm, distinguishing between newly embedded news and long-run information that has already been fully absorbed by the market. Exploiting the wavelet decomposition into short- and long-run covariance regimes, we introduce an approach to focus on particular covariance components. Using generated data, we demonstrate that short-run covariance regimes comprise the relevant information for periodical portfolio management. In an empirical application to US stocks and other international markets for weekly, monthly, quarterly, and yearly holding periods (and rebalancing), we present evidence that the application of wavelet-based covariance estimates from short-run information outperforms portfolio allocations that are based on covariance estimates from historical data.

Bernardi, M., Bonaccolto, G., Caporin, M., and Costola, M. (2020). "Volatility Forecasting in a Data Rich Environment." In: *Macroeconomic Forecasting in the Era of Big Data*. Springer International Publishing, pp. 127–160.
This Chapter reviews the main classes of models that incorporate volatility, with a focus on the most recent advancements in the financial econometrics literature and on the challenges posed by the increased availability of data. There are limits to the feasibility of all models when the cross-sectional dimension diverges, unless strong restrictions are imposed on the model's dynamics. In the latter case, the models might become feasible

at the expense of reduced economic intuition that can be recovered from the model fit. In turn, this could have a negative impact on the forecast and the identification of its drivers.

Boileau, P., Hejazi, N. S., Laan, M. J. van der, and Dudoit, S. (2021). "Cross-Validated Loss-Based Covariance Matrix Estimator Selection in High Dimensions." In: *arXiv e-Print*.

The covariance matrix plays a fundamental role in many modern exploratory and inferential statistical procedures, including dimensionality reduction, hypothesis testing, and regression. In low-dimensional regimes, where the number of observations far exceeds the number of variables, the optimality of the sample covariance matrix as an estimator of this parameter is well-established. High-dimensional regimes do not admit such a convenience, however. As such, a variety of estimators have been derived to overcome the shortcomings of the sample covariance matrix in these settings. Yet, the question of selecting an optimal estimator from among the plethora available remains largely unaddressed. Using the framework of cross-validated loss-based estimation, we develop the theoretical underpinnings of just such an estimator selection procedure. In particular, we propose a general class of loss functions for covariance matrix estimation and establish finite-sample risk bounds and conditions for the asymptotic optimality of the cross-validated estimator selector with respect to these loss functions. We evaluate our proposed approach via a comprehensive set of simulation experiments and demonstrate its practical benefits by application in the exploratory analysis of two single-cell transcriptome sequencing datasets. A free and open-source software implementation of the proposed methodology, the cvCovEst R package, is briefly introduced.

Bollerslev, T., Hood, B., Huss, J., and Pedersen, L. H. (2018). "Risk Everywhere: Modeling and Managing Volatility." In: *The Review of Financial Studies* 31(7), pp. 2729–2773.

Based on high-frequency data for more than fifty commodities, currencies, equity indices, and fixed-income instruments spanning more than two decades, we document strong similarities in realized volatility patterns within and across asset classes. Exploiting these similarities through panel-based estimation of new realized volatility models results in superior out-of-sample risk forecasts, compared to forecasts from existing models and conventional procedures that do not incorporate the similarities in volatilities. We develop a utility-based framework for evaluating risk models that shows significant economic gains from our new risk model. Lastly, we evaluate the effects of transaction costs and trading speed in implementing different risk models.

Bongiorno, C. and Challet, D. (2020a). "Covariance matrix filtering with bootstrapped hierarchies." In: *arXiv e-Print*.

Statistical inference of the dependence between objects often relies on covariance matrices. Unless the number of features (e.g. data points) is much larger than the number of objects, covariance matrix cleaning is necessary to reduce estimation noise. We propose a method that is robust yet flexible enough to account for fine details of the structure covariance matrix. Robustness comes from using a hierarchical ansatz and dependence averaging between clusters; flexibility comes from a bootstrap procedure. This method finds several possible hierarchical structures in DNA microarray gene expression data, and leads to lower realized risk in global minimum variance portfolios than current filtering methods when the number of data points is relatively small.

Bongiorno, C. and Challet, D. (2020b). "Reactive Global Minimum Variance Portfolios with k-BAHC covariance cleaning." In: *arXiv e-Print*.

We introduce a k-fold boosted version of our Boostrapped Average Hierarchical Clustering cleaning procedure for correlation and covariance matrices. We then apply this method to global minimum variance portfolios for various values of k and compare their performance with other state-of-the-art methods. Generally, we find that our method yields better Sharpe ratios after transaction costs than competing filtering methods, despite requiring a larger turnover.

Bongiorno, C. and Challet, D. (2021). "The Oracle estimator is suboptimal for global minimum variance portfolio optimisation." In: *arXiv e-Print*.

A common misconception is that the Oracle eigenvalue estimator of the covariance matrix yields the best realized portfolio performance. In reality, the Oracle estimator simply modifies the empirical covariance matrix eigenvalues so as to minimize the Frobenius distance between the filtered and the realized covariance matrices. This leads to the best portfolios only when the in-sample eigenvectors coincide with the out-of-sample ones. In all the other cases, the optimal eigenvalue correction can be obtained from the solution of a Quadratic-Programming problem. Solving it shows that the Oracle estimators only yield the best portfolios in the limit of infinite data points per asset and only in stationary systems.

Bun, J., Bouchaud, J.-P., and Potters, M. (2017). "Cleaning large correlation matrices: Tools from Random Matrix Theory." In: *Physics Reports* 666, pp. 1–109.

This review covers recent results concerning the estimation of large covariance matrices using tools from Random Matrix Theory (RMT). We introduce several RMT methods and analytical techniques, such as the Replica formalism and Free Probability, with an emphasis on the Marenko-Pastur equation that provides information on the resolvent of multiplicatively corrupted noisy matrices. Special care is devoted to the statistics of the eigenvectors of the empirical correlation matrix, which turn out to be crucial for many applications. We show in particular how these results can be used to build consistent "Rotationally Invariant"estimators (RIE) for large correlation matrices when there is no prior on the structure of the underlying process. The last part of this review is dedicated to some real-world applications within financial markets as a case in point. We establish empirically the efficacy of the RIE framework, which is found to be superior in this case to all previously proposed methods. The case of additively (rather than multiplicatively) corrupted noisy matrices is also dealt with in a special Appendix. Several open problems and interesting technical developments are discussed throughout the paper.

Chang, S. M. (2015). "Double shrinkage estimators for large sparse covariance matrices." In: *Journal of Statistical Computation and Simulation* 85(8), pp. 1497–1511.

Covariance matrices play an important role in many multivariate techniques and hence a good covariance estimation is crucial in this kind of analysis. In many applications a sparse covariance matrix is expected due to the nature of the data or for simple interpretation. Hard thresholding, soft thresholding, and generalized thresholding were therefore developed to this end. However, these estimators do not always yield well-conditioned covariance estimates. To have sparse and well-conditioned estimates, we propose doubly shrinkage estimators: shrinking small covariances towards zero and then shrinking covariance matrix towards a diagonal matrix. Additionally, a richness index is defined to evaluate how rich a covariance matrix is. According to our simulations, the richness index serves as a good indicator to choose relevant covariance estimator.

Choi, Y.-G., Lim, J., and Choi, S. (2019). "High-dimensional Markowitz portfolio optimization problem: empirical comparison of covariance matrix estimators." In: *Journal of Statistical Computation and Simulation* 89(7), pp. 1278–1300.

We compare the performance of recently developed regularized covariance matrix estimators for Markowitz's portfolio optimization and of the minimum variance portfolio (MVP) problem in particular. We focus on seven estimators that are applied to the MVP problem in the literature; three regularize the eigenvalues of the sample covariance matrix, and the other four assume the sparsity of the true covariance matrix or its inverse. Comparisons are made with two sets of long-term S&P 500 stock return data that represent two extreme scenarios of active and passive management. The results show that the MVPs with sparse covariance estimators have high Sharpe ratios but that the naive diversification (also known as the (on market share) portfolio) still performs well in terms of wealth growth.

Clements, A. and Doolan, M. B. (2020). "Combining multivariate volatility forecasts using weighted losses." In: *Journal of Forecasting* 39(4), pp. 628–641.

The ability to improve out-of-sample forecasting performance by combining forecasts is well established in the literature. This paper advances this literature in the area of multivariate volatility forecasts by developing two combination weighting schemes that exploit volatility persistence to emphasise certain losses within the combination estimation period. A comprehensive empirical analysis of the out-of-sample forecast performance across varying dimensions, loss functions, sub-samples and forecast horizons show that new approaches significantly outperform their counterparts in terms of statistical accuracy. Within the financial applications considered, significant benefits from combination forecasts relative to the individual candidate models are observed. Although the more sophisticated combination approaches consistently rank higher relative to the equally weighted approach, their performance is statistically indistinguishable given the relatively low power of these loss functions. Finally, within the applications, further analysis highlights how combination forecasts dramatically reduce the variability in the parameter of interest, namely the portfolio weight or beta.

Coqueret, G. and Milhau, V. (2014). "Estimating Covariance Matrices for Portfolio Optimisation." In: *SSRN e-Print*.

We compare 12 estimators of the covariance matrix: the sample covariance matrix, the identity matrix, the constant-correlation estimator, three estimators derived from an explicit factor model, three obtained from an implicit factor model, and three shrunk estimators. Following the literature, we conduct the comparison by computing the volatility of estimated Minimum Variance portfolios. We do this in two frameworks: first, an ideal situation where the true covariance matrix would be known, and second, a real-world situation where it is unknown. In each of these two cases, we perform the tests with and without short-sale constraints, and we assess the impact of the universe and sample sizes on the results. Our findings are in line with those of Ledoit and Wolf (2003), in that we confirm that in the absence of short-sale constraints, shrunk estimators lead, in general, to the lowest volatilities. With long-only constraints, however, their performance is similar to that of principal component estimators. Moreover, the latter estimators tend to imply lower levels of turnover, which is an important practical consideration.

De Nard, G. (2020). "Oops! I Shrunk the Sample Covariance Matrix Again: Blockbuster Meets Shrinkage." In: *Journal of Financial Econometrics*.
Existing shrinkage techniques struggle to model the covariance matrix of asset returns in the presence of multiple-asset classes. Therefore, we introduce a Blockbuster shrinkage estimator that clusters the covariance matrix accordingly. Besides the definition and derivation of a new asymptotically optimal linear shrinkage estimator, we propose an adaptive Blockbuster algorithm that clusters the covariance matrix even if the (number of) asset classes are unknown and change over time. It displays superior all-around performance on historical data against a variety of state-of-the-art linear shrinkage competitors. Additionally, we find that for small- and medium-sized investment universes the proposed estimator outperforms even recent nonlinear shrinkage techniques. Hence, this new estimator can be used to deliver more efficient portfolio selection and detection of anomalies in the cross-section of asset returns. Furthermore, due to the general structure of the proposed Blockbuster shrinkage estimator, the application is not restricted to financial problems.

De Nard, G., Hediger, S., and Leippold, M. (2020). "Subsampled factor models for asset pricing: the rise of VASA." In: *SSRN e-Print*.
We propose a new method, VASA, based on variable subsample aggregation of model predictions for equity returns using a large-dimensional set of factors. To demonstrate the effectiveness, robustness, and dimension reduction power of VASA, we perform a comparative analysis between state-of-the-art machine learning algorithms. As a performance measure, we explore not only the global predictive but also the stock-specific R2's and their distribution. While the global R2 indicates the average forecasting accuracy, we find that high variability in the stock-specific R2's can be detrimental for the portfolio performance, due to the higher prediction risk. Since VASA shows minimal variability, portfolios formed on this method outperform the portfolios based on more complicated methods like random forests and neural nets.

De Nard, G., Ledoit, O., and Wolf, M. (2021). "Factor Models for Portfolio Selection in Large Dimensions: The Good, the Better and the Ugly." In: *Journal of Financial Econometrics* 19(2), pp. 236–257.
This paper injects factor structure into the estimation of time-varying, large-dimensional covariance matrices of stock returns. Existing factor models struggle to model the covariance matrix of residuals in the presence of time-varying conditional heteroskedasticity in large universes. Conversely, rotation-equivariant estimators of large-dimensional time-varying covariance matrices forsake directional information embedded in market-wide risk factors. We introduce a new covariance matrix estimator that blends factor structure with time-varying conditional heteroskedasticity of residuals in large dimensions up to 1000 stocks. It displays superior all-around performance on historical data against a variety of state-of-the-art competitors, including static factor models, exogenous factor models, sparsity-based models, and structure-free dynamic models. This new estimator can be used to deliver more efficient portfolio selection and detection of anomalies in the cross-section of stock returns.

De Nard, G. and Zhao, Z. (2021). "Using, Taming or Avoiding the Factor Zoo? A Double-Shrinkage Estimator for Covariance Matrices." In: *SSRN e-Print*.
Existing factor models struggle to model the covariance matrix for a large number of stocks and factors. Therefore, we introduce a new covariance matrix estimator that first shrinks the factor model coefficients and then applies nonlinear shrinkage to the residuals and factors. The estimator blends a regularized factor structure with conditional heteroskedasticity of residuals and factors and displays superior all-around performance against various competitors. We show that for the proposed double- shrinkage estimator, it is enough to use only the market factor or the most important latent factor(s). Thus there is no need for laboriously taking into account the factor zoo. Supplementary material for this article is available online.

DeMiguel, V., Martin-Utrera, A., and Nogales, F. J. (2013). "Size matters: Optimal calibration of shrinkage estimators for portfolio selection." In: *Journal of Banking and Finance* 37(8), pp. 3018–3034.
We carry out a comprehensive investigation of shrinkage estimators for asset allocation, and we find that size mattersthe shrinkage intensity plays a significant role in the performance of the resulting estimated optimal portfolios. We study both portfolios computed from shrinkage estimators of the moments of asset returns (shrinkage moments), as well as shrinkage portfolios obtained by shrinking the portfolio weights directly. We make several contributions in this field. First, we propose two novel calibration criteria for the vector of means and the inverse covariance matrix. Second, for the covariance matrix we propose a novel calibration criterion that takes the condition number optimally into account. Third, for shrinkage portfolios we study two novel calibration criteria.

Fourth, we propose a simple multivariate smoothed bootstrap approach to construct the optimal shrinkage intensity. Finally, we carry out an extensive out-of-sample analysis with simulated and empirical datasets, and we characterize the performance of the different shrinkage estimators for portfolio selection.

Deshmukh, S. and Dubey, A. (2020). "Improved Covariance Matrix Estimation With an Application in Portfolio Optimization." In: *IEEE Signal Processing Letters* 27, pp. 985–989.

One of the major challenges in multivariate analysis is the estimation of population covariance matrix from the sample covariance matrix (SCM). Most recent covariance matrix estimators use either shrinkage transformations or asymptotic results from Random Matrix Theory (RMT). Both of these techniques try to achieve a similar goal which is to remove noisy correlations and add structure to SCM to overcome the bias-variance trade-off. Both methods have their respective pros and cons. In this paper, we propose an improved estimator which exploits the advantages of these techniques by taking optimally weighted convex combination of covariance matrices estimated by shrinkage transformation and a filter based on RMT. It is a generalized estimator which can adapt to changing sampling noise conditions by performing hyperparameter optimization. Using data from six of the world's biggest stock exchanges, we show that the proposed estimator outperforms the existing estimators in minimizing the out-of-sample risk of the portfolio and hence predicts population statistics more precisely. The proposed estimator can be useful in a wide range of machine learning and signal processing applications.

Dey, K. K. and Stephens, M. (2018). "CorShrink : Empirical Bayes shrinkage estimation of correlations, with applications." In: *BioRxiv e-Print*.

Estimation of correlation matrices and correlations among variables is a ubiquitous problem in statistics. In many cases – especially when the number of observations is small relative to the number of variables – some kind of shrinkage or regularization is necessary to improve estimation accuracy. Here, we propose an Empirical Bayes shrinkage approach, CorShrink, which adaptively learns how much to shrink correlations by combining information across all pairs of variables. One key feature of CorShrink, which distinguishes it from most existing methods, is its flexibility in dealing with missing data. Indeed, CorShrink explicitly accounts for varying amounts of missingness among pairs of variables. Numerical studies suggest CorShrink is competitive with other popular correlation shrinkage methods, even when there is no missing data. We illustrate CorShrink on gene expression data from GTEx project, which suffers from extensive missing observations, and where existing methods struggle. We also illustrate its flexibility by applying it to estimate cosine similarities between word vectors from word2vec models, thereby generating more accurate word similarity rankings.

Ding, X. and Zhou, Z. (2020). "Estimation and inference for precision matrices of nonstationary time series." In: *Annals of Statistics* 48(4), pp. 2455–2477.

We consider the estimation of and inference on precision matrices of a rich class of univariate locally stationary linear and nonlinear time series, assuming that only one realization of the time series is observed. Using a Cholesky decomposition technique, we show that the precision matrices can be directly estimated via a series of least squares linear regressions with smoothly time-varying coefficients. The method of sieves is utilized for the estimation and is shown to be optimally adaptive in terms of estimation accuracy and efficient in terms of computational complexity. We establish an asymptotic theory for a class of L2 tests based on the nonparametric sieve estimators. The latter are used for testing whether the precision matrices are diagonal or banded. A Gaussian approximation result is established for a wide class of quadratic forms of nonstationary and possibly nonlinear processes of diverging dimensions which is of interest by itself.

Ehling, P. and Heyerdahl-Larsen, C. (2017). "Correlations." In: *Management Science* 63(6), pp. 1919–1937.

Correlations of equity returns have varied substantially over time and remain a source of continuing policy debate. This paper studies stock market correlations in an equilibrium model with heterogeneous risk aversion. In the model, preference heterogeneity causes variations in the volatility of aggregate risk aversion from good to bad states. At times of high volatility in aggregate risk aversion, which is a common factor in returns, we see high correlations. The model matches average industry return correlations and changes in correlations from business cycle peaks to troughs and replicates the dynamics of expected excess returns and standard deviations. Model-implied aggregate risk aversion explains average industry correlations, expected excess returns, standard deviations, and turnover volatility in the data. We find supportive evidence for the model's prediction that industries with low dividend-consumption correlation have low average return correlation but experience disproportional increases in return correlations in recessions.

Engle, R. F., Ledoit, O., and Wolf, M. (2019). "Large Dynamic Covariance Matrices." In: *Journal of Business and Economic Statistics* 37(2), pp. 363–375.

AbstractSecond moments of asset returns are important for risk management and portfolio selection. The problem of estimating second moments can be approached from two angles: time series and the cross-section. In time series, the key is to account for conditional heteroskedasticity; a favored model is Dynamic Conditional Correlation (DCC), derived from the ARCH/GARCH family started by Engle (1982). In the cross-section, the key is to correct in-sample biases of sample covariance matrix eigenvalues; a favored model is nonlinear shrinkage, derived from Random Matrix Theory (RMT). The present paper marries these two strands of literature in order to deliver improved estimation of large dynamic covariance matrices.

Fan, J., Liao, Y., and Liu, H. (2016). "An overview of the estimation of large covariance and precision matrices." In: *The Econometrics Journal* 19(1), pp. C1–C32.

The estimation of large covariance and precision matrices is fundamental in modern multivariate analysis. However, problems arise from the statistical analysis of large panel economic and financial data. The covariance matrix reveals marginal correlations between variables, while the precision matrix encodes conditional correlations between pairs of variables given the remaining variables. In this paper, we provide a selective review of several recent developments on the estimation of large covariance and precision matrices. We focus on two general approaches: a rank-based method and a factor-model-based method. Theories and applications of both approaches are presented. These methods are expected to be widely applicable to the analysis of economic and financial data.

Fan, R., Jang, B., Sun, Y., and Zhou, S. (2019). "Precision Matrix Estimation with Noisy and Missing Data." In: *Proceedings of Machine Learning Research* 89, pp. 2810–2819.

Estimating conditional dependence graphs and precision matrices are some of the most common problems in modern statistics and machine learning. When data are fully observed, penalized maximum likelihood-type estimators have become standard tools for estimating graphical models under sparsity conditions. Extensions of these methods to more complex settings where data are contaminated with additive or multiplicative noise have been developed in recent years. In these settings, however, the relative performance of different methods is not well understood and algorithmic gaps still exist. In particular, in high-dimensional settings these methods require using non-positive semidefinite matrices as inputs, presenting novel optimization challenges. We develop an alternating direction method of multipliers (ADMM) algorithm for these problems, providing a feasible algorithm to estimate precision matrices with indefinite input and potentially nonconvex penalties. We compare this method with existing alternative solutions and empirically characterize the tradeoffs between them. Finally, we use this method to explore the networks among US senators estimated from voting records data.

Gortz, C. and Yeromonahos, M. (2019). "Asymmetries in Risk Premia, Macroeconomic Uncertainty and Business Cycles." In: *CESifo* (7959).

A large literature suggests that the expected equity risk premium is countercyclical. Using a variety of different measures for this risk premium, we document that it also exhibits growth asymmetry, i.e. the risk premium rises sharply in recessions and declines much more gradually during the following recoveries. We show that a model with recursive preferences, in which agents cannot perfectly observe the state of current productivity, can generate the observed asymmetry in the risk premium. Key for this result are endogenous fluctuations in uncertainty which induce procyclical variations in agent's nowcast accuracy. In addition to matching moments of the risk premium, the model is also successful in generating the growth asymmetry in macroeconomic aggregates observed in the data, and in matching the cyclical relation between quantities and the risk premium.

Harutyunyan, H., Moyer, D., Khachatrian, H., Steeg, G. V., and Galstyan, A. (2021). "Efficient Covariance Estimation from Temporal Data." In: *arXiv e-Print*.

Estimating the covariance structure of multivariate time series is a fundamental problem with a wide-range of real-world applications – from financial modeling to fMRI analysis. Despite significant recent advances, current state-of-the-art methods are still severely limited in terms of scalability, and do not work well in high-dimensional undersampled regimes. In this work we propose a novel method called Temporal Correlation Explanation, or T-CorEx, that (a) has linear time and memory complexity with respect to the number of variables, and can scale to very large temporal datasets that are not tractable with existing methods; (b) gives state-of-the-art results in highly undersampled regimes on both synthetic and real-world datasets; and (c) makes minimal assumptions about the character of the dynamics of the system. T-CorEx optimizes an information-theoretic objective function to learn a latent factor graphical model for each time period and applies two regularization techniques to induce temporal consistency of estimates. We perform extensive evaluation of T-Corex using both synthetic and real-world data and demonstrate that it can be used for detecting sudden changes in the underlying covariance matrix, capturing transient correlations and analyzing extremely high-dimensional complex multivariate time series such as high-resolution fMRI data.

Higham, N. J., Strabic, N., and Sego, V. (2016). "Restoring Definiteness via Shrinking, with an Application to Correlation Matrices with a Fixed Block." In: *SIAM Review* 58(2), pp. 245–263.

Indefinite approximations of positive semidefinite matrices arise in various data analysis applications involving covariance matrices and correlation matrices. We propose a method for restoring positive semidefiniteness of an indefinite matrix M0 that constructs a convex linear combination S(alpha) = alpha*M1 + (1-alpha)*M0 of M0 and a positive semidefinite target matrix M1. In statistics, this construction for improving an estimate M0 by combining it with new information in M1 is known as shrinking. We make no statistical assumptions about M0 and define the optimal shrinking parameter as alphaStar = min (alpha in [0,1] such that S(alpha) is positive semidefinite). We describe three algorithms for computing alphaStar. One algorithm is based on the bisection method, with the use of Cholesky factorization to test definiteness; a second employs Newton's method; and a third finds the smallest eigenvalue of a symmetric definite generalized eigenvalue problem. We show that weights that reflect confidence in the individual entries of M0 can be used to construct a natural choice of the target matrix M1. We treat in detail a problem variant in which a positive semidefinite leading principal submatrix of M0 remains fixed, showing how the fixed block can be exploited to reduce the cost of the bisection and generalized eigenvalue methods. Numerical experiments show that when applied to indefinite approximations of correlation matrices shrinking can be at least an order of magnitude faster than computing the nearest correlation matrix.

Huang, M. and Yu, S. (2020). "A new procedure for resampled portfolio with shrinkaged covariance matrix." In: *Journal of Applied Statistics* 47(44), pp. 642–652.

Dealing with estimation error is an important issue when we implement the mean-variance paradigm for portfolio construction. To tackle the problem, two approaches are proposed in literature, the portfolio resampling technique introduced by Michuad and the well-known shrinkaged covariance matrix method. There are certain evidences on the advantages of shrinkaged covariance over portfolio resampling, however, it is unclear whether a combination of the two approaches could produce a better performance compared with using shrinkaged covariance alone. In this paper, we propose a new algorithm to integrated linear or nonlinear shrinkage estimation with resampled portfolio to achieve a further improvement. Our method are demonstrated via extensive simulation and application in active portfolio management process.

Husmann, S., Shivarova, A., and Steinert, R. (2020). "Cross-validated covariance estimators for high-dimensional minimum-variance portfolios." In: *arXiv e-Print*.

The global minimum-variance portfolio is a typical choice for investors because of its simplicity and broad applicability. Although it requires only one input, namely the covariance matrix of asset returns, estimating the optimal solution remains a challenge. In the presence of high-dimensionality in the data, the sample estimator becomes ill-conditioned, which negates the positive effect of diversification in an out-of-sample setting. To address this issue, we review recent covariance matrix estimators and extend the literature by suggesting a multi-fold cross-validation technique. In detail, conducting an extensive empirical analysis with four datasets based on the S&P 500, we evaluate how the data-driven choice of specific tuning parameters within the proposed cross-validation approach affects the out-of-sample performance of the global minimum-variance portfolio. In particular, for cases in which the efficiency of a covariance estimator is strongly influenced by the choice of a tuning parameter, we detect a clear relationship between the optimality criterion for its selection within the cross-validation and the evaluated performance measure. Finally, we show that using cross-validation can improve the performance of highly efficient estimators even when the data-driven covariance parameter deviates from its theoretically optimal value.

Jain, P. and Jain, S. (2019). "Can Machine Learning-Based Portfolios Outperform Traditional Risk-Based Portfolios? The Need to Account for Covariance Misspecification." In: *Risks* 7(3), pp. 74+.

The Hierarchical risk parity (HRP) approach of portfolio allocation, introduced by Lopez de Prado (2016), applies graph theory and machine learning to build a diversified portfolio. Like the traditional risk-based allocation methods, HRP is also a function of the estimate of the covariance matrix, however, it does not require its invertibility. In this paper, we first study the impact of covariance misspecification on the performance of the different allocation methods. Next, we study under an appropriate covariance forecast model whether the machine learning based HRP outperforms the traditional risk-based portfolios. For our analysis, we use the test for superior predictive ability on out-of-sample portfolio performance, to determine whether the observed excess performance is significant or if it occurred by chance. We find that when the covariance estimates are crude, inverse volatility weighted portfolios are more robust, followed by the machine learning-based portfolios. Minimum variance and maximum diversification are most sensitive to covariance misspecification. HRP follows the middle ground; it is less sensitive to covariance misspecification when compared with minimum variance or maximum diversification portfolio, while it is not as robust as the inverse volatility weighed portfolio. We also study the impact of the different rebalancing horizon and how the portfolios compare against a market-capitalization weighted portfolio.

Jay, E., Soler, T., Ovarlez, J.-P., Peretti, P. D., and Chorro, C. (2020a). "Robust Covariance Matrix Estimation and Portfolio Allocation: The Case of Non-Homogeneous Assets." In: *ICASSP 2020 - 2020 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. IEEE.
This paper presents how the most recent improvements made on covariance matrix estimation and model order selection can be applied to the portfolio optimization problem. Our study is based on the case of the Maximum Variety Portfolio and may be obviously extended to other classical frameworks with analogous results. We focus on the fact that the assets should preferably be classified in homogeneous groups before applying the proposed methodology which is to whiten the data before estimating the covariance matrix using the robust Tyler M-estimator and the Random Matrix Theory (RMT). The proposed procedure is applied and compared to standard techniques on real market data showing promising improvements.

Jay, E., Soler, T., Terreaux, E., Ovarlez, J.-P., Pascal, F., Peretti, P. D., and Chorro, C. (2020b). "Improving portfolios global performance using a cleaned and robust covariance matrix estimate." In: *Soft Computing* 24(12), pp. 8643–8654.
This paper presents how the use of a cleaned and robust covariance matrix estimate can improve significantly the overall performance of maximum variety and minimum variance portfolios. We assume that the asset returns are modelled through a multi-factor model where the error term is a multivariate and correlated elliptical symmetric noise extending the classical Gaussian assumptions. The factors are supposed to be unobservable and we focus on a recent method of model order selection, based on the random matrix theory to identify the most informative subspace and then to obtain a cleaned (or de-noised) covariance matrix estimate to be used in the maximum variety and minimum variance portfolio allocation processes. We apply our methodology on real market data and show the improvements it brings if compared with other techniques especially for non-homogeneous asset returns.

Ke, Y., Lian, H., and Zhang, W. (2021). "High-Dimensional Dynamic Covariance Matrices With Homogeneous Structure." In: *Journal of Business & Economic Statistics*.
High-dimensional covariance matrices appear in many disciplines. Much literature has devoted to the research in high-dimensional constant covariance matrices. However, constant covariance matrices are not sufficient in applications, for example, in portfolio allocation, dynamic covariance matrices would be more appropriate. As argued in this article, there are two difficulties in the introduction of dynamic structures into covariance matrices: (1) simply assuming each entry of a covariance matrix is a function of time to introduce the dynamic needed would not work; (2) there is a risk of having too many unknowns to estimate due to the high dimensionality. In this article, we propose a dynamic structure embedded with a homogeneous structure. We will demonstrate the proposed dynamic structure makes more sense in applications and avoids, in the meantime, too many unknown parameters/functions to estimate, due to the embedded homogeneous structure. An estimation procedure is also proposed to estimate the proposed high-dimensional dynamic covariance matrices, and asymptotic properties are established to justify the proposed estimation procedure. Intensive simulation studies show the proposed estimation procedure works very well when the sample size is finite. Finally, we apply the proposed high-dimensional dynamic covariance matrices to portfolio allocation. It is interesting to see the resulting portfolio yields much better returns than some commonly used ones.

Ke, Y., Minsker, S., Ren, Z., Sun, Q., and Zhou, W.-X. (2019). "User-Friendly Covariance Estimation for Heavy-Tailed Distributions." In: *Statistical Science* 34(3), pp. 454–471.
We provide a survey of recent results on covariance estimation for heavy-tailed distributions. By unifying ideas scattered in the literature, we propose user-friendly methods that facilitate practical implementation. Specifically, we introduce elementwise and spectrumwise truncation operators, as well as their M-estimator counterparts, to robustify the sample covariance matrix. Different from the classical notion of robustness that is characterized by the breakdown property, we focus on the tail robustness which is evidenced by the connection between nonasymptotic deviation and confidence level. The key insight is that estimators should adapt to the sample size, dimensionality and noise level to achieve optimal tradeoff between bias and robustness. Furthermore, to facilitate practical implementation, we propose data-driven procedures that automatically calibrate the tuning

parameters. We demonstrate their applications to a series of structured models in high dimensions, including the bandable and low-rank covariance matrices and sparse precision matrices. Numerical studies lend strong support to the proposed methods.

Lam, C. (2020). "High-dimensional covariance matrix estimation." In: *WIREs Computational Statistics* 12(22), e1485+.

Covariance matrix estimation plays an important role in statistical analysis in many fields, including (but not limited to) portfolio allocation and risk management in finance, graphical modeling, and clustering for genes discovery in bioinformatics, Kalman filtering and factor analysis in economics. In this paper, we give a selective review of covariance and precision matrix estimation when the matrix dimension can be diverging with, or even larger than the sample size. Two broad categories of regularization methods are presented. The first category exploits an assumed structure of the covariance or precision matrix for consistent estimation. The second category shrinks the eigenvalues of a sample covariance matrix, knowing from random matrix theory that such eigenvalues are biased from the population counterparts when the matrix dimension grows at the same rate as the sample size.

Lancewicki, T. and Aladjem, M. (2014). "Multi-Target Shrinkage Estimation for Covariance Matrices." In: *IEEE Transactions on Signal Processing* 62(24), pp. 6380–6390.

Covariance matrix estimation is problematic when the number of samples is relatively small compared with the number of variables. One way to tackle this problem is through the use of shrinkage estimators that offer a compromise between the sample covariance matrix and a well-conditioned matrix (also known as the -target-) with the aim of minimizing the mean-squared error (MSE). The use of only one target limits the shrinkage estimators' flexibility when minimizing the MSE. In this paper, we propose a multi-target shrinkage estimator (MTSE) for covariance matrices that exploits the Lediot-Wolf (LW) method by utilizing several targets simultaneously. This greatly increases the estimator's flexibility and enables it to attain a lower MSE. We also offer a general target that serves as a framework for designing a wide variety of targets. In consequence, instead of studying individual targets, the general framework can be utilized. We then show that the framework encompasses several targets that already exist in the literature. Numerical simulations demonstrate that the MTSE significantly reduces the MSE and is highly effective in classification tasks.

Ledoit, O. and Wolf, M. (2017). "Nonlinear Shrinkage of the Covariance Matrix for Portfolio Selection: Markowitz Meets Goldilocks." In: *The Review of Financial Studies* 30(12), pp. 4349–4388.

Markowitz (1952) portfolio selection requires an estimator of the covariance matrix of returns. To address this problem, we promote a nonlinear shrinkage estimator that is more flexible than previous linear shrinkage estimators and has just the right number of free parameters (i.e., the Goldilocks principle). This number is the same as the number of assets. Our nonlinear shrinkage estimator is asymptotically optimal for portfolio selection when the number of assets is of the same magnitude as the sample size. In backtests with historical stock return data, it performs better than previous proposals and, in particular, it dominates linear shrinkage.

Ledoit, O. and Wolf, M. (2020a). "Analytical nonlinear shrinkage of large-dimensional covariance matrices." In: *Annals of Statistics* 48(5), pp. 3043–3065.

This paper establishes the first analytical formula for nonlinear shrinkage estimation of large-dimensional covariance matrices. We achieve this by identifying and mathematically exploiting a deep connection between nonlinear shrinkage and nonparametric estimation of the Hilbert transform of the sample spectral density. Previous nonlinear shrinkage methods were of numerical nature: QuEST requires numerical inversion of a complex equation from random matrix theory whereas NERCOME is based on a sample-splitting scheme. The new analytical method is more elegant and also has more potential to accommodate future variations or extensions. Immediate benefits are (i) that it is typically 1000 times faster with basically the same accuracy as QuEST and (ii) that it accommodates covariance matrices of dimension up to 10,000 and more. The difficult case where the matrix dimension exceeds the sample size is also covered.

Ledoit, O. and Wolf, M. (2020b). *Shrinkage Estimation of Large Covariance Matrices: Keep it Simple, Statistician?* Tech. rep. University of Zurich.

Under rotation-equivariant decision theory, sample covariance matrix eigenvalues can be optimally shrunk by recombining sample eigenvectors with a (potentially nonlinear) function of the unobservable population covariance matrix. The optimal shape of this function reflects the loss/risk that is to be minimized. We solve the problem of optimal covariance matrix estimation under a variety of loss functions motivated by statistical precedent, probability theory, and differential geometry. A key ingredient of our nonlinear shrinkage methodology is a new estimator of the angle between sample and population eigenvectors, without making strong assumptions on the population eigenvalues. We also introduce a broad family of covariance matrix estimators that can handle all regular functional transformations of the population covariance matrix under large-dimensional asymptotics. In addition, we compare via Monte Carlo simulations our methodology to two simpler ones from the literature, linear shrinkage and shrinkage based on the spiked covariance model.

Ledoit, O. and Wolf, M. (2021). "The Power of (Non-)Linear Shrinking: A Review and Guide to Covariance Matrix Estimation." In: *Journal of Financial Econometrics*.
Many econometric and data-science applications require a reliable estimate of the covariance matrix, such as Markowitz portfolio selection. When the number of variables is of the same magnitude as the number of observations, this constitutes a difficult estimation problem; the sample covariance matrix certainly will not do. In this article, we review our work in this area, going back 15plus years. We have promoted various shrinkage estimators, which can be classified into linear and nonlinear. Linear shrinkage is simpler to understand, to derive, and to implement. But nonlinear shrinkage can deliver another level of performance improvement, especially if overlaid with stylized facts such as time-varying co-volatility or factor models.

Li, X. and Zakamulin, V. (2019). "The term structure of volatility predictability." In: *SSRN e-Print*.
Volatility forecasting is crucial for portfolio management, risk management, and pricing of derivative securities. Still, little is known about the accuracy of volatility forecasts and the horizon of volatility predictability. This paper aims to fill these gaps in the literature. We begin this paper by introducing the notions of the spot and forward predicted volatilities and propose to describe the term structure of volatility predictability by the spot and forward forecast accuracy curves. Then we perform a comprehensive study on the term structure of volatility predictability in the stock and foreign exchange markets. Our results quantify the volatility forecast accuracy across horizons in the two major markets and suggest that the horizon of volatility predictability is significantly longer than that reported in the earlier studies. Nevertheless, the horizon of volatility predictability is found to be much shorter than the longest maturity of traded derivative contracts.

Li, X. and Zakamulin, V. (2020). "Stock volatility predictability in bull and bear markets." In: *Quantitative Finance* 20(7), pp. 1149–1167.
The recent literature on stock return predictability suggests that it varies substantially across economic states, being strongest during bad economic times. In line with this evidence, we document that stock volatility predictability is also state dependent. In particular, in this paper, we use a large data set of high-frequency data on individual stocks and a few popular time-series volatility models to comprehensively examine how volatility forecastability varies across bull and bear states of the stock market. We find that the volatility forecast horizon is substantially longer when the market is in a bear state than when it is in a bull state. In addition, over all but the shortest horizons, the volatility forecast accuracy is higher when the market is in a bear state. This difference increases as the forecast horizon lengthens. Our study concludes that stock volatility predictability is strongest during bad economic times, proxied by bear market states.

Ma, F., Lu, X., Yang, K., and Zhang, Y. (2019). "Volatility forecasting: long memory, regime switching and heteroscedasticity." In: *Applied Economics* 51(38), pp. 4151–4163.
In this article, we account for the first time for long memory, regime switching and the conditional time-varying volatility of volatility (heteroscedasticity) to model and forecast market volatility using the heterogeneous autoregressive model of realized volatility (HAR-RV) and its extensions. We present several interesting and notable findings. First, existing models exhibit significant nonlinearity and clustering, which provide empirical evidence on the benefit of introducing regime switching and heteroscedasticity. Second, out-of-sample results indicate that combining regime switching and heteroscedasticity can substantially improve predictive power from a statistical viewpoint. More specifically, our proposed models generally exhibit higher forecasting accuracy. Third, these results are widely consistent across a variety of robustness tests such as different forecasting windows, forecasting models, realized measures, and stock markets. Consequently, this study sheds new light on forecasting.

Marti, G., Nielsen, F., Bihkowski, M., and Donnat, P. (2020). "A review of two decades of correlations, hierarchies, networks and clustering in financial markets." In: *arXiv e-Print*.
This document is a preliminary version of an in-depth review on the state of the art of clustering financial time series and the study of correlation networks. This preliminary document is intended for researchers in this field so that they can feedback to allow amendments, corrections and addition of new material unknown to the authors of this review. The aim of the document is to gather in one place the relevant material that can help the researcher in the field to have a bigger picture, the quantitative researcher to play with this alternative modeling of the financial time series, and the decision maker to leverage the insights obtained from these methods. We hope that this document will form a basis for implementation of an open toolbox of standard tools to study

correlations, hierarchies, networks and clustering in financial markets. We also plan to maintain pointers to online material and an updated version of this work at www.datagrapple.com/Tech.

Molstad, A. J. and Rothman, A. J. (2018). "Shrinking characteristics of precision matrix estimators." In: *Biometrika* 105(3), pp. 563–574.

We propose a framework to shrink a user-specified characteristic of a precision matrix estimator that is needed to fit a predictive model. Estimators in our framework minimize the Gaussian negative loglikelihood plus an L1L1L1 penalty on a linear or affine function evaluated at the optimization variable corresponding to the precision matrix. We establish convergence rate bounds for these estimators and propose an alternating direction method of multipliers algorithm for their computation. Our simulation studies show that our estimators can perform better than competitors when they are used to fit predictive models. In particular, we illustrate cases where our precision matrix estimators perform worse at estimating the population precision matrix but better at prediction.

Moura, G. V., Santos, A. A. P., and Ruiz, E. (2020). "Comparing high-dimensional conditional covariance matrices: Implications for portfolio selection." In: *Journal of Banking & Finance* 118, p. 105882.

Portfolio selection based on high-dimensional covariance matrices is a key challenge in data-rich environments with the curse of dimensionality severely affecting most of the available covariance models. We challenge several multivariate Dynamic Conditional Correlation (DCC)-type and Stochastic Volatility (SV)-type models to obtain minimum-variance and mean-variance portfolios with up to 1000 assets. We conclude that, in a realistic context in which transaction costs are taken into account, although DCC-type models lead to portfolios with lower variance, modeling the covariance matrices as latent Wishart processes with a shrinkage towards the diagonal covariance matrix delivers more stable optimal portfolios with lower turnover and higher information ratios. Our results reconcile previous findings in the portfolio selection literature as those claiming for equicorrelations, a smooth dynamic evolution of correlations or correlations close to zero.

Munro, B. and Bradfield, D. (2016). "Putting the squeeze on the sample covariance matrix for portfolio construction." In: *Investment Analysts Journal* 45(1), pp. 47–62.

ABSTRACTPortfolio construction plays a critical role in adding performance to a fund. Central to portfolio construction are the two primary inputs: the vector of forecast returns and the covariance matrix. Our focus is on the covariance matrix. With guidance from the literature we consider the suitability of two simple estimators, four shrinkage estimators and three blended estimators for mean-variance portfolio construction in the South African environment. Our assessment frameworks comprise a risk-centric framework based on minimum variance portfolios (MVPs) as well as a return-centric framework. Our findings based on a South African equity setting reveal that there are notable differences between the compositions of the MVPs of the covariance estimators. Furthermore, we find that alternative covariance estimators do yield better out-of-period performance in terms of lower realised risks than the sample covariance matrix. In our return-based assessment framework, we considered scenarios of perfect skill and less-than-perfect skill at forecasting returns. In the former case, we found that all of the estimators produced optimal portfolios that substantially outperformed the optimal portfolio derived from the sample covariance matrix. Considering the MVP framework, as well as the return-based framework, we conclude that all of the estimators considered performed better than the sample covariance matrix, effectively reducing the sampling error in the sample covariance without introducing too much specification error. However, no one estimator could be singled out as consistently superior in the South African setting over a range of test metrics considered.

Ng, K. K., Agarwal, P., Mullen, N., Du, D., and Pollak, I. (2011). "Comparison of several covariance matrix estimators for portfolio optimization." In: *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. IEEE, pp. 5752–5755.

Modern portfolio theory dates back to a seminal 1952 paper by H. Markowitz and has been very influential both in academic finance and among practitioners in the financial industry. Given a set of assets, the theory can be used to compute the amount to be invested in each asset in order to construct an optimally diversified portfolio. One of the parameters required in this calculation is the covariance matrix of asset returns which, in any practical application, is unknown and must be estimated from historical data. Due to the fact that financial data is often nonstationary, basing the estimates on historical data over a very long time period may not be advisable. This renders the problem of covariance estimation difficult, especially for large portfolios. A large body of literature exists proposing different covariance estimators. We focus on one frequently cited paper by Ledoit and Wolf [5] which proposes a covariance estimation method and purports to show that this method leads to statistically significant improvements over several other methods. We show that this is not the case: in fact, their method does not exhibit statistically significant differences from three other methods.

Nguyen, D. B. B., Prokopczuk, M., and Sibbertsen, P. (2020). "The memory of stock return volatility: Asset pricing implications." In: *Journal of Financial Markets* 47, p. 100487.
We examine long memory volatility in the cross-section of stock returns. We show that long memory volatility is widespread in the United States and that the degree of memory can be related to firm characteristics, such as market capitalization, book-to-market ratio, prior performance, and price jumps. Long memory volatility is negatively priced in the cross-section. Buying stocks with shorter memory and selling stocks with longer memory in volatility generates significant excess returns of 1.71% per annum. Consistent with theory, we find that the volatility of stocks with longer memory is more predictable than stocks with shorter memory. This makes the latter more uncertain, which is compensated for with higher average returns.

Packham, N. and Woebbeking, F. (2021). "Correlation scenarios and correlation stress testing." In: *SSRN e-Print*.
We develop a general approach for stress testing correlations of financial asset portfolios. The correlation matrix of asset returns is specified in a parametric form, where correlations are represented as a function of risk factors, such as country and industry factors. A sparse factor structure linking assets and risk factors is built using Bayesian variable selection methods. Regular calibration yields a joint distribution of economically meaningful stress scenarios of the factors. As such, the method also lends itself as a reverse stress testing framework: using the Mahalanobis distance or highest density regions (HDR) on the joint risk factor distribution allows to infer worst-case correlation scenarios. We give examples of stress tests on a large portfolio of European and North American stocks.

Pantaleo, E., Tumminello, M., Lillo, F., and Mantegna, R. N. (2011). "When do improved covariance matrix estimators enhance portfolio optimization? An empirical comparative study of nine estimators." In: *Quantitative Finance* 11(7), pp. 1067–1080.
The use of improved covariance matrix estimators as an alternative to the sample estimator is considered an important approach for enhancing portfolio optimization. Here we empirically compare the performance of nine improved covariance estimation procedures using daily returns of 90 highly capitalized US stocks for the period 1997?2007. We find that the usefulness of covariance matrix estimators strongly depends on the ratio between the estimation period T and the number of stocks N, on the presence or absence of short selling, and on the performance metric considered. When short selling is allowed, several estimation methods achieve a realized risk that is significantly smaller than that obtained with the sample covariance method. This is particularly true when T/N is close to one. Moreover, many estimators reduce the fraction of negative portfolio weights, while little improvement is achieved in the degree of diversification. On the contrary, when short selling is not allowed and T?>?N, the considered methods are unable to outperform the sample covariance in terms of realized risk, but can give much more diversified portfolios than that obtained with the sample covariance. When T?<?N, the use of the sample covariance matrix and of the pseudo-inverse gives portfolios with very poor performance.

Pollak, I. (2012). "Covariance estimation and related problems in portfolio optimization." In: *Sensor Array and Multichannel Signal Processing Workshop*.
This overview paper reviews covariance estimation problems and related issues arising in the context of portfolio optimization. Given several assets, a portfolio optimizer seeks to allocate a fixed amount of capital among these assets so as to optimize some cost function. For example, the classical Markowitz portfolio optimization framework defines portfolio risk as the variance of the portfolio return, and seeks an allocation which minimizes the risk subject to a target expected return. If the mean return vector and the return covariance matrix for the underlying assets are known, the Markowitz problem has a closed-form solution. In practice, however, the expected returns and the covariance matrix of the returns are unknown and are therefore estimated from historical data. This introduces several problems which render the Markowitz theory impracticable in real portfolio management applications. This paper discusses these problems and reviews some of the existing literature on methods for addressing them.

Posch, P. N. and Ullmann, D. (2016). "Estimation of Large Correlation Matrix with Shrinking Methods." In: *SSRN e-Print*.
An exact estimation of the true correlation matrix is highly desirable in many applications. In practice there will always be an estimation error which, however, can be not only minimized using the shrinking approach but also an invertible correlation matrix can be calculated when there are fewer observations than assets. We compare several shrinking methods regarding their correlation matrix estimation using several data generating processes. We calculate the distance of the estimator to the true matrix and check, if improvements transfer to economic improvement, measured by the Sharpe ratio. Firstly, we find that a more accurate estimation of the covariance matrix leads to a better estimation of the correlation matrix and secondly, better estimations of

the correlation matrix lead to significant economic improvements. Although each shrinking estimator performs differently regarding the shape of the return distribution, the general usage of shrinking estimators leads to a better estimation of the correlation matrix than the standard estimator.

Reinikainen, K. (2020). "Strategic Asset Allocation Using Robust Covariance Estimation and Portfolio Optimization Methods." MA thesis. Aalto University.

Strategic asset allocation is the single most important determinant of portfolio returns. While the drawbacks of using mean-variance optimization and the sample covariance matrix for strategic asset allocation are well-documented in the literature, they are still broadly applied among investment professionals. In this thesis we study two robust alternatives for the sample covariance matrix, shrinkage and hierarchical clustering, and two robust alternatives for mean-variance optimization, resampling optimization and regularized optimization. We develop a generalisable testing framework for comparing the out-of-sample risk-return characteristics of a broad range of covariance estimation and portfolio optimization methods. The testing framework is applied to provide an empirical comparison of the performance of the traditional and robust methods in two testing samples, when asset class volatilities, correlations and expected returns contain uncertainty. The choice of the portfolio optimization method clearly dominated the choice of covariance estimation method as a determinant of out-of-sample portfolio risk-return characteristics. Out of the three portfolio optimization methods, resampling optimization provided the investor portfolios with superior ratios of portfolio return and volatility as well as portfolio return and maximum drawdown. Mean-variance optimization and regularized optimization performed more inconsistently, demonstrating sensitivity to the underlying assumptions made regarding the structure of uncertainty in expected returns. As a non-robust method, the realized returns of portfolios obtained using mean-variance optimization also deteriorated as the level of uncertainty in expected returns was increased.

Schadner, W. (2021). "Feasible Implied Correlation Matrices from Factor Structures." In: *arXiv e-Print*.

Forward-looking correlations are of interest in different financial applications, including factor-based asset pricing, forecasting stock-price movements or pricing index options. With a focus on non-FX markets, this paper defines necessary conditions for option implied correlation matrices to be mathematically and economically feasible and argues, that existing models are typically not capable of guaranteeing so. To overcome this difficulty, the problem is addressed from the underlying factor structure and introduces two approaches to solve it. Under the quantitative approach, the puzzle is reformulated into a nearest correlation matrix problem which can be used either as a stand-alone estimate or to re-establish positive-semi-definiteness of any other model's estimate. From an economic approach, it is discussed how expected correlations between stocks and risk factors (like CAPM, Fama-French) can be translated into a feasible implied correlation matrix. Empirical experiments are carried out on monthly option data of the S&P 100 and S&P 500 index (1996-2020).

Senneret, M., Malevergne, Y., Abry, P., Perrin, G., and Jaffres, L. (2016). "Covariance versus Precision Matrix Estimation for Efficient Asset Allocation." In: *IEEE Journal of Selected Topics in Signal Processing* 10(6).

Asset allocation constitutes one of the most crucial and most challenging tasks in financial engineering, which often requires the estimation of large covariance or precision matrices, from short time span multivariate observations, a mandatory yet difficult step. The present contribution reviews and compares a large selection of estimators for covariance and precision matrices, organized into classes of estimation principles (Direct, Factor, Shrinkage, Sparsity). This includes the theoretical derivation of several additional estimators not available in the literature. Rather than assessing estimation performance from synthetic data based on a priori selected models of questionable practical interest, it is chosen here to evaluate practically the quality of these estimators directly from portfolio selection performance, quantified by financial criteria. Portfolio selection is conducted over two datasets of different natures: a 15-year large subset (within Stoxx Europe 600) of 244 European stock returns, and a 50-year benchmark dataset of 90 US equity portfolios. This large scale comparative study addresses issues such as the relative benefits and difficulties of using robust versus direct estimates, of choosing precision or covariance estimates, of quantifying the impacts of constraints.

Shaik, M. and Maheswaran, S. (2020). "A new unbiased additive robust volatility estimation using extreme values of asset prices." In: *Financial Markets and Portfolio Management* 34(3), pp. 313–347.

We propose a new unbiased robust volatility estimator based on extreme values of asset prices. We show that the proposed Add Extreme Value Robust Volatility Estimator (AEVRVE) is unbiased and is 2-3 times more efficient relative to the Classical Robust Volatility Estimator (CRVE). We put forth a novel procedure to remove the downward bias present in the data even without increasing the number of steps in the stock price path. We perform Monte Carlo simulation experiments to show the properties of unbiasedness and efficiency. The proposed estimator remains exactly unbiased relative to the standard robust volatility estimator in the empirical data based on global stock indices namely CAC 40, DOW, IBOVESPA, NIKKEI, S&P 500 and SET 50.

Shin, D. W. (2018). "Forecasting realized volatility: A review." In: *Journal of the Korean Statistical Society* 47(4), pp. 395–404.

Forecast methods for realized volatilities are reviewed. Basic theoretical and empirical features of realized volatilities as well as versions of estimators of realized volatility are briefly investigated. Major forecast models featuring the empirical aspects of persistency and asymmetry are discussed in terms of forecasting models for which the heterogeneous autoregressive (HAR) model is one of the most basic one in the recent literature. Forecast methods addressing the issues of jump, break, implied volatility, and market microstructure noise are reviewed. Forecasting realized covariance matrix is also considered.

Simon, P. M. and Turkay, C. (2018). "Hunting high and low: visualising shifting correlations in financial markets." In: *Computer Graphics Forum* 37(3), pp. 479–490.

The analysis of financial assets' correlations is fundamental to many aspects of finance theory and practice, especially modern portfolio theory and the study of risk. In order to manage investment risk, in-depth analysis of changing correlations is needed, with both high and low correlations between financial assets (and groups thereof) important to identify. In this paper, we propose a visual analytics framework for the interactive analysis of relations and structures in dynamic, high-dimensional correlation data. We conduct a series of interviews and review the financial correlation analysis literature to guide our design. Our solution combines concepts from multi-dimensional scaling, weighted complete graphs and threshold networks to present interactive, animated displays which use proximity as a visual metaphor for correlation and animation stability to encode correlation stability. We devise interaction techniques coupled with context-sensitive auxiliary views to support the analysis of subsets of correlation networks. As part of our contribution, we also present behaviour profiles to help guide future users of our approach. We evaluate our approach by checking the validity of the layouts produced, presenting a number of analysis stories, and through a user study. We observe that our solutions help unravel complex behaviours and resonate well with study participants in addressing their needs in the context of correlation analysis in finance.

Su, W. (2021). "Volatility of S&P500: Estimation and Evaluation." In: *arXiv e-Print*.

In an era when derivatives is getting popular, risk management has gradually become the core content of modern finance. In order to study how to accurately estimate the volatility of the S&P 500 index, after introducing the theoretical background of several methods, this paper uses the historical volatility method, GARCH model method and implied volatility method to estimate the real volatility respectively. At the same time, two ways of adjusting the estimation window, rolling and increasing, are also considered. The unbiased test and goodness of fit test are used to evaluate these methods. The empirical result shows that the implied volatility is the best estimator of the real volatility. The rolling estimation window is recommended when using the historical volatility. On the contrary, the estimation window is supposed to be increased when using the GARCH model.

Sun, R., Ma, T., Liu, S., and Sathye, M. (2019). "Improved covariance matrix estimation for portfolio risk measurement: A review." In: *Journal of Risk and Financial Management* 12(1), p. 48.

The literature on portfolio selection and risk measurement has considerably advanced in recent years. The aim of the present paper is to trace the development of the literature and identify areas that require further research. This paper provides a literature review of the characteristics of financial data, commonly used models of portfolio selection, and portfolio risk measurement. In the summary of the characteristics of financial data, we summarize the literature on fat tail and dependence characteristic of financial data. In the portfolio selection model part, we cover three models: mean-variance model, global minimum variance (GMV) model and factor model. In the portfolio risk measurement part, we first classify risk measurement methods into two categories: moment-based risk measurement and moment-based and quantile-based risk measurement. Moment-based risk measurement includes time-varying covariance matrix and shrinkage estimation, while moment-based and quantile-based risk measurement includes semi-variance, VaR and CVaR.

Tchernister, A. and Rubisov, D. (2009). "Robust estimation of historical volatility and correlations in risk management." In: *Quantitative Finance* 9(1), pp. 43–54.

Financial time series have two features which, in many cases, prevent the use of conventional estimators of volatilities and correlations: leptokurtotic distributions and contamination of data with outliers. Other techniques are required to achieve stable and accurate results. In this paper, we review robust estimators for volatilities and correlations and identify those best suited for use in risk management. The selection criteria were that the estimator should be stable to both fractionally small departures for all data points (fat tails), and to fractionally large departures for a small number of data points (outliers). Since risk management typically deals with thousands of time series at once, another major requirement was the independence of the approach of any manual correction or data pre-processing. We recommend using volatility t-estimators, for which we derived the estimation error formula for the case when the exact shape of the data distribution is unknown. A convenient robust estimator for correlations is Kendall's tau, whose drawback is that it does not guarantee the positivity of the correlation matrix. We chose to use geometric optimization that overcomes this problem by finding the closest correlation matrix to a given matrix in terms of the Hadamard norm. We propose the weights for the norm and demonstrate the efficiency of the algorithm on large-scale problems.

Tong, J., Yang, J., Xi, J., Yu, Y., and Ogunbona, P. O. (2019). "Tuning the Parameters for Precision Matrix Estimation Using Regression Analysis." In: *IEEE Access* 7, pp. 90585–90596.
Precision matrix, i.e., inverse covariance matrix, is widely used in signal processing, and often estimated from training samples. Regularization techniques, such as banding and rank reduction, can be applied to the covariance matrix or precision matrix estimation for improving the estimation accuracy when the training samples are limited. In this paper, exploiting regression interpretations of the precision matrix, we introduce two data-driven, distribution-free methods to tune the parameter for regularized precision matrix estimation. The numerical examples are provided to demonstrate the effectiveness of the proposed methods and example applications in the design of minimum mean squared error (MMSE) channel estimators for large-scale multiple-input multiple-output (MIMO) communication systems are demonstrated.

Tran, T., Nguyen, N., Nguyen, T., and Mai, A. (2020). "Voting shrinkage algorithm for Covariance Matrix Estimation and its application to portfolio selection." In: *2020 RIVF International Conference on Computing and Communication Technologies (RIVF)*. IEEE.
Reducing errors of covariance matrix estimation plays a very important role in many optimization problems, e.g., portfolio optimization. In this paper we propose a data-driven approach which basically combines the original framework of a popular approach named shrinkage estimation and cross-validation technique to adapt the shrinkage intensity with different levels of uncertainty of real data. Particularly, this approach can be applied well to asset management to enhance the quality of portfolio selection which is known as a huge area of research in modern portfolio theory. Experimental results carried out using the prices and volumes data of the Vietnamese stock market show that our proposed method can practically improve the quality and robustness of portfolio selection. Last but not least, we introduce an automatic backtesting system that can help us evaluate the portfolio based on various financial indicators in a real-time manner.

Trucios, C., Hotta, L. K., and Valls Pereira, P. L. (2019). "On the robustness of the principal volatility components." In: *Journal of Empirical Finance* 52, pp. 201–219.
Abstract In this paper, we analyse the recent principal volatility components analysis procedure. The procedure overcomes several difficulties in modelling and forecasting the conditional covariance matrix in large dimensions arising from the curse of dimensionality. We show that outliers have a devastating effect on the construction of the principal volatility components and on the forecast of the conditional covariance matrix and consequently in economic and financial applications based on this forecast. We propose a robust procedure and analyse its finite sample properties by means of Monte Carlo experiments and also illustrate it using empirical data. The robust procedure outperforms the classical method in simulated and empirical data.

Wang, J., Ma, F., Liang, C., and Chen, Z. (2021). "Volatility forecasting revisited using Markov-switching with time-varying probability transition." In: *International Journal of Finance & Economics*.
This study proposes a novel model, Markov-switching Heterogeneous Autoregressive (MS-HAR) model with jump-driven time-varying transition probabilities (TVTP), to forecast the future volatility in Chinese stock market. The in-sample results show that MS-HAR models are more powerful than HAR-RV-type models; furthermore, the high-volatility regime is short-lived. Moreover, the out-of-sample results indicate that the MS-HAR with TVTP model can achieve a superior forecasting performance and increase the economic value than the competing models including the simple HAR model and the MS-HAR with fixed transition probabilities (FTP) model. The results are robust to several robustness checks including alternative forecast window, alternative evaluation method, alternative predictive model, sub-sample analysis and alternative representative index.

Watagoda, L. C. R. P. and Olive, D. J. (2021). "Comparing six shrinkage estimators with large sample theory and asymptotically optimal prediction intervals." In: *Statistical Papers* 62(5), pp. 2407–2431.
Consider the multiple linear regression model with sample size n. This paper compares the six shrinkage estimators: forward selection, lasso, partial least squares, principal components regression, lasso variable selection, and ridge regression, with large sample theory and two new prediction intervals that are asymptotically optimal if

the estimator is a consistent estimator of . Few prediction intervals have been developed for p>n, and they are not asymptotically optimal. For p fixed, the large sample theory for variable selection estimators like forward selection is new, and the theory shows that lasso variable selection is consistent under much milder conditions than lasso. This paper also simplifies the proofs of the large sample theory for lasso, ridge regression, and elastic net.

Yuan, J. and Yuan, X. (2021). "A Monte Carlo synthetic sample based performance evaluation method for covariance matrix estimators." In: *Applied Economics Letters*.

The evaluation of covariance matrix estimators is very important for portfolio analysis and risk management. The Monte Carlo synthetic sample based performance evaluation method proposed by this article can avoid the main shortcomings of statistical and economic methods which are widely used in the existing literature. The proposed method does not need the true covariance and does not need to introduce the performance of the out-of-sample portfolios. It is an intuitive, effective and robust measure for both simulation and empirical analysis.

Zakamulin, V. (2015). "A Test of Covariance-Matrix Forecasting Methods." In: *The Journal of Portfolio Management* 41(3), pp. 97–108.

Providing a more accurate covariance matrix forecast can substantially improve the performance of optimized portfolios. Using out-of-sample tests we evaluate alternative covariance matrix forecasting methods by looking at (1) their forecast accuracy (2) their ability to track the volatility of the minimum-variance portfolio (3) their ability to keep the volatility of the minimum-variance portfolio at a target level. We find large differences between the methods. Our results suggest that shrinkage of the sample covariance matrix improves neither the forecast accuracy nor the performance of minimum-variance portfolios. In contrast, switching from the sample covariance matrix forecast to a multivariate GARCH forecast reduces forecasting error and portfolio tracking error by at least half. Our findings also reveal that the exponentially weighted covariance matrix forecast performs only slightly worse than the multivariate GARCH forecast.

Zitelli, G. L. (2020). "Random matrix models for datasets with fixed time horizons." In: *Quantitative Finance* 20(5), pp. 769–781.

This paper examines the use of random matrix theory as it has been applied to model large financial datasets, especially for the purpose of estimating the bias inherent in Mean-Variance portfolio allocation when a sample covariance matrix is substituted for the true underlying covariance. Such problems were observed and modeled in the seminal work of Laloux et al. [Noise dressing of financial correlation matrices. Phys. Rev. Lett., 1999, 83, 1467] and rigorously proved by Bai et al. [Enhancement of the applicability of Markowitz's portfolio optimization by utilizing random matrix theory. Math. Finance, 2009, 19, 639-667] under minimal assumptions. If the returns on assets to be held in the portfolio are assumed independent and stationary, then these results are universal in that they do not depend on the precise distribution of returns. This universality has been somewhat misrepresented in the literature, however, as asymptotic results require that an arbitrarily long time horizon be available before such predictions necessarily become accurate. In order to reconcile these models with the highly non-Gaussian returns observed in real financial data, a new ensemble of random rectangular matrices is introduced, modeled on the observations of independent Levy processes over a fixed time horizon.