Examples and Intuitions II

The $\Theta^{(1)}$ matrices for AND, NOR, and OR are:

$$
\begin{aligned}
&AND:\\
&\quad \Theta^{(1)} = [-30 \quad 20 \quad 20]\\
&NOR:\\
&\quad \Theta^{(1)} = [10 \quad -20 \quad -20]\\
&\quad OR:\\
&\quad \Theta^{(1)} = [-10 \quad 20 \quad 20]
\end{aligned}
$$

We can combine these to get the XNOR logical operator (which gives 1 if x_1 and x_2 are both 0 or both 1).

$$
\begin{bmatrix} x_0 \\ x_1 \\ x_2 \end{bmatrix} \rightarrow \begin{bmatrix} a_1^{(2)} \\ a_2^{(2)} \end{bmatrix} \rightarrow [a^{(3)}] \rightarrow h_\Theta(x)
$$

For the transition between the first and second layer, we'll use a $\Theta^{(1)}$ matrix that combines the values for AND and NOR:

$$
\Theta^{(1)} = \begin{bmatrix} -30 & 20 & 20 \\ 10 & -20 & -20 \end{bmatrix}
$$

For the transition between the second and third layer, we'll use a $\Theta^{(2)}$ matrix that uses the value for OR:

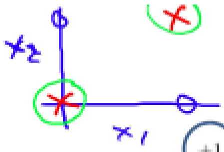
$$
\Theta^{(2)} = [-10 \quad 20 \quad 20]
$$

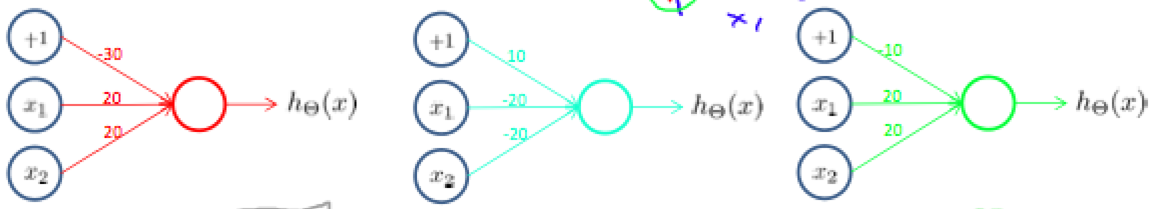
Let's write out the values for all our nodes:

$$
\begin{aligned}
&a^{(2)} = g(\Theta^{(1)} \cdot x)\\
&a^{(3)} = g(\Theta^{(2)} \cdot a^{(2)})\\
&h_\Theta(x) = a^{(3)}
\end{aligned}
$$

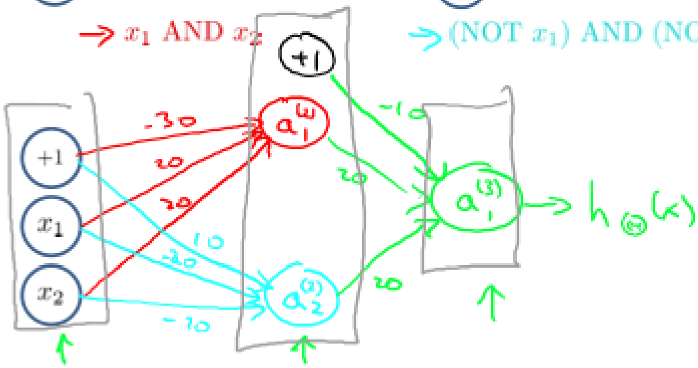
And there we have the XNOR operator using a hidden layer with two nodes! The following summarizes the above algorithm:

Putting it together: x_1 XNOR x_2





$\rightarrow x_1$ AND x_2 $\rightarrow$ (NOT x_1) AND (NOT x_2) $\rightarrow x_1$ OR x_2



x_1	x_2	$a_1^{(2)}$	$a_2^{(2)}$	$h_\Theta(x)$
0	0	0	1	1
0	1	0	0	0
1	0	0	0	0
1	1	1	0	1