HW3

ex 1

(a) $P(A) = 0.106 + 0.141 + 0.200 = 0.447$

$P(C) = 0.215 + 0.200 + 0.065 + 0.02 = 0.5$

$P(A \cap C) = 0.2$

No points if calculation is missing.

(b) $P(A|C) = \frac{P(A \cap C)}{P(C)} = \frac{0.2}{0.5} = 0.4$.

$P(C|A) = \frac{P(A \cap C)}{P(A)} = \frac{0.2}{0.447} = 0.4474$

No points if calculation is missing.

$P(A|C)$ is the probability of blood group A given ethnic group is 3

$P(C|A)$ is the probability of ethnic group 3 given blood group A

(c) $P(\text{ethnic group } 1 \mid O, A, AB) = \frac{0.082 + 0.106 + 0.004}{1 - 0.008 - 0.018 - 0.065} = \frac{0.192}{0.909} = 0.2112$

9

ex 2

(a) $P(M, Pr, long) = 0.05$

(b) $P(M, Pr) = P(M, Pr, long) + P(M, Pr, short)$

$= 0.05 + 0.07 = 0.12$

(c) $P(\text{Short-sleeved}) = 0.04 + 0.02 + 0.05 + 0.08 + 0.07 + 0.12 + 0.03 + 0.07 + 0.08 = 0.56$

$P(\text{long-sleeved}) = 0.03 + 0.02 + 0.03 + 0.10 + 0.05 + 0.07 + 0.04 + 0.02 + 0.08 = 0.44$

(d) $P(M) = 0.10 + 0.05 + 0.07 + 0.08 + 0.07 + 0.12 = 0.49$

$P(Pr) = 0.02 + 0.05 + 0.02 + 0.02 + 0.07 + 0.07 = 0.25$

(e) $P(M \mid short, Pl) = \dfrac{0.08}{0.04 + 0.08 + 0.03} = \dfrac{0.08}{0.15} = 53.33\%$

(f) $P(short \mid M, Pl) = \dfrac{P(short, M, Pl)}{P(M, Pl)} = \dfrac{0.08}{0.08 + 0.10} = 44.44\%$

$P(long \mid M, Pl) = \dfrac{P(long, M, Pl)}{P(M, Pl)} = \dfrac{0.10}{0.10 + 0.08} = 55.56\%$

ex 3 $P(\text{has disease} \mid \text{both test positive})$

$= \dfrac{P(\text{has disease, both test positive})}{P(\text{both test positive})}$

$P(\text{both test positive, has disease}) = P(\text{disease})\, P(\text{test 1 positive} \mid \text{disease})\, P(\text{test 2 positive} \mid \text{disease})$

$= 0.05 \times 0.98^2 = 0.04802$

T

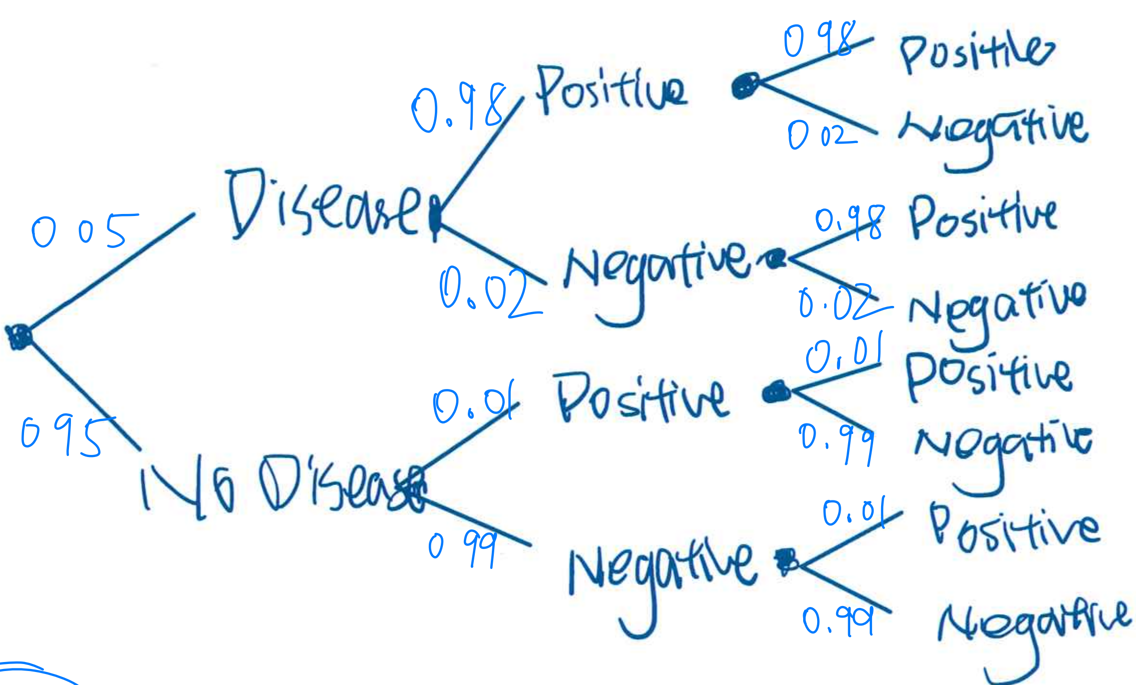
$P(\text{no disease, both test positive})$ T

$= P(\text{no disease})\, P(\text{test1 positive} \mid \text{no disease})\, P(\text{test 2 positive} \mid \text{no disease})$

$= (1-0.05) \times (1-0.99) \times (1-0.99) = 9.5\times10^{-5}$

$\therefore P(\text{has disease} \mid \text{both test positive}) = \dfrac{0.04802}{0.04802 + 9.5\times10^{-5}} = 0.9980$

For the checks marked with a "T", instead of writing the formula, it is equally fine to use a tree diagram



8

ex #4 (a) 365^n

(b) A' : all students have different birthdays

(c) $P(A') = P(A_2)P(A_3|A_2)\cdots P(A_n|A_{n-1},\cdots A_2)$

$A_i = \{$ person i has different birthday from person $1 \ldots i-1\}$

$$P(A_i|A_{i-1},\cdots A_2) = \frac{365-(i-1)}{365} \quad i=3,4,\cdots n$$

$$P(A_2) = \frac{364}{365}$$

$$\therefore P(A') = \frac{\frac{365!}{(365-n)!}}{365^n}$$

(d) $P(A) = 1 - P(A') = 1 - \frac{\frac{365!}{(365-n)!}}{365^n}$

(e) Top 10: couples tend to conceive around Christmas time

Bottom 10:

These are around major holidays on which planned hospital births are less frequent than on usual days.

Ex 5:

a) $\binom{8}{3} \times 3! = \frac{8!}{5!}$

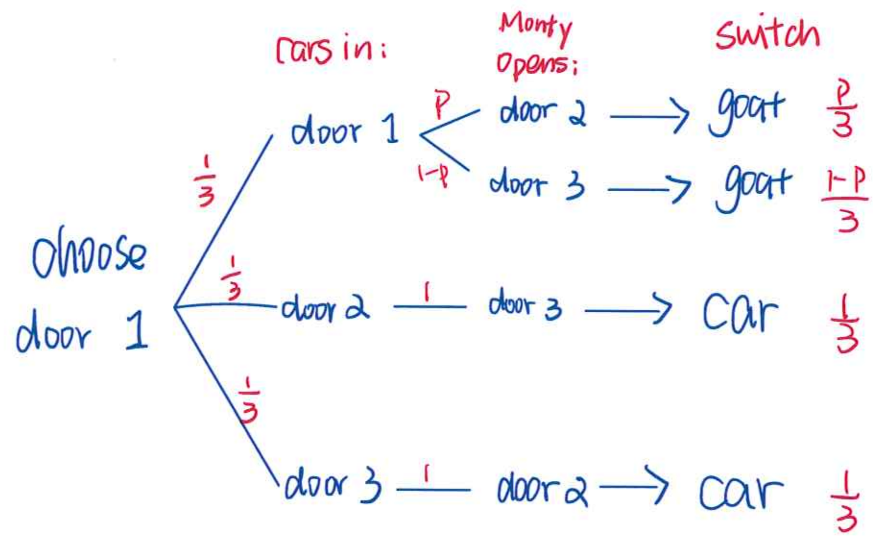
b) $\binom{30}{6} = \frac{30!}{6!\,24!}$

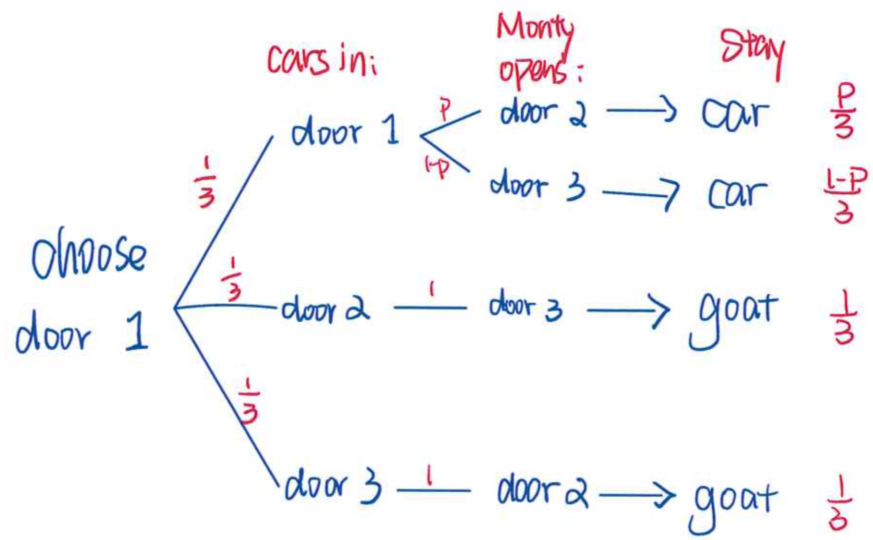
c) $\binom{8}{2}\binom{10}{2}\binom{12}{2} = \frac{\cancel{8!}}{2!\,6!} \times \frac{\cancel{10!}}{2!\,\cancel{8!}} \times \frac{12!}{2!\,\cancel{10!}} = \frac{12!}{2!\,2!\,2!\,6!}$

d) $\frac{\binom{8}{2}\binom{10}{2}\binom{12}{2}}{\binom{30}{6}} = \frac{\frac{12!}{2!\,2!\,2!\,\cancel{6!}}}{\frac{30!}{24!\,\cancel{6!}}} = \frac{12!\,24!}{2!\,2!\,2!\,30!}$

e) $\frac{\binom{8}{6}+\binom{10}{6}+\binom{12}{6}}{\binom{30}{6}} = \frac{\frac{8!}{6!\,2!} + \frac{10!}{6!\,4!} + \frac{12!}{6!\,6!}}{\binom{30}{6}}$

Ex 6:





a) $P(\text{switch is success}) = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

b) $P(\text{switch is success} \mid \text{Monty opens door 2})$

$$= \frac{P(\text{switch is success, Monty opens door 2})}{P(\text{Monty opens door 2})}$$

$$= \frac{1}{3} \div \left(\frac{p}{3} + \frac{1}{3}\right) = \frac{1}{p+1}$$

c) P(switch is success | Monty opens door 3)

$$= \frac{P(\text{switch is success, Monty opens door 3})}{P(\text{Monty opens door 3})}$$

$$= \frac{1}{3} \div \left(\frac{1-p}{3} + \frac{1}{3} \right)$$

$$= \frac{1}{2-p}$$

d)

① From b), we have $P(\text{switch is success} \mid \text{Monty opens door 2}) = \frac{1}{p+1}$

② P(stay is success | Monty opens door 2)

$$= \frac{P(\text{stay is success, Monty opens door 2})}{P(\text{Montey opens door 2})}$$

$$= \frac{p}{3} \div \left(\frac{p}{3} + \frac{1}{3} \right) = \frac{p}{p+1}$$

$$\frac{1}{p+1} < \frac{p}{p+1} \Rightarrow 1 < p \quad \text{but } \frac{1}{2} \leq p \leq 1$$

$\therefore$ No

e)

From c), we have $P(\text{switch is success} \mid \text{Monty opens door 3}) = \frac{1}{2-p}$

P(stay is success | Monty opens door 3)

$$= \frac{P(\text{stay is success, Monty opens door 3})}{P(\text{Montey opens door 3})}$$

$$= \frac{1-p}{3} \div \left(\frac{1-p}{3} + \frac{1}{3} \right)$$

$$= \frac{1-p}{2-p}$$

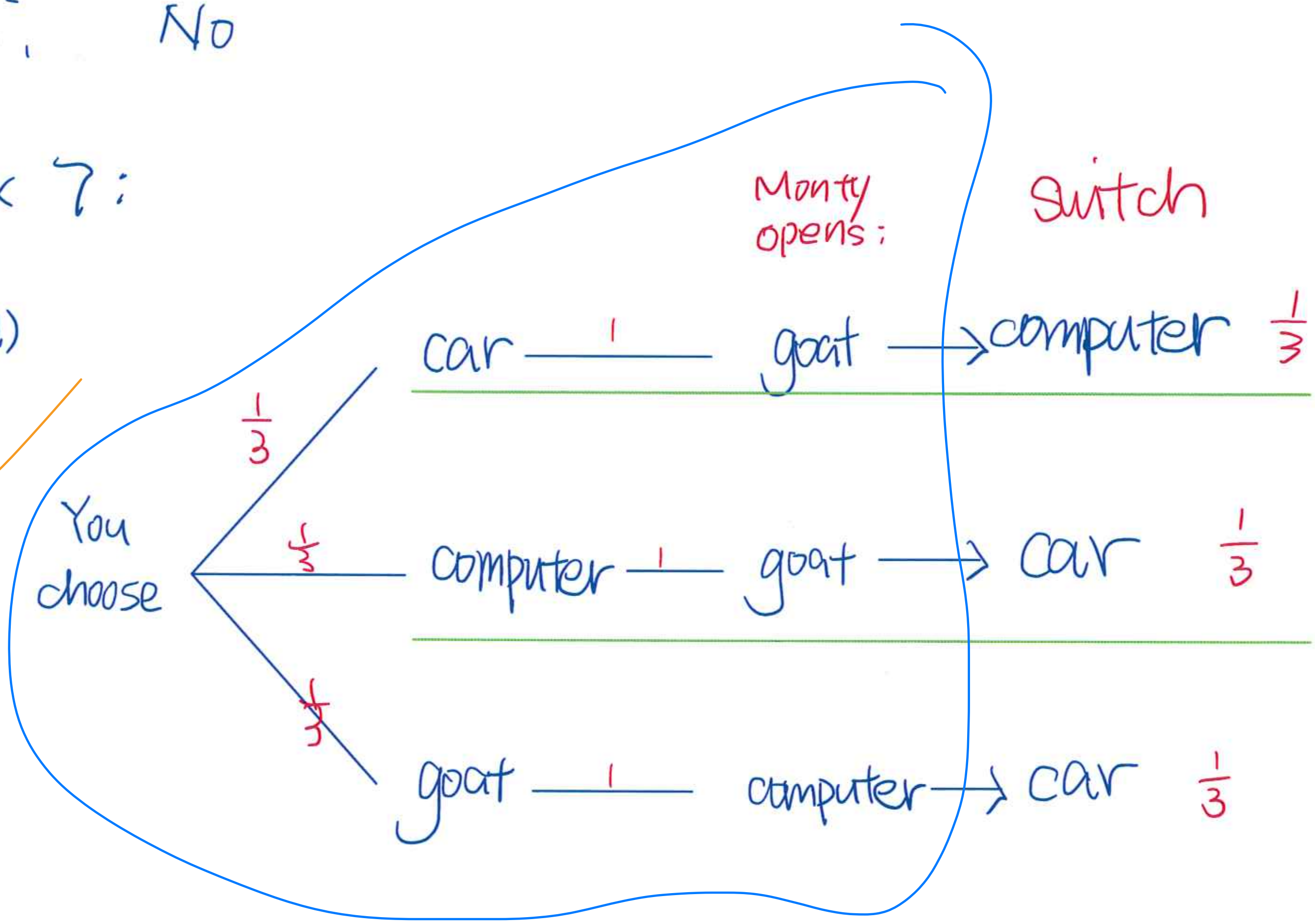
$$\frac{1}{2-p} < \frac{1-p}{2-p} \Rightarrow 1 < 1-p \Rightarrow p < 0$$

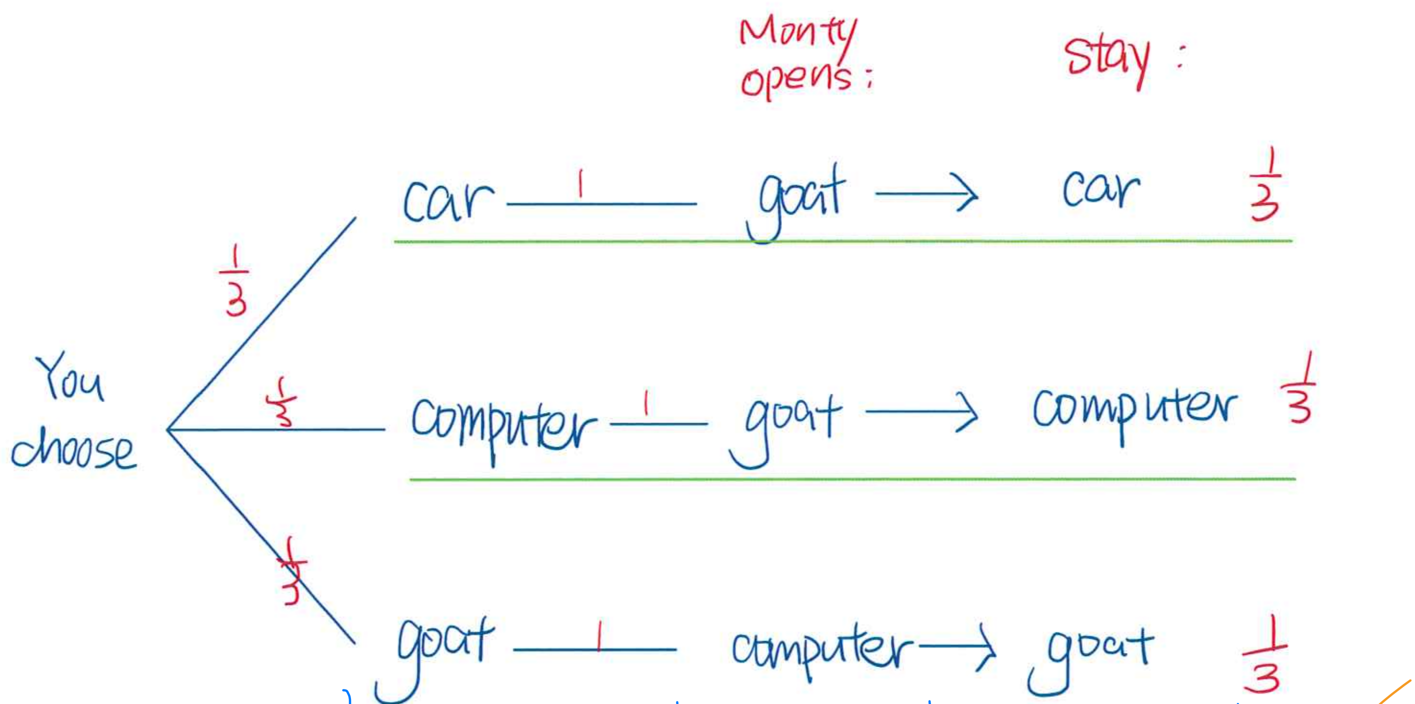
but $\frac{1}{2} \le p \le 1$

$\therefore$ No

Ex 7:

a)





A = {Monty reveals a goat}:

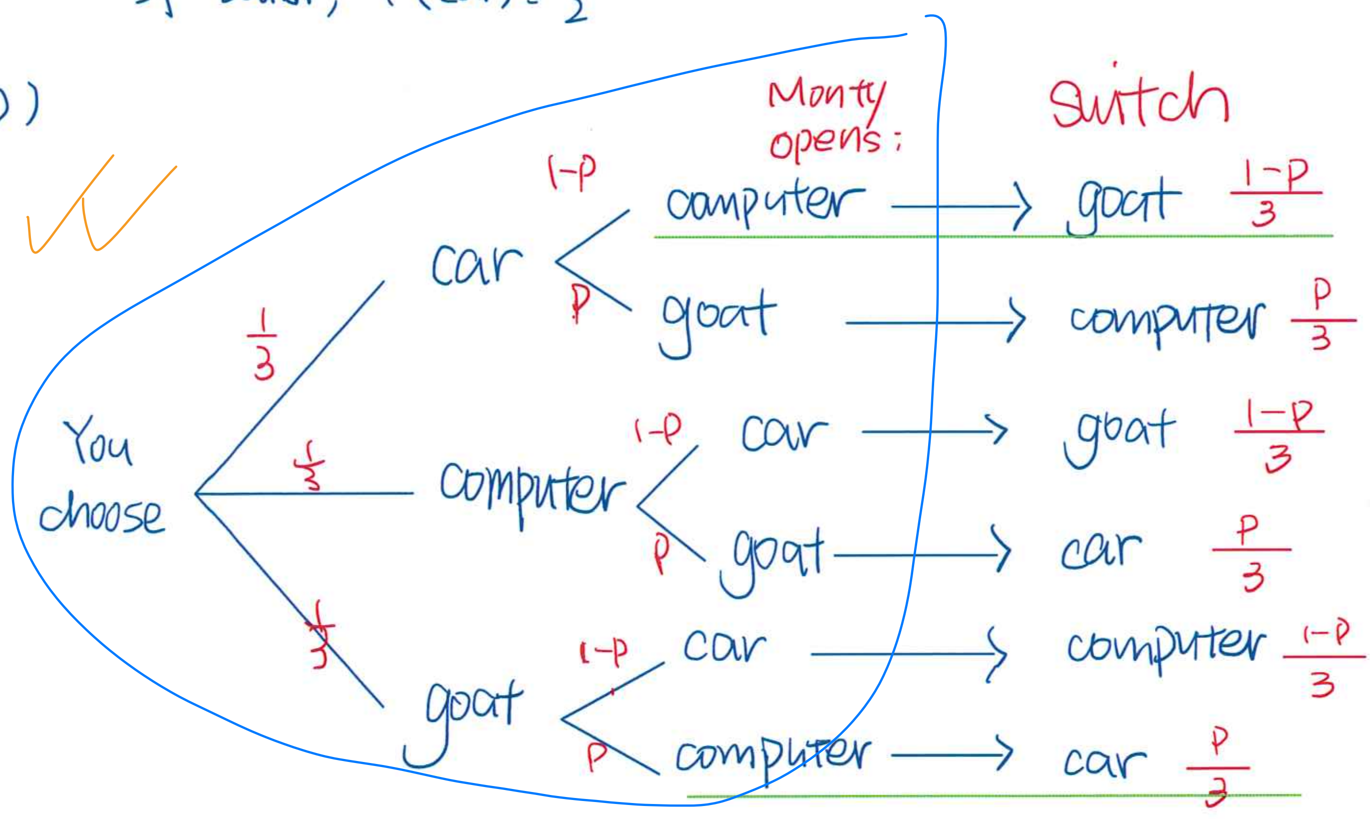
if switch: $P(\text{getting computer} \mid A) = P(\text{getting car} \mid A) = \frac{1}{2}$ $\quad \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{3}} =$

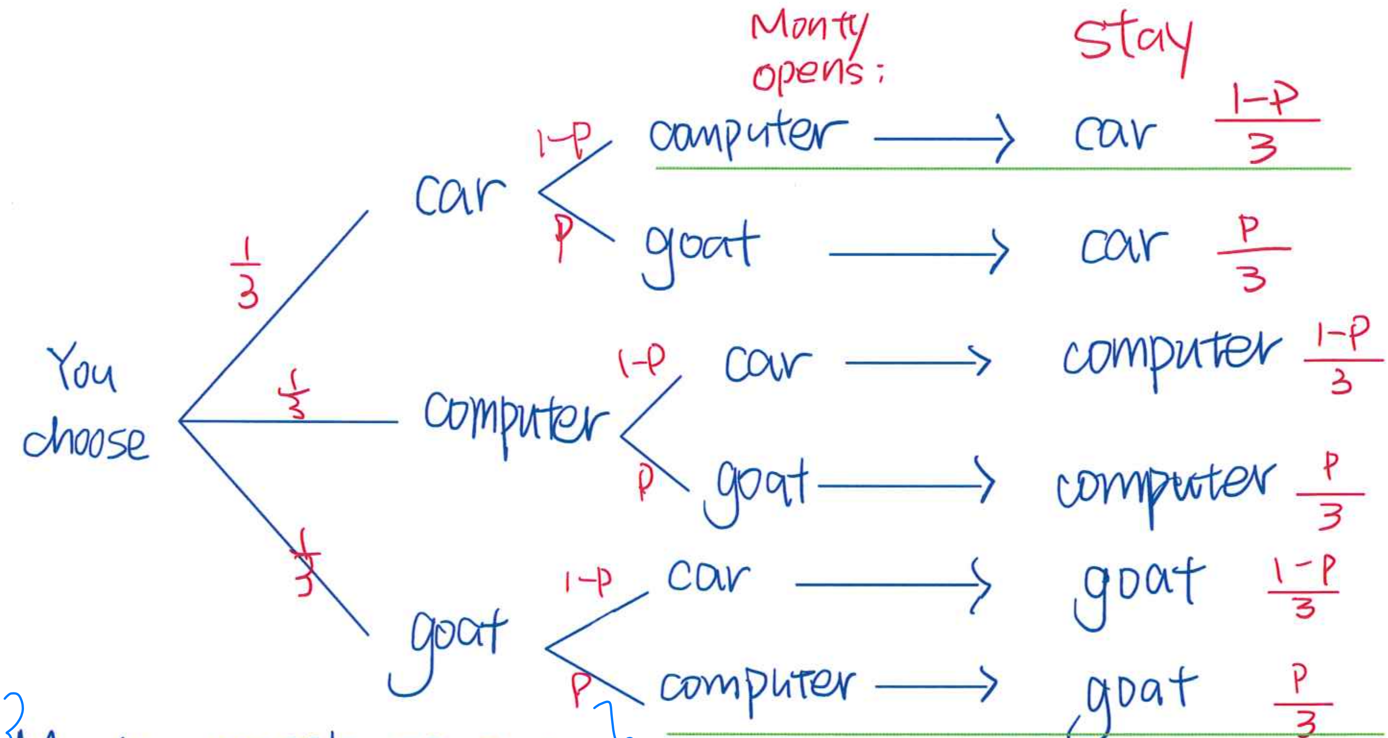
if stay: $P(\text{getting computer} \mid A) = P(\text{getting car} \mid A) = \frac{1}{2}$

$\therefore$ It does not matter if you switch or not

If swith, $P(\text{getting car} \mid A) = \frac{1}{2}$

b)





$A = \{\text{Monty reveals computer}\}$

If switch: $P(\text{getting goat} \mid A) = 1-p$, $P(\text{getting car} \mid A) = p$

If stay: $P(\text{getting goat} \mid A) = p$, $P(\text{getting car} \mid A) = 1-p$

$\therefore$ If $p > \frac{1}{2}$, then switch

If $p < \frac{1}{2}$, then stay

If $p = \frac{1}{2}$, it does not matter

$$\frac{\frac{1-p}{3}}{\frac{1-p}{3} + \frac{p}{3}}$$

If switch: $P(\text{getting car} \mid A) = p$

$$\frac{\frac{p}{3}}{\frac{1-p}{3} + \frac{p}{3}}$$

14

Ex. 8:

A: severe case

B: vaccinated

C: age < 50

C and C' are inter-changeable.

b) $\frac{P(A|B)}{P(A|B')} = \frac{5.3/100k}{16.5/100k} = 0.321 > \frac{P(A|B,C)}{P(A|B',C)} = \frac{0.3/100k}{3.8/100k} = 0.079$

$\frac{P(A|B)}{P(A|B')} = \frac{5.3/100k}{16.5/100k} = 0.321 > \frac{P(A|B,C')}{P(A|B',C')} = \frac{13.6/100k}{95/100k} = 0.142$

Satisfied.

c)

$P(A|B,C) = 0.3/100k = 3 \times 10^{-6}$

$P(A|B',C) = 3.8/100k = 3.8 \times 10^{-5}$ $\Rightarrow$ $P(A|B,C) < P(A|B',C)$

$P(A|B,C') = 13.6/100k = 1.36 \times 10^{-4}$

$P(A|B',C') = 95/100k = 9.5 \times 10^{-4}$ $\Rightarrow$ $P(A|B,C') < P(A|B',C')$

$P(A|B) = 5.3/100k = 5.3 \times 10^{-5}$

$P(A|B') = 16.5/100k = 1.65 \times 10^{-4}$ $\Rightarrow$ $P(A|B) < P(A|B')$

Since $P(A|B) < P(A|B)$, it does not satisfy the stronger definition

5

Exercise 9

(a) $\binom{33}{30}$

(b) $\binom{29}{26}$

(c) $\binom{40}{30}$