Homework 1: Applied Machine Learning - Linear | Logisitc | SVM

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In [1]:
```
import numpy as np
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
from numpy.linalg import inv
%matplotlib inline
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler, MinMaxScaler
from sklearn.preprocessing import OrdinalEncoder
from sklearn.svm import LinearSVC, SVC
from sklearn.metrics import accuracy_score
```

In [2]:
```
import warnings

def fxn():
    warnings.warn("deprecated", DeprecationWarning)

with warnings.catch_warnings():
    warnings.simplefilter("ignore")
    fxn()
```

In [3]:
```
pd.options.mode.chained_assignment = None
```

Part 1: Linear Regression

In part 1, we will use **two datasets** to train and evaluate our linear regression model.

The first dataset will be a synthetic dataset sampled from the following equations:

ϵ ~ Normal(0,3)

z = 3x + 10y + 10 + ϵ

In [4]:

```
np.random.seed(0)
epsilon = np.random.normal(0, 3, 100)
x = np.linspace(0, 10, 100)
y = np.linspace(0, 5, 100)
z = 3 * x + 10 * y + 10 + epsilon
```

To apply linear regression, we need to first check if the assumptions of linear regression are not violated.

Assumptions of Linear Regression:

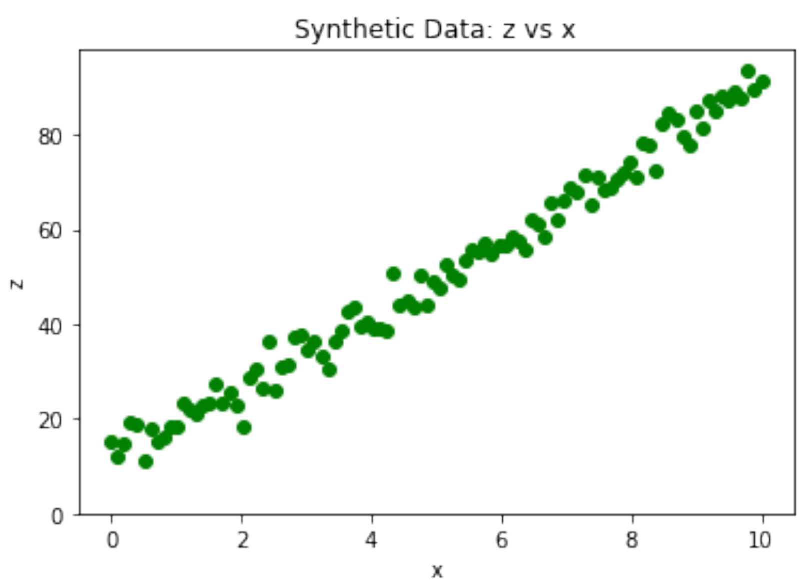
- Linearity: y is a linear (technically affine) function of x.
- Independence: the x's are independently drawn, and not dependent on each other.
- Homoscedasticity: the ϵ's, and thus the y's, have constant variance.
- Normality: the ϵ's are drawn from a Normal distribution (i.e. Normally-distributed errors)

These properties, as well as the simplicity of this dataset, will make it a good test case to check if our linear regression model is working properly.

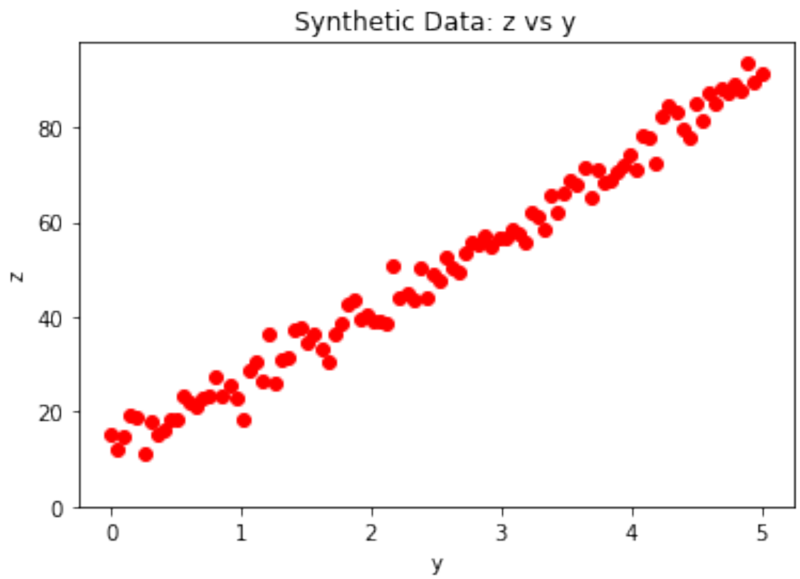
1.1. Plot z vs x and z vs y in the synthetic dataset as scatter plots. Label your axes and make sure your y-axis starts from 0. Do the independent and dependent features have linear relationship?

In [5]:

```
plt.scatter(x, z, c = "green")
plt.ylim(ymin = 0)
plt.xlabel("x")
plt.ylabel("z")
plt.title("Synthetic Data: z vs x")
plt.show()
```



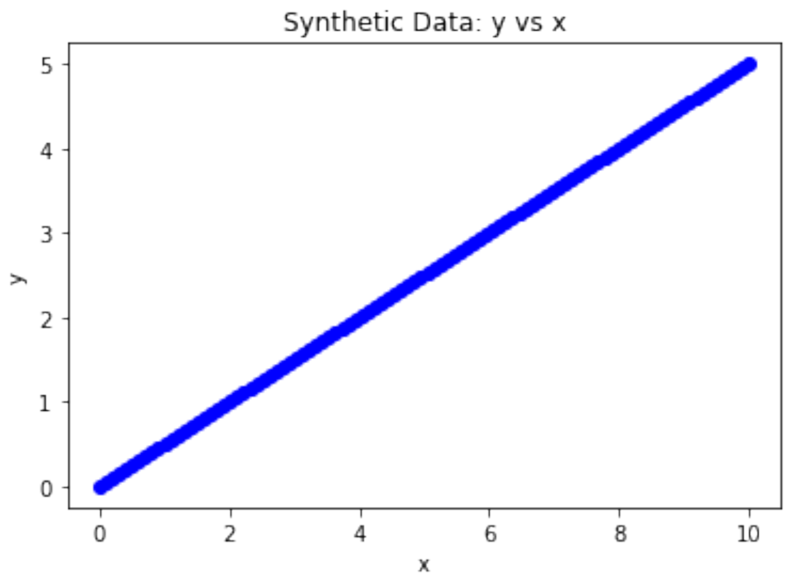
In [6]:
```
plt.scatter(y, z, c = "red")
plt.ylim(ymin = 0)
plt.xlabel("y")
plt.ylabel("z")
plt.title("Synthetic Data: z vs y")
plt.show()
```



Yes, the independent and dependent features have a linear relationship above.

1.2. Are the independent variables correlated? Use pearson correlation to verify? What would be the problem if linear regression is applied to correlated features?

In [7]:
```
plt.scatter(x, y, c = "blue")
plt.xlabel("x")
plt.ylabel("y")
plt.title("Synthetic Data: y vs x")
plt.show()
```



Looking at the scatter plot above, it looks like that the independent variables are correlated. Let's do this with pearson correlation now.

In [8]:
```
# verification using pearson correlation which is the covariance between the
# divided by the multiplication of the standard deviations of both the variab
covariances = np.cov(x, y)
covariance_x_y = covariances[0, 1]
pearson_correlation_coeff = covariance_x_y / (np.std(x) * np.std(y))
print(f"Pearson Correlation Coefficient = {pearson_correlation_coeff}")
```

```
Pearson Correlation Coefficient = 1.01010101010101
```

As the pearson correlation coefficient is approximately 1 which states that they are highly positively correlated. Thus, the independent variables in the dataset are highly correlated!

The problem that arises when linear regression is applied to correlated features is the problem of "Multicollinearity". When independent variables are highly correlated, change in one variable would cause change to another and so, the model would fluctuate significantly. This would result in a highly unstable model with unstable parameters and could vary a lot for some small change in the data or the model.

The second dataset we will be using is an auto MPG dataset. This dataset contains various characteristics for around 8128 cars. We will use linear regression to predict the selling_price label

In [9]:
```
auto_mpg_df = pd.read_csv('Car details v3.csv')
# Dropping Torque column, there is information in this column but it will tak
# The idea of the exercise is to familarize yourself with the basics of Linea
auto_mpg_df = auto_mpg_df.drop(['torque'], axis = 1)
```

In [10]:

```
auto_mpg_df
```

Out[10]:

	name	year	selling_price	km_driven	fuel	seller_type	transmission	owner	mile
0	Maruti Swift Dzire VDI	2014	450000	145500	Diesel	Individual	Manual	First Owner	k
1	Skoda Rapid 1.5 TDI Ambition	2014	370000	120000	Diesel	Individual	Manual	Second Owner	2 k
2	Honda City 2017-2020 EXi	2006	158000	140000	Petrol	Individual	Manual	Third Owner	k
3	Hyundai i20 Sportz Diesel	2010	225000	127000	Diesel	Individual	Manual	First Owner	k
4	Maruti Swift VXI BSIII	2007	130000	120000	Petrol	Individual	Manual	First Owner	k
...	...	...	...	...	...	...	...	...	
8123	Hyundai i20 Magna	2013	320000	110000	Petrol	Individual	Manual	First Owner	k
8124	Hyundai Verna CRDi SX	2007	135000	119000	Diesel	Individual	Manual	Fourth & Above Owner	k
8125	Maruti Swift Dzire ZDi	2009	382000	120000	Diesel	Individual	Manual	First Owner	k
8126	Tata Indigo CR4	2013	290000	25000	Diesel	Individual	Manual	First Owner	2 k
8127	Tata Indigo CR4	2013	290000	25000	Diesel	Individual	Manual	First Owner	2 k

8128 rows × 12 columns

1.3. Missing Value analysis - Auto mpg dataset.

Are there any missing values in the dataset? If so, what can be done about it? Jusify your approach.

In [11]:
```
columns_with_units = ["mileage", "engine", "max_power"]

numerical_data = auto_mpg_df.select_dtypes(include=[np.number])
categorical_data = auto_mpg_df.select_dtypes(exclude=[np.number])
categorical_data = categorical_data.drop(columns_with_units, axis = 1)
numerical_data_with_units = auto_mpg_df[columns_with_units]

# Removing units from the numerical data that has units
for column in columns_with_units:
    numerical_data_with_units[column] = numerical_data_with_units[column].map


numerical_data_with_units = numerical_data_with_units.replace(r'^\s*$', np.na
```

In [12]:
```
print("Numerical Data is as follows:")
numerical_data
```

```
Numerical Data is as follows:
```

Out[12]:

	year	selling_price	km_driven	seats
0	2014	450000	145500	5.0
1	2014	370000	120000	5.0
2	2006	158000	140000	5.0
3	2010	225000	127000	5.0
4	2007	130000	120000	5.0
...	...	...	...	...
8123	2013	320000	110000	5.0
8124	2007	135000	119000	5.0
8125	2009	382000	120000	5.0
8126	2013	290000	25000	5.0
8127	2013	290000	25000	5.0

8128 rows × 4 columns

In [13]:

```
print("Categorical Data is as follows:")
categorical_data
```

```
Categorical Data is as follows:
```

Out[13]:

	name	fuel	seller_type	transmission	owner
0	Maruti Swift Dzire VDI	Diesel	Individual	Manual	First Owner
1	Skoda Rapid 1.5 TDI Ambition	Diesel	Individual	Manual	Second Owner
2	Honda City 2017-2020 EXi	Petrol	Individual	Manual	Third Owner
3	Hyundai i20 Sportz Diesel	Diesel	Individual	Manual	First Owner
4	Maruti Swift VXI BSIII	Petrol	Individual	Manual	First Owner
...	...	...	...	...	...
8123	Hyundai i20 Magna	Petrol	Individual	Manual	First Owner
8124	Hyundai Verna CRDi SX	Diesel	Individual	Manual	Fourth & Above Owner
8125	Maruti Swift Dzire ZDi	Diesel	Individual	Manual	First Owner
8126	Tata Indigo CR4	Diesel	Individual	Manual	First Owner
8127	Tata Indigo CR4	Diesel	Individual	Manual	First Owner

8128 rows × 5 columns

In [14]:

```
print("Numerical Data is as follows:")
numerical_data_with_units
```

```
Numerical Data is as follows:
```

Out[14]:

	mileage	engine	max_power
0	23.4	1248	74
1	21.14	1498	103.52
2	17.7	1497	78
3	23.0	1396	90
4	16.1	1298	88.2
...	...	...	...
8123	18.5	1197	82.85
8124	16.8	1493	110
8125	19.3	1248	73.9
8126	23.57	1396	70
8127	23.57	1396	70

8128 rows × 3 columns

In [15]:

```
print("Number of Missing Values in Numerical data (without units): ")
print(numerical_data.isna().sum())
```

```
Number of Missing Values in Numerical data (without units):
year               0
selling_price      0
km_driven          0
seats            221
dtype: int64
```

In [16]:

```
print("Number of Missing Values in Categorical data: ")
print(categorical_data.isnull().sum())
```

```
Number of Missing Values in Categorical data:
name            0
fuel            0
seller_type     0
transmission    0
owner           0
dtype: int64
```

In [17]:

```
print("Number of Missing Values in Numerical data (with units): ")
print(numerical_data_with_units.isna().sum())
```

```
Number of Missing Values in Numerical data (with units):
mileage      221
engine       221
max_power    216
dtype: int64
```

There are no missing values for the Categorical variables in the dataset. The only missing values are in the Numerical data (variables) namely "seats", "mileage", "engine", "max_power".

As we can see above, the number of missing entries for each of the following columns are:

1. seats - 221 (2.718996%)
2. mileage - 221 (2.718996%)
3. engine - 221 (2.718996%)
4. max_power - 216 (2.657480%)

In [23]:
```
print("Percentage of Missing Values in Numerical data (without units): ")
print((numerical_data.isna().sum() / len(auto_mpg_df)) * 100)
```

```
Percentage of Missing Values in Numerical data (without units):
year             0.000000
selling_price    0.000000
km_driven        0.000000
seats            2.718996
dtype: float64
```

In [24]:
```
print("Percentage of Missing Values in Categorical data: ")
print((categorical_data.isnull().sum() / len(auto_mpg_df)) * 100)
```

```
Percentage of Missing Values in Categorical data:
name            0.0
fuel            0.0
seller_type     0.0
transmission    0.0
owner           0.0
dtype: float64
```

In [25]:
```
print("Percentage of Missing Values in Numerical data (with units): ")
print((numerical_data_with_units.isna().sum() / len(auto_mpg_df)) * 100)
```

```
Percentage of Missing Values in Numerical data (with units):
mileage      2.718996
engine       2.718996
max_power    2.657480
dtype: float64
```

In [26]:
```
partially_pre_processed_data = pd.concat([numerical_data, categorical_data, n
partially_pre_processed_data
```

Out[26]:

	year	selling_price	km_driven	seats	name	fuel	seller_type	transmission	owne
0	2014	450000	145500	5.0	Maruti Swift Dzire VDI	Diesel	Individual	Manual	Firs Owne
1	2014	370000	120000	5.0	Skoda Rapid 1.5 TDI Ambition	Diesel	Individual	Manual	Secon Owne
2	2006	158000	140000	5.0	Honda City 2017-2020 EXi	Petrol	Individual	Manual	Thir Owne
3	2010	225000	127000	5.0	Hyundai i20 Sportz Diesel	Diesel	Individual	Manual	Firs Owne
4	2007	130000	120000	5.0	Maruti Swift VXI BSIII	Petrol	Individual	Manual	Firs Owne
...	...	...	...	...	...	...	...	...	.
8123	2013	320000	110000	5.0	Hyundai i20 Magna	Petrol	Individual	Manual	Firs Owne
8124	2007	135000	119000	5.0	Hyundai Verna CRDi SX	Diesel	Individual	Manual	Fourt Abov Owne
8125	2009	382000	120000	5.0	Maruti Swift Dzire ZDi	Diesel	Individual	Manual	Firs Owne
8126	2013	290000	25000	5.0	Tata Indigo CR4	Diesel	Individual	Manual	Firs Owne
8127	2013	290000	25000	5.0	Tata Indigo CR4	Diesel	Individual	Manual	Firs Owne

8128 rows × 12 columns

In [27]:
```
print("Rows with Missing Values in Numerical data (without units): ")
is_nan_data = partially_pre_processed_data.isnull().any(axis = 1)
rows_with_nan = partially_pre_processed_data[is_nan_data]
rows_with_nan
```

```
Rows with Missing Values in Numerical data (without units):
```

Out[27]:

	year	selling_price	km_driven	seats	name	fuel	seller_type	transmission	owne
13	2007	200000	80000	NaN	Maruti Swift 1.3 VXi	Petrol	Individual	Manual	Secon Owne
31	2003	70000	50000	NaN	Fiat Palio 1.2 ELX	Petrol	Individual	Manual	Secon Owne
78	2003	50000	70000	NaN	Tata Indica DLS	Diesel	Individual	Manual	Firs Owne
87	2015	475000	78000	NaN	Maruti Swift VDI BSIV W ABS	Diesel	Dealer	Manual	Firs Owne
119	2010	300000	120000	NaN	Maruti Swift VDI BSIV	Diesel	Individual	Manual	Secon Owne
...	...	...	...	...	...	...	...	...	.
7846	2000	200000	100000	NaN	Toyota Qualis Fleet A3	Diesel	Individual	Manual	Firs Owne
7996	2000	140000	50000	NaN	Hyundai Santro LS zipPlus	Petrol	Individual	Manual	Secon Owne
8009	2006	145000	80000	NaN	Hyundai Santro Xing XS eRLX Euro III	Petrol	Individual	Manual	Secon Owne
8068	2017	580000	165000	NaN	Ford Figo Aspire Facelift	Diesel	Individual	Manual	Firs Owne
8103	2006	130000	100000	NaN	Maruti Swift 1.3 VXi	Petrol	Individual	Manual	Secon Owne

222 rows × 12 columns

As we can see the percentage of missing values is quite less in comparison to the number of total entries in the dataset. Also, the total number of rows with any missing entry is just 222. So, most probably, we can remove the rows with any entries as null or nan (missing). This wouldn't affect much as it's a small piece of our data. In case we don't want to remove that data, we can also put mean of each of those corresponding column entries but I would prefer to remove those rows that introducing noise in my dataset unnecessarily.

1.4. The features engine, max_power and mileage have units in the dataset. In the real world if we have such datasets, we generally remove the units from each feature. After doing so, convert the datatype of these columns to float. For example: 1248 CC engine is 1248, 23.4 kmpl is 23.4 and so on.

Hint: Check for distinct units in each of these features. A feature might have multiple units as well. Also, a feature could have no value but have unit. For example 'CC' without any value. Remove such rows.

In [28]:

```
# At first we would see what all units are possible for the columns under con
columns_with_units = ["mileage", "engine", "max_power"]
units_dict = dict()

# Removing units from the numerical data that has units
for column in columns_with_units:
    units = set()
    for row in auto_mpg_df[column]:
        if row is not(np.nan):
            splitted_data = str(row).split(' ')
            if len(splitted_data) > 1:
                units.add(splitted_data[1])
    units_dict[column] = units

print(units_dict)
```

```
{'mileage': {'km/kg', 'kmpl'}, 'engine': {'CC'}, 'max_power': {'bhp'}}
```

As we can see above that there are 2 units present for mileage, 1 unit for engine and 1 for max_power. Thus, we would need to convert all the values in the mileage to one of the units so, as to generalize it. I am going to take kmpl as the default unit and thus, convert all mileage values in km/kg into kmpl. After, that I am going to remove all the units for each of the columns with units and remove any rows with nan (missing values) values. Also, after doing so I am going to convert all the numerical features with units to float.

In [29]:

```
# In this step, we will  remove the units, convert km/kg to kmpl when applica
# and remove any unwanted (only unit) entries as well. And also, convert all
for column in columns_with_units:
    for idx, row in enumerate(auto_mpg_df[column]):
        if row is not(np.nan):
            splitted_data = str(row).split(' ')
            if len(splitted_data) > 1:
                data = float(splitted_data[0]) if splitted_data[0] != '' else
                unit = splitted_data[1]
                if unit == "km/kg":
                    data *= 1.4
                auto_mpg_df.loc[idx, column] = data
            else:
                auto_mpg_df.loc[idx, column] = np.NaN
auto_mpg_df
```

Out[29]:

	name	year	selling_price	km_driven	fuel	seller_type	transmission	owner	mile
0	Maruti Swift Dzire VDI	2014	450000	145500	Diesel	Individual	Manual	First Owner	2
1	Skoda Rapid 1.5 TDI Ambition	2014	370000	120000	Diesel	Individual	Manual	Second Owner	2
2	Honda City 2017-2020 EXi	2006	158000	140000	Petrol	Individual	Manual	Third Owner	
3	Hyundai i20 Sportz Diesel	2010	225000	127000	Diesel	Individual	Manual	First Owner	2
4	Maruti Swift VXI BSIII	2007	130000	120000	Petrol	Individual	Manual	First Owner	
...	...	...	...	...	...	...	...	...	
8123	Hyundai i20 Magna	2013	320000	110000	Petrol	Individual	Manual	First Owner	
8124	Hyundai Verna CRDi SX	2007	135000	119000	Diesel	Individual	Manual	Fourth & Above Owner	
8125	Maruti Swift Dzire ZDi	2009	382000	120000	Diesel	Individual	Manual	First Owner	
8126	Tata Indigo CR4	2013	290000	25000	Diesel	Individual	Manual	First Owner	2
8127	Tata Indigo CR4	2013	290000	25000	Diesel	Individual	Manual	First Owner	2

8128 rows × 12 columns

In [30]:

```
# Remove all the rows with any missing values
columns_with_missing_values = ["seats", "mileage", "engine", "max_power"]
auto_mpg_df.dropna(subset=columns_with_missing_values, how="any", inplace=Tru
auto_mpg_df
```

Out[30]:

	name	year	selling_price	km_driven	fuel	seller_type	transmission	owner	mile
0	Maruti Swift Dzire VDI	2014	450000	145500	Diesel	Individual	Manual	First Owner	
1	Skoda Rapid 1.5 TDI Ambition	2014	370000	120000	Diesel	Individual	Manual	Second Owner	2
2	Honda City 2017-2020 EXi	2006	158000	140000	Petrol	Individual	Manual	Third Owner	
3	Hyundai i20 Sportz Diesel	2010	225000	127000	Diesel	Individual	Manual	First Owner	
4	Maruti Swift VXI BSIII	2007	130000	120000	Petrol	Individual	Manual	First Owner	
...	...	...	...	...	...	...	...	...	
8123	Hyundai i20 Magna	2013	320000	110000	Petrol	Individual	Manual	First Owner	
8124	Hyundai Verna CRDi SX	2007	135000	119000	Diesel	Individual	Manual	Fourth & Above Owner	
8125	Maruti Swift Dzire ZDi	2009	382000	120000	Diesel	Individual	Manual	First Owner	
8126	Tata Indigo CR4	2013	290000	25000	Diesel	Individual	Manual	First Owner	2
8127	Tata Indigo CR4	2013	290000	25000	Diesel	Individual	Manual	First Owner	2

7906 rows × 12 columns

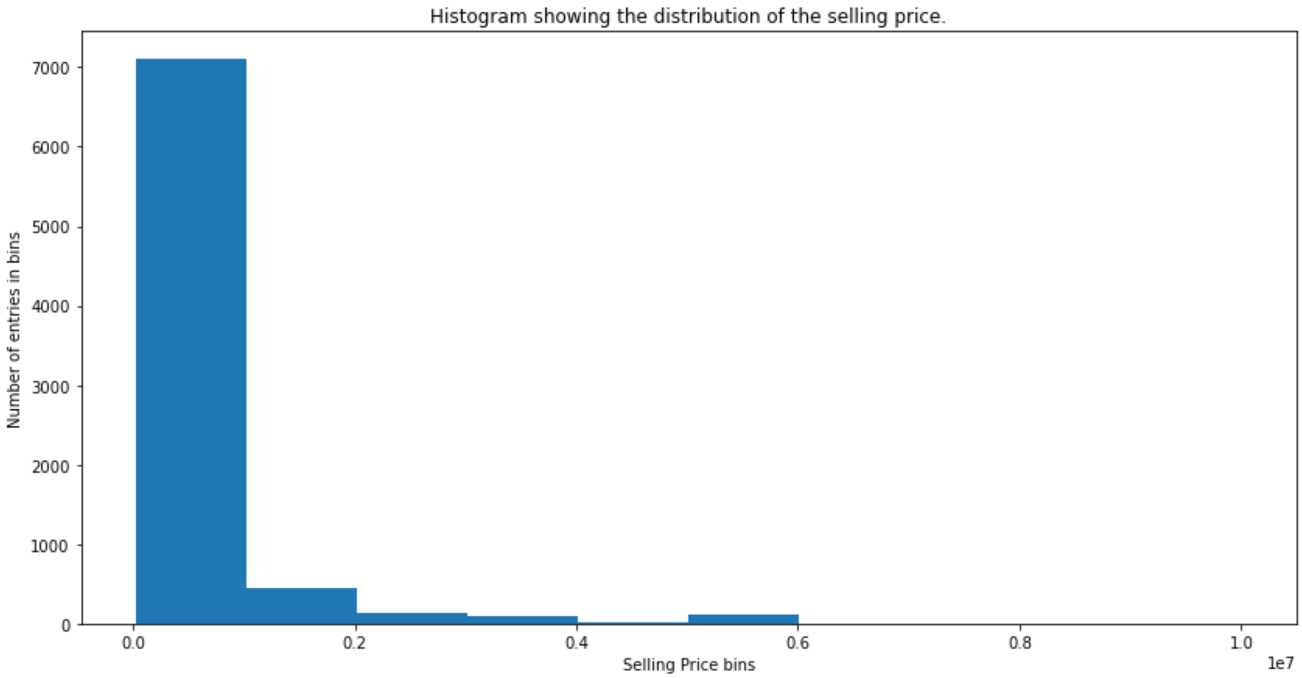
In [31]:

```
auto_mpg_X = auto_mpg_df.drop(columns=['selling_price'])
auto_mpg_y = auto_mpg_df['selling_price']
```

1.5. Plot the distribution of the label (selling_price) using a histogram. Make multiple plots with different binwidths. Make sure to label your axes while plotting.

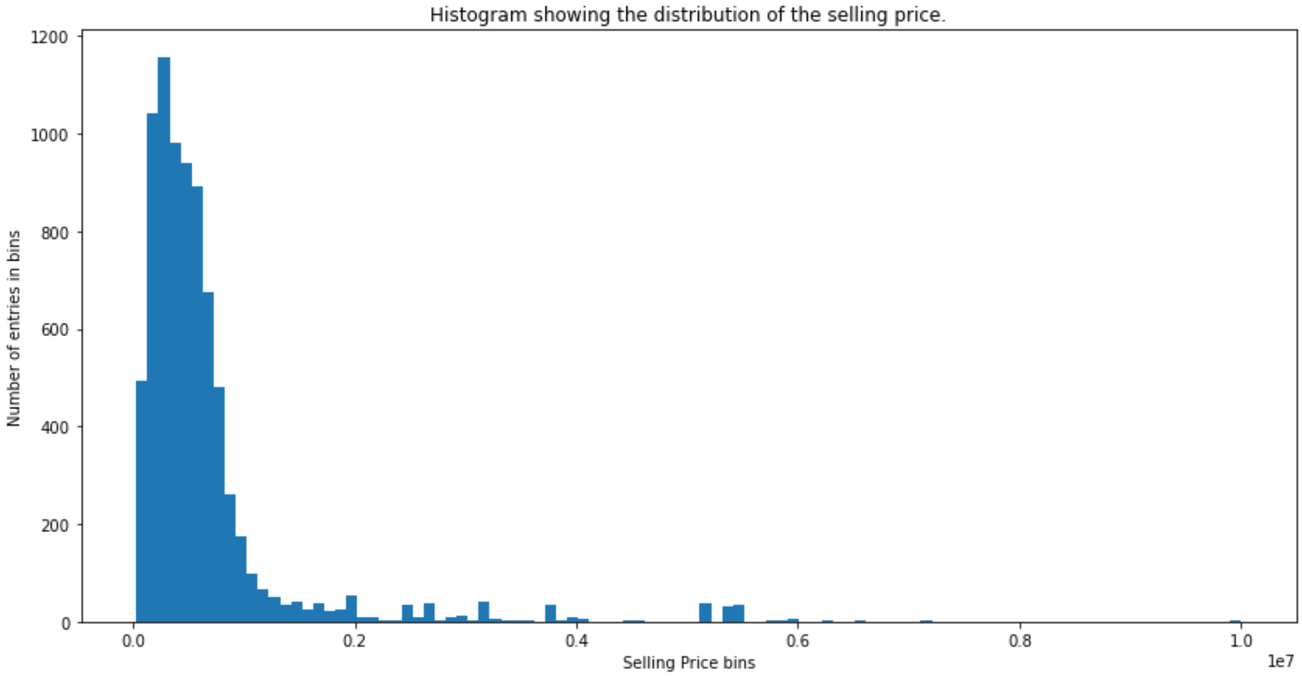
In [32]:

```
plt.rcParams["figure.figsize"] = (14,7)
plt.hist(auto_mpg_y, bins = 10)
plt.ylabel("Number of entries in bins")
plt.xlabel("Selling Price bins")
plt.title("Histogram showing the distribution of the selling price.")
plt.show()
```



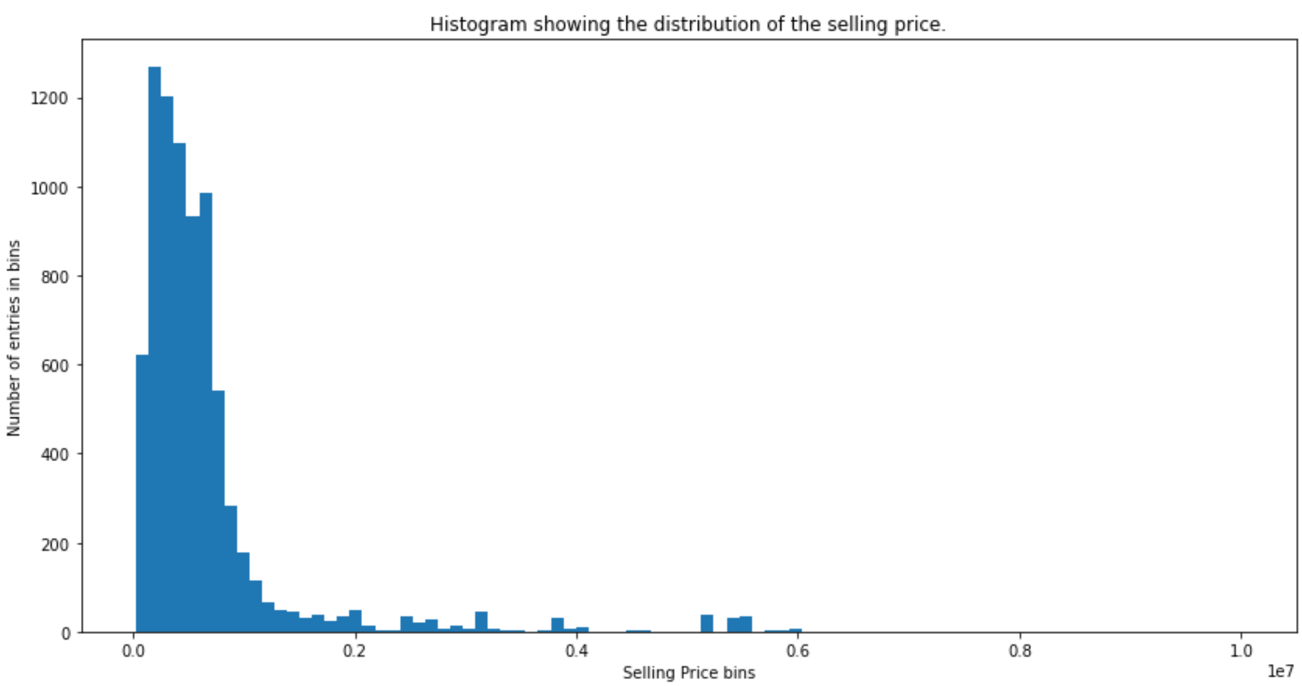
In [33]:

```
plt.rcParams["figure.figsize"] = (14,7)
plt.hist(auto_mpg_y, bins = 100)
plt.ylabel("Number of entries in bins")
plt.xlabel("Selling Price bins")
plt.title("Histogram showing the distribution of the selling price.")
plt.show()
```

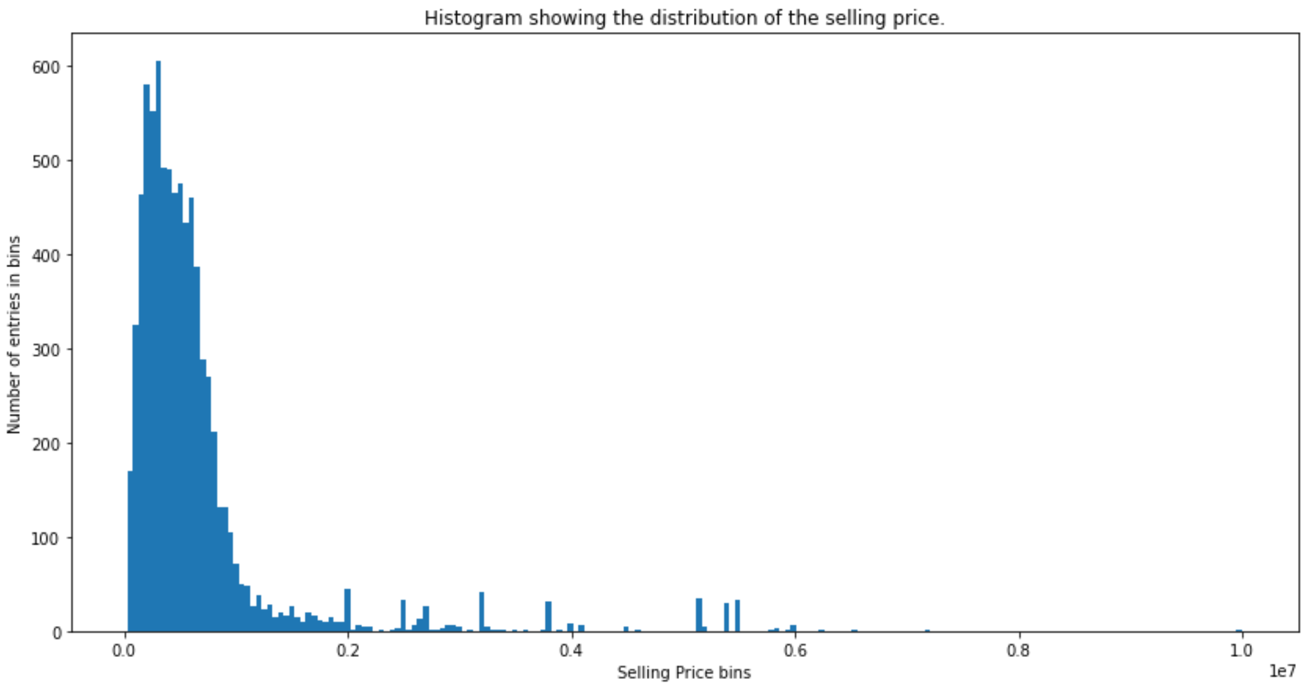


In [34]:

```
plt.rcParams["figure.figsize"] = (14,7)
plt.hist(auto_mpg_y, bins = int(np.sqrt(len(auto_mpg_y))))
plt.ylabel("Number of entries in bins")
plt.xlabel("Selling Price bins")
plt.title("Histogram showing the distribution of the selling price.")
plt.show()
```

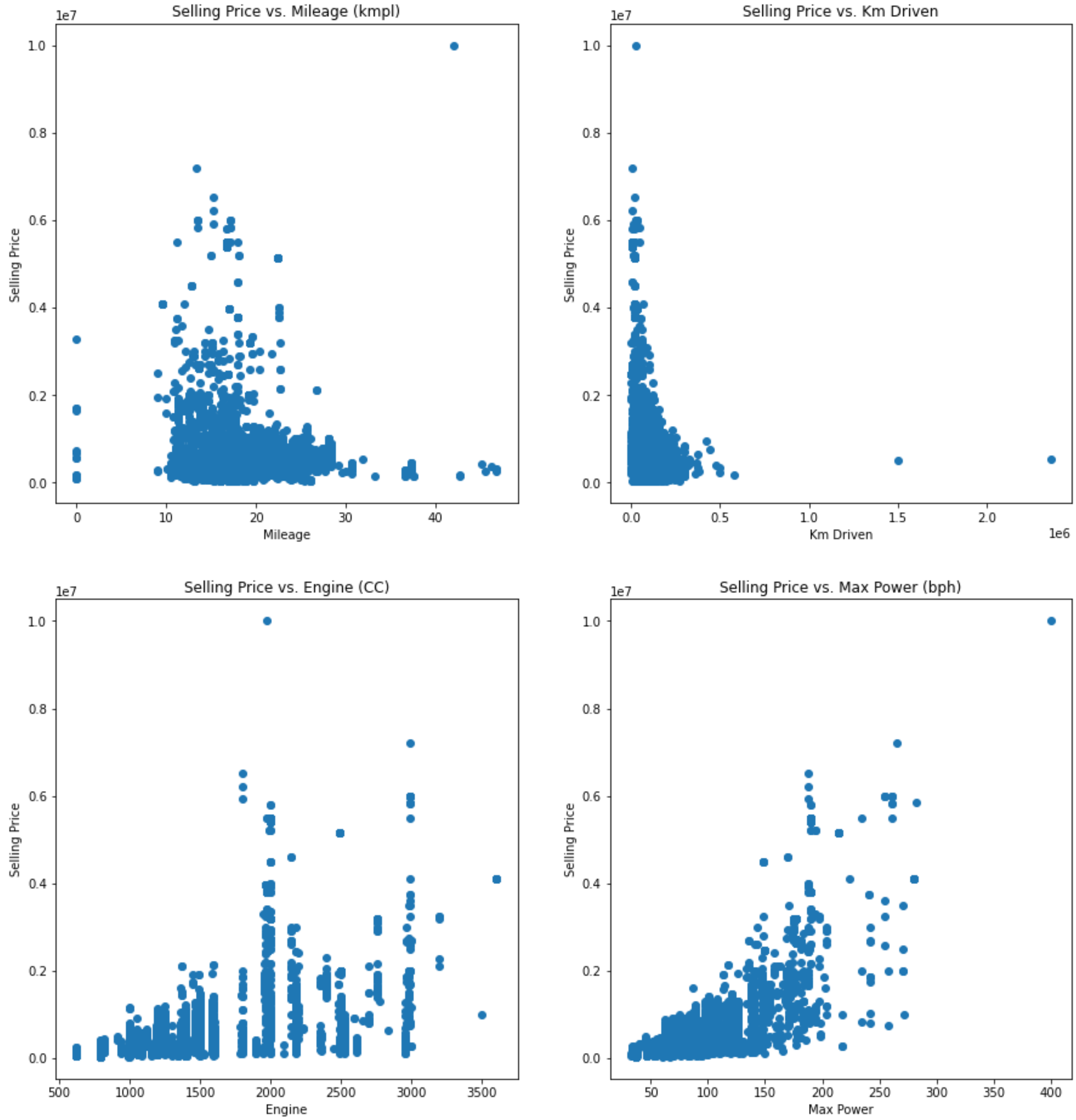


In [35]:
```
plt.rcParams["figure.figsize"] = (14,7)
plt.hist(auto_mpg_y, bins = 200)
plt.ylabel("Number of entries in bins")
plt.xlabel("Selling Price bins")
plt.title("Histogram showing the distribution of the selling price.")
plt.show()
```



1.6. Plot the relationships between the label (Selling Price) and the continuous features (Mileage, km driven, engine, max power) using a small multiple of scatter plots. Make sure to label the axes. Do you see something interesting about the distributions of these features.

In [36]:
```
plt.rcParams["figure.figsize"] = (15,16)
fig, axs = plt.subplots(2, 2)

axs[0, 0].scatter(auto_mpg_X["mileage"], auto_mpg_y)
axs[0, 0].set_title('Selling Price vs. Mileage (kmpl)')
axs[0, 0].set_xlabel("Mileage")
axs[0, 0].set_ylabel("Selling Price")

axs[0, 1].scatter(auto_mpg_X["km_driven"], auto_mpg_y)
axs[0, 1].set_title('Selling Price vs. Km Driven')
axs[0, 1].set_xlabel("Km Driven")
axs[0, 1].set_ylabel("Selling Price")

axs[1, 0].scatter(auto_mpg_X["engine"], auto_mpg_y)
axs[1, 0].set_title('Selling Price vs. Engine (CC)')
axs[1, 0].set_xlabel("Engine")
axs[1, 0].set_ylabel("Selling Price")

axs[1, 1].scatter(auto_mpg_X["max_power"], auto_mpg_y)
axs[1, 1].set_title('Selling Price vs. Max Power (bph)')
axs[1, 1].set_xlabel("Max Power")
axs[1, 1].set_ylabel("Selling Price")

plt.show()
```

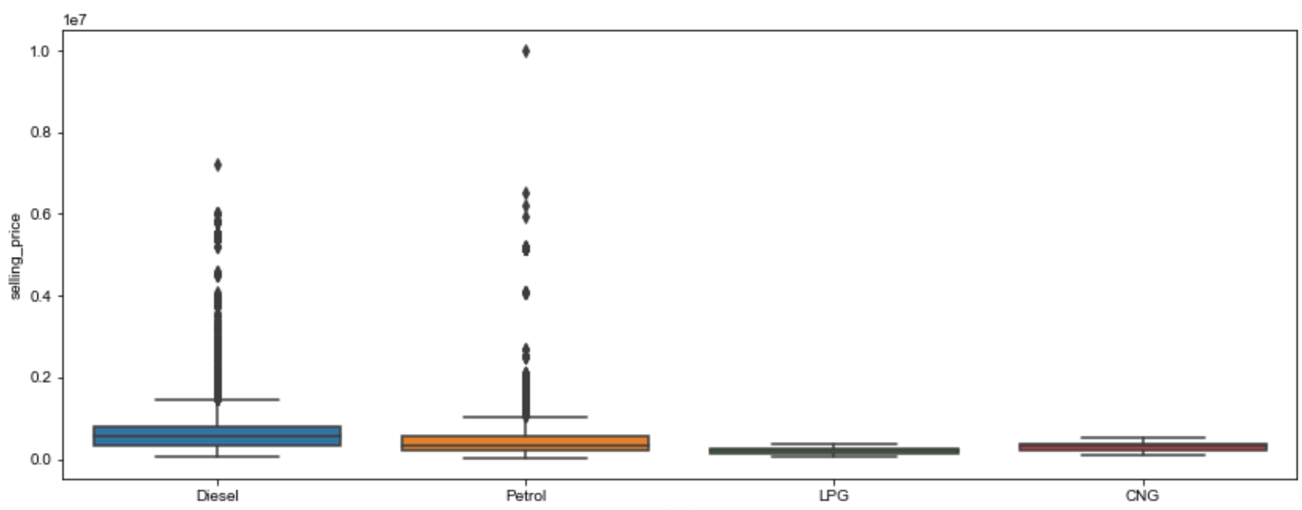

Selling Price vs. Mileage (kmpl)
1e7
Selling Price
Mileage
Selling Price vs. Km Driven
1e7
Selling Price
Km Driven
1e6
Selling Price vs. Engine (CC)
1e7
Selling Price
Engine
Selling Price vs. Max Power (bph)
1e7
Selling Price
Max Power

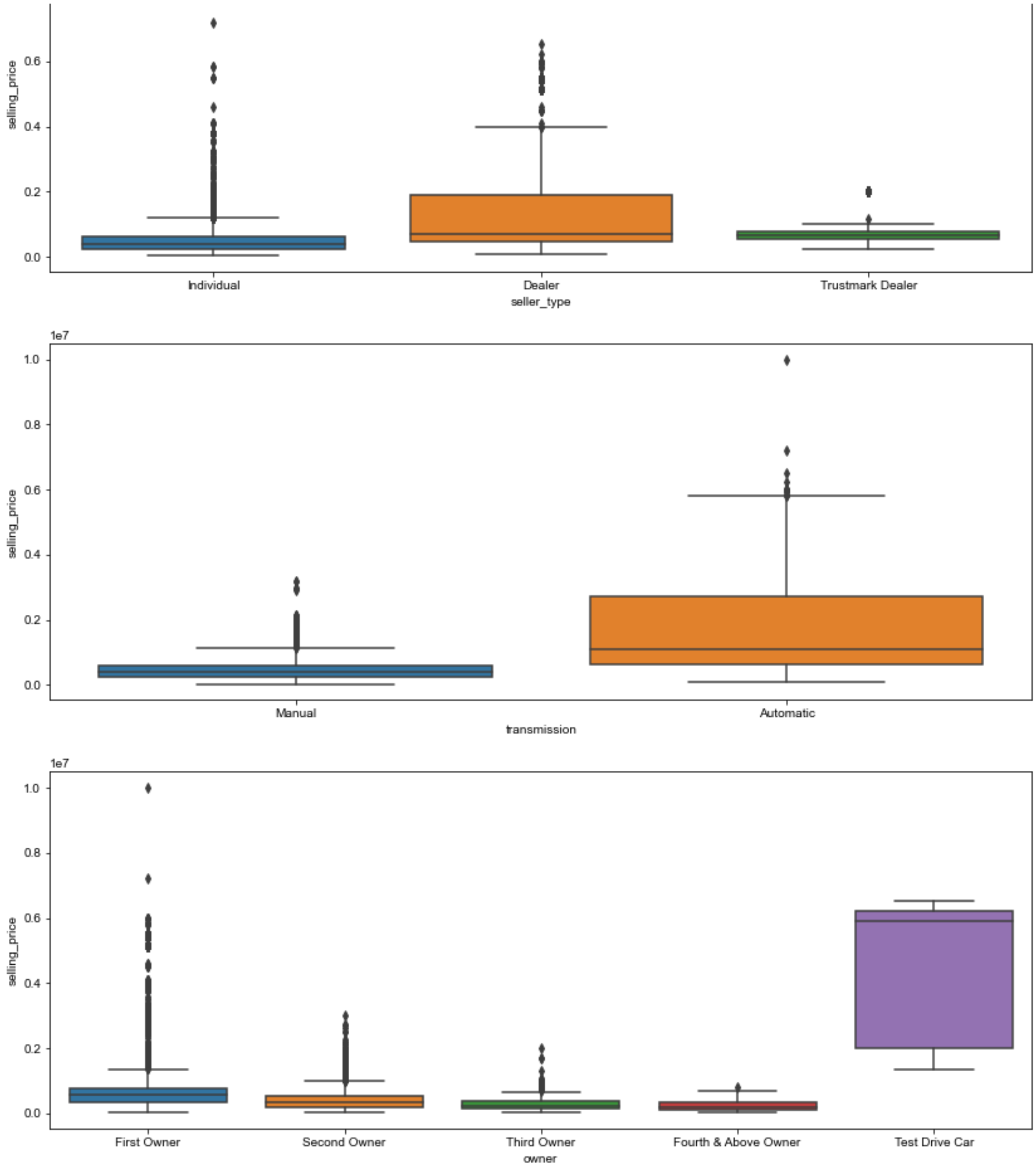
- Mileage: It seems like it follows a skewed normal distribution w.r.t the selling price with some outliers on both the ends (tails) of the normal distribution. And there is not much correlation between the data points for selling price and mileage other than that.
- Km Driven: Looking at the plot above, it's pretty evident that this feature forms a heavily positively skewed normal distribution with some outliers. Other than that there is no linear correlation between the data for the "Km Driven" and "Selling Price".
- Max Power: It seems like this feature has some positive correlation (although not a strong one) w.r.t the selling price. As the trend here suggests that as the max power increases, the selling price would increase by some factor. This distribution also has some outliers.
- Engine: It looks like this feature doesn't have a strong correlation w.r.t the selling price. Also, for most of the engine power ranges, the selling price varies a lot.

1.7. Plot the relationships between the label (Selling Price) and the discrete features (fuel type, Seller type, transmission) using a small multiple of box plots. Make sure to label the axes.

In [37]:

```
plt.rcParams["figure.figsize"] = (14,24)
fig, axes = plt.subplots(4, 1)
categorical_cols = ["fuel", "seller_type", "transmission", "owner"]
sns.set_style("whitegrid")

for idx, categorical_col in enumerate(categorical_cols):
    sns.boxplot(x = categorical_col, y = "selling_price", \
                data = pd.concat([auto_mpg_X, auto_mpg_y], axis = 1), \
                orient = 'v', ax = axes[idx])
```





1.8. From the visualizations above, do you think linear regression is a good model for this problem? Why and/or why not?

No, Linear regression would not be a good model for this problem. This is because there is not a very strong linear relationship between the data and the labels (selling price). That would mean that if we need to fit a linear regression model for this, we will have to preprocess the data so, as to form a distribution such that we can apply linear regression.

In [38]:
```
auto_mpg_X['year'] =  2020 - auto_mpg_X['year']
```

In [39]:
```
#dropping the car name as it is irrelevant.
auto_mpg_X.drop(['name'],axis = 1,inplace=True)

#check out the dataset with new changes
auto_mpg_X.head()
```

Out[39]:

	year	km_driven	fuel	seller_type	transmission	owner	mileage	engine	max_power	s
0	6	145500	Diesel	Individual	Manual	First Owner	23.4	1248.0	74.0	
1	6	120000	Diesel	Individual	Manual	Second Owner	21.14	1498.0	103.52	
2	14	140000	Petrol	Individual	Manual	Third Owner	17.7	1497.0	78.0	
3	10	127000	Diesel	Individual	Manual	First Owner	23.0	1396.0	90.0	
4	13	120000	Petrol	Individual	Manual	First Owner	16.1	1298.0	88.2	

Data Pre-processing

1.9. Before we can fit a linear regression model, there are several pre-processing steps we should apply to the datasets:

1. Encode categorial features appropriately.
2. Split the dataset into training (60%), validation (20%), and test (20%) sets.
3. Standardize the columns in the feature matrices X_train, X_val, and X_test to have zero mean and unit variance. To avoid information leakage, learn the standardization parameters (mean, variance) from X_train, and apply it to X_train, X_val, and X_test.
4. Add a column of ones to the feature matrices X_train, X_val, and X_test. This is a common trick so that we can learn a coefficient for the bias term of a linear model.

In [40]:

```
# For synthetic dataset:
seed = 2102

independent_vars_data_df = pd.DataFrame(data = pd.concat([pd.Series(x), pd.Se
# 2. Split the dataset into training (60%), validation (20%), and test (20%)
X_synth_dev, X_synth_test, y_synth_dev, y_synth_test = train_test_split(indep
                                                                         test_
X_synth_train, X_synth_val, y_synth_train, y_synth_val = train_test_split(X_s
                                                                           tes

print(f"Training Data Size: {X_synth_train.shape}")
print(f"Validation Data Size: {X_synth_val.shape}")
print(f"Test Data Size: {X_synth_test.shape}")

# 3. Standardize the columns in the feature matrices
std_synth_scaler = StandardScaler()

# Train data
X_synth_train_scaled = pd.DataFrame(data = std_synth_scaler.fit_transform(X_s

# Validation Data
X_synth_val_scaled = pd.DataFrame(data = std_synth_scaler.transform(X_synth_v

# Test Data
X_synth_test_scaled = pd.DataFrame(data = std_synth_scaler.transform(X_synth_

# 4. Add a column of ones to the feature matrices
X_synth_train_scaled["Bias"] = 1
X_synth_val_scaled["Bias"] = 1
X_synth_test_scaled["Bias"] = 1

# Convert all the dataframes to matrices and vectors
synth_X_train = X_synth_train_scaled.values
synth_X_val = X_synth_val_scaled.values
synth_X_test = X_synth_test_scaled.values

print(synth_X_train, synth_X_val, synth_X_test)
print(y_synth_train, y_synth_val, y_synth_test)
```

```
Training Data Size: (60, 2)
Validation Data Size: (20, 2)
Test Data Size: (20, 2)
[[ 6.29834946e-01  6.29834946e-01  1.00000000e+00]
 [-1.43845421e+00 -1.43845421e+00  1.00000000e+00]
 [ 8.40169437e-01  8.40169437e-01  1.00000000e+00]
 [-1.16852495e-03 -1.16852495e-03  1.00000000e+00]
 [-1.78901169e+00 -1.78901169e+00  1.00000000e+00]
 [-1.40339846e+00 -1.40339846e+00  1.00000000e+00]
 [ 1.39054469e-01  1.39054469e-01  1.00000000e+00]
 [-7.72394990e-01 -7.72394990e-01  1.00000000e+00]
 [-9.47673732e-01 -9.47673732e-01  1.00000000e+00]
 [-2.81614512e-01 -2.81614512e-01  1.00000000e+00]
 [-8.07450738e-01 -8.07450738e-01  1.00000000e+00]
```

```
 [ 3.38872235e-02  3.38872235e-02  1.00000000e+00]
 [ 1.03998720e-01  1.03998720e-01  1.00000000e+00]
 [ 4.19500456e-01  4.19500456e-01  1.00000000e+00]
 [-4.21837506e-01 -4.21837506e-01  1.00000000e+00]
 [-1.64878870e+00 -1.64878870e+00  1.00000000e+00]
 [-1.19306397e+00 -1.19306397e+00  1.00000000e+00]
 [ 3.49388959e-01  3.49388959e-01  1.00000000e+00]
 [ 9.10280933e-01  9.10280933e-01  1.00000000e+00]
 [-1.68384445e+00 -1.68384445e+00  1.00000000e+00]
 [-9.82729480e-01 -9.82729480e-01  1.00000000e+00]
 [ 5.24667701e-01  5.24667701e-01  1.00000000e+00]
 [-5.62060499e-01 -5.62060499e-01  1.00000000e+00]
 [ 2.79277462e-01  2.79277462e-01  1.00000000e+00]
 [ 1.29589417e+00  1.29589417e+00  1.00000000e+00]
 [-1.71890020e+00 -1.71890020e+00  1.00000000e+00]
 [-4.91949003e-01 -4.91949003e-01  1.00000000e+00]
 [ 1.26083842e+00  1.26083842e+00  1.00000000e+00]
 [ 1.19072692e+00  1.19072692e+00  1.00000000e+00]
 [ 1.50622866e+00  1.50622866e+00  1.00000000e+00]
 [ 7.70057940e-01  7.70057940e-01  1.00000000e+00]
 [-1.76447267e-01 -1.76447267e-01  1.00000000e+00]
 [ 1.36600566e+00  1.36600566e+00  1.00000000e+00]
 [ 9.45336682e-01  9.45336682e-01  1.00000000e+00]
 [-4.56893254e-01 -4.56893254e-01  1.00000000e+00]
 [-3.16670261e-01 -3.16670261e-01  1.00000000e+00]
 [ 4.89611953e-01  4.89611953e-01  1.00000000e+00]
 [ 1.57634015e+00  1.57634015e+00  1.00000000e+00]
 [-1.29823122e+00 -1.29823122e+00  1.00000000e+00]
 [ 1.64645165e+00  1.64645165e+00  1.00000000e+00]
 [-1.61373295e+00 -1.61373295e+00  1.00000000e+00]
 [ 1.08555968e+00  1.08555968e+00  1.00000000e+00]
 [ 5.94779198e-01  5.94779198e-01  1.00000000e+00]
 [-3.51726009e-01 -3.51726009e-01  1.00000000e+00]
 [ 8.75225185e-01  8.75225185e-01  1.00000000e+00]
 [-9.12617983e-01 -9.12617983e-01  1.00000000e+00]
 [-7.12800217e-02 -7.12800217e-02  1.00000000e+00]
 [-5.97116248e-01 -5.97116248e-01  1.00000000e+00]
 [-1.41391519e-01 -1.41391519e-01  1.00000000e+00]
 [-1.15800822e+00 -1.15800822e+00  1.00000000e+00]
 [ 1.40106141e+00  1.40106141e+00  1.00000000e+00]
 [ 1.43611716e+00  1.43611716e+00  1.00000000e+00]
 [ 3.84444707e-01  3.84444707e-01  1.00000000e+00]
 [ 7.35002191e-01  7.35002191e-01  1.00000000e+00]
 [ 1.33094991e+00  1.33094991e+00  1.00000000e+00]
 [-8.77562235e-01 -8.77562235e-01  1.00000000e+00]
 [ 2.44221714e-01  2.44221714e-01  1.00000000e+00]
 [ 3.14333211e-01  3.14333211e-01  1.00000000e+00]
 [ 1.01544818e+00  1.01544818e+00  1.00000000e+00]
 [-1.57867720e+00 -1.57867720e+00  1.00000000e+00]] [[-0.84250649 -0.84250649
1.        ]
 [ 0.80511369  0.80511369  1.        ]
 [ 0.20916597  0.20916597  1.        ]
 [ 0.55972345  0.55972345  1.        ]
 [ 1.5412844   1.5412844   1.        ]
 [-1.75395594 -1.75395594  1.        ]
 [-1.47350996 -1.47350996  1.        ]
 [-1.05284098 -1.05284098  1.        ]
```

```
 [-1.01778523 -1.01778523  1.        ]
 [-1.36834271 -1.36834271  1.        ]
 [ 0.4545562   0.4545562   1.        ]
 [-1.54362145 -1.54362145  1.        ]
 [-0.24655876 -0.24655876  1.        ]
 [ 1.15567117  1.15567117  1.        ]
 [-1.12295247 -1.12295247  1.        ]
 [-0.70228349 -0.70228349  1.        ]
 [-0.21150302 -0.21150302  1.        ]
 [-0.73733924 -0.73733924  1.        ]
 [-1.33328696 -1.33328696  1.        ]
 [ 0.17411022  0.17411022  1.        ]] [[ 1.12061542  1.12061542  1.        ]
 [-0.52700475 -0.52700475  1.        ]
 [-1.22811972 -1.22811972  1.        ]
 [-1.26317547 -1.26317547  1.        ]
 [-1.50856571 -1.50856571  1.        ]
 [-1.82406744 -1.82406744  1.        ]
 [-0.66722774 -0.66722774  1.        ]
 [ 1.05050393  1.05050393  1.        ]
 [ 0.06894297  0.06894297  1.        ]
 [ 1.6113959   1.6113959   1.        ]
 [-1.08789673 -1.08789673  1.        ]
 [ 0.69994644  0.69994644  1.        ]
 [ 0.66489069  0.66489069  1.        ]
 [-0.10633577 -0.10633577  1.        ]
 [ 1.22578267  1.22578267  1.        ]
 [-0.03622427 -0.03622427  1.        ]
 [ 1.47117291  1.47117291  1.        ]
 [-0.632172   -0.632172    1.        ]
 [-0.38678176 -0.38678176  1.        ]
 [ 0.98039243  0.98039243  1.        ]]
[68.75292825 23.25170941 68.80174997 50.48778661 12.00855243 21.98008287
 55.45207692 34.70726652 25.83892318 44.02659901 37.84241974 49.28638628
 55.72944006 62.24945014 39.17757343 11.1085704  25.48465765 57.66157125
 72.09564543 18.8349972  36.20320327 65.52976091 42.78178113 56.46746714
 85.0825471  19.14692202 39.54509025 77.89885325 83.03128821 88.88739627
 71.00500472 50.31226905 87.20268875 74.00687987 40.60824326 50.59980093
 58.44273829 93.74144987 23.1222351  91.20596809 17.6987501  72.46197601
 65.91341194 38.82058337 70.48567323 31.14737656 47.71764072 38.75187519
 43.94618525 22.79124814 84.96825928 88.08143226 55.7302431  65.28542153
 81.51774189 31.25663027 56.58854418 58.21426981 78.15702492 15.20249403] [37.
22460027 68.55163033 54.96572059 62.22759986 87.60725764 14.55237557
 18.51293879 30.37108637 26.35936352 20.87007555 61.31990972 16.15498991
 45.04941346 84.37453621 18.50264671 33.19522862 43.41333109 36.18499261
 22.64472101 56.96802175] [82.34354446 43.50612945 23.12189895 27.41153015 18.
50452278 15.29215704
 30.72427726 77.65961358 53.55181695 89.57265547 28.93055276 71.60002024
 67.76068611 48.95773876 79.7632558  52.37282871 87.02869515 36.43101103
 38.87125932 71.15101512]
```

In [41]:

```
# For MPG dataset:
categorical_cols = ["fuel", "seller_type", "transmission", "owner"]
numerical_cols = list(set(auto_mpg_X.columns) - set(categorical_cols))
seed = 2102

# 1. Encode categorial features appropriately.
auto_mpg_X = pd.get_dummies(auto_mpg_X, columns = categorical_cols)
print(f"Original DataSet Size: {auto_mpg_X.shape}")
auto_mpg_X
```

```
Original DataSet Size: (7906, 20)
```

Out[41]:

	year	km_driven	mileage	engine	max_power	seats	fuel_CNG	fuel_Diesel	fuel_LPG
0	6	145500	23.4	1248.0	74.0	5.0	0	1	0
1	6	120000	21.14	1498.0	103.52	5.0	0	1	0
2	14	140000	17.7	1497.0	78.0	5.0	0	0	0
3	10	127000	23.0	1396.0	90.0	5.0	0	1	0
4	13	120000	16.1	1298.0	88.2	5.0	0	0	0
...	...	...	...	...	...	...	...	...	...
8123	7	110000	18.5	1197.0	82.85	5.0	0	0	0
8124	13	119000	16.8	1493.0	110.0	5.0	0	1	0
8125	11	120000	19.3	1248.0	73.9	5.0	0	1	0
8126	7	25000	23.57	1396.0	70.0	5.0	0	1	0
8127	7	25000	23.57	1396.0	70.0	5.0	0	1	0

7906 rows × 20 columns

In [42]:

```
# 2. Split the dataset into training (60%), validation (20%), and test (20%)
auto_mpg_y = np.log(auto_mpg_y)
X_dev, X_test, y_dev, y_test = train_test_split(auto_mpg_X, auto_mpg_y, test_
X_train, X_val, y_train, y_val = train_test_split(X_dev, y_dev, test_size = 0

print(f"Training Data Size: {X_train.shape}")
print(f"Validation Data Size: {X_val.shape}")
print(f"Test Data Size: {X_test.shape}")
```

```
Training Data Size: (4743, 20)
Validation Data Size: (1581, 20)
Test Data Size: (1582, 20)
```

In [43]:
```
# 3. Standardize the columns in the feature matrices
new_categorical_cols = list(set(X_train.columns) - set(numerical_cols))
std_scaler = StandardScaler()

# Train data
X_train_scaled = std_scaler.fit_transform(X_train)

# Validation Data
X_val_scaled = std_scaler.transform(X_val)

# Test Data
X_test_scaled = std_scaler.transform(X_test)
```

In [44]:
```
# 4. Add a column of ones to the feature matrices
auto_mpg_X_train = np.hstack([np.ones((X_train_scaled.shape[0], 1)), X_train_
auto_mpg_X_val = np.hstack([np.ones((X_val_scaled.shape[0], 1)), X_val_scaled
auto_mpg_X_test = np.hstack([np.ones((X_test_scaled.shape[0], 1)), X_test_sca
```

In [45]:
```
# Convert all the dataframes to matrices and vectors
auto_mpg_X_train = np.matrix(auto_mpg_X_train)
auto_mpg_X_val = np.matrix(auto_mpg_X_val)
auto_mpg_X_test = np.matrix(auto_mpg_X_test)
auto_mpg_y_train = np.transpose(np.matrix(y_train.values))
auto_mpg_y_val = np.transpose(np.matrix(y_val.values))
auto_mpg_y_test = np.transpose(np.matrix(y_test.values))
```

In [46]:
```
print(auto_mpg_X_train, auto_mpg_X_val, auto_mpg_X_test, auto_mpg_y_train, au
```

```
[[ 1.         -0.25865838  0.69979235 ... -0.5852223  -0.03248534
  -0.25753047]
 [ 1.         -1.04073446 -0.32746808 ... -0.5852223  -0.03248534
  -0.25753047]
 [ 1.          1.04480176  0.87100242 ... -0.5852223  -0.03248534
  -0.25753047]
 ...
 [ 1.          0.52341771  0.18616213 ...  1.70875238 -0.03248534
  -0.25753047]
 [ 1.          2.86964595  0.3573722  ... -0.5852223  -0.03248534
  -0.25753047]
 [ 1.         -1.04073446 -0.92670333 ... -0.5852223  -0.03248534
  -0.25753047]] [[ 1.         -0.5193504  -0.66988822 ...  1.70875238 -0.03248
534
  -0.25753047]
 [ 1.         -1.04073446 -0.58428319 ... -0.5852223  -0.03248534
  -0.25753047]
 [ 1.          0.52341771  0.52858227 ...  1.70875238 -0.03248534
  -0.25753047]
 ...
 [ 1.         -0.78004243 -0.72125125 ...  1.70875238 -0.03248534
  -0.25753047]
 [ 1.          0.26272568  0.87100242 ...  1.70875238 -0.03248534
  -0.25753047]
 [ 1.          1.30549379  0.18616213 ... -0.5852223  -0.03248534
  -0.25753047]] [[ 1.         -0.5193504   0.18616213 ... -0.5852223  -0.03248
534
  -0.25753047]
 [ 1.          0.52341771  1.43599565 ... -0.5852223  -0.03248534
  -0.25753047]
 [ 1.         -0.25865838 -0.49867815 ... -0.5852223  -0.03248534
  -0.25753047]
 ...
 [ 1.          0.78410973 -1.18350132 ... -0.5852223  -0.03248534
  -0.25753047]
 [ 1.          0.52341771  0.18616213 ...  1.70875238 -0.03248534
  -0.25753047]
 [ 1.          1.30549379  0.61418731 ...  1.70875238 -0.03248534
  -0.25753047]] [[13.12834545]
 [12.89921983]
 [13.12236338]
 ...
 [12.5776362 ]
 [10.71441777]
 [14.15554786]] [[13.38472764]
 [14.07787482]
 [12.92391   ]
 ...
 [13.9552725 ]
 [12.89921983]
 [12.50617724]] [[13.52782849]
 [13.77468856]
 [13.30468493]
 ...
 [12.25008953]
 [13.11231304]
 [12.4292162 ]]
```

In [47]:
```
feature_columns = list(auto_mpg_X.columns)
feature_columns.insert(0, "bias")
print(feature_columns)
```

```
['bias', 'year', 'km_driven', 'mileage', 'engine', 'max_power', 'seats', 'fuel
_CNG', 'fuel_Diesel', 'fuel_LPG', 'fuel_Petrol', 'seller_type_Dealer', 'seller
_type_Individual', 'seller_type_Trustmark Dealer', 'transmission_Automatic', '
transmission_Manual', 'owner_First Owner', 'owner_Fourth & Above Owner', 'owne
r_Second Owner', 'owner_Test Drive Car', 'owner_Third Owner']
```

At the end of this pre-processing, you should have the following vectors and matrices:

- Auto MPG dataset: auto_mpg_X_train, auto_mpg_X_val, auto_mpg_X_test, auto_mpg_y_train, auto_mpg_y_val, auto_mpg_y_test

Implement Linear Regression

Now, we can implement our linear regression model! Specifically, we will be implementing ridge regression, which is linear regression with L2 regularization. Given an (m x n) feature matrix X, an (m x 1) label vector y, and an (n x 1) weight vector w, the hypothesis function for linear regression is:

$$y = Xw$$

Note that we can omit the bias term here because we have included a column of ones in our X matrix, so the bias term is learned implicitly as a part of w. This will make our implementation easier.

Our objective in linear regression is to learn the weights w which best fit the data. This notion can be formalized as finding the optimal w which minimizes the following loss function:

$$\min_{w} \|Xw - y\|_2^2 + \alpha\|w\|_2^2$$

This is the ridge regression loss function. The $\|Xw - y\|_2^2$ term penalizes predictions Xw which are not close to the label y. And the $\alpha\|w\|_2^2$ penalizes large weight values, to favor a simpler, more generalizable model. The α hyperparameter, known as the regularization parameter, is used to tune the complexity of the model - a higher α results in smaller weights and lower complexity, and vice versa. Setting $\alpha = 0$ gives us vanilla linear regression.

Conveniently, ridge regression has a closed-form solution which gives us the optimal w without having to do iterative methods such as gradient descent. The closed-form solution, known as the Normal Equations, is given by:

$$w = (X^T X + \alpha I)^{-1} X^T y$$

1.10. Implement a `LinearRegression` class with two methods: `train` and `predict`. You may NOT use sklearn for this implementation. You may, however, use `np.linalg.solve` to find the closed-form solution. It is highly recommended that you vectorize your code.

In [48]:

```
class LinearRegression():
    '''
    Linear regression model with L2-regularization (i.e. ridge regression).

    Attributes
    ----------
    alpha: regularization parameter
    w: (n x 1) weight vector
    '''

    def __init__(self, alpha=0):
        self.alpha = alpha
        self.w = None

    def train(self, X, y):
        '''Trains model using ridge regression closed-form solution
        (sets w to its optimal value).

        Parameters
        ----------
        X : (m x n) feature matrix
        y: (m x 1) label vector

        Returns
        -------
        None
        '''
        part_1 = np.add(np.dot(np.transpose(X), X), self.alpha * np.eye(X.sha
        part_2 = np.dot(np.transpose(X), y)
        self.w = np.linalg.solve(part_1, part_2)

    def predict(self, X):
        '''Predicts on X using trained model.

        Parameters
        ----------
        X : (m x n) feature matrix

        Returns
        -------
        y_pred: (m x 1) prediction vector
        '''
        y_pred = np.dot(X, self.w)
        return y_pred
```

Train, Evaluate, and Interpret Linear Regression Model

1.11. A) Train a linear regression model ($\alpha = 0$) on the auto MPG training data. Make predictions and report the mean-squared error (MSE) on the training, validation, and test sets. Report the first 5 predictions on the test set, along with the actual labels.

In [49]:

```
def calc_mse(y_actual, y_pred):
    return np.square(np.subtract(y_actual, y_pred)).mean()
```

In [50]:

```
model_lr_mpg = LinearRegression()
model_lr_mpg.train(auto_mpg_X_train, auto_mpg_y_train)

auto_mpg_y_train_pred = model_lr_mpg.predict(auto_mpg_X_train)
print(f"MSE for Training data: {calc_mse(auto_mpg_y_train, auto_mpg_y_train_p

auto_mpg_y_val_pred = model_lr_mpg.predict(auto_mpg_X_val)
print(f"MSE for Validation data: {calc_mse(auto_mpg_y_val, auto_mpg_y_val_pre

auto_mpg_y_test_pred = model_lr_mpg.predict(auto_mpg_X_test)
print(f"MSE for Test data: {calc_mse(auto_mpg_y_test, auto_mpg_y_test_pred)}"
```

```
MSE for Training data: 0.08582221347134225
MSE for Validation data: 0.0894739098447177
MSE for Test data: 0.09347415923787637
```

In [51]:

```
print("First 5 predictions on Test data:")
print(auto_mpg_y_test_pred[:5])
print("First 5 actual labels on Test data:")
print(np.array(auto_mpg_y_test[:5]))
```

```
First 5 predictions on Test data:
[[13.26689697]
 [13.13107023]
 [13.11775945]
 [12.39515423]
 [12.84369699]]
First 5 actual labels on Test data:
[[13.52782849]
 [13.77468856]
 [13.30468493]
 [12.4490149 ]
 [13.12236338]]
```

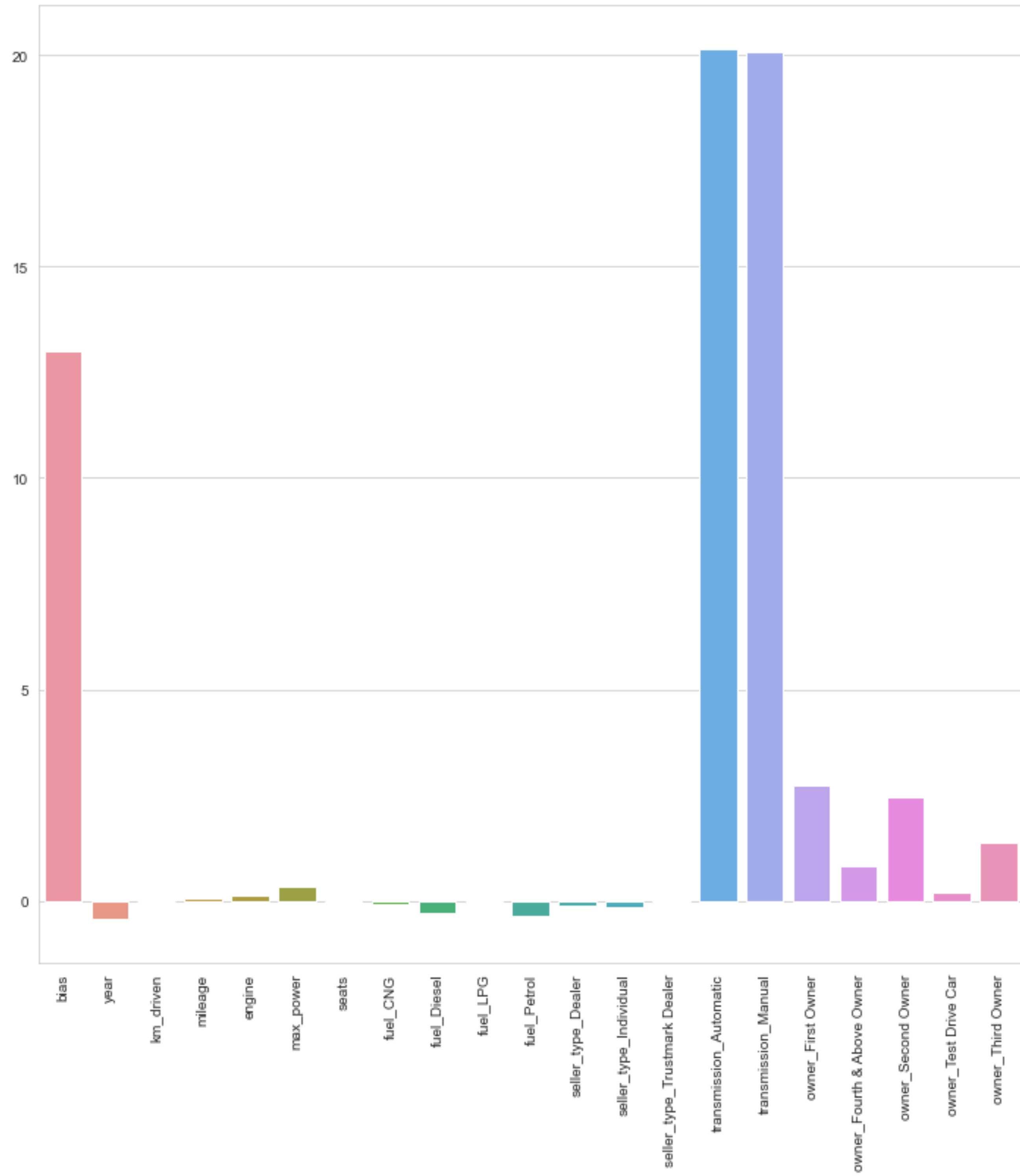
B) As a baseline model, use the mean of the training labels (auto_mpg_y_train) as the prediction for all instances. Report the mean-squared error (MSE) on the training, validation, and test sets using this baseline. This is a common baseline used in regression problems and tells you if your model is any good. Your linear regression MSEs should be much lower than these baseline MSEs.

In [52]:
```
auto_mpg_y_train_mean_vals = np.full((auto_mpg_y_train.shape), np.mean(auto_m
auto_mpg_y_val_mean_vals = np.full((auto_mpg_y_val.shape), np.mean(auto_mpg_y
auto_mpg_y_test_mean_vals = np.full((auto_mpg_y_test.shape), np.mean(auto_mpg

print(f"MSE for Training data w.r.t Baseline: {calc_mse(auto_mpg_y_train, aut
print(f"MSE for Validation data w.r.t Baseline: {calc_mse(auto_mpg_y_val, aut
print(f"MSE for Test data w.r.t Baseline: {calc_mse(auto_mpg_y_test, auto_mpg
```

```
MSE for Training data w.r.t Baseline: 0.670713963055895
MSE for Validation data w.r.t Baseline: 0.7005930201266619
MSE for Test data w.r.t Baseline: 0.7119101307269784
```

1.12. Interpret your model trained on the auto MPG dataset using a bar chart of the model weights. Make sure to label the bars (x-axis) and don't forget the bias term! Use lecture 3, slide 15 as a reference. According to your model, which features are the greatest contributors to the selling price

In [53]:
```
plt.rcParams["figure.figsize"] = (12,12)
ax = sns.barplot(x = feature_columns, y = np.array(model_lr_mpg.w)[:, 0])
ax.tick_params(axis = 'x', rotation = 90)
```



Transmission (Automatic and Manual), Owner (First Owner and Second Owner) are the features (+ Bias) that are the greatest contributors to the selling price.

Tune Regularization Parameter α

Now, let's do ridge regression and tune the α regularization parameter on the auto MPG dataset.

1.13. Sweep out values for α using `alphas = np.logspace(-2, 1, 10)`. Perform a grid search over these α values, recording the training and validation MSEs for each α. A simple grid search is fine, no need for k-fold cross validation. Plot the training and validation MSEs as a function of α on a single figure. Make sure to label the axes and the training and validation MSE curves. Use a log scale for the x-axis.

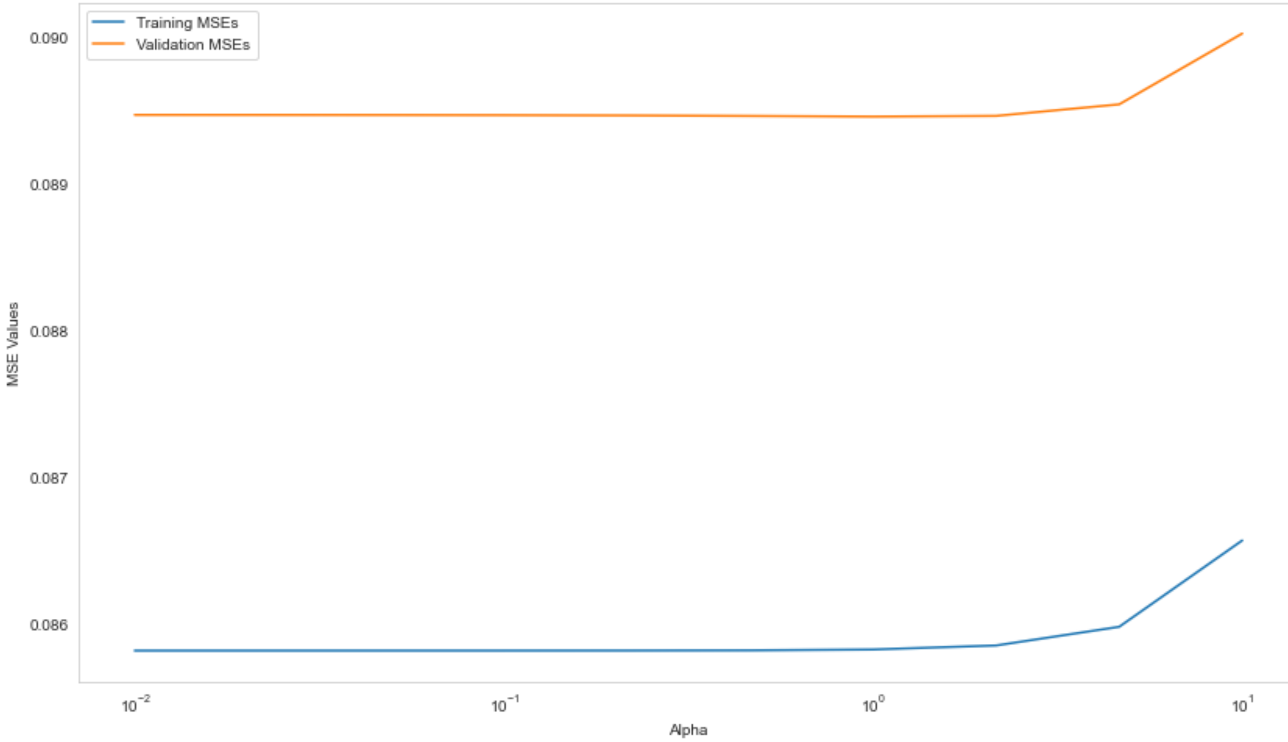
In [54]:

```
import pprint

alphas = np.logspace(-2, 1, 10)
training_MSEs = dict()
validation_MSEs = dict()
curr_min_training_mse = np.inf
best_alpha = np.inf

for alpha in alphas:
    model_lr_mpg = LinearRegression(alpha)
    model_lr_mpg.train(auto_mpg_X_train, auto_mpg_y_train)

    auto_mpg_y_train_pred = model_lr_mpg.predict(auto_mpg_X_train)
    training_mse = calc_mse(auto_mpg_y_train, auto_mpg_y_train_pred)

    auto_mpg_y_val_pred = model_lr_mpg.predict(auto_mpg_X_val)
    validation_mse = calc_mse(auto_mpg_y_val, auto_mpg_y_val_pred)

    training_MSEs[alpha] = training_mse
    validation_MSEs[alpha] = validation_mse

    if training_mse < curr_min_training_mse:
        curr_min_training_mse = training_mse
        best_alpha = alpha

print(f"The Best Alpha is: {best_alpha} with the optimal minimum training MSE
print("\n")
print(f"The Training MSEs are as follows:")
pprint.pprint(training_MSEs)
print("\n")
print(f"The Validation MSEs are as follows:")
pprint.pprint(validation_MSEs)
```

```
The Best Alpha is: 0.01 with the optimal minimum training MSE of 0.08582221422
530054


The Training MSEs are as follows:
{0.01: 0.08582221422530054,
 0.021544346900318832: 0.0858222169708894,
 0.046415888336127774: 0.08582222971463023,
 0.1: 0.08582228886429794,
 0.21544346900318834: 0.08582256339736195,
 0.46415888336127775: 0.08582383751325384,
 1.0: 0.08582974989863232,
 2.154434690031882: 0.08585717740219874,
 4.6415888336127775: 0.08598433120182429,
 10.0: 0.08657299697681935}


The Validation MSEs are as follows:
{0.01: 0.08947371634734722,
 0.021544346900318832: 0.08947349483682585,
 0.046415888336127774: 0.08947302441489335,
 0.1: 0.08947204252039061,
 0.21544346900318834: 0.08947007374960347,
 0.46415888336127775: 0.08946651276232442,
 1.0: 0.08946199871999667,
 2.154434690031882: 0.08946691909378986,
 4.6415888336127775: 0.08954537904686093,
 10.0: 0.09002820008473657}
```

In [55]:

```
# Plot the training and validation MSEs for the alphas
plt.rcParams["figure.figsize"] = (14, 8)
plt.plot(training_MSEs.keys(), training_MSEs.values(), label = "Training MSEs
plt.plot(validation_MSEs.keys(), validation_MSEs.values(), label = "Validatio
plt.xlabel("Alpha")
plt.ylabel("MSE Values")
plt.xscale('log')
plt.legend()
plt.grid(False)
plt.show()
```



Explain your plot above. How do training and validation MSE behave with decreasing model complexity (increasing α)?

As we decrease the model complexity by increasing alpha values, the (Training and Validation) MSEs slowly decrease and then increase at a growing pace when the alpha values reaches the limit of 10 in our experiment. This makes sense as we get the best model when the alpha is 0.01. As we decrease the model complexity, the model is not able to fit the data well and as efficiently as possible and thus, the errors would increase as we increase the value of alpha. (Underfitting)

1.14. Using the α which gave the best validation MSE above, train a model on the training set. Report the value of α and its training, validation, and test MSE. This is the final tuned model which you would deploy in production.

In [57]:

```
model_rr_mpg = LinearRegression(alpha = best_alpha)
model_rr_mpg.train(auto_mpg_X_train, auto_mpg_y_train)

print(f"At the Best Alpha of {best_alpha}:")

auto_mpg_y_train_pred = model_rr_mpg.predict(auto_mpg_X_train)
print(f"MSE for Training data: {calc_mse(auto_mpg_y_train, auto_mpg_y_train_p

auto_mpg_y_val_pred = model_rr_mpg.predict(auto_mpg_X_val)
print(f"MSE for Validation data: {calc_mse(auto_mpg_y_val, auto_mpg_y_val_pre

auto_mpg_y_test_pred = model_rr_mpg.predict(auto_mpg_X_test)
print(f"MSE for Test data: {calc_mse(auto_mpg_y_test, auto_mpg_y_test_pred)}"

print("\n")
```

```
At the Best Alpha of 0.01:
MSE for Training data: 0.08582221422530054
MSE for Validation data: 0.08947371634734722
MSE for Test data: 0.09347429978146067
```

Part 2: Logistic Regression

Gender Recognition by Voice and Speech Analysis

This dataset is used to identify a voice as male or female, based upon acoustic properties of the voice and speech.

In [96]:

```
voice_df = pd.read_csv("voice-classification.csv")
voice_df.head()
```

Out[96]:

	meanfreq	sd	median	Q25	Q75	IQR	skew	kurt	s
0	0.059781	0.064241	0.032027	0.015071	0.090193	0.075122	12.863462	274.402906	0.89
1	0.066009	0.067310	0.040229	0.019414	0.092666	0.073252	22.423285	634.613855	0.89
2	0.077316	0.083829	0.036718	0.008701	0.131908	0.123207	30.757155	1024.927705	0.84
3	0.151228	0.072111	0.158011	0.096582	0.207955	0.111374	1.232831	4.177296	0.96
4	0.135120	0.079146	0.124656	0.078720	0.206045	0.127325	1.101174	4.333713	0.9

5 rows × 21 columns

Data - Checking Rows & Columns

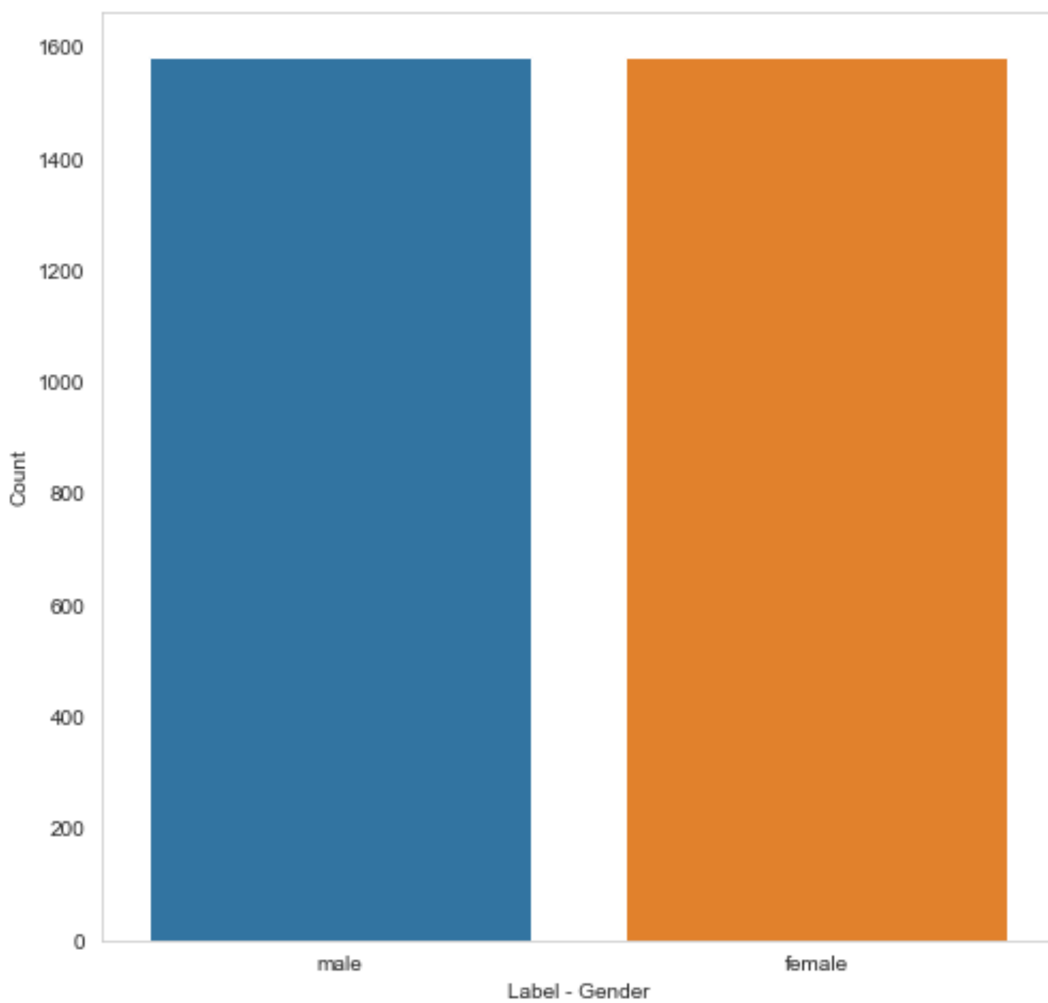
In [97]:
```
#Number of Rows & Columns
print(voice_df.shape)
```

```
(3168, 21)
```

2.1 What is the probability of observing different categories in the Label feature of the dataset?

This is mainly to check class imbalance in the dataset, and to apply different techniques to balance the dataset, which we will learn later.

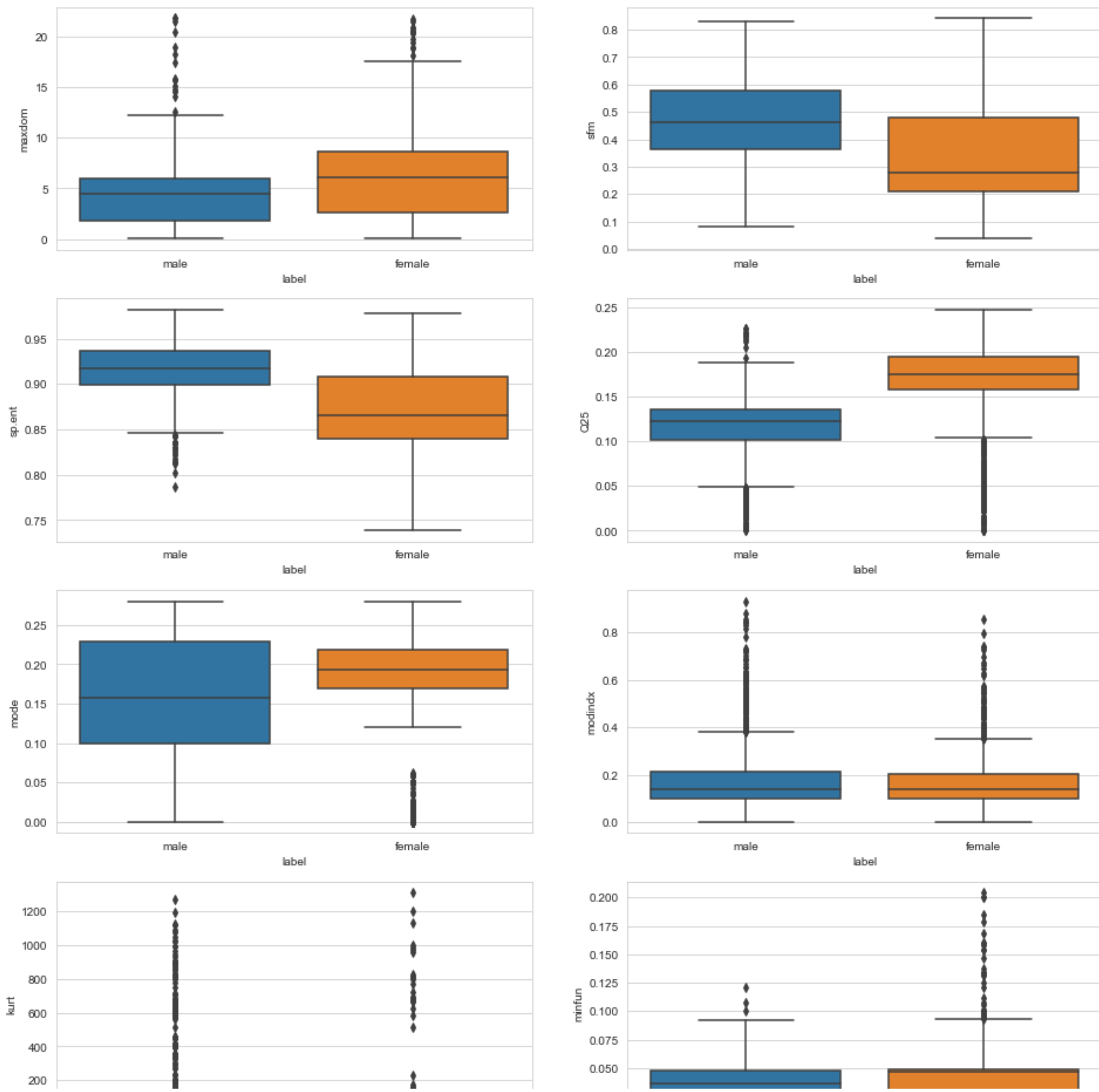
In [98]:
```
import warnings
warnings.filterwarnings("ignore")
plt.rcParams["figure.figsize"] = (8, 8)
sns.countplot(voice_df["label"])
plt.grid(False)
plt.xlabel("Label - Gender")
plt.ylabel("Count")
plt.show()
```

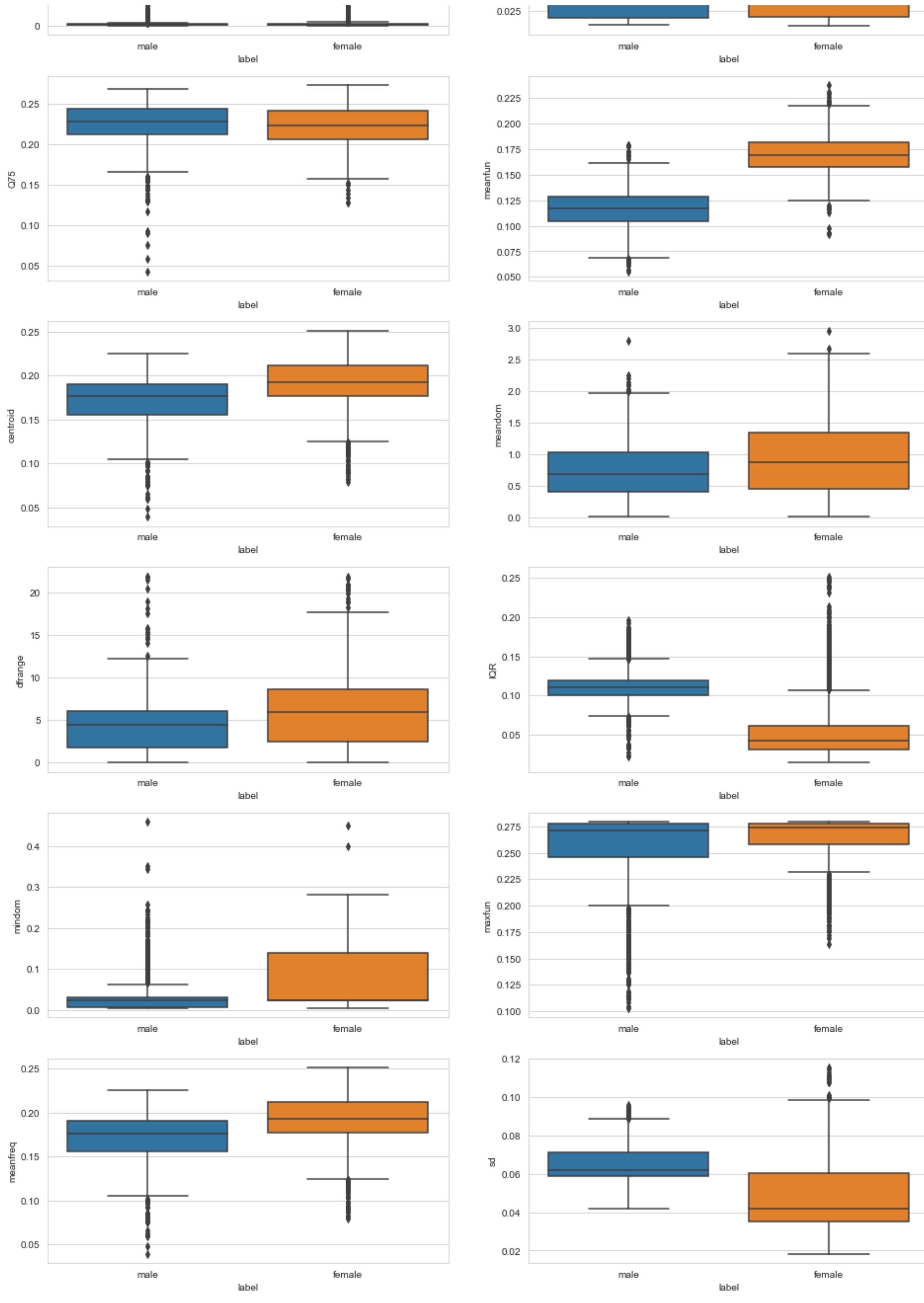


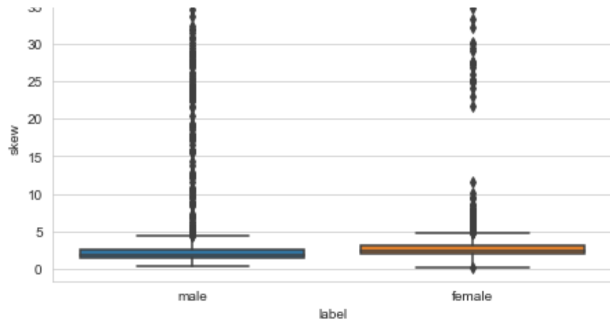
2.2 Plot the relationships between the label and the 20 numerical features using a small multiple of box plots. Make sure to label the axes. What useful information do this plot provide?

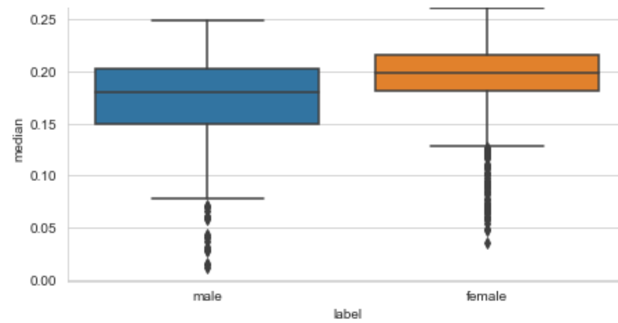
In [99]:

```
plt.rcParams["figure.figsize"] = (16,45)
fig, axes = plt.subplots(10, 2)
numerical_cols = set(voice_df.columns) - set(["label"])
sns.set_style("whitegrid")

for numerical_col, curr_ax in zip(numerical_cols, axes.flatten()):
    sns.boxplot(x = "label", y = numerical_col, \
                data = voice_df, \
                orient = 'v', ax = curr_ax)
```




male
female
label
Q75
meanfun
centroid
meandom
dfrange
IQR
mindom
maxfun
meanfreq
sd





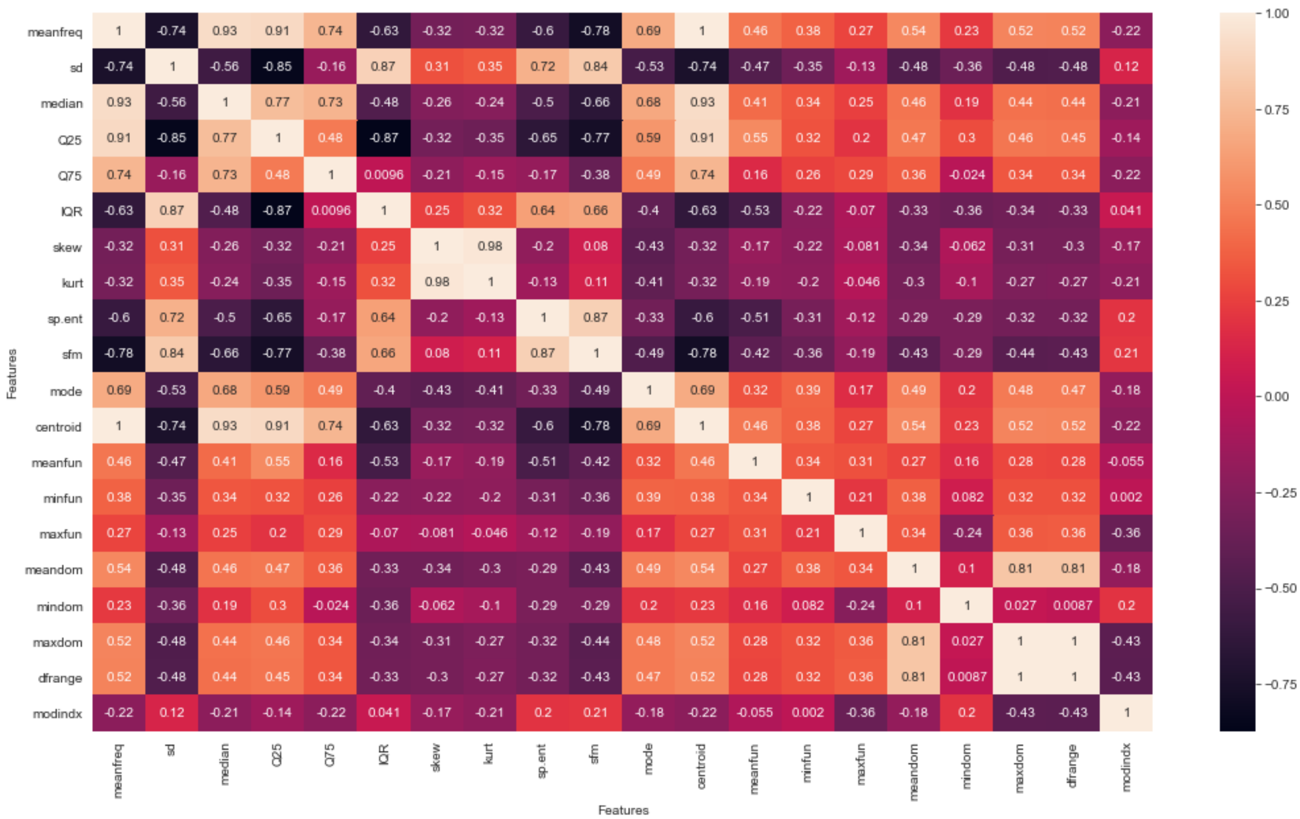
The important information from the plots above can be summarized as follows:

1. Most of the features in the dataset follow a normal distribution with some skewness (positive or negative). For example, "Q25", "meanfun", "meanfreq", etc.
2. There are a lot of outliers in the dataset. For example, "kurt", "maxfun", "skew", etc.
3. One of the columns in the dataset namely "sfm" is very clean and has no outliers.
4. Some of the positively skewed distributions are "maxdom", modindx", "dfrange", "skew". Some of the negatively skewed distributions are "maxfun", "Q75", "median".
5. Some of the features follow similar distributions like "kurt and skew", "centroid and Q25".

2.3 Plot the correlation matrix, and check if there is high correlation between the given numerical features (Threshold >=0.9). If yes, drop those highly correlated features from the dataframe. Why is necessary to drop those columns before proceeding further?

In [100…

```
plt.rcParams["figure.figsize"] = (18,10)
correlation_matrix = voice_df.corr()
sns.heatmap(correlation_matrix, annot = True)
plt.xlabel("Features")
plt.ylabel("Features")
plt.show()
```



In [101…

```
# Finding highly correlated features
correlated_pairs = correlation_matrix.unstack()
sorted_correlated_pairs = correlated_pairs.sort_values(kind = "quicksort")
highly_correlated_pairs = sorted_correlated_pairs[sorted_correlated_pairs >=
highly_correlated_features = set()

print("Highly Correlated Features with their Correlations: ")
for index, corr_val in zip(highly_correlated_pairs.index, highly_correlated_p
    index1, index2 = index
    if index1 != index2:
        highly_correlated_features.add(index1)
        highly_correlated_features.add(index2)
        print(index1, index2, corr_val)

print("\n")
print("Highly correlated features are: ")
highly_correlated_features = list(highly_correlated_features)
print(highly_correlated_features)
```

```
Highly Correlated Features with their Correlations:
centroid Q25 0.9114163463244436
Q25 centroid 0.9114163463244436
Q25 meanfreq 0.9114163463244436
meanfreq Q25 0.9114163463244436
centroid median 0.9254453730463191
median centroid 0.9254453730463191
median meanfreq 0.9254453730463191
meanfreq median 0.9254453730463191
kurt skew 0.9770204562201018
skew kurt 0.9770204562201018
maxdom dfrange 0.9998384146229784
dfrange maxdom 0.9998384146229784
meanfreq centroid 1.0
centroid meanfreq 1.0


Highly correlated features are:
['centroid', 'maxdom', 'dfrange', 'Q25', 'meanfreq', 'kurt', 'skew', 'median']
```

In [102…

```python
# Now we need to drop one of those highly correlated features (transitive rel
voice_df.drop(["Q25", "meanfreq", "skew", "median", "maxdom"], axis = 1, inpl
```

In [103…

```python
voice_df
```

Out[103…

	sd	Q75	IQR	kurt	sp.ent	sfm	mode	centroid	r
0	0.064241	0.090193	0.075122	274.402906	0.893369	0.491918	0.000000	0.059781	0
1	0.067310	0.092666	0.073252	634.613855	0.892193	0.513724	0.000000	0.066009	(
2	0.083829	0.131908	0.123207	1024.927705	0.846389	0.478905	0.000000	0.077316	0
3	0.072111	0.207955	0.111374	4.177296	0.963322	0.727232	0.083878	0.151228	0
4	0.079146	0.206045	0.127325	4.333713	0.971955	0.783568	0.104261	0.135120	C
...	...	...	...	...	...	...	...	...	
3163	0.084734	0.201144	0.151859	6.630383	0.962934	0.763182	0.200836	0.131884	(
3164	0.089221	0.204911	0.162193	2.503954	0.960716	0.709570	0.013683	0.116221	C
3165	0.095798	0.224360	0.190936	6.604509	0.946854	0.654196	0.008006	0.142056	C
3166	0.090628	0.219943	0.176435	5.388298	0.950436	0.675470	0.212202	0.143659	(
3167	0.092884	0.250827	0.180756	5.769115	0.938829	0.601529	0.267702	0.165509	(

3168 rows × 16 columns

The problem that arises when linear regression is applied to correlated features is the problem of "Multicollinearity". When independent variables are highly correlated, change in one variable would cause change to another and so, the model would fluctuate significantly. This would result in a highly unstable model with unstable parameters and could vary a lot for some small change in the data or the model.

Separating Features & Y variable from the processed dataset

Please note to replace the dataframe below with the new dataframe created after removing highly correlated features

In [104…

```
# Split data into features and labels
voice_X = voice_df.drop(columns=['label']) #replace "voice_df1" with your dat
voice_y = voice_df['label']
print(voice_X.columns)
```

```
Index(['sd', 'Q75', 'IQR', 'kurt', 'sp.ent', 'sfm', 'mode', 'centroid',
       'meanfun', 'minfun', 'maxfun', 'meandom', 'mindom', 'dfrange',
       'modindx'],
      dtype='object')
```

In [105…

```
voice_X
```

Out[105…

	sd	Q75	IQR	kurt	sp.ent	sfm	mode	centroid	r
0	0.064241	0.090193	0.075122	274.402906	0.893369	0.491918	0.000000	0.059781	0
1	0.067310	0.092666	0.073252	634.613855	0.892193	0.513724	0.000000	0.066009	(
2	0.083829	0.131908	0.123207	1024.927705	0.846389	0.478905	0.000000	0.077316	0
3	0.072111	0.207955	0.111374	4.177296	0.963322	0.727232	0.083878	0.151228	0
4	0.079146	0.206045	0.127325	4.333713	0.971955	0.783568	0.104261	0.135120	C
...	...	...	...	...	...	...	...	...	
3163	0.084734	0.201144	0.151859	6.630383	0.962934	0.763182	0.200836	0.131884	C
3164	0.089221	0.204911	0.162193	2.503954	0.960716	0.709570	0.013683	0.116221	C
3165	0.095798	0.224360	0.190936	6.604509	0.946854	0.654196	0.008006	0.142056	C
3166	0.090628	0.219943	0.176435	5.388298	0.950436	0.675470	0.212202	0.143659	(
3167	0.092884	0.250827	0.180756	5.769115	0.938829	0.601529	0.267702	0.165509	C

3168 rows × 15 columns

In [106…

```
voice_y
```

Out[106…

```
0          male
1          male
2          male
3          male
4          male
          ...
3163    female
3164    female
3165    female
3166    female
3167    female
Name: label, Length: 3168, dtype: object
```

2.4 Apply the following pre-processing steps:

1) Use OrdinalEncoding to encode the label in the dataset (male & female)

2) Convert the label from a Pandas series to a Numpy (m x 1) vector. If you don't do this, it may cause problems when implementing the logistic regression model.

3)Split the dataset into training (60%), validation (20%), and test (20%) sets.

4) Standardize the columns in the feature matrices. To avoid information leakage, learn the standardization parameters from training, and then apply training, validation and test dataset.

5) Add a column of ones to the feature matrices of train, validation and test dataset. This is a common trick so that we can learn a coefficient for the bias term of a linear model.

In [107…

```
# Step - 1 & 2

label_types = list(set(voice_y))
ord_encoder = OrdinalEncoder(categories = [label_types])
enc_labels = ord_encoder.fit_transform(voice_y.to_numpy().reshape(-1, 1))
print(f"The shape of the labels data is: {enc_labels.shape}")
```

```
The shape of the labels data is: (3168, 1)
```

In [108…

```
# Step -  3
seed = 2102

X_dev, X_test, y_dev, y_test = train_test_split(voice_X, enc_labels, test_siz
X_train, X_val, y_train, y_val = train_test_split(X_dev, y_dev, test_size = 0

print(f"Training Data Size: {X_train.shape}")
print(f"Validation Data Size: {X_val.shape}")
print(f"Test Data Size: {X_test.shape}")
```

```
Training Data Size: (1900, 15)
Validation Data Size: (634, 15)
Test Data Size: (634, 15)
```

In [109…

```
# Step - 4

# Pre-Processing these 2 columns with the min-max scaler because the data for
# and the distribution of the data is driven by the tail of the data and the
# preserve the distribution of the data in these columns.
min_max_scale_cols = ["maxfun", "mindom", "kurt"]
std_scale_cols = list(set(X_train.columns) - set(min_max_scale_cols))

std_scaler = StandardScaler()
min_max_scaler = MinMaxScaler()

# Train data
X_train_std_scaled = std_scaler.fit_transform(X_train[std_scale_cols])
X_train_min_max_scaled = min_max_scaler.fit_transform(X_train[min_max_scale_c
X_train_scaled = np.hstack([X_train_std_scaled, X_train_min_max_scaled])

# Validation Data
X_val_std_scaled = std_scaler.transform(X_val[std_scale_cols])
X_val_min_max_scaled = min_max_scaler.transform(X_val[min_max_scale_cols])
X_val_scaled = np.hstack([X_val_std_scaled, X_val_min_max_scaled])

# Test Data
X_test_std_scaled = std_scaler.transform(X_test[std_scale_cols])
X_test_min_max_scaled = min_max_scaler.transform(X_test[min_max_scale_cols])
X_test_scaled = np.hstack([X_test_std_scaled, X_test_min_max_scaled])
```

In [110…

```
# Step - 5
voice_X_train = np.hstack([X_train_scaled, np.ones((X_train_scaled.shape[0],
voice_X_val = np.hstack([X_val_scaled, np.ones((X_val_scaled.shape[0], 1))])
voice_X_test = np.hstack([X_test_scaled, np.ones((X_test_scaled.shape[0], 1))
```

In [111…

```
# Convert all the dataframes to matrices and vectors
voice_X_train = np.matrix(voice_X_train)
voice_X_val = np.matrix(voice_X_val)
voice_X_test = np.matrix(voice_X_test)
voice_y_train = np.matrix(y_train)
voice_y_val = np.matrix(y_val)
voice_y_test = np.matrix(y_test)
```

In [112…

```
print(voice_X_train, voice_X_val, voice_X_test)
```

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[[-7.01221083e-01  1.03714277e+00  1.25212000e+00 ...  4.08602151e-02
   1.86969632e-02  1.00000000e+00]
 [-1.56337380e+00 -8.29969275e-01  1.78365483e+00 ...  4.08602151e-02
   2.84964087e-03  1.00000000e+00]
 [ 1.77085825e+00 -1.51893707e-01 -1.68421980e+00 ...  4.08602151e-02
   1.49507000e-02  1.00000000e+00]
 ...
 [ 3.43654805e-01  9.05597070e-01 -6.72949203e-02 ...  4.08602151e-02
   1.30743357e-03  1.00000000e+00]
 [ 2.13017844e-01 -1.07284537e+00 -9.43687143e-01 ...  3.33333333e-01
   6.96012430e-03  1.00000000e+00]
 [-4.23642731e-01  5.99001522e-01  1.38268616e+00 ...  4.08602151e-02
   1.50541816e-03  1.00000000e+00]] [[ 2.34993253e-01  1.91430549e+00 -5.13667
566e-01 ...  4.08602151e-02
   4.72345573e-03  1.00000000e+00]
 [-2.66016510e+00 -2.10620562e-01  1.58752040e+00 ...  6.45161290e-03
   3.09194670e-03  1.00000000e+00]
 [-2.58620886e-01  1.04578596e+00  8.19882646e-01 ...  1.95698925e-01
   4.99369562e-03  1.00000000e+00]
 ...
 [ 1.15014341e-01 -7.80150727e-02 -1.33755564e+00 ...  6.45161290e-03
   1.20001120e-03  1.00000000e+00]
 [ 1.91795635e+00  3.17437320e+00 -1.28178126e+00 ...  4.08602151e-02
   3.45514483e-03  1.00000000e+00]
 [ 3.17995276e-01 -3.40554511e-01 -1.58822801e+00 ...  6.45161290e-03
   5.59422127e-03  1.00000000e+00]] [[ 0.51392825 -1.20079012  0.65742206 ...
0.09247312  0.00461973
   1.        ]
 [ 1.53601023  2.20570565 -0.66102515 ...  0.04086022  0.00435182
   1.        ]
 [-1.01606053 -0.04888053  1.25184776 ...  0.04086022  0.0064443
   1.        ]
 ...
 [-0.76311976 -1.10903936  1.04049212 ...  0.00645161  0.00619482
   1.        ]
 [-0.63927222 -0.45144587  0.29082202 ...  0.3655914   0.05228704
   1.        ]
 [-0.71096891 -1.1756977   0.31952692 ...  0.          0.01214536
   1.        ]]
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In [113…

```
print(voice_y_train, voice_y_val, voice_y_test)
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2.5 Implement Logistic Regression

We will now implement logistic regression with L2 regularization. Given an (m x n) feature matrix X, an (m x 1) label vector y, and an (n x 1) weight vector w, the hypothesis function for logistic regression is:

$$y = \sigma(Xw)$$

where $\sigma(x) = \frac{1}{1+e^{-x}}$, i.e. the sigmoid function. This function scales the prediction to be a probability between 0 and 1, and can then be thresholded to get a discrete class prediction.

Just as with linear regression, our objective in logistic regression is to learn the weights w which best fit the data. For L2-regularized logistic regression, we find an optimal w to minimize the following loss function:

$$\min_{w} \ -y^T \log(\sigma(Xw)) \ - \ (\mathbf{1} - y)^T \log(\mathbf{1} - \sigma(Xw)) \ + \ \alpha \|w\|_2^2$$

Unlike linear regression, however, logistic regression has no closed-form solution for the optimal w. So, we will use gradient descent to find the optimal w. The (n x 1) gradient vector g for the loss function above is:

$$g = X^T \Big(\sigma(Xw) - y\Big) + 2\alpha w$$

Below is pseudocode for gradient descent to find the optimal w. You should first initialize w (e.g. to a (n x 1) zero vector). Then, for some number of epochs t, you should update w with $w - \eta g$, where η is the learning rate and g is the gradient. You can learn more about gradient descent here.

> $w = \mathbf{0}$
>
> $\text{for } i = 1, 2, \ldots, t$
>
> $\quad w = w - \eta g$

Implement a LogisticRegression class with five methods: train, predict, calculate_loss, calculate_gradient, and calculate_sigmoid. **You may NOT use sklearn for this implementation. It is highly recommended that you vectorize your code.**

In [114…

```
class LogisticRegression():
    '''
    Logistic regression model with L2 regularization.

    Attributes
    ----------
    alpha: regularization parameter
    t: number of epochs to run gradient descent
    eta: learning rate for gradient descent
    w: (n x 1) weight vector
    '''

    def __init__(self, alpha, t, eta):
        self.alpha = alpha
        self.t = t
        self.eta = eta
        self.w = None

    def train(self, X, y):
        '''Trains logistic regression model using gradient descent
        (sets w to its optimal value).

        Parameters
        ----------
        X : (m x n) feature matrix
        y: (m x 1) label vector

        Returns
        -------
        losses: (t x 1) vector of losses at each epoch of gradient descent
        '''
        curr_epoch = 0
        self.w = np.zeros((X.shape[1], 1))
        losses = list()

        while curr_epoch < self.t:
            losses.append(self.calculate_loss(X, y))
            self.w = self.w - self.eta * self.calculate_gradient(X, y)
            curr_epoch += 1

        return losses

    def predict(self, X):
        '''Predicts on X using trained model. Make sure to threshold
        the predicted probability to return a 0 or 1 prediction.

        Parameters
        ----------
        X : (m x n) feature matrix

        Returns
        -------
        y_pred: (m x 1) 0/1 prediction vector
```

```
        '''
        y_preds = self.calculate_sigmoid(np.dot(X, self.w))
        threshold = 0.5

        for idx, y_pred in enumerate(y_preds):
            if y_pred <= threshold:
                y_preds[idx] = 0
            else:
                y_preds[idx] = 1

        return y_preds

    def calculate_loss(self, X, y):
        '''Calculates the logistic regression loss using X, y, w,
        and alpha. Useful as a helper function for train().

        Parameters
        ----------
        X : (m x n) feature matrix
        y: (m x 1) label vector

        Returns
        -------
        loss: (scalar) logistic regression loss
        '''
        activation_label_1 = self.calculate_sigmoid(np.dot(X, self.w))
        activation_label_2 = np.subtract(1, activation_label_1)
        loss_term_1_label_1 = np.dot(np.transpose(y), np.log(activation_label_
        loss_term_2_label_2 = np.dot(np.transpose(np.subtract(1, y)), np.log(
        reg_term_3 = self.alpha * np.sum(np.square(self.w))
        loss = - loss_term_1_label_1 - loss_term_2_label_2 + reg_term_3

        return loss

    def calculate_gradient(self, X, y):
        '''Calculates the gradient of the logistic regression loss
        using X, y, w, and alpha. Useful as a helper function
        for train().

        Parameters
        ----------
        X : (m x n) feature matrix
        y: (m x 1) label vector

        Returns
        -------
        gradient: (n x 1) gradient vector for logistic regression loss
        '''
        diff_pred_actual = np.subtract(self.calculate_sigmoid(np.dot(X, self.
        scaled_val_acc_error = np.dot(np.transpose(X), diff_pred_actual)
        regularization_term = np.dot(2 * self.alpha, self.w)
        gradient = np.add(scaled_val_acc_error, regularization_term)
```

```
        return gradient

    def calculate_sigmoid(self, x):
        '''Calculates the sigmoid function on each element in vector x.
        Useful as a helper function for predict(), calculate_loss(),
        and calculate_gradient().

        Parameters
        ----------
        x: (m x 1) vector

        Returns
        -------
        sigmoid_x: (m x 1) vector of sigmoid on each element in x
        '''
        sigmoid_x = 1 / (1 + np.exp(- x.reshape(-1, 1)))

        return sigmoid_x
```

2.6 Plot Loss over Epoch and Search the space randomly to find best hyperparameters.

A: Using your implementation above, train a logistic regression model **(alpha=0, t=100, eta=1e-3)** on the voice recognition training data. Plot the training loss over epochs. Make sure to label your axes. You should see the loss decreasing and start to converge.

B: Using **alpha between (0,1), eta between(0, 0.001) and t between (0, 100)**, find the best hyperparameters for LogisticRegression. You can randomly search the space 20 times to find the best hyperparameters.

C. Compare accuracy on the test dataset for both the scenarios.

In [115…

```
# For A part when alpha is 0
```

In [116…

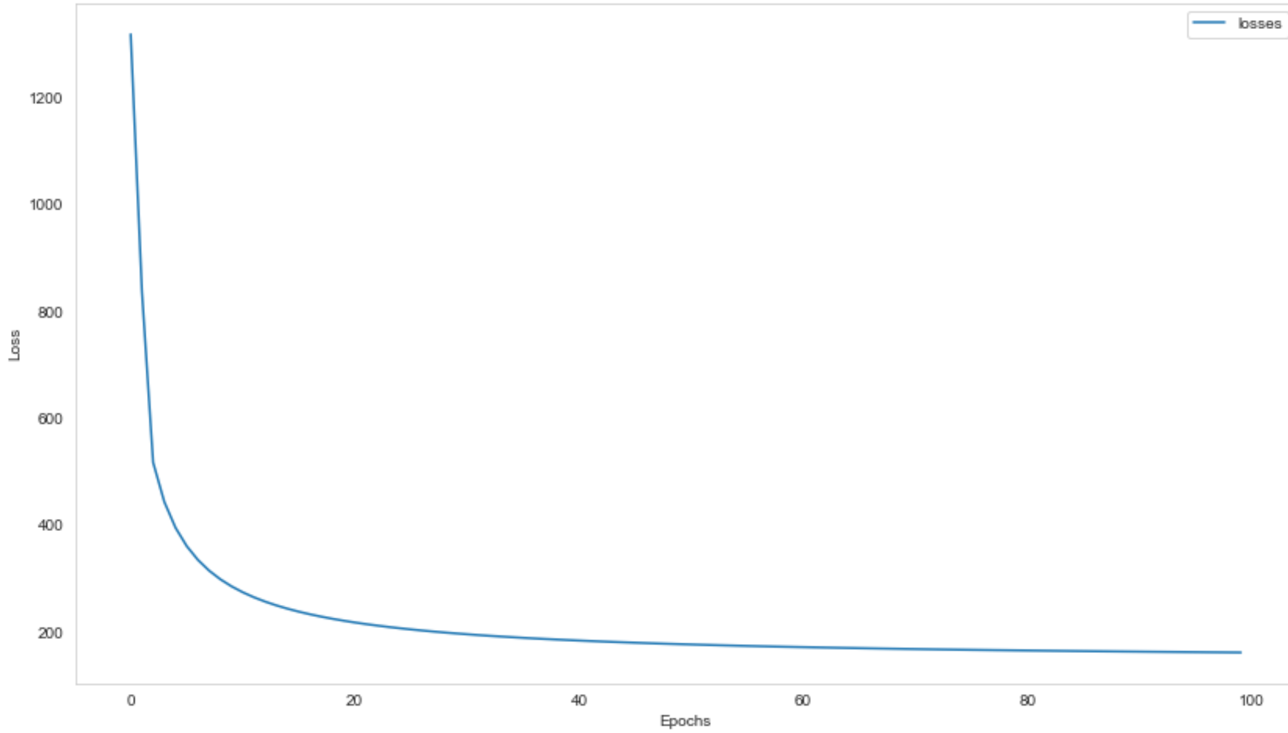
```
# Training a logistic regression model using the parameters specified
model_log_reg_alpha_0 = LogisticRegression(0, 100, 1e-3)
log_reg_alpha_0_losses = model_log_reg_alpha_0.train(voice_X_train, voice_y_t
validation_loss_alpha_0 = model_log_reg_alpha_0.calculate_loss(voice_X_val, vo
test_loss_alpha_0 = model_log_reg_alpha_0.calculate_loss(voice_X_test, voice_

print(f"Validation Loss: {validation_loss_alpha_0}")
print(f"Test Loss: {test_loss_alpha_0}")
```

```
Validation Loss: [[72.68464679]]
Test Loss: [[60.48815139]]
```

In [117…

```
# Training Loss over epochs plot - PART A
log_reg_alpha_0_losses_arr = [loss[0, 0] for loss in log_reg_alpha_0_losses]
plt.rcParams["figure.figsize"] = (14, 8)
plt.plot(list(range(100)), log_reg_alpha_0_losses_arr, label = "losses")
plt.xlabel("Epochs")
plt.ylabel("Loss")
plt.legend()
plt.grid(False)
plt.show()
```



In [118…

```
# For B part with some non zero alphas
```

In [119…

```
# Finding best hyperparameters
import random
random.seed(21)

counter = 1
random_search_count = 200
min_loss = np.inf
optimal_alpha = np.inf
optimal_eta = np.inf
optimal_t = np.inf

while counter <= random_search_count:
    curr_alpha = float(random.randrange(0, 100) / 100.0)
    curr_eta = float(random.randrange(0, 1000) / 1000000.0)
    curr_t = random.randint(1, 101)

    print(f"At current experiment {counter} with hyperparameter values as:")
    print(f"alpha = {curr_alpha}, eta = {curr_eta}, t = {curr_t}")

    model_log_reg = LogisticRegression(curr_alpha, curr_t, curr_eta)
    losses = model_log_reg.train(voice_X_train, voice_y_train)
    curr_val_loss = model_log_reg.calculate_loss(voice_X_val, voice_y_val)[0,

    print(f"Validation Loss is: {curr_val_loss}")

    if curr_val_loss < min_loss:
        min_loss = curr_val_loss
        optimal_alpha = curr_alpha
        optimal_eta = curr_eta
        optimal_t = curr_t

    counter += 1

print(f"The best hyperparameter values are: alpha = {optimal_alpha}, eta = {o
print(f"Validation Loss at this was: {min_loss}")
```

```
At current experiment 1 with hyperparameter values as:
alpha = 0.21, eta = 0.000428, t = 89
Validation Loss is: 78.11775376478961
At current experiment 2 with hyperparameter values as:
alpha = 0.53, eta = 0.00065, t = 37
Validation Loss is: 85.37733392545037
At current experiment 3 with hyperparameter values as:
alpha = 0.61, eta = 0.000863, t = 28
Validation Loss is: 86.2304794349043
At current experiment 4 with hyperparameter values as:
alpha = 0.6, eta = 0.000827, t = 66
Validation Loss is: 84.78495956894729
At current experiment 5 with hyperparameter values as:
alpha = 0.23, eta = 0.000517, t = 68
Validation Loss is: 78.78996216465812
At current experiment 6 with hyperparameter values as:
alpha = 0.3, eta = 0.000807, t = 1
```

```
Validation Loss is: 254.92720294463797
At current experiment 7 with hyperparameter values as:
alpha = 0.01, eta = 0.000379, t = 75
Validation Loss is: 77.0461192065779
At current experiment 8 with hyperparameter values as:
alpha = 0.54, eta = 7e-05, t = 19
Validation Loss is: 219.84703900307017
At current experiment 9 with hyperparameter values as:
alpha = 0.96, eta = 0.000237, t = 30
Validation Loss is: 114.94124640533694
At current experiment 10 with hyperparameter values as:
alpha = 0.88, eta = 4.3e-05, t = 56
Validation Loss is: 176.32314887003332
At current experiment 11 with hyperparameter values as:
alpha = 0.94, eta = 0.000789, t = 53
Validation Loss is: 89.9648215707089
At current experiment 12 with hyperparameter values as:
alpha = 0.78, eta = 0.000452, t = 5
Validation Loss is: 174.31782145389806
At current experiment 13 with hyperparameter values as:
alpha = 0.42, eta = 0.000553, t = 64
Validation Loss is: 81.76149900686639
At current experiment 14 with hyperparameter values as:
alpha = 0.88, eta = 0.000118, t = 84
Validation Loss is: 104.1185769948682
At current experiment 15 with hyperparameter values as:
alpha = 0.47, eta = 2.3e-05, t = 20
Validation Loss is: 300.7339835164702
At current experiment 16 with hyperparameter values as:
alpha = 0.94, eta = 0.000837, t = 12
Validation Loss is: 102.02482455383858
At current experiment 17 with hyperparameter values as:
alpha = 0.15, eta = 0.000828, t = 3
Validation Loss is: 158.80413262473476
At current experiment 18 with hyperparameter values as:
alpha = 0.58, eta = 0.000164, t = 71
Validation Loss is: 97.66676074016021
At current experiment 19 with hyperparameter values as:
alpha = 0.93, eta = 0.000351, t = 52
Validation Loss is: 92.69853996138981
At current experiment 20 with hyperparameter values as:
alpha = 0.62, eta = 0.000154, t = 24
Validation Loss is: 146.21377901837383
At current experiment 21 with hyperparameter values as:
alpha = 0.85, eta = 3.8e-05, t = 61
Validation Loss is: 179.06333250242935
At current experiment 22 with hyperparameter values as:
alpha = 0.3, eta = 7.6e-05, t = 100
Validation Loss is: 109.61802904439658
At current experiment 23 with hyperparameter values as:
alpha = 0.25, eta = 0.000213, t = 87
Validation Loss is: 85.44830978651424
At current experiment 24 with hyperparameter values as:
alpha = 0.84, eta = 0.000772, t = 98
Validation Loss is: 90.79922941071018
At current experiment 25 with hyperparameter values as:
alpha = 0.74, eta = 0.000747, t = 10
```

```
Validation Loss is: 109.49157038663948
At current experiment 26 with hyperparameter values as:
alpha = 0.15, eta = 0.000573, t = 61
Validation Loss is: 77.52444210442096
At current experiment 27 with hyperparameter values as:
alpha = 0.71, eta = 0.000846, t = 50
Validation Loss is: 86.24611311713821
At current experiment 28 with hyperparameter values as:
alpha = 0.25, eta = 0.000349, t = 19
Validation Loss is: 113.56854522094676
At current experiment 29 with hyperparameter values as:
alpha = 0.68, eta = 0.000419, t = 38
Validation Loss is: 91.9199967719427
At current experiment 30 with hyperparameter values as:
alpha = 0.46, eta = 0.000977, t = 58
Validation Loss is: 82.21524942902472
At current experiment 31 with hyperparameter values as:
alpha = 0.64, eta = 0.000395, t = 41
Validation Loss is: 91.30253323963174
At current experiment 32 with hyperparameter values as:
alpha = 0.62, eta = 0.000395, t = 35
Validation Loss is: 93.82579994095869
At current experiment 33 with hyperparameter values as:
alpha = 0.03, eta = 0.000854, t = 12
Validation Loss is: 94.80213971324532
At current experiment 34 with hyperparameter values as:
alpha = 0.2, eta = 0.000735, t = 92
Validation Loss is: 77.09969517044294
At current experiment 35 with hyperparameter values as:
alpha = 0.63, eta = 2e-06, t = 63
Validation Loss is: 380.4237182404852
At current experiment 36 with hyperparameter values as:
alpha = 0.08, eta = 0.000563, t = 77
Validation Loss is: 75.3139788899897
At current experiment 37 with hyperparameter values as:
alpha = 0.95, eta = 0.000534, t = 29
Validation Loss is: 94.74810902478796
At current experiment 38 with hyperparameter values as:
alpha = 0.73, eta = 0.000623, t = 41
Validation Loss is: 87.54072137665824
At current experiment 39 with hyperparameter values as:
alpha = 0.71, eta = 0.000637, t = 15
Validation Loss is: 102.42327096536214
At current experiment 40 with hyperparameter values as:
alpha = 0.23, eta = 0.000685, t = 22
Validation Loss is: 88.11736143643711
At current experiment 41 with hyperparameter values as:
alpha = 0.8, eta = 0.000356, t = 43
Validation Loss is: 93.74135611420353
At current experiment 42 with hyperparameter values as:
alpha = 0.44, eta = 0.00078, t = 77
Validation Loss is: 81.92112108334442
At current experiment 43 with hyperparameter values as:
alpha = 0.39, eta = 0.000251, t = 14
Validation Loss is: 147.6674500488674
At current experiment 44 with hyperparameter values as:
alpha = 0.21, eta = 6.7e-05, t = 23
```

```
Validation Loss is: 208.2792014624677
At current experiment 45 with hyperparameter values as:
alpha = 0.99, eta = 0.000439, t = 48
Validation Loss is: 91.98171491114552
At current experiment 46 with hyperparameter values as:
alpha = 0.2, eta = 0.000699, t = 5
Validation Loss is: 141.83151562649934
At current experiment 47 with hyperparameter values as:
alpha = 0.86, eta = 0.000532, t = 85
Validation Loss is: 88.83656456189682
At current experiment 48 with hyperparameter values as:
alpha = 0.3, eta = 0.000416, t = 21
Validation Loss is: 103.24959171567191
At current experiment 49 with hyperparameter values as:
alpha = 0.94, eta = 0.000604, t = 2
Validation Loss is: 213.2035576679704
At current experiment 50 with hyperparameter values as:
alpha = 0.44, eta = 0.000502, t = 65
Validation Loss is: 82.37975170175045
At current experiment 51 with hyperparameter values as:
alpha = 0.48, eta = 0.000682, t = 32
Validation Loss is: 85.59615420798077
At current experiment 52 with hyperparameter values as:
alpha = 0.31, eta = 0.000166, t = 72
Validation Loss is: 94.91658341559413
At current experiment 53 with hyperparameter values as:
alpha = 0.08, eta = 0.000449, t = 53
Validation Loss is: 79.86206862542781
At current experiment 54 with hyperparameter values as:
alpha = 0.99, eta = 0.000759, t = 62
Validation Loss is: 91.153673718406
At current experiment 55 with hyperparameter values as:
alpha = 0.07, eta = 0.000857, t = 37
Validation Loss is: 76.77851935001175
At current experiment 56 with hyperparameter values as:
alpha = 0.06, eta = 0.000873, t = 34
Validation Loss is: 77.12379889732146
At current experiment 57 with hyperparameter values as:
alpha = 0.84, eta = 0.000296, t = 5
Validation Loss is: 207.34802309228738
At current experiment 58 with hyperparameter values as:
alpha = 0.53, eta = 0.000254, t = 22
Validation Loss is: 122.97645405557185
At current experiment 59 with hyperparameter values as:
alpha = 0.5, eta = 7.1e-05, t = 44
Validation Loss is: 157.37719298779612
At current experiment 60 with hyperparameter values as:
alpha = 0.82, eta = 0.00092, t = 19
Validation Loss is: 91.45872637684894
At current experiment 61 with hyperparameter values as:
alpha = 0.19, eta = 0.000763, t = 94
Validation Loss is: 76.93929213544543
At current experiment 62 with hyperparameter values as:
alpha = 0.61, eta = 5e-05, t = 33
Validation Loss is: 203.7995118484433
At current experiment 63 with hyperparameter values as:
alpha = 0.17, eta = 0.000981, t = 84
```

```
Validation Loss is: 76.69088614114659
At current experiment 64 with hyperparameter values as:
alpha = 0.67, eta = 0.000933, t = 94
Validation Loss is: 88.16616679106939
At current experiment 65 with hyperparameter values as:
alpha = 0.61, eta = 2.8e-05, t = 28
Validation Loss is: 261.3572509096047
At current experiment 66 with hyperparameter values as:
alpha = 0.46, eta = 0.000814, t = 20
Validation Loss is: 88.85252948935269
At current experiment 67 with hyperparameter values as:
alpha = 0.47, eta = 0.000615, t = 99
Validation Loss is: 82.53465472887225
At current experiment 68 with hyperparameter values as:
alpha = 0.54, eta = 0.000496, t = 75
Validation Loss is: 83.536789714582
At current experiment 69 with hyperparameter values as:
alpha = 0.61, eta = 0.000315, t = 18
Validation Loss is: 122.25607753510111
At current experiment 70 with hyperparameter values as:
alpha = 0.68, eta = 0.000602, t = 36
Validation Loss is: 88.0943745193389
At current experiment 71 with hyperparameter values as:
alpha = 0.17, eta = 0.000786, t = 93
Validation Loss is: 76.51089565385334
At current experiment 72 with hyperparameter values as:
alpha = 0.01, eta = 0.000391, t = 88
Validation Loss is: 75.42963738098089
At current experiment 73 with hyperparameter values as:
alpha = 0.16, eta = 0.000976, t = 92
Validation Loss is: 76.61303837370993
At current experiment 74 with hyperparameter values as:
alpha = 0.03, eta = 0.00027, t = 76
Validation Loss is: 81.37203680490522
At current experiment 75 with hyperparameter values as:
alpha = 0.41, eta = 0.000659, t = 32
Validation Loss is: 85.13743027652039
At current experiment 76 with hyperparameter values as:
alpha = 0.74, eta = 0.000431, t = 33
Validation Loss is: 94.30895752439365
At current experiment 77 with hyperparameter values as:
alpha = 0.87, eta = 0.000415, t = 97
Validation Loss is: 88.74829694920344
At current experiment 78 with hyperparameter values as:
alpha = 0.17, eta = 0.000136, t = 19
Validation Loss is: 168.7903030378317
At current experiment 79 with hyperparameter values as:
alpha = 0.12, eta = 0.000406, t = 13
Validation Loss is: 122.97137812388547
At current experiment 80 with hyperparameter values as:
alpha = 0.23, eta = 2.5e-05, t = 83
Validation Loss is: 186.41932395774182
At current experiment 81 with hyperparameter values as:
alpha = 0.57, eta = 0.000805, t = 19
Validation Loss is: 90.91312678752814
At current experiment 82 with hyperparameter values as:
alpha = 0.09, eta = 0.000888, t = 16
```

```
Validation Loss is: 87.46474141685087
At current experiment 83 with hyperparameter values as:
alpha = 0.83, eta = 0.000288, t = 17
Validation Loss is: 130.29625411924357
At current experiment 84 with hyperparameter values as:
alpha = 0.38, eta = 0.000551, t = 42
Validation Loss is: 83.8942975215863
At current experiment 85 with hyperparameter values as:
alpha = 0.5, eta = 0.000505, t = 29
Validation Loss is: 91.49826041558327
At current experiment 86 with hyperparameter values as:
alpha = 0.79, eta = 0.00017, t = 24
Validation Loss is: 140.7934663857432
At current experiment 87 with hyperparameter values as:
alpha = 0.23, eta = 0.000878, t = 49
Validation Loss is: 77.984654439711
At current experiment 88 with hyperparameter values as:
alpha = 0.25, eta = 0.000727, t = 96
Validation Loss is: 78.22867125916018
At current experiment 89 with hyperparameter values as:
alpha = 0.7, eta = 0.000775, t = 6
Validation Loss is: 127.82949398107205
At current experiment 90 with hyperparameter values as:
alpha = 0.77, eta = 0.000727, t = 65
Validation Loss is: 87.45035478392438
At current experiment 91 with hyperparameter values as:
alpha = 0.95, eta = 0.000792, t = 8
Validation Loss is: 116.00058061048506
At current experiment 92 with hyperparameter values as:
alpha = 0.01, eta = 0.000396, t = 60
Validation Loss is: 79.01548964580601
At current experiment 93 with hyperparameter values as:
alpha = 0.08, eta = 0.000407, t = 98
Validation Loss is: 75.70686955236862
At current experiment 94 with hyperparameter values as:
alpha = 0.72, eta = 0.000179, t = 92
Validation Loss is: 92.07587804400025
At current experiment 95 with hyperparameter values as:
alpha = 0.29, eta = 0.000258, t = 90
Validation Loss is: 82.92937367825786
At current experiment 96 with hyperparameter values as:
alpha = 0.77, eta = 0.000321, t = 96
Validation Loss is: 87.39392942134587
At current experiment 97 with hyperparameter values as:
alpha = 0.05, eta = 0.00025, t = 50
Validation Loss is: 91.49444002758996
At current experiment 98 with hyperparameter values as:
alpha = 0.19, eta = 0.000316, t = 93
Validation Loss is: 79.38057956757638
At current experiment 99 with hyperparameter values as:
alpha = 0.37, eta = 0.000837, t = 65
Validation Loss is: 80.40909065809296
At current experiment 100 with hyperparameter values as:
alpha = 0.7, eta = 0.000501, t = 74
Validation Loss is: 86.05039615182523
At current experiment 101 with hyperparameter values as:
alpha = 0.12, eta = 0.000558, t = 77
```

```
Validation Loss is: 76.07013565639588
At current experiment 102 with hyperparameter values as:
alpha = 0.07, eta = 0.00052, t = 77
Validation Loss is: 75.49067900261777
At current experiment 103 with hyperparameter values as:
alpha = 0.94, eta = 0.000331, t = 67
Validation Loss is: 91.0973995474252
At current experiment 104 with hyperparameter values as:
alpha = 0.39, eta = 0.000298, t = 90
Validation Loss is: 82.87892993204807
At current experiment 105 with hyperparameter values as:
alpha = 0.79, eta = 0.000538, t = 98
Validation Loss is: 88.10730362575518
At current experiment 106 with hyperparameter values as:
alpha = 0.71, eta = 0.000194, t = 9
Validation Loss is: 197.0478747669047
At current experiment 107 with hyperparameter values as:
alpha = 0.78, eta = 0.000834, t = 42
Validation Loss is: 87.2905875888987
At current experiment 108 with hyperparameter values as:
alpha = 0.25, eta = 0.000363, t = 9
Validation Loss is: 150.5473754892188
At current experiment 109 with hyperparameter values as:
alpha = 0.4, eta = 0.000673, t = 54
Validation Loss is: 81.34116578570912
At current experiment 110 with hyperparameter values as:
alpha = 0.13, eta = 0.000888, t = 10
Validation Loss is: 99.46568735300333
At current experiment 111 with hyperparameter values as:
alpha = 0.83, eta = 7.5e-05, t = 81
Validation Loss is: 121.52281202076892
At current experiment 112 with hyperparameter values as:
alpha = 0.97, eta = 0.000458, t = 65
Validation Loss is: 90.21194013635598
At current experiment 113 with hyperparameter values as:
alpha = 0.79, eta = 0.000969, t = 3
Validation Loss is: 147.9021723253709
At current experiment 114 with hyperparameter values as:
alpha = 0.87, eta = 0.00066, t = 91
Validation Loss is: 90.11697226328553
At current experiment 115 with hyperparameter values as:
alpha = 0.56, eta = 0.000482, t = 67
Validation Loss is: 84.20665983764711
At current experiment 116 with hyperparameter values as:
alpha = 0.65, eta = 0.000398, t = 52
Validation Loss is: 88.31598181901151
At current experiment 117 with hyperparameter values as:
alpha = 0.09, eta = 0.00023, t = 4
Validation Loss is: 243.33808490431764
At current experiment 118 with hyperparameter values as:
alpha = 0.06, eta = 0.000677, t = 56
Validation Loss is: 75.55931072912766
At current experiment 119 with hyperparameter values as:
alpha = 0.05, eta = 0.000252, t = 36
Validation Loss is: 100.99509036169307
At current experiment 120 with hyperparameter values as:
alpha = 0.9, eta = 0.000603, t = 52
```

```
Validation Loss is: 89.1537140300676
At current experiment 121 with hyperparameter values as:
alpha = 0.92, eta = 0.000437, t = 59
Validation Loss is: 89.98062123575576
At current experiment 122 with hyperparameter values as:
alpha = 0.3, eta = 0.000408, t = 50
Validation Loss is: 84.46521942771608
At current experiment 123 with hyperparameter values as:
alpha = 0.41, eta = 0.000498, t = 30
Validation Loss is: 90.3105880191271
At current experiment 124 with hyperparameter values as:
alpha = 0.8, eta = 0.000936, t = 84
Validation Loss is: 90.25325454161347
At current experiment 125 with hyperparameter values as:
alpha = 0.1, eta = 3.8e-05, t = 62
Validation Loss is: 176.63873935750814
At current experiment 126 with hyperparameter values as:
alpha = 0.22, eta = 0.000503, t = 48
Validation Loss is: 81.47174796564605
At current experiment 127 with hyperparameter values as:
alpha = 0.91, eta = 0.000619, t = 28
Validation Loss is: 92.83171019307382
At current experiment 128 with hyperparameter values as:
alpha = 0.99, eta = 0.00046, t = 81
Validation Loss is: 90.48574785989692
At current experiment 129 with hyperparameter values as:
alpha = 0.8, eta = 0.000776, t = 80
Validation Loss is: 88.99385178411356
At current experiment 130 with hyperparameter values as:
alpha = 0.85, eta = 0.000748, t = 78
Validation Loss is: 89.6287954716896
At current experiment 131 with hyperparameter values as:
alpha = 0.89, eta = 0.000839, t = 88
Validation Loss is: 91.64675243272698
At current experiment 132 with hyperparameter values as:
alpha = 0.33, eta = 0.000251, t = 62
Validation Loss is: 89.17140897662185
At current experiment 133 with hyperparameter values as:
alpha = 0.63, eta = 0.000395, t = 40
Validation Loss is: 91.58975302192957
At current experiment 134 with hyperparameter values as:
alpha = 0.78, eta = 0.000391, t = 68
Validation Loss is: 88.05859312204808
At current experiment 135 with hyperparameter values as:
alpha = 0.91, eta = 0.00093, t = 1
Validation Loss is: 256.89327057746266
At current experiment 136 with hyperparameter values as:
alpha = 0.2, eta = 0.00032, t = 26
Validation Loss is: 104.67549777696756
At current experiment 137 with hyperparameter values as:
alpha = 0.39, eta = 0.000896, t = 51
Validation Loss is: 80.7554193124416
At current experiment 138 with hyperparameter values as:
alpha = 0.91, eta = 0.000148, t = 12
Validation Loss is: 196.86236393338868
At current experiment 139 with hyperparameter values as:
alpha = 0.7, eta = 0.000179, t = 11
```

```
Validation Loss is: 188.47614373386847
At current experiment 140 with hyperparameter values as:
alpha = 0.66, eta = 0.000309, t = 80
Validation Loss is: 86.97749376696744
At current experiment 141 with hyperparameter values as:
alpha = 0.48, eta = 0.00076, t = 96
Validation Loss is: 83.28456307213766
At current experiment 142 with hyperparameter values as:
alpha = 0.59, eta = 0.00011, t = 22
Validation Loss is: 174.53172199739564
At current experiment 143 with hyperparameter values as:
alpha = 0.9, eta = 0.000443, t = 27
Validation Loss is: 98.9479758029807
At current experiment 144 with hyperparameter values as:
alpha = 0.86, eta = 0.000343, t = 48
Validation Loss is: 93.26361845401657
At current experiment 145 with hyperparameter values as:
alpha = 0.82, eta = 0.000595, t = 88
Validation Loss is: 88.6211306535396
At current experiment 146 with hyperparameter values as:
alpha = 0.24, eta = 0.000267, t = 11
Validation Loss is: 158.51971299074953
At current experiment 147 with hyperparameter values as:
alpha = 0.03, eta = 0.000329, t = 66
Validation Loss is: 80.50302267528014
At current experiment 148 with hyperparameter values as:
alpha = 0.32, eta = 0.000967, t = 20
Validation Loss is: 84.75493251150077
At current experiment 149 with hyperparameter values as:
alpha = 0.87, eta = 0.000519, t = 26
Validation Loss is: 96.23045434730045
At current experiment 150 with hyperparameter values as:
alpha = 0.3, eta = 0.000612, t = 89
Validation Loss is: 79.04581805888293
At current experiment 151 with hyperparameter values as:
alpha = 0.35, eta = 0.000733, t = 49
Validation Loss is: 80.56824707237482
At current experiment 152 with hyperparameter values as:
alpha = 0.2, eta = 0.000963, t = 51
Validation Loss is: 77.16628178848529
At current experiment 153 with hyperparameter values as:
alpha = 0.59, eta = 0.000411, t = 56
Validation Loss is: 86.61447018018053
At current experiment 154 with hyperparameter values as:
alpha = 0.58, eta = 0.000631, t = 86
Validation Loss is: 84.38139018497354
At current experiment 155 with hyperparameter values as:
alpha = 0.35, eta = 0.000685, t = 23
Validation Loss is: 88.49615772378246
At current experiment 156 with hyperparameter values as:
alpha = 0.85, eta = 0.000393, t = 80
Validation Loss is: 88.46602042191229
At current experiment 157 with hyperparameter values as:
alpha = 0.34, eta = 0.000101, t = 25
Validation Loss is: 171.20789386382282
At current experiment 158 with hyperparameter values as:
alpha = 0.27, eta = 3.5e-05, t = 45
```

```
Validation Loss is: 207.2738915193139
At current experiment 159 with hyperparameter values as:
alpha = 0.8, eta = 0.000957, t = 91
Validation Loss is: 90.84826631781661
At current experiment 160 with hyperparameter values as:
alpha = 0.47, eta = 1.9e-05, t = 10
Validation Loss is: 358.41578720384433
At current experiment 161 with hyperparameter values as:
alpha = 0.64, eta = 0.000593, t = 6
Validation Loss is: 143.65260724948806
At current experiment 162 with hyperparameter values as:
alpha = 0.5, eta = 0.000622, t = 57
Validation Loss is: 82.9966495194571
At current experiment 163 with hyperparameter values as:
alpha = 0.89, eta = 0.00046, t = 31
Validation Loss is: 95.55424698873217
At current experiment 164 with hyperparameter values as:
alpha = 0.0, eta = 0.000964, t = 70
Validation Loss is: 72.6348247025868
At current experiment 165 with hyperparameter values as:
alpha = 0.18, eta = 9.6e-05, t = 40
Validation Loss is: 143.22025675638085
At current experiment 166 with hyperparameter values as:
alpha = 0.24, eta = 0.000465, t = 48
Validation Loss is: 82.60977832358537
At current experiment 167 with hyperparameter values as:
alpha = 0.0, eta = 0.000378, t = 22
Validation Loss is: 103.25106017536525
At current experiment 168 with hyperparameter values as:
alpha = 0.17, eta = 0.000584, t = 56
Validation Loss is: 78.23191632781048
At current experiment 169 with hyperparameter values as:
alpha = 0.67, eta = 0.00086, t = 7
Validation Loss is: 115.94288549855254
At current experiment 170 with hyperparameter values as:
alpha = 0.39, eta = 0.000192, t = 54
Validation Loss is: 99.28761816760505
At current experiment 171 with hyperparameter values as:
alpha = 0.51, eta = 0.000903, t = 71
Validation Loss is: 83.4975272814653
At current experiment 172 with hyperparameter values as:
alpha = 0.31, eta = 0.00023, t = 74
Validation Loss is: 87.42393627829148
At current experiment 173 with hyperparameter values as:
alpha = 0.74, eta = 0.000947, t = 60
Validation Loss is: 87.52865675882595
At current experiment 174 with hyperparameter values as:
alpha = 0.02, eta = 0.000674, t = 75
Validation Loss is: 73.6355062058941
At current experiment 175 with hyperparameter values as:
alpha = 0.09, eta = 8.2e-05, t = 56
Validation Loss is: 132.61394831898573
At current experiment 176 with hyperparameter values as:
alpha = 0.95, eta = 0.000777, t = 85
Validation Loss is: 92.123094170175
At current experiment 177 with hyperparameter values as:
alpha = 0.24, eta = 0.000155, t = 8
```

```
Validation Loss is: 223.07942275720836
At current experiment 178 with hyperparameter values as:
alpha = 0.63, eta = 0.000834, t = 68
Validation Loss is: 85.4540220035252
At current experiment 179 with hyperparameter values as:
alpha = 0.77, eta = 0.000548, t = 80
Validation Loss is: 87.2765989741556
At current experiment 180 with hyperparameter values as:
alpha = 0.62, eta = 0.000482, t = 87
Validation Loss is: 84.74279847584299
At current experiment 181 with hyperparameter values as:
alpha = 0.86, eta = 0.000346, t = 20
Validation Loss is: 114.9441170385836
At current experiment 182 with hyperparameter values as:
alpha = 0.34, eta = 0.000157, t = 46
Validation Loss is: 111.48761561531848
At current experiment 183 with hyperparameter values as:
alpha = 0.36, eta = 0.000305, t = 82
Validation Loss is: 83.05761691916182
At current experiment 184 with hyperparameter values as:
alpha = 0.8, eta = 9e-06, t = 10
Validation Loss is: 394.27442631080464
At current experiment 185 with hyperparameter values as:
alpha = 0.85, eta = 6.9e-05, t = 24
Validation Loss is: 203.44546169468978
At current experiment 186 with hyperparameter values as:
alpha = 0.65, eta = 0.0001, t = 87
Validation Loss is: 106.66649901442
At current experiment 187 with hyperparameter values as:
alpha = 0.24, eta = 0.000368, t = 35
Validation Loss is: 92.10202062826308
At current experiment 188 with hyperparameter values as:
alpha = 0.45, eta = 6e-05, t = 21
Validation Loss is: 224.1450183830839
At current experiment 189 with hyperparameter values as:
alpha = 0.04, eta = 0.000599, t = 64
Validation Loss is: 75.17879619597394
At current experiment 190 with hyperparameter values as:
alpha = 0.35, eta = 0.000697, t = 84
Validation Loss is: 80.08238887131559
At current experiment 191 with hyperparameter values as:
alpha = 0.51, eta = 0.000161, t = 5
Validation Loss is: 255.3357621958032
At current experiment 192 with hyperparameter values as:
alpha = 0.13, eta = 0.000238, t = 46
Validation Loss is: 95.72475599629557
At current experiment 193 with hyperparameter values as:
alpha = 0.68, eta = 0.000256, t = 53
Validation Loss is: 94.93604428881581
At current experiment 194 with hyperparameter values as:
alpha = 0.25, eta = 0.000707, t = 25
Validation Loss is: 85.59056879423275
At current experiment 195 with hyperparameter values as:
alpha = 0.93, eta = 0.000976, t = 68
Validation Loss is: 91.82028630217249
At current experiment 196 with hyperparameter values as:
alpha = 0.54, eta = 8e-05, t = 42
```

```
Validation Loss is: 152.6586210261989
At current experiment 197 with hyperparameter values as:
alpha = 0.28, eta = 0.000218, t = 65
Validation Loss is: 90.6151765073421
At current experiment 198 with hyperparameter values as:
alpha = 0.05, eta = 0.000745, t = 46
Validation Loss is: 75.97587298864973
At current experiment 199 with hyperparameter values as:
alpha = 0.77, eta = 0.000545, t = 82
Validation Loss is: 87.31158163167213
At current experiment 200 with hyperparameter values as:
alpha = 0.8, eta = 0.000658, t = 23
Validation Loss is: 93.50423808743078
The best hyperparameter values are: alpha = 0.0, eta = 0.000964, t = 70
Validation Loss at this was: 72.6348247025868
```

In [120…

```
# Train the model using the best found hyperparameters
model_log_reg = LogisticRegression(optimal_alpha, optimal_t, optimal_eta)
log_reg_train_losses = model_log_reg.train(voice_X_train, voice_y_train)
```

In [121…

```
# Accuracy Comparisons for PART A and PART B
y_test_preds_alpha_0 = model_log_reg_alpha_0.predict(voice_X_test)
y_test_preds_optimal_alpha = model_log_reg.predict(voice_X_test)

print(f"Accuracy Score of Logistic Regression at Alpha 0 is: {accuracy_score(
print(f"Accuracy Score of Logistic Regression at Alpha {optimal_alpha} is: \
    {accuracy_score(y_test_preds_optimal_alpha, voice_y_test)}")
```

```
Accuracy Score of Logistic Regression at Alpha 0 is: 0.9747634069400631
Accuracy Score of Logistic Regression at Alpha 0.0 is:     0.973186119873817
```

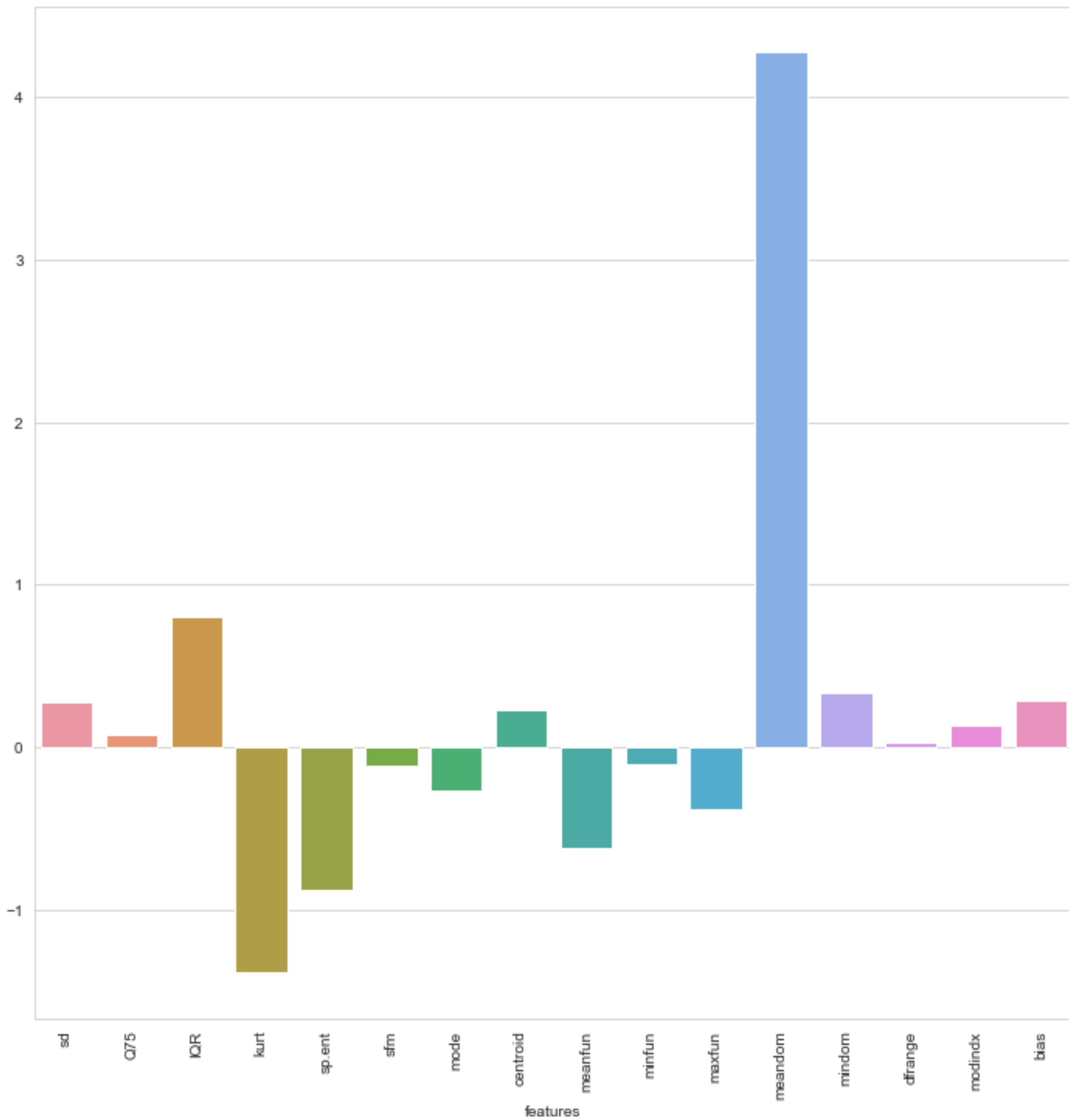
After hyperparameter tuning, there is no improvement. Even so, it becomes a little less than before.

2.7 Feature Importance

Interpret your trained model using a bar chart of the model weights. Make sure to label the bars (x-axis) and don't forget the bias term!

In [122…

```
feature_columns = list(voice_X.columns)
feature_columns.append("bias")
plt.rcParams["figure.figsize"] = (12,12)
plt.xlabel("features")
ax = sns.barplot(x = feature_columns, y = np.array(model_log_reg.w)[:, 0])
ax.tick_params(axis = 'x', rotation = 90)
```



The most important features as we can see above are:

1. meandom
2. kurt
3. sp.ent
4. IQR
5. meanfun

Part 3: Support Vector Machines - with the same Dataset

3.1 Dual SVM

A) Train a dual SVM (with default parameters) for both kernel=“linear” and kernel=“rbf”) on the Voice Recognition training data.

B) Make predictions and report the accuracy on the training, validation, and test sets. Which kernel gave better accuracy on test dataset and why do you think that was better?

C) Please report the support vectors in both the cases and what do you observe? Explain

In [123…

```
# PART A - Linear SVM
model_dual_lin_svm = SVC(kernel = "linear")
model_dual_lin_svm.fit(voice_X_train, voice_y_train)
```

Out[123…

```
SVC(kernel='linear')
```

In [124…

```
# PART A - RBF SVM
model_dual_rbf_svm = SVC(kernel = "rbf")
model_dual_rbf_svm.fit(voice_X_train, voice_y_train)
```

Out[124…

```
SVC()
```

In [125…

```
# PART B - Linear SVM
print("For Linear SVM:")

lin_svm_y_train_preds = model_dual_lin_svm.predict(voice_X_train)
print(f"Accuacy Score for Training Data: {accuracy_score(lin_svm_y_train_pred

lin_svm_y_val_preds = model_dual_lin_svm.predict(voice_X_val)
print(f"Accuacy Score for Validation Data: {accuracy_score(lin_svm_y_val_pred

lin_svm_y_test_preds = model_dual_lin_svm.predict(voice_X_test)
print(f"Accuacy Score for Testing Data: {accuracy_score(lin_svm_y_test_preds,
```

```
For Linear SVM:
Accuacy Score for Training Data: 0.9763157894736842
Accuacy Score for Validation Data: 0.9716088328075709
Accuacy Score for Testing Data: 0.9794952681388013
```

In [126…

```
# PART B - RBF SVM
print("For RBF SVM:")

rbf_svm_y_train_preds = model_dual_rbf_svm.predict(voice_X_train)
print(f"Accuacy Score for Training Data: {accuracy_score(rbf_svm_y_train_pred

rbf_svm_y_val_preds = model_dual_rbf_svm.predict(voice_X_val)
print(f"Accuacy Score for Validation Data: {accuracy_score(rbf_svm_y_val_pred

rbf_svm_y_test_preds = model_dual_rbf_svm.predict(voice_X_test)
print(f"Accuacy Score for Testing Data: {accuracy_score(rbf_svm_y_test_preds,
```

```
For RBF SVM:
Accuacy Score for Training Data: 0.9842105263157894
Accuacy Score for Validation Data: 0.9794952681388013
Accuacy Score for Testing Data: 0.9810725552050473
```

RBF Kernel has better accuracy on the test data than the linear kernel because RBF Kernel converts the data into higher dimensional plane and creates more hyperplanes to separate the data points. Due to this increase in complexity of the hyperplanes and the increase in dimensions and support vectors, more complex data can be fit well using the RBF Kernel.

In [129…

```
# PART C - Linear SVM
decision_function = model_dual_lin_svm.decision_function(voice_X_train)
support_vector_indices = np.where(np.abs(decision_function) <= 1 + 1e-15)[0]
support_vectors = voice_X_train[support_vector_indices]
print(f"Number of support vectors with Linear SVM are: {support_vectors.shape
support_vectors
```

```
Number of support vectors with Linear SVM are: 127
```

Out[129…

```
matrix([[ 6.79382435e-01, -1.58515171e-03,  4.55344662e-01, ...,
          6.45161290e-03,  1.86814345e-03,  1.00000000e+00],
        [-1.45019774e+00, -2.20057687e-01,  1.16040261e+00, ...,
          6.45161290e-03,  4.99181215e-03,  1.00000000e+00],
        [-1.65538809e+00, -8.37171666e-01,  1.29881208e+00, ...,
          6.45161290e-03,  5.37501450e-02,  1.00000000e+00],
        ...,
        [-1.74423088e+00, -8.13622029e-01,  1.89871824e+00, ...,
          6.45161290e-03,  2.79160532e-02,  1.00000000e+00],
        [ 9.81778061e-01,  2.22384135e+00, -5.30444804e-01, ...,
          4.53763441e-01,  3.15787672e-03,  1.00000000e+00],
        [ 4.44892847e-01,  8.86167694e-01, -4.44589313e-02, ...,
          4.08602151e-02,  2.02538800e-03,  1.00000000e+00]])
```

In [130…

```
# PART C - RBF SVM
print(f"Number of support vectors with RBF SVM are: {model_dual_rbf_svm.suppo
model_dual_rbf_svm.support_vectors_
```

```
Number of support vectors with RBF SVM are: 243
```

Out[130…

```
array([[ 1.46044014,  0.12571299, -1.55823414, ...,  0.00645161,
         0.01154799,  1.        ],
       [-0.14291593, -0.24794541,  0.25367505, ...,  0.04086022,
         0.00426603,  1.        ],
       [ 0.93152034, -0.5262627 , -1.83598916, ...,  0.47096774,
         0.0051018 ,  1.        ],
       ...,
       [ 0.98177806,  2.22384135, -0.5304448 , ...,  0.45376344,
         0.00315788,  1.        ],
       [-0.22937925, -0.07754958,  1.05141168, ...,  0.34408602,
         0.00587675,  1.        ],
       [-0.51958109, -1.2512546 ,  1.60053777, ...,  0.10967742,
         0.20100826,  1.        ]])
```

Observation: The number of support vectors increases for the RBF kernel and is approximately doubled and thus, would be suited well for data with higher complexity. As the RBF kernel increases the dimensionality of the data and produces non-linear hyperplanes and support vectors, potentially it can do so in more ways when compared to the Linear SVM. Due to this increase in complexity, it tries to fit the data in a better manner which in turn means having more support vectors to do the same.

3.2 Using Kernel “rbf”, tune the hyperparameter “C” using the Grid Search & k-fold cross validation. You may take k=5 and assume values in grid between 1 to 100 with interval range of your choice.

In [131…

```
from sklearn.model_selection import GridSearchCV, cross_val_score
from sklearn.pipeline import make_pipeline
```

In [132…

```
# Using Grid Search and K-fold Cross Validation
voice_X_dev = np.vstack((voice_X_train, voice_X_val))
voice_y_dev = np.vstack((voice_y_train, voice_y_val))

pipe = make_pipeline(GridSearchCV(SVC(kernel = "rbf"), \
                                  param_grid = {"C": list(range(1, 101))}, \
                                  cv=5,
                                  return_train_score = True))
pipe.fit(voice_X_dev, voice_y_dev)

grid_search_results = pipe.named_steps["gridsearchcv"]
grid_search_result_C = grid_search_results.best_params_['C']
print(f"Best value of hyperparameter C is: {grid_search_result_C}")
print(f"Best Accuracy Score: {grid_search_results.best_score_}")
print(f"Accuracy Score for Testing Data: {pipe.score(voice_X_test, voice_y_te
```

```
Best value of hyperparameter C is: 10
Best Accuracy Score: 0.9857933593719548
Accuracy Score for Testing Data: 0.9810725552050473
```