HW 2 Chandan Suri – CS4090

1. for each of the three vehicles, 3 possible outcomes are there (turn right (R), turn left (L), turn or go straight (S)).

If all of them are moving simultaneously, then total number of possible outcomes will be =>

$\boxed{3}$ directions $\boxed{3}$ directions $\boxed{3}$ directions (3 vehicles)

$\Rightarrow 3 \times 3 \times 3 = 3^3 = \underline{\underline{27}}$

a) When all 3 vehicles go in the same direction, So, we can first choose a direction from 3 possible directions in 3 ways. Then, other 2 vehicles have only 1 possible direction.

So, $\boxed{3}$ $\boxed{1}$ $\boxed{1}$

$\Rightarrow 3 \times 1 \times 1 = 3$ outcomes.

Outcomes (A) => $\{(R,R,R), (L,L,L), (S,S,S)\}$

Ans.

b) Event B => When all 3 vehicles take different directions, thus, no 2 (or 3) vehicles can have the same direction.

So, $\boxed{3}$ $\boxed{2}$ $\boxed{1}$

$\Rightarrow 3 \times 2 \times 1 = 6$ outcomes.

So, B => $\{(R,S,L), (R,L,S), (L,S,R), (L,R,S), (S,R,L), (S,L,R)\}$

Ans.

(c) Event C => When exactly 2 of the 3 vehicles turn right.

Firstly, we choose 2 out of the 3 vehicles which will turn right. The ways to choose will be =>

$$^{3}C_2 \Rightarrow \frac{3!}{2!1!} = \underline{3 \text{ ways}}$$

Then, we can choose the direction for the third vehicle left from 2 other possible directions (L and S).

So,

$$3 \times 2 \Rightarrow \underline{6 \text{ total ways}}.$$

Thus,

$$C = \{(R,R,L), (R,R,S), (R,L,R), (R,S,R), (L,R,R), (S,R,R)\}$$

Ans.

(d) Event D => When exactly two vehicles go in the same direction.

Firstly, we choose 2 out of the 3 vehicles to go in the same direction. The ways to choose will be =>

$$^{3}C_2 \Rightarrow \frac{3!}{2!1!} = \underline{3 \text{ ways}}$$

Then, for each of those 2 vehicles, we can choose the same direction in 3 ways.

Also, for the third vehicle, we can choose from 2 other possible directions (other then the direction of the 2 vehicles).

So,

$$3 \times 3 \times 2 \Rightarrow \underline{18 \text{ ways}}$$

Thus,

$$D = \{(R,R,L), (R,R,S), (R,L,R), (R,S,R), (L,R,R), (S,R,R), (L,L,R), (L,L,S), (L,R,L), (L,S,L), (R,L,L), (S,L,L), (S,S,L), (S,L,S), (S,R,S), (S,S,R), (R,S,S), (L,S,S)\}$$ — Ans.

(e.) All possible outcomes when 3 vehicles can turn in a direction $\Rightarrow$

[Events when none of the vehicles have the same direction]
+ [Event when exactly 2 of the vehicles have the same direction] +
[Event when all 3 vehicles move in the same direction]

$\Rightarrow$ Event B + Event D + Event A

$\Rightarrow$ $A \cup B \cup D$

Thus,

$D' \Rightarrow$ All possible outcomes − Event D outcomes.

$\Rightarrow$ $\underline{\underline{A \cup B}}$

So,

$D' = \{ (R,S,L), (R,L,S), (L,S,R), (L,R,S), (S,R,L), (S,L,R), (R,R,R), (L,L,L), (S,S,S) \}$

Ans.

$C \cup D \Rightarrow$ All outcomes from Event C + All outcomes from Event D.

As outcomes in event C is a subpart of outcomes in event D.

This is equivalent to $\Rightarrow$

$\underline{C \cup D = D}$

$C \cup D = \{ (R,R,L), (R,R,S), (R,L,R), (R,S,R), (L,R,R), (S,R,R), (L,L,R), (L,L,S), (L,R,L), (L,S,L), (R,L,L), (S,L,L), (S,S,L), (S,L,S), (S,R,S), (S,S,R), (R,S,S), (L,S,S) \}$

Ans

$C \cap D \Rightarrow$ Outcomes in event C $\cap$ outcomes in event D.

= Common outcomes in events C and D.

Thus,

$C \cap D = C$ (as C is a perfect subset of D).

So,

$C \cap D = \{(R,R,L), (R,R,S), (R,L,R), (R,S,R), (L,R,R), (S,R,R)\}$

Ans.

(2.) first printing copies ⇒ 1, 2

Second printing copies ⇒ 3, 4, 5

Cases of selecting printing copies of the text ⇒

Case (1) :- Event when 2nd printing copy is selected at first instance itself while choosing ⇒

$\boxed{3}$ → 3 2nd printing copies to choose from.

⇒ 3 ways.

Case (2) :- Event when 2nd printing copy isn't selected at first step but is selected at the second step.

$\boxed{2} \times \boxed{3}$

2 1st printing copies to select from.

3, 2nd printing copies to select from.

⇒ 2×3 ⇒ 6 ways

Case (3) :- Event when 2nd printing copy is selected at the third step.

(As there can be no repetitions, this is the last case).

$\boxed{2} \times \boxed{1} \times \boxed{3}$

2 1st printing copies to select from

1 1st printing copy to select from when 1 was already selected

3 2nd printing copies to select from.

$\Rightarrow \quad 2 \times 1 \times 3 \Rightarrow \underline{6 \text{ ways}}$

(a) $S \Rightarrow$ All possible outcomes

= Outcomes from case 1 U Outcomes from case 2 U Outcomes from case 3.

$S \Rightarrow$

Outcomes from Case 1 = { (3), (4), (5) }

U

Outcomes from case 2 = { (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5) }

U

Outcomes from case 3 = { (1, 2, 3), (2, 1, 3), (1, 2, 4), (2, 1, 4), (1, 2, 5), (2, 1, 5) }

Thus,

S = { (3), (4), (5), (1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5), (1, 2, 3), (2, 1, 3), (1, 2, 4), (2, 1, 4), (1, 2, 5), (2, 1, 5) }

— Ans.

(b) Event A $\Rightarrow$ Exactly one book must be examined.

This would happen when the 1st selection itself was one of the three 2nd printing copies.

So, 3 ways to do this. (Case 1).

A = { (3), (4), (5) }

— Ans.

(c.) Event B ⇒ Book 5 is the one selected.

⇒ Book 5 is selected at the first step ∪
Book 5 is selected at the second step ∪
Book 5 is selected at the third step.

$\Rightarrow \{(5)\} \cup \{(1,5),(2,5)\} \cup \{(1,2,5),(2,1,5)\}$

$B = \{(5),(1,5),(2,5),(1,2,5),(2,1,5)\}$

Ans.

(d.) Out of S ⇒

C ⇒ Event that book 1 is not examined.

So,
C = S - Event that book 1 is examined.

$C = S - \{(1,3),(1,4),(1,5),(1,2,3),(2,1,3),(1,2,4),(2,1,4),(1,2,5),(2,1,5)\}$

$C = \{(3),(4),(5),(2,3),(2,4),(2,5)\}$

Ans

3.

a)

Event A = { Chevrolet, Buick }

Event B = { ford, Lincoln }

Event C = { Toyota }

Let's create 3 sets of events that are mutually exclusive.

1st :-

A = { Chevrolet, Buick }

C = { Toyota }

These are mutually exclusive as no outcome is common to events A and C.

[$\because A \cap C = \phi$]

2nd :-

B = { ford, Lincoln }

C = { Toyota }

These are mutually exclusive as no outcome is common to events B and C.

[$\because B \cap C = \phi$]

3rd :- Let's combine (union) events A and B.

A ∪ B = { Chevrolet, Buick, ford, Lincoln }

C = { Toyota }

As we can see above, no outcome is common to the events A ∪ B and C. So, these two are also mutually exclusive.

[$\because (A \cup B) \cap C = \phi$]

Ans.

(b) If no outcome is common to all three events A, B and C, it does not necessarily mean that these events are mutually exclusive.

For example,

Let 3 events be defined as=>

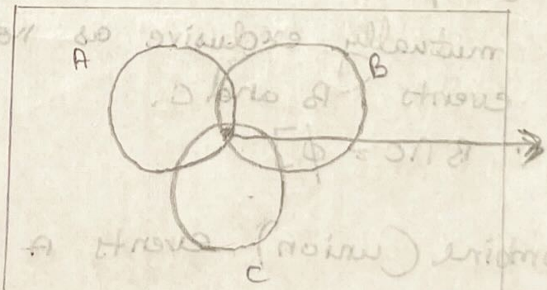
3 different automobile dealers who deal with particular Companies.

A = { Chevrolet, Buick, Ford }

B = { Ford, Lincoln }

C = { Toyota }

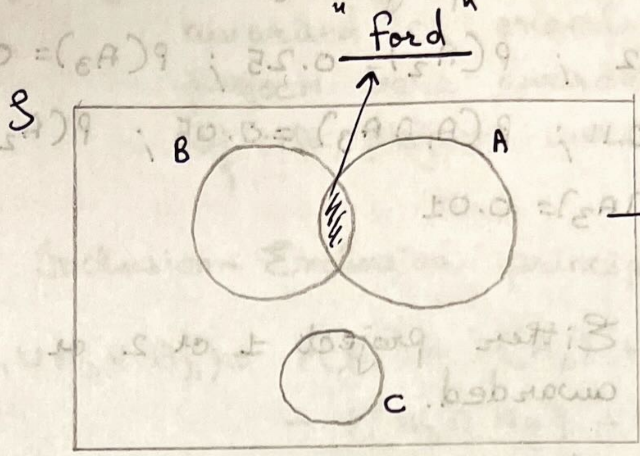
As we can see above, nothing is common to all the three events simultaneously, thus, the area shown below is not possible.



This area is not valid for our example.

But, for our example above, A and B events have something in common although all the three events have nothing in common. So, all three events are not mutually exclusive even though no outcome is common to all the three events.

The venn diagram showing the same below:-



As we can see here, no area is common to all of them but because A & B have an outcome common, these are not mutually exclusive.

4.

$A_i = \{\text{awarded project } i\}$

$P(A_1) = 0.22$; $P(A_2) = 0.25$; $P(A_3) = 0.28$;

$P(A_1 \cap A_2) = 0.11$; $P(A_1 \cap A_3) = 0.05$; $P(A_2 \cap A_3) = 0.07$;

$P(A_1 \cap A_2 \cap A_3) = 0.01$

a) $A_1 \cup A_2 \Rightarrow$ Either project 1 or 2 or both were awarded.

Using the inclusion-exclusion principle,

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$$

$$= 0.22 + 0.25 - 0.11$$

$$= 0.47 - 0.11 = \underline{0.36}$$

Ans.

b) $A_1' \cap A_2' \Rightarrow$ Project 1 was not awarded and Project 2 was not awarded. (Neither of the projects 1 and 2 was awarded).

$= A_1' \cap A_2'$

Using De-Morgan's law,

$= (A_1 \cup A_2)'$

As $P(A_1 \cup A_2) = 0.36$

$[\because P(x) + P(x') = 1]$

so, $$P((A_1 \cup A_2)') = 1 - P(A_1 \cup A_2)$$

$$= 1 - 0.36$$

$$= \underline{0.64}$$

Ans.

(c.) $A_1 \cup A_2 \cup A_3 \Rightarrow$ Either of the projects 1 or 2 or 3 was awarded OR exactly two of the three projects were awarded OR all three of the projects were awarded.

Using Inclusion-Exclusion principle,

$$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_2 \cap A_3) - P(A_1 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$$

$$= 0.22 + 0.25 + 0.28 - 0.11 - 0.07 - 0.05 + 0.01$$

$$= \underline{\underline{0.53}}$$

Ans.

(d.) $A_1' \cap A_2' \cap A_3' \Rightarrow$ Project 1 was not awarded and Project 2 was not awarded and Project 3 was not awarded.
[None of the project 1,2 or 3 was awarded].

$$\boxed{\because P(x) + P(x') = 1}$$

So,

$$P(A_1 \cup A_2 \cup A_3) + P((A_1 \cup A_2 \cup A_3)') = 1$$

Using de-morgan's law,

$$(A_1 \cup A_2 \cup A_3)' = A_1' \cap A_2' \cap A_3'$$

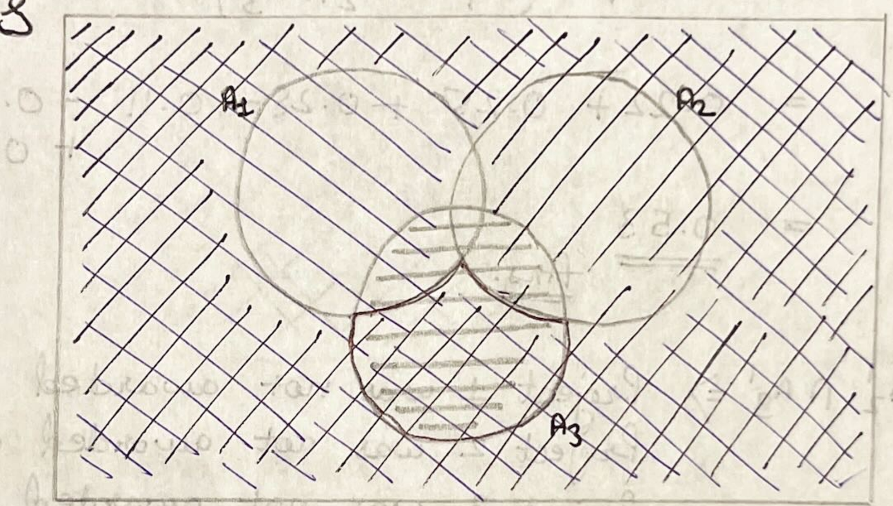
So, $P(A_1' \cap A_2' \cap A_3') = 1 - P(A_1 \cup A_2 \cup A_3) = 1 - 0.53$
$= \underline{0.47}$ Ans

(e.) $A_1' \cap A_2' \cap A_3 \Rightarrow$ Project 1 was not awarded and Project 2 was not awarded but Project 3 was awarded.

[Neither Project 1 nor 2 was awarded but project 3 is awarded].

Using Venn diagram, let's show the overlapping area under consideration $\Rightarrow$

$A_1' \cap A_2' \cap A_3 = (A_1 \cup A_2)' \cap A_3$ [can be re-written as this!]



$A_1' \rightarrow$ shown in black

$A_2' \rightarrow$ shown in blue

$A_3 \rightarrow$ shown in grey.

The marked area is the intersection!

The intersection of all the areas is shown in the diagram above $\Rightarrow$

from the diagram above, we can write $\Rightarrow$

$$P(A_1' \cap A_2' \cap A_3) = P(A_3) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$$

$$= 0.28 - 0.05 - 0.07 + 0.01$$

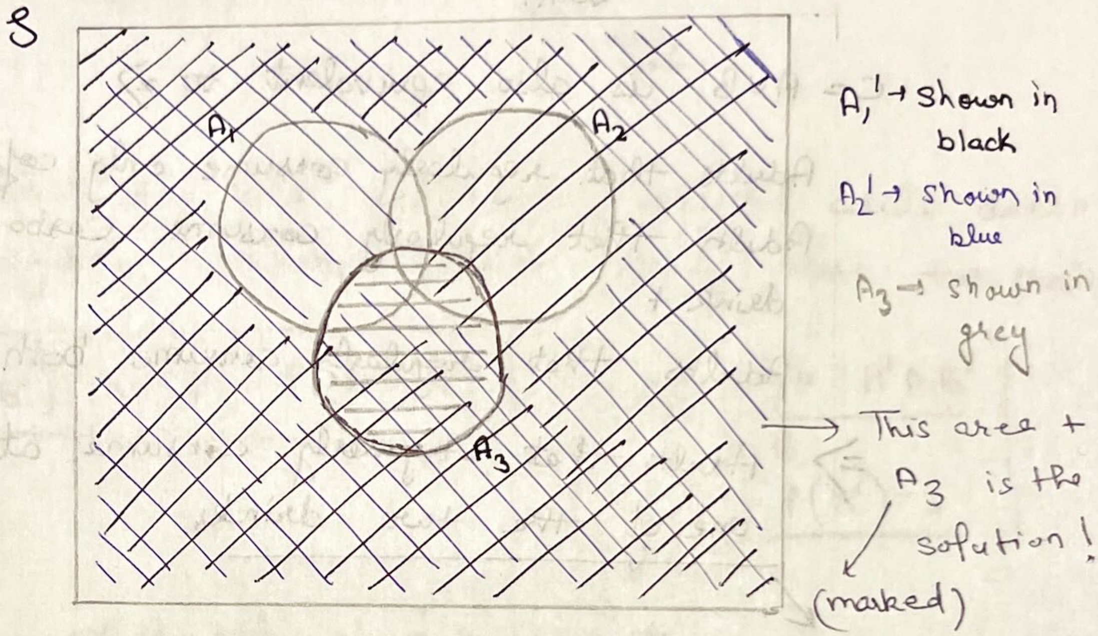
$$= \underline{0.17}$$ Ans.

(f.) $(A_1' \cap A_2') \cup A_3 \Rightarrow$ [Project 1 was not awarded and Project 2 was not awarded] or Project 3 was awarded or both.

[Neither Project 1 nor Project 2 was awarded or Project 3 was awarded or both.]

Using Venn diagram, let's show the overlapping area under consideration ⇒

$$(A_1' \cap A_2') \cup A_3 = (A_1 \cup A_2)' \cup A_3$$



The areas above show that ⇒ (from the diagram)

$$P(A_1' \cap A_2' \cup A_3) = P(A_1' \cap A_2' \cap A_3') + P(A_3)$$

from (d),

$$P(A_1' \cap A_2' \cap A_3') = 0.47$$

So,

$$P(A_1' \cap A_2' \cup A_3) = 0.47 + 0.28$$

$$= \underline{\underline{0.75}}$$

Ans.

(5.) $A \Rightarrow$ Event when adults regularly consume coffee.

$B \Rightarrow$ Event when adults regularly consume carbonated soda.

$C \Rightarrow A \cup B \Rightarrow$ Event when adults regularly consume either coffee or carbonated soda or both.

$C = A \cup B$ is also equivalent to $\Rightarrow$

Adults that regularly consume only coffee ~~and~~ +
Adult that regularly consume carbonated drink +
Adults that regularly consume both

$\Rightarrow$ Adults that regularly consume atleast one of the two drinks.

This is given!

So,

$P(A \cup B) = 0.7$

$P(A) = 0.55$

$P(B) = 0.45$

(a). Probability that a randomly selected adult regularly consumes both coffee and soda $\Rightarrow$

$P(A \cap B) = ?$

Using Inclusion-Exclusion Principle,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

~~P(A ∪ B) =~~

Putting in the values,

$$0.7 = 0.55 + 0.45 - P(A \cap B)$$

$$P(A \cap B) = 1 - 0.7 = \underline{\underline{0.3}}$$

Ans.

(b.) Probability that a randomly selected adult doesn't regularly consume atleast one of these two products ⇒

$$\boxed{P(A' \cap B') = 1 - P(A \cup B)} \quad \left\{ \because (A \cup B)' = A' \cap B' \text{ and } P(x) + P(x') = 1 \right\}$$

⇒

$$\underline{P(A' \cap B') = 1 - 0.7 = 0.3}$$

Ans.

(6.) (a) A and $B \cap A'$ are disjoint.

↓

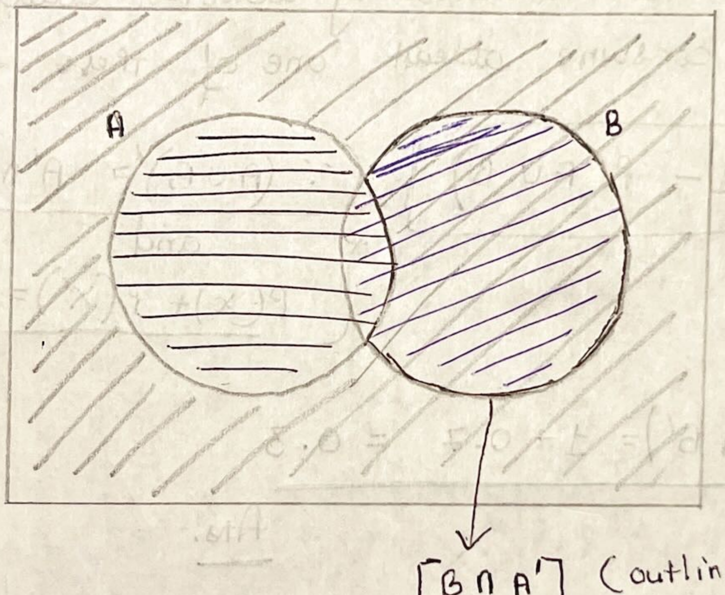
Verify

This will be true when, (if)

$$A \cap (B \cap A') = \phi$$

(there is no intersection of areas of A and $B \cap A'$).

&



A → shown in black.

B → shown in blue

A' → shown in grey

As we can see from above,
there is no intersection of areas for A and $B \cap A'$,

thus,

A and $B \cap A'$ have nothing in common.

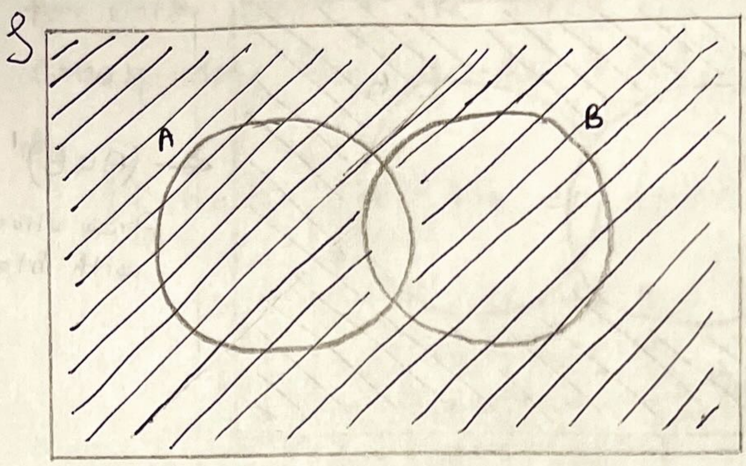
Thus,

A and $B \cap A'$ are disjoint.

Hence, Proved

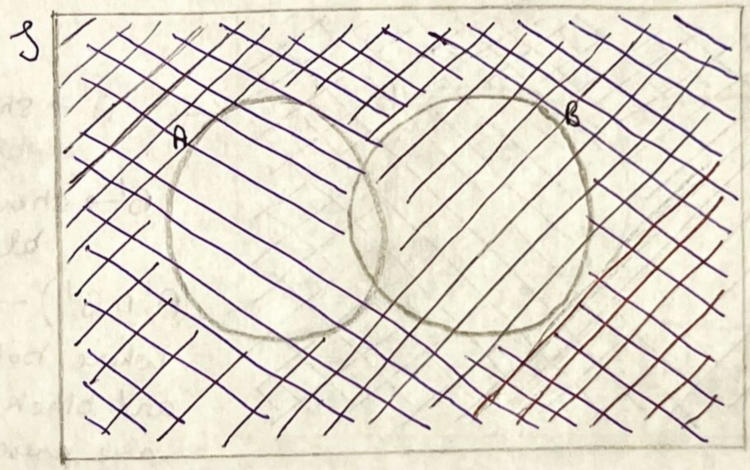
Ans.

(b) $(A \cap B)' = A' \cup B'$



$\leftarrow (A \cap B)'$

Area shown with black



$\leftarrow$ $A' \rightarrow$ shown in black.

$B' \rightarrow$ shown in blue

$(A' \cup B')$

$\hookrightarrow$ Union of both the areas is shown.

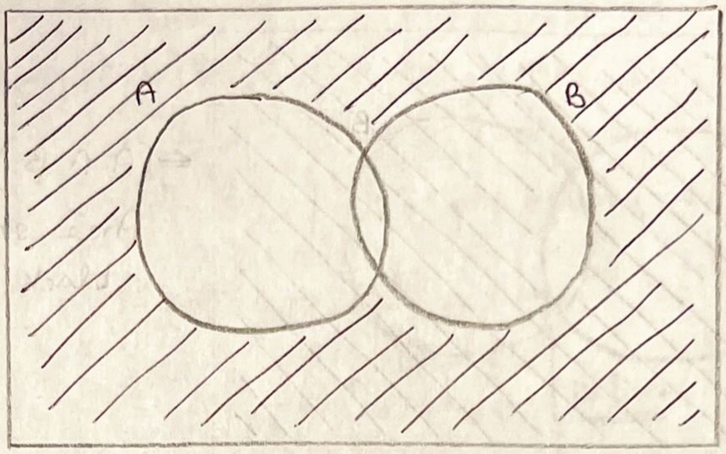
As we can see above, the area for $(A \cap B)'$ is basically everything else except the intersection of A and B. Also, the area for $(A' \cup B')$ is basically the union of A' (everything but A) and B' (everything but B). When we take the union of these, we get the shaded area to be $S - (A \cap B)$ which is the same as that of $(A \cap B)'$.

As the final shaded regions are the same for $(A \cap B)'$ and $A' \cup B'$. This shows that:-

$(A \cap B)' = A' \cup B'$ [Using venn diagrams above].

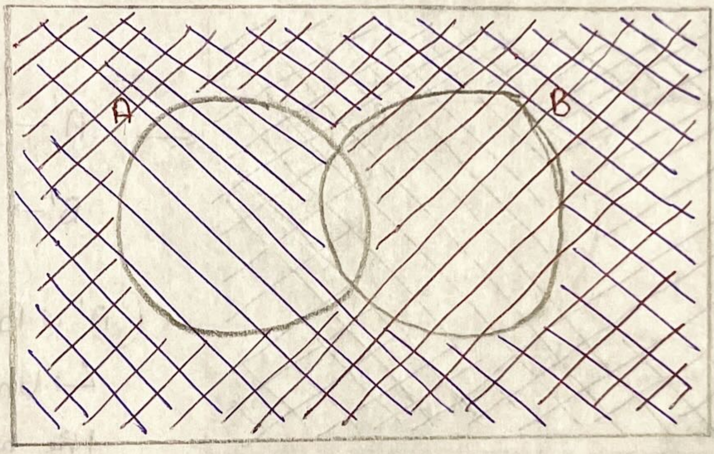
Hence, Proved. Ans

(c) $(A \cup B)' = A' \cap B'$

S


$\leftarrow (A \cup B)'$

Area shown with black

S


$\leftarrow$ $A' \rightarrow$ shown in black

$B' \rightarrow$ shown in blue

$(A' \cap B') \rightarrow$ Area where both blue and black lines are present.

As we can see above, there area for $(A \cup B)'$ is essentially everything but the area spanned by A ~~and~~ or B. (or both).
Basically, it's $S - (A \cup B)$.
Also, in the second venn diagram, area shown with black is A' and area shown in blue is B'. So, intersection of these 2 areas is the one where both blue and black lines are present which is also essentially $S - (A \cup B)$.
As the final shaded regions are the same for both, thus, it's shown that:-

$(A \cup B)' = A' \cap B'$ [Using venn diagrams above].

Hence, Proved. Ans.

7.

(a) for creating a sonet of 14 lines from 10 sonets in the book,

for each line of the sonet can be taken from the corresponding line of the other 10 sonets,

So,
for each line has 10 options to choose from.

$$\left.\begin{array}{c}\underline{\quad 10 \quad}\\ \underline{\quad 10 \quad}\\ \underline{\quad 10 \quad}\\ \vdots \\ \underline{\quad 10 \quad}\end{array}\right] \text{14 lines}$$

Thus, the number of sonets that can be created from the 10 in the book are ⇒

$$\Rightarrow \underbrace{10 \times 10 \times 10 \times \ldots \times 10}_{14 \text{ times}}$$

$$\Rightarrow \underline{\underline{10^{14}}}$$

Ans. $\left[(\text{Number of sonnets to choose from})^{\#\text{lines}}\right]$

(b.) Total number of outcomes (sonnets) that can be created $= \underline{\underline{10^{14}}}$

When we cannot choose any of the lines from either the first or last sonnet in the book, we have 8 sonnets left that we can use to choose the 14 lines from.

$$\left.\begin{array}{c}\underline{\quad 8 \quad}\\ \underline{\quad 8 \quad}\\ \vdots \\ \underline{\quad 8 \quad}\end{array}\right\} \text{14 lines}$$

∴ Probability that none of its lines came from either the first or last sonnet in the book =>

$$\frac{\text{Number of sonnets possible when first \& last one are not considered}}{\text{Total number of sonnets possible}}$$

$$\Rightarrow \frac{8^{14}}{10^{14}} \Rightarrow \left(\frac{8}{10}\right)^{14} = \boxed{\left(\frac{4}{5}\right)^{14}} = \underline{0.04}$$

Ans

(8.) Total number of students interested in taking a particular course = $\underline{120}$.

Default capacity of the course = $\underline{86}$

$\therefore$ Students potentially in the waitlist = $\underline{34}$. $\begin{pmatrix}120 - 86 \\ = 34\end{pmatrix}$

Firstly, potentially choosing the 34 students for the waitlist out of 120 will have the following number of ways $\Rightarrow$

$$\boxed{{}^{120}C_{34}}$$

Then, those 34 students can be placed in the waitlist in the following ways (as the order in the waitlist matters) $\Rightarrow$

$\underline{34!}$ (34 factorial).

$\therefore$ The number of different waitlists the instructor can potentially see will be $\Rightarrow$

$$= {}^{120}C_{34} \times 34!$$

$$= \frac{120!}{86!\,\cancel{34!}} \times \cancel{34!}$$

$$= \boxed{\frac{120!}{86!}}$$

Ans.