ex 1:

(a) $P(Y \le 50) = 0.05 + 0.10 + 0.12 + 0.14 + 0.25 + 0.17$
$= 0.83$

(b) $P(Y > 50) = 1 - 0.83 = 0.17$

(c) $P(Y \le 49) = P(Y \le 50) - P(Y = 50) = 0.66$

$P(Y \le 47) = 0.05 + 0.1 + 0.12 = 0.27$

ex 2 (a) $p(y) = ky$

$\sum_{y=1}^{5} p(y) = 1 \Rightarrow k(1+2+3+4+5) = 1 \Rightarrow k = \frac{1}{15}$

(b) $p(1) + p(2) + p(3) = \frac{1}{15}(1+2+3) = 0.4$

(c) $p(2) + p(3) + p(4) = \frac{1}{15}(2+3+4) = 0.6$

(d) $p(y) = \frac{y^2}{50}$

$\sum_{y=1}^{5} p(y) = \frac{1}{50}(1^2 + 2^2 + 3^2 + 4^2 + 5^2) = \frac{55}{50} > 1$

No

10

Ex 3:

(a)

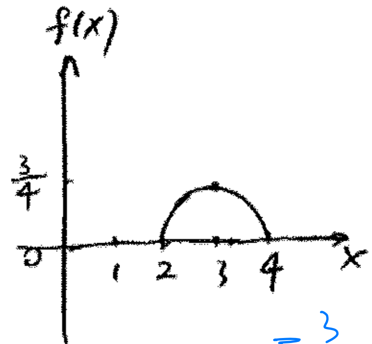
$$\int_2^4 f(x)\,dx = 1$$

$$\therefore \frac{1}{k} = \int_2^4 (-x^2+6x-8)\,dx$$

$$\therefore \frac{1}{k} = \frac{4}{3}$$

$$k = \frac{3}{4}$$

(b)



$$= \frac{3}{4}\int_3^4 (-x^2+6x-8)\,dx = \frac{3}{4}\left(-\frac{x^3}{3} + 6\frac{x^2}{2} - 8x \Big|_3^4\right)$$

(c) $P(X>3) = \int_3^4 f(x)\,dx = \frac{1}{2}$

$$= \frac{3}{4}\left(-\frac{x^3}{3} + 6\frac{x^2}{2} - 8x \Big|_{2.75}^{3.25}\right)$$

(d) $P(2.75 \le X \le 3.25) = \int_{2.75}^{3.25} \frac{3}{4}\left(1-(x-3)^2\right) dx = 0.3672$

(e) $P(X<2.5 \text{ or } X>3.5) = 1 - \int_{2.5}^{3.5} \frac{3}{4}\left(1-(x-3)^2\right) dx = 0.3125$

$$= 1 - \frac{3}{4}\left(-\frac{x^3}{3} + 6\frac{x^2}{2} - 8x \Big|_{2.5}^{3.5}\right)$$

Subtract 1 point if intermediate steps are missing

8 Ex 4:

(a) $P(X \le 1) = F(1) = \frac{1}{4}$

(b) $P(0.5 \le X \le 1) = F(1) - F(0.5) = \frac{1}{4} - \frac{1}{16} = \frac{3}{16}$

(c) $P(X > 1.5) = 1 - F(1.5) = \frac{7}{16}$

(d) $F(\tilde{\mu}) = \frac{1}{2}$

$\frac{\tilde{\mu}^2}{4} = \frac{1}{2}$

$\tilde{\mu} = \sqrt{2}$

(e) $f(x) = 0$ if $x < 0$ or $x \geqslant 2$

$f(x) = \frac{1}{2}x$ if $0 \le x < 2$

7 ex5

$X \sim B(6, 0.1)$ $p = 0.1$

(a) $P(X=1) = \binom{6}{1} p^1 (1-p)^{6-1} = 0.3543$

(b) $P(X \geqslant 2) = 1 - (P(X=0) + P(X=1))$

[If someone sums up 2 to 6, it's fine, too]

$= 1 - \left(\binom{6}{0} 0.1^0 0.9^6 + \binom{6}{1} 0.1^1 0.9^5 \right)$

$= 0.1143$

(c) 4 or 5 goblets must be selected

select 4 goblets with 0 defects: $P(X=0) = \binom{4}{0} 0.1^0 0.9^4 = 0.6561$

select 4 goblets, one of which is defected, 5th is good

$\binom{4}{1} 0.1^1 0.9^3 \times 0.9 = 0.26244$

$\therefore$ $0.6561 + 0.26244 = 0.91854$

ex 6:

X = number of boxes that do not contain a prize until you find 2 prizes. $X \sim NB(2, 0.2)$

(a) $P(X=x) = \binom{x+2-1}{2-1} 0.2^2 (1-0.2)^x = (x+1)\, 0.2^2\, 0.8^x$

(b) $P(4 \text{ boxes purchased}) = P(2 \text{ boxes without prizes})$
$= NB(2, 2, 0.2)$
$= (2+1)\, 0.2^2\, 0.8^2 = 0.0768$

(c) $P(X \le 2) = \sum_{x=0}^{2} nb(x; 2, 0.8) = 0.04 + 0.064 + 0.0768 = 0.1808$

5 ex 7:

$X \in \{3, 4, 5\}$

$P(X=3) = (\frac{1}{2})^3 \cdot 2 = \frac{1}{4}$ ✓

$P(X=5) = \binom{4}{2} (\frac{1}{2})^2 (\frac{1}{2})^2 = \frac{3}{8}$ ✓

$P(X=4) = 1 - P(X=3) - P(X=5) = \frac{3}{8}$ ✓

pmf:

x	1	2	3	4	5
p(x)	0	0	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{3}{8}$

(a) $P(X=0) = \dfrac{e^{-2}2^{0}}{0!} = 0.135$

(b) S = an operator who receives no requests

the number of operators that receive no requests follow a $Bin(n=5, p=0.135)$ distribution

$P(4\ S\text{'s in } 5 \text{ trials}) = \binom{5}{4} 0.135^{4}\, 0.865^{1} = 0.00144$

(c) $P($ all operators receive exactly x requests$)$

$$= \left(\frac{e^{-2}2^{x}}{x!}\right)^{5} = \frac{e^{-10}2^{5x}}{(x!)^{5}}$$

$\therefore$ $P($all receive the same number$) = \displaystyle\sum_{x=0}^{\infty} \frac{e^{-10}2^{5x}}{(x!)^{5}}$

!h

Figure 1: Solution for Exercise 8

Exercise 8 See Figure 1.

Exercise 9

(a) $X > n$ means that there are at least $n+1$ failures before the first success. The probability for this is $(1-p)^{n+1}$, hence the claim follows by taking complement.

(b) $P(\frac{X_n}{n} \leq x) = P(X_n \leq nx) = 1 - (1 - \frac{\lambda}{n})^{nx+1} \to 1 - e^{-\lambda x} = P(X \leq x)$.

(c) The geometric distribution models the number of failures before the first success in a binomial experiment. The binomial experiment with success parameter λ/n approximates the Poisson process. Hence, the exponential distribution is the distribution of the waiting time until the first event occurs in a Poisson process.