Columbia University
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STAT UN1201
Fall 2021

Sheet 2: Introduction to Probability Theory

Graded exercises: 2 (4 points), 5 (6 points), 7 (7 points), 8 (3 points)

NOTE: All results should be rounded to two decimal places unless otherwise stated. If a number or result has fewer decimal places, it is okay to keep fewer.

Exercise 1

[D, Section 2.1, Exercise 2]
a) A={RRR,LLL,SSS}
b) B={RLS,RSL,SRL,SLR,LSR,LRS}
c) C={RRL,RRS,SRR,LRR,RSR,RLR }
d) D={RRL,RRS,SRR,LRR,RSR,RLR,LLR,LLS,RLL,SLL,LRL,LSL,SSR,SSL,LSS,RSS,SRS,SLS}
e) D'= A ∪ B = {RRR,LLL,SSS,RLS,SRL,SLR,RSL,LRS,LSR} ; C ∪ D = D ; C ∩ D = C

Exercise 2

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[D, Section 2.1, Exercise 6]

Could be different order, of course

a) S={3,4,5,13,14,15,23,24,25,123,124,125,213,214,215}
b) A={3,4,5}
c) B={5,15,25,125,215}
d) C={3,4,5,23,24,25}

Exercise 3

Read [D, Example 2.10] and solve [D, Section 2.1, Exercise 10]
a) A=[Chevrolets, Buicks]; B=[Ford, Lincoln]; C=[Toyota]
b) No. Consider examples below: they have no common outcome to all three events but not mutually exclusive.

Exercise 4

$(A_{ic} = A_i^c = A_i')$

[D, Section 2.2, Exercise 13]
a) $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) = 0.36$
awarded project 1 or 2
b) $P(A_{1c} \cap A_{2c}) = 1 - P(A_1 \cup A_2) = 0.64$
not awarded project 1 and not awarded project 2

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c) $P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3) = 0.53$
awarded project 1 or 2 or 3
d) $P(A_{1c} \cap A_{2c} \cap A_{3c}) = 1 - P(A_1 \cup A_2 \cup A_3) = 0.47$
not awarded any project of 1,2,3
e) $P(A_{1c} \cap A_{2c} \cap A_3) = P(A_3) - P(A_3 \cap A_1) - P(A_3 \cap A_2) + P(A_1 \cap A_2 \cap A_3) = 0.17$
awarded project 3 and not awarded project 1 and not awarded project 2
f) $P(A_{1c} \cap A_{2c} \cup A_3) = 1 - [P(A_1 \setminus A_3) + P(A_2 \setminus A_3) - [P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3)]] = 0.75$
(not awarded project 1 and not awarded project 2) or awarded project 3

For each subexercise, 1 point for correct formula (there can be different correct formulas!) and 1 point for result. Do not grade the descriptio

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Exercise 5
[D, Section 2.2, Exercise 14]
A1: coffee
A2: soda
a) $P(A_1 \cap A_2) = 0.55 + 0.45 - 0.7 = 0.3$
b) $P(A_1^c \cap A_2^c) = 1 - P(A_1 \cup A_2) = 0.3$

Exercise 6
Given two events A and B, consider the following statements:

(a) A and $B \cap A'$ are disjoint.

(b) $(A \cap B)' = A' \cup B'$.

(c) $(A \cup B)' = A' \cap B'$.

Verify these statements by drawing a Venn diagram for each (a), (b) and (c) and covering

- for (a), the interiors of A and $B \cap A'$;
- for (b), the interiors of $(A \cap B)'$, A' and B';
- for (c), the interiors of $(A \cup B)'$, A' and B'

with lines of different colors.
Please see the figure.

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Exercise 7
[D, Section 2.3, Exercise 38]
It suffices to state the results as a sum/product of numbers, factorials, binomial coefficients ... For example, $6 \times 9!$ is sufficient, no need to calculate this further.
a)10^{14}
b)$\frac{8^{14}}{10^{14}}$

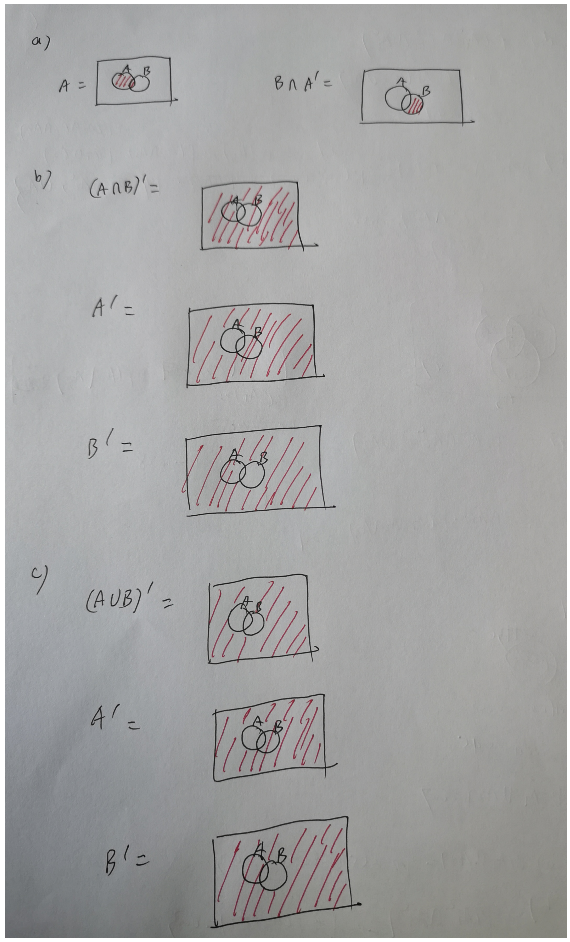

Figure 1: exercise 6

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Exercise 8

$\binom{120}{34} \times 34!$(Note that the order matters in this problem)

or $\frac{120!}{(120-34)!} = \frac{120!}{86!}$ ✓✓ (only 1 point for $\binom{120}{34}$

$\frac{120!}{34!}$

120^{34}

0 points for anything else)