Sheet 8: Confidence Intervals

NOTE: All results should be rounded to two decimal places unless otherwise stated. If a number or result has fewer decimal places, it is okay to keep fewer. For probabilities, give two decimal places when expressed in percentage (e.g., 12.34%) and four decimal places when expressed as numbers (e.g., 0.1234).

In all exercises, unless otherwise stated, "construct a CI" means "construct a balanced CI". Also, if numbers are given, first construct the CI (L_n, R_n) with statistics L_n and R_n and then compute their actual values by plugging in the numbers.

Exercise 1
[D, Section 6.2, Exercise 22b]

Exercise 2
[D, Section 6.2, Exercise 23]

Exercise 3

(a) Read Example 1-3 under https://online.stat.psu.edu/stat415/lesson/1/1.2

(b) Let $X_1, \ldots, X_n$ be iid with pdf

$$f(x) = \frac{1}{x\sqrt{2\pi\theta_2}} e^{-\frac{(\log(x)-\theta_1)^2}{2\theta_2}}, \qquad -\infty < x < \infty,$$

and unknown parameters θ_1 and θ_2. Find the maximum likelihood estimators for θ_1 and θ_2, respectively.

Exercise 4

Let $X_1, ..., X_n$ be iid and $U([a, b])$-distributed with unknown parameters $\theta_1 = a$ and $\theta_2 = b$. Find the MLE's ofθ_1 and θ_2. Argue carefully using indicator functions.

Exercise 5

Let $X_1, \ldots, X_n$ be iid pdf

$$f(x) = \begin{cases} K_{a,c}\, x^{-a-1}, & x \geq c, \\ 0, & x < c. \end{cases}$$

and unknown parameters a and c.

(a) Determine $K_{a,c}$.

(b) Find the maximum likelihood estimator of c.

(c) Find the maximum likelihood estimator of a.

Exercise 6 Let $X_1, \ldots, X_n$ be iid and $U([0,\theta])$-distributed with unknown parameter θ. In class we have shown that the MLE is $\widehat{\theta}_{MLE} = \max X_i$.

(a) Find the MOM estimator $\widehat{\theta}_{MOM}$.

(b) Are $\widehat{\theta}_{MLE}$ and $\widehat{\theta}_{MOM}$ unbiased? You may want to take another look at Exercise 4 from HW 7.

(c) Calculate the MSE of $\widehat{\theta}_{MLE}$ and $\widehat{\theta}_{MOM}$.

(d) Which one has a smaller MSE?