

All answers written below in text boxes

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Calculus Assessment Test

Mark or fill in the correct answers. No justifications needed!

1. Rewrite the expression as a single logarithm:

$$4 \ln y - 2 \ln x^3$$

- A. $\ln(y^4 - x^6)$
- B. $\ln \frac{4y}{2x^3}$
- C. $\ln(4y - 2x^3)$
- D. $\ln \frac{y^4}{x^6}$

Solution: D

2. Determine the set D of x -values for which

$$2x^2 - 3x + 1 > 7x(2x - 1)$$

- A. $D = (-\infty, -\frac{1}{3}) \cup (\frac{1}{5}, \infty)$
- B. $D = (-\frac{1}{3}, \frac{1}{5})$
- C. $D = (-\infty, -\frac{1}{6}) \cup (\frac{1}{2}, \infty)$
- D. $D = (-\frac{1}{6}, \frac{1}{2})$

Solution: D

3. Find $x < y$ such that

$$\begin{aligned} 4 &= \frac{1}{2}(x + y), \\ 19 &= \frac{1}{3}(x^2 + xy + y^2). \end{aligned}$$

Answer:

$x =$

1

$y =$

7

4. Calculate the following limit

$$\lim_{x \rightarrow \infty} x^5 e^{3x} e^{-\frac{x^2}{2}}$$

- A. $-\infty$
- B. $+\infty$
- C. 0
- D. $e^{-\frac{x^2}{2}}$

Solution: C

5. Calculate the following limit

$$\lim_{x \rightarrow 0^+} \frac{3(e^x - 1 - x)}{10x^3}$$

- A. $\frac{1}{20}$
- B. $\frac{3}{10}$
- C. $-\infty$
- D. $+\infty$
- E. 0

Solution: D

6. What is $f'(1)$ if

$$f(x) = \frac{xe^x}{x^2 + 1}$$

- A. e
- B. $e^2/2$
- C. $e/2$
- D. $2e$

Solution: C

7. The following table lists the values of functions g and h and their derivatives at several points.

x	1	2	3
$g(x)$	0	4	3
$g'(x)$	-2	4	4
$h(x)$	5	3	2
$h'(x)$	6	1	2

Let $f(x) = g(h(x))$ and $f'(x)$ be its derivative. What is $f'(2)$?

- A. 4
- B. 8
- C. 2
- D. 3

Solution: A

8. Given $x > 0$, for which value of $\theta > 0$ is the following expression maximized?

$$-4\theta - 3 + 2 \ln(\theta)$$

- A. $\theta = \frac{1}{2}$
- B. $\theta = \frac{1}{3}$
- C. $\theta = \frac{1}{4}$
- D. $\theta = \frac{1}{5}$

Solution: A

9. Evaluate the integral

$$\int_0^2 x\sqrt{1+2x^2} \, dx$$

- A. $26/9$
- B. $26/3$
- C. $13/3$
- D. $9/2$

Solution: C

10. Evaluate the integral

$$\int_0^2 \left(\frac{1}{2}x^3 - \frac{2}{\sqrt{x}} - 1 \right) dx$$

- A. $-\sqrt{2}$
- B. $-2\sqrt{2}$
- C. $-3\sqrt{2}$
- D. $-4\sqrt{2}$

Solution: D

11. Evaluate the integral

$$\int_0^1 \int_0^1 (xy^2 + 3x^2y^3) \, dy \, dx$$

- A. $\frac{1}{12}$
- B. $\frac{5}{12}$
- C. $\frac{7}{12}$
- D. $\frac{11}{12}$

Solution: B

12. Evaluate the integral

$$\iint_A f(x, y) \, dy \, dx$$

where $f(x, y) = 24xy$ and $A = \{(x, y) \in [0, 1] \times [0, 1] : x + y \leq \frac{1}{2}\}$.

- A. $\frac{1}{10}$
- B. $\frac{1}{12}$
- C. $\frac{1}{14}$
- D. $\frac{1}{16}$

Solution: D

13. Compute

$$\sum_{n=1}^{\infty} \frac{3}{4^n}$$

- A. 1
- B. 2
- C. 3
- D. 4

Solution: A

14. Compute

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$

A. e

B. e^{-1}

C. 1

D. $\log 2$

Solution: B