Calculus Assessment Test

Mark or fill in the correct answers. No justifications needed!

1. Rewrite the expression as a single logarithm:

$$4 \ln y - 2 \ln x^3$$

   A. $\ln(y^4 - x^6)$
   B. $\ln \frac{4y}{2x^3}$
   C. $\ln(4y - 2x^3)$
   D. $\ln \frac{y^4}{x^6}$

2. Determine the set D of x-values for which

$$2x^2 - 3x + 1 > 7x(2x - 1)$$

   A. $D = (-\infty, -\frac{1}{3}) \cup (\frac{1}{5}, \infty)$
   B. $D = (-\frac{1}{3}, \frac{1}{5})$
   C. $D = (-\infty, -\frac{1}{6}) \cup (\frac{1}{2}, \infty)$
   D. $D = (-\frac{1}{6}, \frac{1}{2})$

3. Find $x < y$ such that

$$4 = \frac{1}{2}(x + y),$$
$$19 = \frac{1}{3}(x^2 + xy + y^2).$$

   Answer:

   $x =$ [ ] $y =$ [ ]

4. Calculate the following limit

$$\lim_{x \to \infty} x^5 e^{3x} e^{-\frac{x^2}{2}}$$

   A. $-\infty$
   B. $+\infty$
   C. 0
   D. $e^{-\frac{x^2}{2}}$

5. Calculate the following limit

$$\lim_{x \to 0^+} \frac{3(e^x - 1 - x)}{10x^3}$$

A. $\frac{1}{20}$

B. $\frac{3}{10}$

C. $-\infty$

D. $+\infty$

E. 0

6. What is $f'(1)$ if

$$f(x) = \frac{xe^x}{x^2 + 1}$$

A. e

B. $e^2/2$

C. $e/2$

D. $2e$

7. The following table lists the values of functions g and h and their derivatives at several points.

x	1	2	3
$g(x)$	0	4	3
$g'(x)$	-2	4	4
$h(x)$	5	3	2
$h'(x)$	6	1	2

Let $f(x) = g(h(x))$ and $f'(x)$ be its derivative. What is $f'(2)$?

A. 4

B. 8

C. 2

D. 3

8. Given $x > 0$, for which value of $\theta > 0$ is the following expression maximized?

$$-4\theta - 3 + 2\ln(\theta)$$

A. $\theta = \frac{1}{2}$

B. $\theta = \frac{1}{3}$

C. $\theta = \frac{1}{4}$

D. $\theta = \frac{1}{5}$

9. Evaluate the integral
$$\int_0^2 x\sqrt{1+2x^2}\,\mathrm{d}x$$

   A. 26/9
   B. 26/3
   C. 13/3
   D. 9/2

10. Evaluate the integral
$$\int_0^2 \left(\frac{1}{2}x^3 - \frac{2}{\sqrt{x}} - 1\right)\mathrm{d}x$$

   A. $-\sqrt{2}$
   B. $-2\sqrt{2}$
   C. $-3\sqrt{2}$
   D. $-4\sqrt{2}$

11. Evaluate the integral
$$\int_0^1 \int_0^1 (xy^2 + 3x^2 y^3)\,\mathrm{d}y\,\mathrm{d}x$$

   A. $\frac{1}{12}$
   B. $\frac{5}{12}$
   C. $\frac{7}{12}$
   D. $\frac{11}{12}$

12. Evaluate the integral
$$\iint_A f(x,y)\,\mathrm{d}y\,\mathrm{d}x$$
where $f(x,y) = 24xy$ and $A = \{(x,y) \in [0,1] \times [0,1] : x + y \leq \frac{1}{2}\}$.

   A. $\frac{1}{10}$
   B. $\frac{1}{12}$
   C. $\frac{1}{14}$
   D. $\frac{1}{16}$

13. Compute
$$\sum_{n=1}^{\infty} \frac{3}{4^n}$$

   A. 1
   B. 2
   C. 3
   D. 4

14. Compute

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$

A. e

B. e^{-1}

C. 1

D. $\log 2$