Sheet 9: Confidence Intervals

NOTE: All results should be rounded to two decimal places unless otherwise stated. If a number or result has fewer decimal places, it is okay to keep fewer. For probabilities, give two decimal places when expressed in percentage (e.g., 12.34%) and four decimal places when expressed as numbers (e.g., 0.1234).

In all exercises, unless otherwise stated, "construct a CI" means "construct a balanced CI". Also, if numbers are given, first construct the CI (L_n, R_n) with statistics L_n and R_n and then compute their actual values by plugging in the numbers.

Exercise 1
[D, Section 7.1, Exercise 5]. For this exercise, you may use the final formula for the CI we derived in class. For c and d, first read Example 7.4 (see also my scanned notes). Do NOT use the black box formula after Example 7.4.

Exercise 2

(a) Solve [D, page 307, Exercise 48a].

(b) Calculate a 99%-CI of the form $(-\infty, R_n)$ for the true average compressive strength.

For this exercise, I want you to go through all Steps 1–4 in detail. Do not use any black box formulas for CI's.

Exercise 3
[D, page 308, Exercise 59]

Exercise 4
Let $X_1, \ldots, X_n$ be iid $N(\mu, \sigma^2)$-distributed with unknown μ and σ^2. Find a 99%-confidence interval for σ^2 of the form $(0, R_n)$. Calculate this interval explicitly if $n = 15$ and the sample variance is 20. For this exercise, I want you to go through all Steps 1–4 in detail. Do not use any black box formulas for CI's.

Exercise 5
Solve [D, Section 7.2, Exercise 13]. Assume that $n = 50$ is large. For this exercise, I want you to go through all steps in the construction of CI's in detail. Do not use any black box formulas for CI's. Note: in this exercise, you need to find an approximate pivot and an approximate CI.

Exercise 6
Suppose that$X_1, \cdots, X_n$ form a random sample from the exponential distribution with unknown parameter λ. Describe a method for constructing a confidence interval for λ with a specified confidence coefficient $\gamma (0 < \gamma < 1)$. Hint: Go through the sampling distributions we covered in class to find an appropriate pivot. For this exercise, I want you to go through all steps in the construction of CI's in detail.

Exercise 7
Suppose that the sample mean of 100 iid Poisson variables is 3.2. Find an approximate 95%-confidence interval for the unknown parameter θ of the Poisson distribution. For this exercise, I want you to go through all steps in the construction of CI's in detail.