Sheet 4: Random Variables

NOTE: All results should be rounded to two decimal places unless otherwise stated. If a number or result has fewer decimal places, it is okay to keep fewer. For probabilities, give two decimal places when expressed in percentage (e.g., 12.34%) and four decimal places when expressed as numbers (e.g., 0.1234).

Exercise 1
[D, Section 3.2, Exercise 12]

Exercise 2
[D, Section 3.2, Exercise 14]

Exercise 3
[D, Section 4.1, Exercise 6 (switch the order of a and b)]

Exercise 4
[D, Section 4.2, Exercise 11a–e]

Exercise 5
[D, Section 3.4, Exercise 49]

Exercise 6
Read [D, pages 128-129 (Negative binomial distribution)] and solve [D, Section 3.5, Exercise 75a-c]

Exercise 7
[D, Section 3.5, Exercise 76]

Exercise 8
[D, p. 138, Exercise 109]

Exercise 9

(a) Suppose that X has a geometric distribution with parameter $p \in (0,1)$ (see formula (3.17) on page 129 for the pmf). For any natural number n show that $P(X \leq n) = 1 - (1-p)^{n+1}$.

(b) Suppose now that X_n has a geometric distribution with parameter λ/n, where $\lambda \in (0,1)$. Show that the distribution function of X_n/n converges to the distribution function of an exponential random variable X with parameter λ, that is, for every $x > 0$,

$$P(\tfrac{X_n}{n} \leq x) \longrightarrow P(X \leq x).$$

You can pretend as if nx is always a natural number.

(c) What does the geometric distribution model in a binomial process? From here, explain what the exponential distribution models in a Poisson process.