DOMS | Page No.
Date / /

Hands ON -12 :

(a) Aggregrate Method :

=) In the aggregrate method, we analyze the total cost for a sequence of operations and then divide it by the no. of operations to get the average cost (amortized cost) per operation.

1. Inserting an element without resizing : For each insertion where there is enough space in the table, the cost is constant, $O(1)$.
2. Resizing : Every time the table doubles in size, the cost is proportional to the no. of elements being copied to the new table. If the table size is k before doubling, copying all k elements takes $O(k)$ time.

Total cost of n Insertions :

=) The cost of inserting n elements : $O(n)$ for the insertions.

=) The cost of resizing : The first doubling involves copying 1 element, the next involves copying 2, then 4 and so on.

=) The sum of these doubling operations is :

$O(1+2+4+8+\cdots 2^{k}) \sim O(n)$

=) Amortized cost per Insertion : $\frac{O(n)+O(n)}{n} \doteq O(1)$

So, using aggregate method, the amortized time complexity for inserting n elements is $O(1)$.

(b) Accounting Method :

=) The accounting method assigns "credits" to each operation to account for the costs of future expensive operations.

1. Assigning Credits :

Each insertion will be charged 3 credits

=) 2 credits for the insertion itself, which pays constant time $O(1)$ operation.

=) 1 credit to help pay for the cost of future resizing operations.

2. Cost of Insertion :

=) When no resizing happens, the cost is exactly 1 credit for the insertion.

=) When a resizing happens, it costs O(c) for copying c elements, but since we have 1 credit saved for each previous insertion, we have enough credits to cover the resizing.

Resizing & cost:

=) When the table doubles the cost of copying elements doubles as well.

=) The total no. of credits that we collect is 3 credits per insertion, for n insertions, resulting in 3n credits.

=) Each resizing is covered by the saved credits. The total no. of resizing operations is proportional to the no. of doublings (about log n times).

Final Amortized cost:

- Inserting n elements costs 1 credit each.
- Total credits collected = 3n
- Cost of each resizing operation is already covered by saved credits.

Amortized cost per Insertion:

$$= \frac{3n}{n}$$
$$= O(1)$$

∴ O(1) is the amortized time complexity for inserting n elements using accounting method for a dynamic table that doubles in size.