

Hands-On (3)

① function $x=f(n)$

$x=1;$

for $i=1:n$

for $j=1:n$

$x=x+1;$

$$T(n) = \sum_{i=1}^n \sum_{j=1}^n (1)$$

$$= \sum_{i=1}^n n$$

$$= 1+2+3+\dots+n$$

$$= \frac{n(n+1)}{2}$$

$$\therefore T(n) = \frac{n(n+1)}{2}$$

③ Using limit function

~~n~~ $\frac{n(n+1)}{2}$ (Upper Bound Function)

$$0 \leq f(n) \leq c g(n) \quad n \geq n_0$$

$$0 \leq \frac{n(n+1)}{2} \leq c g(n) \quad g(n)=n^2$$

$$0 \leq \frac{n^2+n}{2} \leq c n^2 \quad \text{if } c=1$$

$$0 \leq \frac{n^2+n}{2} \leq n^2 \quad n_0=0$$

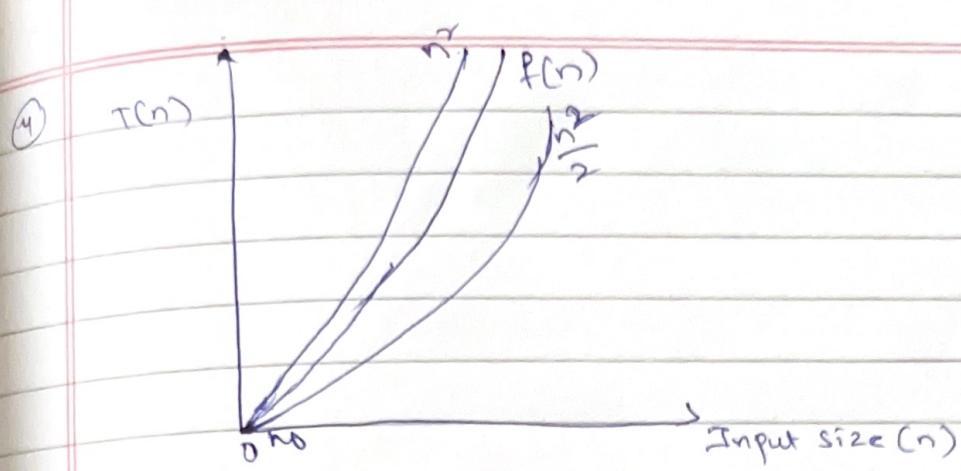
\therefore Upper Bound Function is $n^2 \quad O(n^2)$

Lower Bound $f(n) \geq c g(n)$

$$g(n)=n^2 \quad c=\frac{1}{2}$$

$$\frac{n^2+n}{2} \geq \frac{n^2}{2}$$

\therefore Lower Bound Function is $\frac{n^2}{2} \quad \Omega(n^2)$



$n_0 = 0$, The value n_0 satisfies both the conditions i.e., ~~at~~ 0 the values are equal and as n grows $f(n)$ grows with n^2 but less than it is greater than $\frac{n^2}{2}$.

⑤ It doesn't affect the asymptotic results from 1. If it is a constant factor, it slightly increase the curve compared to the first, but it too follows the quadratic shape. The overall growth rate is still quadratic.