Capacity Analysis(ET2594) - Project Report

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Scenario:

A passport verification centre provides 3 stages of service called stage A, stage B, and stage C with a single server at each stage. The services at the three stages are provided systematically and in a sequential manner. At stage A, the server checks the required documents in two sections, collecting the documents and verifying them, these two services are with service rates μ_{A1} and $\mu_{A2,}$ considering the stage A service rate as μ_A. If all the requirements are met the applicant goes to stage B where the server takes the required credentials and biometrics of the applicant with service rate μ_B. Now the applicant moves to stage C here, another server verifies all the details and takes the interview of the applicant with service rate μ_C. We simulate the whole passport office system if the arrival rate of the applicants is lambda (λ).

Goals:

- Our Goal is to prevent the probability for queue for the whole system, with queue length -k, to exceed a given threshold value (ε).
- Find out the distributions of the response times of each subsystem.

System Overview:

- In the given scenario, we have a System which includes 3 subsystems A,B and C which are arranged in a Cascaded design, with different Service rates for each sub-system.
- But it is also stated that, the Sub-system A has 2 sequential services given to each applicant.
- And then sub-system A is connected to B, and then B is connected to C.
- This arrangement gives us of many states in the state diagram which is given below.

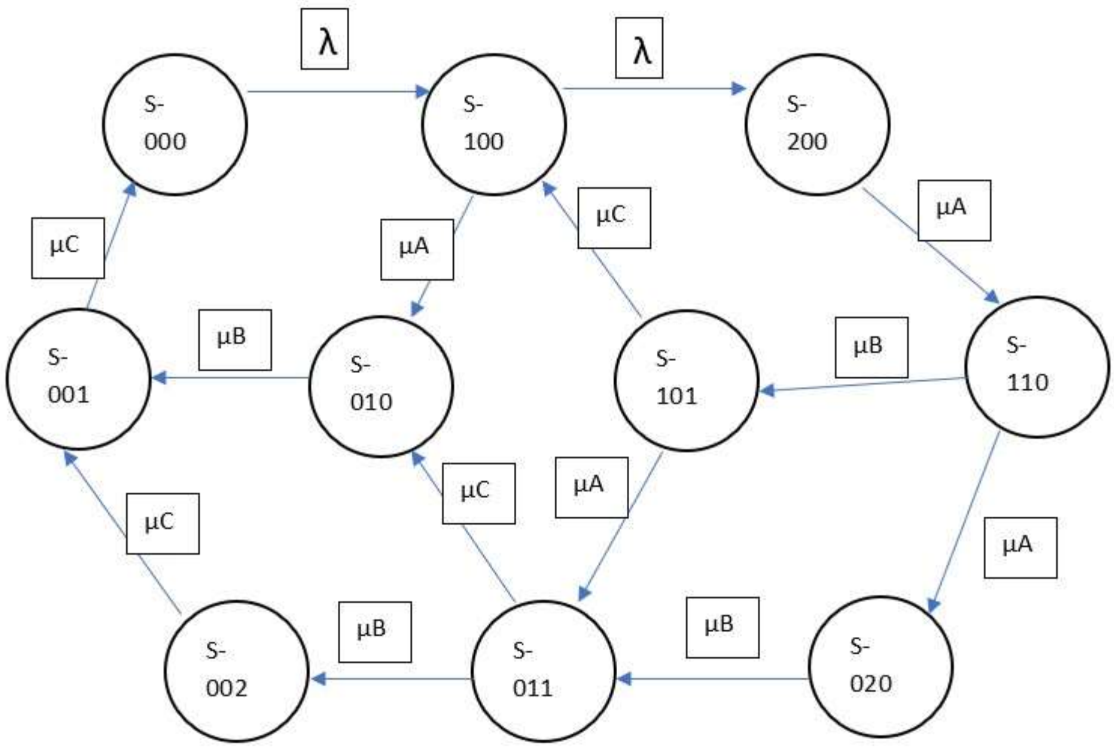


Figure- state diagram

Simulation:

To simulate the given Scenario, the system is Cascaded system with sub-systems A, B&C. So, we modified the code for plain M/M/1 system, those modifications and concepts are explained below:

System A:

- For System A, the Arrival rate is lambda(λ), because applicants as soon as they enter, they enter to system A.
- And System A consists of 2 sequential services, with 2 corresponding service rates, So, in M/M/1 system instead of generating 1 random service times, we generate , 2 service times with corresponding mu values and they are added.
- We could also consider, the total service time of system A, as mu(a)= mu(a1)+mu(a2);

System B & C:

- In System B, the arrival rate for this system is dependent on the service rate of previous sub-system(A).
- So, we designed system B’s function such that, it takes ‘array of departed times of system A’ as input instead of arrival rate(λ) and remaining is similar to direct M/M/1 system.
- System C, functions same as system B, but here it takes ‘array of departed times of system **B** as input.

Dimensioning:

Theoretical Method

Here, we determine the minimum service rate for the service at each section

- K refers to the queue size. K=1,2,3,4,5
- Average Arrival rates considered are lambda(λ) 0.5/min, 1/min and 5/min
- Threshold level values(ε) are 0.1 and 0.01
- Solving the equation P(K) = (1- ρ) ρ*k
- Consider P(K) = [0.1 0.01] and for the range of k from 1 to 5 i.e. K=[1:5]
- The results have been calculated and tabulated in the tables below for different lambda (λ), Threshold level values(ε) and the number of people (K) in the queue

Table 1

λ = 0.5								
	ε = 0.01				**ε = 0.1**			
K	**ρ(max)**	**μ_A(min)**	**μ_B(min)**	**μ_C(min)**	**ρ(max)**	**μ_A(min)**	**μ_B(min)**	**μ_C(min)**
1	0.9899	0.505	0.5102	0.5154	0.8873	0.5635	0.6351	0.7158
2	0.9950	0.502	0.5045	0.5070	0.9472	0.5279	0.5573	0.5884
3	0.9960	0.5017	0.5034	0.5051	0.9655	0.5179	0.5364	0.5556
4	0.9975	0.5013	0.5026	0.5039	0.9743	0.5132	0.5267	0.5406
5	0.9980	0.5010	0.5020	0.5030	0.9796	0.5104	0.5210	0.5319

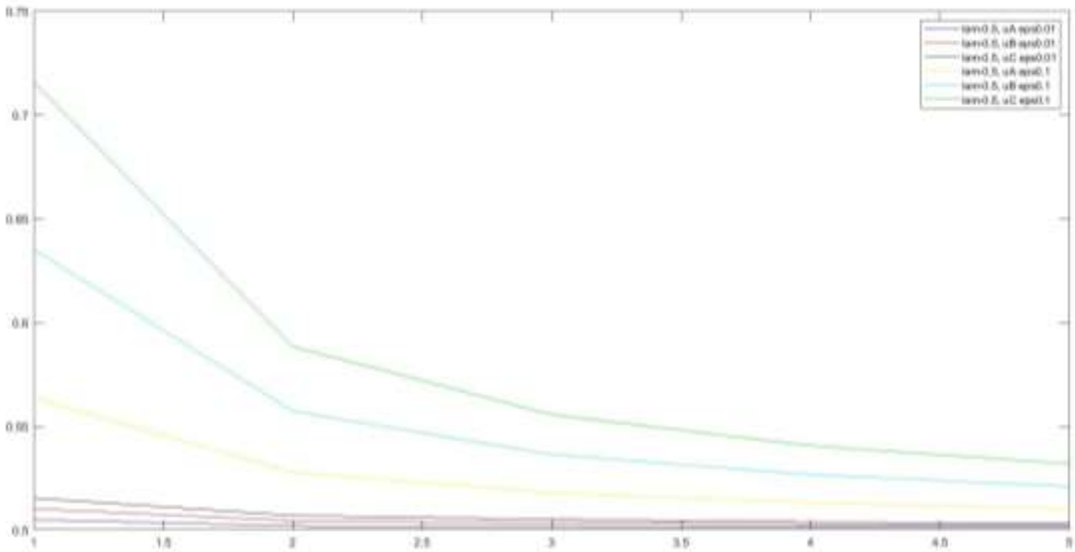


Figure 1

The x axis represents the number of people in the queue(K).It varies from 1 to 5.The y axis represents the service rate (/min).

Table 2

λ = 1								
ε = 0.01					ε = 0.1			
K	ρ(max)	μ_A(min)	μ_B(min)	μ_C(min)	ρ(max)	μ_A(min)	μ_B(min)	μ_C(min)
1	0.9899	1.0102	1.0205	1.0309	0.8873	1.1270	1.2701	1.4314
2	0.9950	1.0051	1.0102	1.0153	0.9472	1.0557	1.1145	1.7166
3	0.9960	1.0034	1.0068	1.0102	0.9655	1.0358	1.0728	1.1112
4	0.9975	1.0025	1.0050	1.0075	0.9743	1.0263	1.0533	1.0810
5	0.9980	1.0020	1.0040	1.0060	0.9796	1.0208	1.0421	1.0638

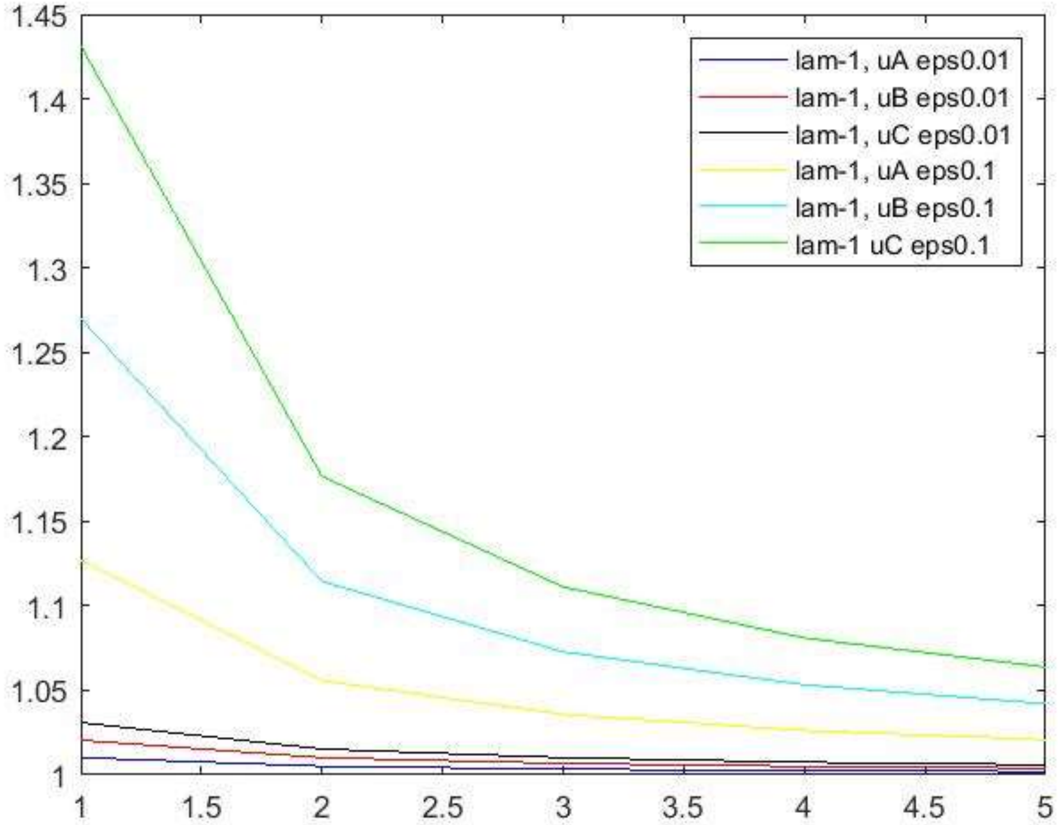


Figure 2

Table 3

λ = 5								
	ε = 0.01				**ε = 0.1**			
K	ρ(max)	μ_A(min)	μ_B(min)	μ_C(min)	ρ(max)	μ_A(min)	μ_B(min)	μ_C(min)
1	0.9899	5.0510	5.1025	5.1546	0.8873	5.6351	6.3509	7.1576
2	0.9950	5.0253	5.0507	5.0762	0.9472	5.2786	5.5728	5.8834
3	0.9960	5.0168	5.0336	5.0505	0.9655	5.1788	5.3640	5.5558
4	0.9975	5.0126	5.0252	5.0378	0.9743	5.1317	5.2668	5.4055
5	0.9980	5.0100	5.0201	5.0302	0.9796	5.1042	5.2106	5.3192

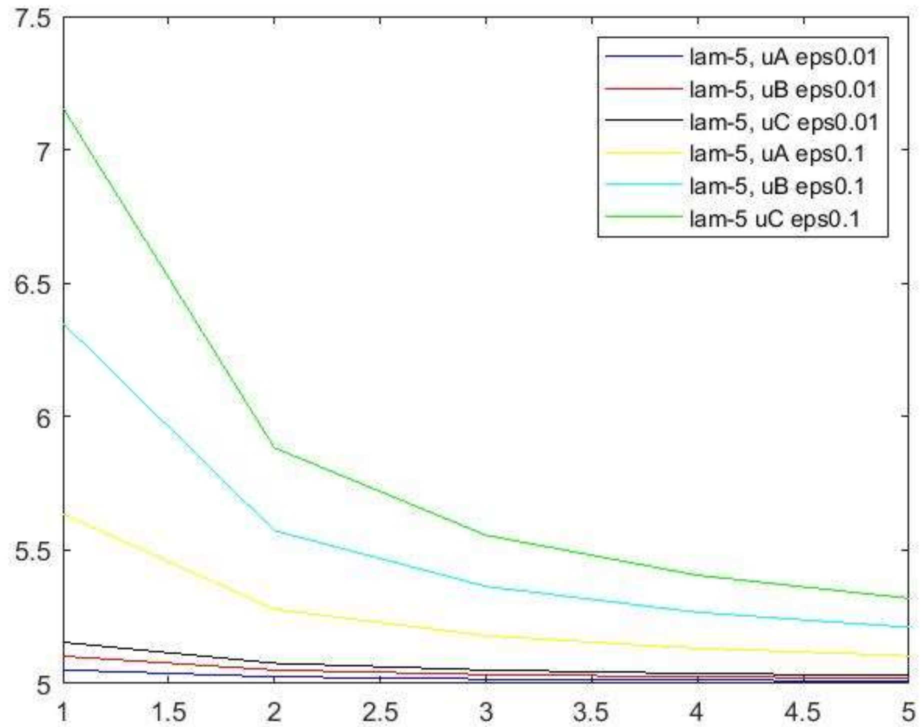


Figure 3

Optimisation Task

Optimization task: Here, we determine the maximal arrival rate for the service at each section

- K refers to the queue size. K=1,2,3,4,5
- Average service rate
    - ♦ case 1- μA=μB=μC=0.5/min
    - ♦ case2- μA=1/min, μB=1/min, μC=1.5/min
- Threshold level values(ε) are 0.1 and 0.01.

Note- Here in optimization , we considered the combinations of μA ,μB & μC.

Table 4

μ_A μ_B μ_C = 0.5				
ε = 0.01			**ε = 0.1**	
K	ρ (max)	λ (max)	ρ (max)	λ (max)
1	0.9899	0.4949	0.8873	0.4436
2	0.9950	0.4975	0.9472	0.4736
3	0.9960	0.4983	0.9655	0.4827
4	0.9975	0.4987	0.9743	0.4872
5	0.9980	0.4990	0.9796	0.4898

Table 5

μ_A = 1 μ_B = 1 μ_C = 1.5				
ε = 0.01			**ε = 0.1**	
K	ρ (max)	λ (max)	ρ (max)	λ (max)
1	0.9899	0.9899	0.8873	0.8873
2	0.9950	0.9950	0.9472	0.9472
3	0.9960	0.9967	0.9655	0.9655
4	0.9975	0.9975	0.9743	0.9743
5	0.9980	0.9980	0.9796	0.9796

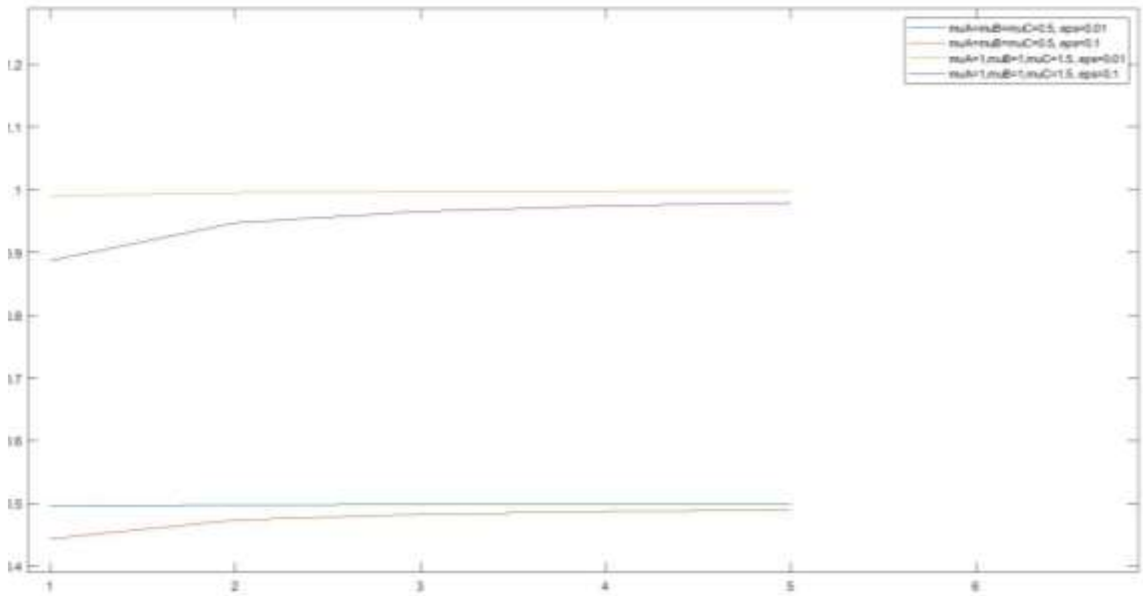
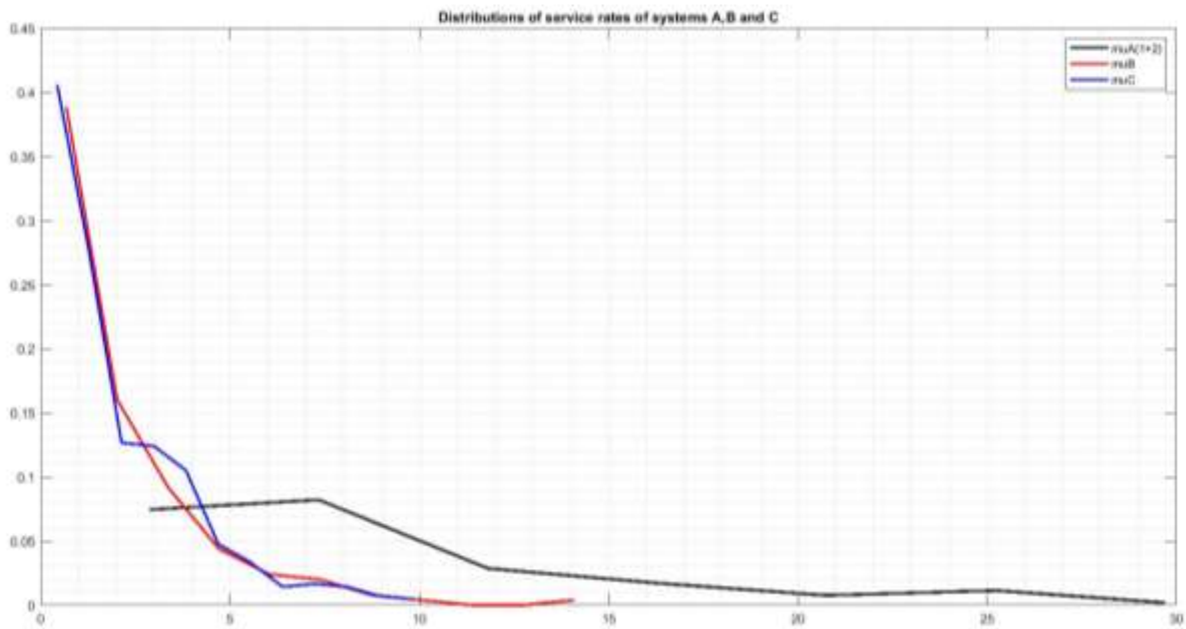


Figure-4

Observations from Simulation:

- After simulating the whole scenario, we tried to generate and plot the envelope of service rates of each sub system- A,B and C. which is given below
- These envelopes represent the Service time distributions of each sub-systems.



Discussion:

In this Project, in the first stage we tried to make a theoretical model of the given scenario and we tried to estimate the utilization parameter(rho) and with the help of that, we calculated the min value of mu(A,B,C), for given value of lambda, in dimensioning task. And also , max value of lambda, for given mu conditions.

From simulation, we calculated the distribution of service rates(mu(A,B,C)). And we also calculated the min value of mu(A) . The theoretical and simulated values are quite different from each other. We are tending to believe that the simulated values are more accurate because of the huge time period for which we are simulating the scenario and in theoretical we restricted the queue size.

Possible Extension of this Idea:

The given scenario can be further extended to many different types of simulation works. Few of the possibilities are :

- Here we considered this systems process as Poisson process (i.e M/M/1 systems), instead of that we could make the service rates as Gaussian-ly or Uniformly distributed(i.e M/U/1) systems
- We considered multiple services in system A, similarly we can extend this idea to B, and C systems, and also more Cascaded systems if they are attached in the future.

References:

1) Problem sets-7,8,10, Capacity Analysis.
2) Matlab Documentation.
3) https://www.oreilly.com/library/view/quantitative-techniques-theory/9789332512085/xhtml/ch9sec20.xhtml