CBCS SCHEME

USN | | | | | | | | | | |

17CS36

Third Semester B.E. Degree Examination, Dec.2018/Jan.2019
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing one full question from each module.

Module-1

1 a. Define proposition, tautology, contradiction. Determine whether the following compound statement is a tautology or not.
$\{(p \vee q) \rightarrow r\} \leftrightarrow \{\neg r \rightarrow \neg(p \vee q)\}$ **(06 Marks)**

b. Using the laws of logic, show that $(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$ **(07 Marks)**

c. Establish the validity of the following argument

$$\begin{array}{l} \forall x,\ p(x) \vee q(x) \\ \exists x,\ \neg p(x) \\ \forall x,\ \neg q(x) \vee r(x) \\ \forall x,\ s(x) \rightarrow \neg r(x) \\ \hline \therefore\ \exists x,\ \neg s(x) \end{array}$$

(07 Marks)

OR

2 a. Define converge, inverse and contra positive of a conditional. Find converse, inverse and contra positive of $\forall x, (x > 3) \rightarrow (x^2 > 9)$, where universal set is R. **(06 Marks)**

b. Test the validity of the following arguments:

i) If there is a strike by students, the exam will be postponed but the exam was not postponed.
___
$\therefore$ there was no strike by students.

ii) If Ravi studies, then he will pass in DMS.
If Ravi doesn't play cricket, then he will study.
Ravi failed in DMS.
___
$\therefore$ Ravi played cricket

(06 Marks)

c. Define dual of logical statement. Write the dual of the statement $(p \vee T_0) \wedge (q \vee F_0) \vee (r \wedge s \wedge T_0)$. **(02 Marks)**

d. Let $p(x) : x \geq 0$
$q(x) : x^2 \geq 0$ and $r(x) : x^2 - 3x - 4 = 0$
Then, for the universe completing of all real numbers, find the truth values of :
i) $\exists x\{p(x) \wedge q(x)\}$ ii) $\forall x\{p(x) \rightarrow q(x)\}$ iii) $\exists x\{p(x) \wedge r(x)\}$ **(06 Marks)**

Module-2

3 a. Prove that for any positive integer n, $\sum_{i=1}^{n} \frac{F_{i-1}}{2^i} = 1 - \frac{F_{n+2}}{2^n}$, F_n denote the Fibonacci number. **(06 Marks)**

b. How many positive integers n can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want n to exceed 5,000,000? **(07 Marks)**

c. Determine the coefficient of $a^2b^3c^2d^5$ in the expansion of $(a + 2b - 3c + 2d + 5)^{16}$. **(07 Marks)**

OR

4 a. Prove by using principle of mathematical induction

$$\sum_{i=1}^{n} i \cdot 2^{i} = 2 + (n-1)\cdot 2^{n+1}$$ **(06 Marks)**

b. A committee of 12 is to be selected from 10 men and 10 women. In how many ways can the selection be carries out if
   i) There are no restrictions
   ii) There must be six men and six women
   iii) There must be an even number of women. **(07 Marks)**

c. Determine the number of integer solutions of $x_1 + x_2 + x_3 + x_4 = 32$ where $x_i \geq 0$, $1 \leq i \leq 4$. **(07 Marks)**

Module-3

5 a. If A = {1, 2, 3, 4, 5} and there are 6720 injective functions $f : A \to B$, what is $|B|$? **(03 Marks)**

b. Let m, n be positive integers with $1 < n \leq m$ then prove that,
$s(m+1, n) = s(m, n-1) + ns(m, n)$ **(05 Marks)**

c. If $f : R \to R$ defined by $f(x) = x^2$, determine whether the function is one-to-one and whether it is onto. If it is not onto, find the range. **(06 Marks)**

d. Let A = {1, 2, 3, 4, 5} × {1, 2, 3, 4, 5} and define R on A by (x_1, y_1) R (x_2, y_2) if $x_1 + y_1 = x_2 + y_2$, verify that R is an equivalence relation on A. **(06 Marks)**

OR

6 a. If $f : R \to R$ defined by $f(x) = x^3$, determine whether f is invertible and if determine f^{-1}. **(05 Marks)**

b. Define the relation R for two lines ℓ_1 and ℓ_2 by ℓ_1 R ℓ_2 if ℓ_1 is perpendicular to ℓ_2. Determine whether the relation is reflexive, symmetric, antisymmetric or transitive. **(05 Marks)**

c. Let A = {1, 2, 3, 6, 9, 18} and R on A by xRy if x|y. Draw the Hasse diagram for the poset(A, R). **(05 Marks)**

d. For A = {1, 2, 3, 4}, let R{(1, 1) (1, 2) (2, 3) (3, 3) (3, 4)} be a relation on A. Draw the directed graph G on A that is associated with R. Do likewise for R^2, R^3. **(05 Marks)**

Module-4

7 a. Determine the number of positive integers n where $1 \leq n \leq 100$ and n is not divisible by 2, 3 or 5. **(06 Marks)**

b. How many derangements are there for 1, 2, 3, 4 and 5? **(07 Marks)**

c. Solve the recurrence relation $2a_{n+3} = a_{n+2} + 2a_{n+1} - a_n$, $n \geq 0$, $a_0 = 0$, $a_1 = 1$, $a_2 = 2$. **(07 Marks)**

OR

8 a. In how many ways can the 26 letters of the alphabet be permuted so that none of the patterns car, dog, pun or byte occurs? **(06 Marks)**

b. Find the root polynomial for 3 × 3 board using the expansion formula. **(07 Marks)**

c. The number of bacteria in a culture is 1000 (approximately) and this number increases 250% every two hours. Use a recurrence relation to determine the number of bacteria present after one day. **(07 Marks)**

Module-5

9 a. Show that the graphs Fig.Q9(a)(i) and (ii) are isomorphic.

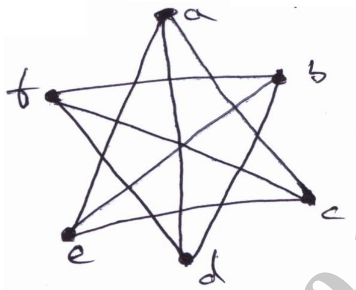

Fig.Q9(a)(i)

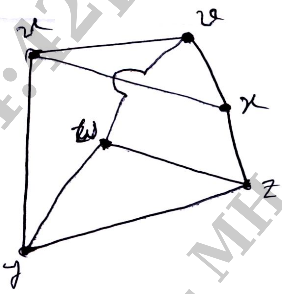

Fig.Q9(a)(ii) **(06 Marks)**

b. Let G = (V, E) be an undirected graph or multigraph with no isolated vertices. Then prove that G has an Euler circuit if and only if G is connected and every vertex in G has even degree. **(07 Marks)**

c. Construct an optimal prefix code for the symbols a, b, c, d, e, f, g, h, i, j that occur with respective frequencies 78, 16, 30, 35, 125, 31, 20, 50, 80, 3. **(07 Marks)**

OR

10 a. Let G = (V, E) be a connected undirected graph. What is the largest possible value for |V| if |E| = 19 and $\deg(v) \geq 4$ for all $v \in V$? **(06 Marks)**

b. For every tree T = (V, E) if $|V| \geq 2$, then prove that T has atleast two pendant vertices. **(07 Marks)**

c. For the tree shown in Fig.Q10(c), list the vertices according to a preorder and a postorder traversal.

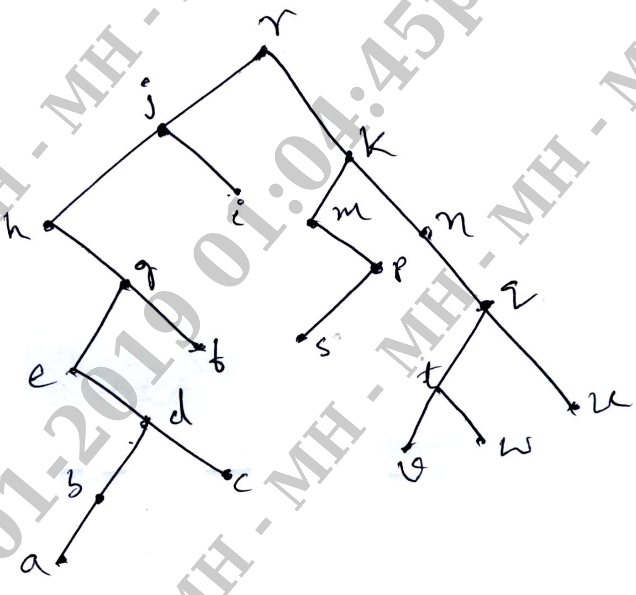

Fig.Q10(c) **(07 Marks)**

* * * * *