CBCS SCHEME

USN

17CS/IS36

Third Semester B.E. Degree Examination, Dec.2019/Jan.2020
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Define tautology and contradiction. Prove that for any propositions p, q, r the compound proposition $\{p \wedge (p \wedge r) \rightarrow s\} \rightarrow (r \rightarrow s)$ is tautology. **(06 Marks)**

b. Establish the validity of the argument: If A get the superwiser's position and work hard, then he will get raise.
If he gets a raise, then he will buy a new car.
He has not bought a new car.
Therefore, he does not get a superwiser's position or he did not work hard. **(07 Marks)**

c. Determine the truth value of each of the following quantified statements; if the universe being the set of all non-zero integers.
i) $\exists x, \exists y\,[xy = 2]$
ii) $\exists x, \forall y\,[xy = 2]$
iii) $\forall x, \exists y\,[xy = 2]$
iv) $\exists x, \exists y, [(3x + y = 8) \wedge (2x - y = 7)]$
v) $\exists x, \exists y\,[(4x + 2y = 3) \wedge (x - y = 1)]$ **(07 Marks)**

OR

2 a. Define dual of a logical statement and prove the logical equivalence using laws of logic $[(\neg p \vee q) \wedge (p \wedge (p \wedge q))] \Leftrightarrow p \wedge q$ **(06 Marks)**

b. Establish the validity of the argument : All Engineering students study physics. All engineering students of computer science study logic.
Ravi is an engineering student who does not study logic
Sachin studies logic but does not study physics.
Therefore, Ravi is not a student of computer science and Sachin is not an engineering student. **(07 Marks)**

c. Give: i) Direct proof ii) Indirect proof and "If n is an odd integer, then n + 7 is an even integer". **(07 Marks)**

Module-2

3 a. Prove that every positive integer greater than or equal to 14 may be written as sum of 3's and /or 8's. **(06 Marks)**

b. Find the number of arrangements of all the letters in TALLAHASSEE. How many of these arrangements have no adjacent A's? **(07 Marks)**

c. In how many ways can one distribute eight identical balls into four distinct containers so that i) No container is left empty ii) the fourth container gets an odd number of balls. **(07 Marks)**

OR

4 a. If $L_0, L_1, L_2 \ldots\ldots$ are Lucas numbers, then prove that $L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$. **(06 Marks)**

b. A question paper contains 10 questions of which 7 are to be answered. In how many ways a student can select the 7 questions
i) If he can choose any seven?
ii) If he should select three questions from first five and four questions from the last five?
iii) If he should select at least three from the first five? **(07 Marks)**

c. Find the coefficient of $x^2y^2z^3$ and the number of distinct terms in the expansion of $(3x - 2y - 4z)^7$. **(07 Marks)**

Module-3

5 a. If $f: R \to R$ is defined by $f(x) = x^2 + 5$, find $f(-1)$; $f(2/3)$; $f^{-1}(1)$; $f^{-1}([6, 10])$; $f^{-1}([-4, 5))$; $f^{-1}([-4, 5])$. **(06 Marks)**

b. State Pigeonhole principle. ABC is an equilateral triangle whose sides are of length 3cm each. If we select 10 points inside the triangle, prove that at least two of these points are such that the distance between them is less than 1cm. **(07 Marks)**

c. Let $A = \{1, 2, 3, 4, 5\}$. Define a relation R on $A \times A$ by $(x_1, y_1) R (x_2, y_2)$ if and one if $x_1 + y_1 = x_2 + y_2$. Then verify that R is an equivalence relation on $A \times A$ and hence find the equivalence classes $[(1, 3)]$, $[(2, 4)]$ and $[(1, 1)]$. **(07 Marks)**

OR

6 a. Let $A = B = R$, the set of all real numbers, and the functions $f : A \to B$ and $g : B \to A$ be defined by $f(x) = 2x^3 - 1$, $\forall\, x \in A$; $g(y) = \left\{\frac{1}{2}(y+1)\right\}^{1/3}$, $\forall\, y \in B$. Show that each of f and g is the inverse of the other. **(06 Marks)**

b. Define one-to-one function and on to function. Determine in each of the following cases where f is one-to-one or onto or both or neither $[f : A \to B]$.
i) $A = B = \{1, 2, 3, 4\}$;
$f = \{(1, 1), (2, 3), (3, 4), (4, 2)\}$
ii) $A = \{a, b, c\}$, $B = \{1, 2, 3, 4\}$
$f = \{(a, 1), (b, 1), (c, 3)\}$
iii) $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5\}$
$f = \{(1, 1), (2, 3), (3, 4)\}$
iv) $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5\}$
$f = \{(1, 1), (2, 3), (3, 3)\}$
v) $A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d\}$
$f = \{(1, a)\ (2, a), (3, d), (4, c)\}$ **(07 Marks)**

c. Draw Hasse diagram representing the positive divisors of 36. **(07 Marks)**

Module-4

7 a. Determine the number of positive integers n such that $1 \le n \le 300$ and n is not divisible by 5, 6, 8 and divisible by at least one of 5, 6, 8. **(06 Marks)**

b. Four persons P_1, P_2, P_3, P_4 who arrive late for a dinner party, find that only one chair at each of five tables T_1, T_2, T_3, T_4 and T_5 is vacant, P_1 will not sit at T_1 or T_2, P_2 will not sit at T_2, P_3 will not sit at T_3 or T_4 and P_4 will not sit at T_4 or T_5. Find the number of ways they can occupy the vacant chair. **(07 Marks)**

c. Define Homogeneous and non-homogeneous recurrence relations of first order and solve the recurrence relation $a_n - 3a_{n-1} = 5 \times 3^n$ for $n \ge 1$ given that $a_0 = 2$. **(07 Marks)**

OR

8 a. Find the number of nonnegative integer solutions of the equation $x_1 + x_2 + x_3 + x_4 = 18$ under the condition $x_i \leq 7$, for $i = 1, 2, 3, 4$. **(06 Marks)**

b. Define derangements and determine the rook polynomial of the board in Fig.Q.8(b) using expansion formula by selecting square 1 as ⊛ **(07 Marks)**

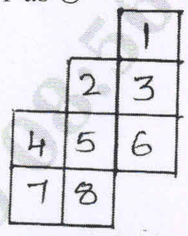

Fig.Q.8(b)

c. Solve the recurrence relation $a_n = a_{n-1} + a_{n-2}$ $a_1 = 1, a_2 = 3$. **(07 Marks)**

Module-5

9 a. Define complete graph and complete bipartite graph. Draw Kuratowski's first graph K_5 and second graph $K_{3,3}$ and hence find the number of edges in them. **(06 Marks)**

b. State Handshaking property. Show that $\delta \leq \frac{2m}{n} < \Delta$, for a given graph with n vertices and m edges, if δ is the minimum and Δ is the maximum of the degree of vertices. **(07 Marks)**

c. Obtain an optimal prifix code for the message MISSION SUCCESSFUL. Indicate the code and find the optimal weight. **(07 Marks)**

OR

10 a. Define circuit and Euler circuit in graphs and discuss the solution of Konigsberg bridge problem. **(06 Marks)**

b. Define isomorphism. Verify the two graphs are isomorphic in Fig.Q.10(b). **(07 Marks)**

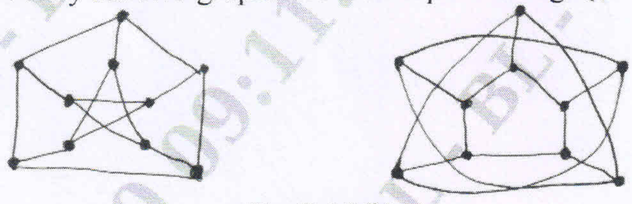

Fig.Q.10(b)

c. Define tree and show that a tree with n vertices has n – 1 edges. **(07 Marks)**

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