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Third Semester B.E. Degree Examination, Dec.2018/Jan.2019 Engineering Mathematics – III

Time: 3 hrs.

Max. Marks: 100

**Note: Answer any FIVE full questions, choosing
ONE full question from each module.**

Module-1

- 1 a. Find the Fourier series expansion for the periodic function $f(x)$, if in one second
- $$f(x) = \begin{cases} 0; & -\pi < x < 0 \\ x; & 0 < x < \pi \end{cases} \quad (08 \text{ Marks})$$
- b. Expand the function $f(x) = x(\pi - x)$ over the interval $(0, \pi)$ in half range Fourier cosine series. (06 Marks)
- c. The following value of function y gives the displacement in inches of a certain machine part for rotations x of a flywheel. Expand y -in terms of Fourier series upto the second harmonic.

Rotations	x	0	$\pi/6$	$2\pi/6$	$3\pi/6$	$4\pi/6$	$5\pi/6$	π
Displacement	y	0	9.2	14.4	17.8	17.3	11.7	0

(06 Marks)

OR

- 2 a. Find the Fourier series expansion for the function :
- $$f(x) = \begin{cases} \pi x; & 0 \leq x \leq 1 \\ \pi(2 - x); & 1 \leq x \leq 2 \end{cases}$$
- and deduce $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$. (08 Marks)
- b. Expand in Fourier series $f(x) = (\pi - x)^2$ over the interval $0 \leq x \leq 2\pi$. (06 Marks)
- c. The following table gives the variations of periodic current over a period T .

t (secs)	0	$T/6$	$T/3$	$T/2$	$2T/3$	$5T/6$	T
A (Amps)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Expand the function (periodic current) by Fourier series and show that there is a direct current part of 0.75 amp and also obtain amplitude of first harmonic. (06 Marks)

Module-2

- 3 a. Find Fourier transform of $f(x) = \begin{cases} 1-x^2; & |x| < 1 \\ 0; & |x| > 1 \end{cases}$
- and hence evaluate $\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} dx$. (08 Marks)
- b. Find Fourier Cosine transform of the function :
- $$f(x) = \begin{cases} 4x; & 0 < x < 1 \\ 4-x; & 1 < x < 4 \\ 0; & x > 4 \end{cases} \quad (06 \text{ Marks})$$
- c. Find z-transforms of : i) $a^n \sin n\theta$ ii) $a^{-n} \cos n\theta$. (06 Marks)

OR

- 4 a. Find Fourier sine transform of $f(x) = e^{-|x|}$ and hence evaluate : $\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx, m > 0$. (08 Marks)
- b. Find z-transform of $u_n = \cos h\left(\frac{n\pi}{2} + \theta\right)$. (06 Marks)
- c. Solve the difference equation using z-transforms $u_{n+2} + 6u_{n+1} + 9u_n = 2^n$. Given $u_0 = u_1 = 0$. (06 Marks)

Module-3

- 5 a. If θ - is the acute angle between the two regression lines relating the variables x and y , show that $\tan \theta = \left(\frac{1-r^2}{r} \right) \left(\frac{\sigma_x \sigma_y}{\sigma_x^2 \sigma_y^2} \right)$. (08 Marks)

Indicate the significance of the cases $r = \pm 1$ and $r = 0$.

- b. Fit a straight line $y = ax + b$ for the data.

x	12	15	21	25
y	50	70	100	120

- c. Find a real root of the equation by using Newton-Raphson method near $x = 0.5$, $xe^x = 2$, perform three iterations. (06 Marks)

OR

- 6 a. Compute the coefficient of correlation and equation of regression of lines for the data :

x	1	2	3	4	5	6	7
y	9	8	10	12	11	13	14

- b. The Growth of an organism after x - hours is given in the following table :

x (hours)	5	15	20	30	35	40
y (Growth)	10	14	25	40	50	62

Find the best values of a and b in the formula $y = ae^{bx}$ to fit this data. (06 Marks)

- c. Find a real root of the equation $\cos x = 3x - 1$ correct to three decimals by using Regula - False position method, given that root lies in between 0.6 and 0.7. Perform three iterations. (06 Marks)

Module-4

- 7 a. Find $y(8)$ from $y(1) = 24$, $y(3) = 120$, $y(5) = 336$, $y(7) = 720$ by using Newton's backward difference interpolation formula. (08 Marks)
- b. Define $f(x)$ - as a polynomial in x for the following data using Newton's divided difference formula. (06 Marks)

x	-4	-1	0	2	5
f(x)	1245	33	5	9	1335

- c. Evaluate the integral $I = \int_0^6 \frac{dx}{4x+5}$ using Simpson's $\frac{1}{3}$ rd rule using 7 ordinates. (06 Marks)

OR

- 8 a. For the following data calculate the differences and obtain backward difference interpolation polynomial. Hence find $f(0.35)$. (08 Marks)

x	0.1	0.2	0.3	0.4	0.5
f(x)	1.40	1.56	1.76	2.0	2.28

- b. Using Lagrange's interpolation find y when $x = 10$.

x	5	6	9	11
y	12	13	14	16

(06 Marks)

- c. Evaluate $\int_0^1 \frac{x}{1+x^2} dx$ by Weddle's rule considering seven ordinates. (06 Marks)

Module-5

- 9 a. Verify the Green's theorem in the plane for $\int_C (x^2 + y^2)dx + 3x^2y dy$ where C – is the circle $x^2 + y^2 = 4$ traced in positive sense. (08 Marks)

- b. Evaluate $\int_C (\sin z dx - \cos x dy + \sin y dx)$ by using Stokes theorem, where C – is the boundary of the rectangle $0 \leq x \leq \pi$, $0 \leq y \leq 1$ and $z = 3$. (06 Marks)

- c. Find the curve on which the functional $\int_0^1 [y'^2 + 12xy]dx$ with $y(0) = 0$, $y(1) = 1$ can be extremised. (06 Marks)

OR

- 10 a. Given $f = (3x^2 - y)i + xzj + (yz - x)k$ evaluate $\int_C f \cdot dr$ from $(0, 0, 0)$ to $(1, 1, 1)$ along the paths $x = t$, $y = t^2$ and $z = t^3$. (08 Marks)

- b. Derive Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0$. (06 Marks)

- c. Prove that the shortest distance between two points in a plane is a straight line. (06 Marks)

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