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15MAT41

Fourth Semester B.E. Degree Examination, Dec.2018/Jan.2019
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Using Taylor's series method solve $\frac{dy}{dx} = x^2 + y^2$ with y(0) = 1 and hence find y(0.1) and consider upto 3rd degree. **(06 Marks)**

b. Using modified Euler's method solve $\frac{dy}{dx} = 1 + \frac{y}{x}$ with y(1) = 2 then find y(1.2) in two steps. **(05 Marks)**

c. Given $\frac{dy}{dx} = \frac{x+y}{2}$, give that y(0) = 2, y(0.5) = 2.636, y(1) = 3.595 and y(1.5) = 4.968 then find value of y at x = 2 using Milne's predictor and corrector formulae. **(05 Marks)**

OR

2 a. Using modified Euler's method solve $\frac{dy}{dx} = x + \sqrt{y}$, with y(0) = 1 then find y(0.2) with h = 0.2. **(06 Marks)**

b. Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$, with y(0) = 1 and hence find y(0.1) by taking one steps using Runge-Kutta method of fourth order. **(05 Marks)**

c. Given $\frac{dy}{dx} = \frac{(1+x^2)y^2}{2}$, given that y(0) = 1, y(0.1) = 1.06. y(0.2) = 1.12 and y(0.3) = 1.21 then evaluate y(0.4) using Adam's – Bash forth method. **(05 Marks)**

Module-2

3 a. Given $\frac{d^2y}{dx^2} = \frac{2dy}{dx} - y$, y(0) = 1, y′(0) = 2, evaluate y(0.1) and y′(0.1) using Runge-Kutta method of fourth order. **(06 Marks)**

b. Solve the Bessel's differential equation : $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y = 0$ leading to $J_n(x)$. **(05 Marks)**

c. Express $x^3 + 2x^2 - 4x + 5$ in terms of Legendre polynomials. **(05 Marks)**

OR

4 a. Using Milne's method. obtain an approximate solution at the point x = 0.8 of the problem $\frac{d^2y}{dx^2} = 1 - 2y\frac{dy}{dx}$ using the following data :

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y′	0	0.1996	0.3937	0.5689

(06 Marks)

b. If α and β are two distinct roots of $J_n(x) = 0$ then P-T $\int_0^1 x\, J_n(\alpha x)\, J_n(\beta x)\, dx = \{0$ if $\alpha \neq \beta$. **(05 Marks)**

c. With usual notation, prove that $J+\frac{1}{2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$. **(05 Marks)**

Module-3

5 a. State and prove Cauchy-Riemann equation in Cartesian form. **(06 Marks)**

b. Find analytic function f(z) whose imaginary part is $v = \left(r - \frac{1}{r}\right)\sin\theta$. **(05 Marks)**

c. Discuss the transformation of $\omega = e^z$. **(05 Marks)**

OR

6 a. State and prove Cauchy's integral formula. **(06 Marks)**

b. Emulate $\oint_c \frac{e^{2z}}{(z+1)(z-2)} dz$ where c is |z| = 3 using Cauchy's residue theorem. **(05 Marks)**

c. Find the bilinear transformation which maps z = –1, 0, 1 into ω = 0, i, 3i. **(05 Marks)**

Module-4

7 a. Derive mean and variance of the binomial distribution. **(06 Marks)**

b. A random variable x has the following probability mass function.

x	0	1	2	3	4	5
P(x)	k	3k	5k	7k	9k	11k

i) find k ii) find p(x < 3) iii) find $p(3 < x \leq 5)$. **(05 Marks)**

c. The joint distribution of two random variable x and y as follows :

x \ y	–4	2	7
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

Compute : i) E(x) and E(y) ii) E(xy) iii) cov(xy). **(05 Marks)**

OR

8 a. 2% of the fuses manufactured by a firm are found defective. Find the probability that a box containing 200 fuses contains. i) no defective fuses ii) 3 or more defective fuses. **(06 Marks)**

b. In a test on 2000 electric bulbs. It was found that the life of a particular brand was distributed normally with an average life of 2040 hours and S.D 60 hours. Estimate the number of bulbs likely to burn $(P(0 < z < 1.83) = 0.4664 \ \ P(1.33) = 0.4082, P(2) = 0.4772)$ i) more than 2150 ii) less than 1960 iii) more than 1920 but less than 2160 hours. **(05 Marks)**

c. The joint probability distribution of two random variable X and Y given by the following table:

X \ Y	1	3	9
2	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$
4	$\frac{1}{4}$	$\frac{1}{4}$	0
6	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$

Find marginal distribution of X and Y and evaluate cov(XY). **(05 Marks)**

Module-5

9 a. Define: i) Null hypothesis ii) significance level iii) Type–I and Type–II error. **(06 Marks)**

b. Ten individual are chosen at random from a population and their height in inches are found to be 63, 63, 66, 67, 68, 69, 70, 70, 71, 71. Test the hypothesis that mean height of the universe is 66 inches. Given that ($t_{0.05} = 2.262$ for 9d.f) **(05 Marks)**

c. Find the unique fixed probability vector for the regular stochastic matrix :

$$A = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix}.$$ **(05 Marks)**

OR

10 a. A coin is tossed 1000 times and head turns up 540 times. Decide on the hypothesis that the coin is unbiased. **(06 Marks)**

b. Four coins are tossed 100 times and following results were obtained :

No. of heads	0	1	2	3	4
Frequency	5	29	36	25	5

Fit a binomial distribution for the data and test the goodness of fit $\left(\chi^2_{0.05} = 9.49\right)$. **(05 Marks)**

c. A student's study habit are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand if he does not study one night he is 60% sure not to study the next night. In the long run how often does he study? **(05 Marks)**

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