

Fourth Semester B.E. Degree Examination, Dec.2019/Jan.2020
Engineering Mathematics - IV

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. From Taylor's series method, find $y(0.1)$, considering upto fourth degree term if $y(x)$ satisfying the equation $\frac{dy}{dx} = x - y^2$, $y(0) = 1$. (06 Marks)
- b. Using Runge-Kutta method of fourth order $\frac{dy}{dx} + y = 2x$ at $x = 1.1$ given that $y = 3$ at $x = 1$ initially. (07 Marks)
- c. If $\frac{dy}{dx} = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.040$ and $y(0.3) = 2.090$, find $y(0.4)$ correct upto four decimal places by using Milne's predictor-corrector formula. (07 Marks)

OR

- 2 a. Using modified Euler's method find y at $x = 0.2$ given $\frac{dy}{dx} = 3x + \frac{1}{2}y$ with $y(0) = 1$ taking $h = 0.1$. (06 Marks)
- b. Given $\frac{dy}{dx} + y + zy^2 = 0$ and $y(0) = 1$, $y(0.1) = 0.9008$, $y(0.2) = 0.8066$, $y(0.3) = 0.722$. Evaluate $y(0.4)$ by Adams-Bashforth method. (07 Marks)
- c. Using Runge-Kutta method of fourth order, find $y(0.2)$ for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ taking $h = 0.2$. (07 Marks)

Module-2

- 3 a. Apply Milne's method to compute $y(0.8)$ given that $\frac{d^2y}{dx^2} = 1 - 2y \frac{dy}{dx}$ and the following table of initial values.
- | x | 0 | 0.2 | 0.4 | 0.6 |
|----|---|--------|--------|--------|
| y | 0 | 0.02 | 0.0795 | 0.1762 |
| y' | 0 | 0.1996 | 0.3937 | 0.5689 |
- (06 Marks)
- b. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre polynomials. (07 Marks)
- c. Obtain the series solution of Bessel's differential equation $x^2y'' + xy' + (x^2 + n^2)y = 0$ leading to $J_n(x)$. (07 Marks)

OR

- 4 a. Given $y'' - xy' - y = 0$ with the initial conditions $y(0) = 1$, $y'(0) = 0$, compute $y(0.2)$ and $y'(0.2)$ using fourth order Runge-Kutta method. (06 Marks)
- b. Prove $J_{-1/2}(k) = \sqrt{\frac{2}{\pi k}} \cos k$. (07 Marks)
- c. Prove the Rodfigues formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n y}{dx^n} (x^2 - 1)^n$ (07 Marks)

Module-3

- 5 a. Derive Cauchy-Riemann equations in Cartesian form. (06 Marks)
- b. Discuss the transformation $w = z^2$. (07 Marks)
- c. By using Cauchy's residue theorem, evaluate $\int_C \frac{e^{2z}}{(z+1)(z+2)} dz$ if C is the circle $|z| = 3$. (07 Marks)

OR

- 6 a. Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) |f(z)|^2 = 4|f'(z)|^2$ (06 Marks)
- b. State and prove Cauchy's integral formula. (07 Marks)
- c. Find the bilinear transformation which maps $z = \infty, i, 0$ into $w = -1, -i, 1$. (07 Marks)

Module-4

- 7 a. Find the mean and standard of Poisson distribution. (06 Marks)
- b. In an examination 7% of students score less than 35 marks and 89% of the students score less than 60 marks. Find the mean and standard deviation if the marks are normally distributed given $A(1.2263) = 0.39$ and $A(1.4757) = 0.43$ (07 Marks)
- c. The joint probability distribution table for two random variables X and Y is as follows:

	Y X	-2	-1	4	5
X	1	0.1	0.2	0	0.3
	2	0.2	0.1	0.1	0

Determine:

- i) Marginal distribution of X and Y
 ii) Covariance of X and Y
 iii) Correlation of X and Y (07 Marks)

OR

- 8 a. A random variable X has the following probability function:

x	0	1	2	3	4	5	6	7
P(x)	0	K	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find K and evaluate $P(x \geq 6)$, $P(3 < x \leq 6)$. (06 Marks)

- b. The probability that a pen manufactured by a factory be defective is $1/10$. If 12 such pens are manufactured, what is the probability that
- i) Exactly 2 are defective
 - ii) Atleast two are defective
 - iii) None of them are defective. (07 Marks)
- c. The length of telephone conversation in a booth has been exponential distribution and found on an average to be 5 minutes. Find the probability that a random call made
- i) Ends in less than 5 minutes
 - ii) Between 5 and 10 minutes. (07 Marks)

Module-5

- 9 a. A die is thrown 9000 times and a throw of 3 or 4 was observed 3240 times. Show that the die cannot be regarded as an unbiased die. (06 Marks)
- b. A group of 10 boys fed on diet A and another group of 8 boys fed on a different disk B for a period of 6 months recorded the following increase in weight (lbs):

Diet A:	5	6	8	1	12	4	3	9	6	10
Diet B:	2	3	6	8	10	1	2	8		

Test whether diets A and B differ significantly $t_{0.05} = 2.12$ at 16df. (07 Marks)

- c. Find the unique fixed probability vector for the regular stochastic matrix

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$$

(07 Marks)

OR

- 10 a. Define the terms:
 i) Null hypothesis
 ii) Type-I and Type-II error
 iii) Confidence limits (06 Marks)
- b. The t.p.m. of a Markov chain is given by $P = \begin{bmatrix} 1/2 & 0 & 1/2 \\ 1 & 0 & 0 \\ 1/4 & 1/2 & 1/4 \end{bmatrix}$. Find the fixed probabilities vector. (07 Marks)
- c. Two boys B_1 and B_2 and two girls G_1 and G_2 are throwing ball from one to another. Each boy throws the ball to the other boy with probability $1/2$ and to each girl with probability $1/4$. On the other hand each girl throws the ball to each boy with probability $1/2$ and never to the other girl. In the long run how often does each receive the ball? (07 Marks)

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