Branch & bound.

Set of feasible solutions are Partitioned & the subsets do not have optimal solution are deleted for further consideration

- branch is the first step which involves division of a given problem into two or more sub problems.
- Subproblems are exclusive & independent Problems.

bounding step aids in limiting the growth of state space.

- best solution of the subproblem is identified and used for lower bound of the given Problem.

- lower bound is the optimistic estimate of the best solution

upper bound implies the best feasible solution

- FIFO BB.
- LIFO BB
- LCBB

– CBB

Alg LC (P, root, goal)
// i/p problem P, root & goal node.
// o/p solution node.

```
Begin
    Q = null
    enqueue(Q, root)
    best = root value.
    while (Q != null) then.
        V = Extractmin(Q)  /. Remove best node.
        if bound of V is better than best then.
            for all children u to v do.
                If bound of u is better than best then.
                    enqueue (Q, u)
                    update the best Value.
                Endif
            Endfor
        End if
    End while
End
```

$T(n) = O(n^n)$

Assignment Problem.

Assign unique job to every worker such that total cost is minimized. The constraint of the problem is

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ or } \sum_{j=1}^{n} x_{ij} = 1$$

$$x_{ij} = \begin{cases} 1 & \text{if assigned} \\ 0 & \text{if not assigned.} \end{cases} \quad \forall \, i = 1 \dots n, \; j = 1 \dots n.$$

Objective function is to minimize the following function

$$f = \sum_{j=1}^{n} \sum_{i=1}^{n} c_{ij} \cdot x_{ij}$$

$c_{ij} \rightarrow$ Cost.

$f \rightarrow$ total assignment cost.

9	$\boxed{2}$	7	8
6	4	$\boxed{3}$	7
5	8	$\boxed{1}$	8
7	6	9	$\boxed{4}$

$2+3+1+4$

$lb = 10$

Start
$lb = 10$

$a \to 1$ $a \to 2$ $a \to 3$ $a - 4$.

$lb = 9+3+1+4$ $2+3+1+4$ $7+4+5+4$ $8+3+1+6$.

17   10   20   18

$b \to 1$ $b \to 3$ $b \to 4$

$a \to 2$
$b \to 1$

$6+2+1+4$ $3+2+5+4$ $7+2+1+\cancel{7}$

13   14   17.

$c \to 3$ $c \to 4$
$d \to 4$ $d \to 3$

13 ✓   25

Job assignment.

18	3	15
4	7	14
13	12	7.

lb = 3 + 4 + 7 = 14.

Start

$a \to 1$ $a \to 2$ $a \to 3$

18 + 7 + 7   3 + 4 + 7   15 + 4 + 12.

14   16

32   14   3.1

$b \to 1$ $c \to 1$

$c \to 3$ $b \to 3$

4 + 3 + 7   13 + 3 + 14

14. ✓   3.0

$a \to 2$

$b \to 1$

$c \to 3$ soln = 14.

Knapsack

- fractional Knapsack
- 0/1 knapsack.

Fractional knapsack.

Knapsack need to be filled with items in such a way that capacity is not exceeded the maximum Profit.

Fractional	1/0 Knapsack.
- Items can be spilted	- items cannot be spilted.
- not Several combination of Solution	Several combination of Solution will be obtained.
- optimistic problem solving using greedy	- Combinatoric. optimization solving using bb.
- polynomial complexity.	- exponential complexity.

Ex 1: $W = 6$.

items	w_i	v_i
1	2	8
2	3	6
3	2	4.

items	w_i	v_i	v_i/w_i
1	2	8	$8/2 = 4$
2	3	6	$6/3 = 2$
3	2	4	$4/2 = 2$

Arrange v_i/w_i in descending order.

items	w_i	v_i	v_i/w_i
1	2	8	4
2	3	6	2
3	2	4	2.

$$ub = v + (W - w) * \frac{v_{i+1}}{w_{i+1}}$$

$$= 0 + (6 - 0) * 4$$

$$0 + 6 * 4$$

$$ub = 24.$$

with 1

$w = 2$

$v = 8$

$v_{i+1}/w_{i+1} = 2.$

$ub = v + (W - w) * v_{i+1}/w_{i+1}$

$= 8 + (6-2) * 2$

$8 + (4) * 2.$

$8 + 8 = 16$

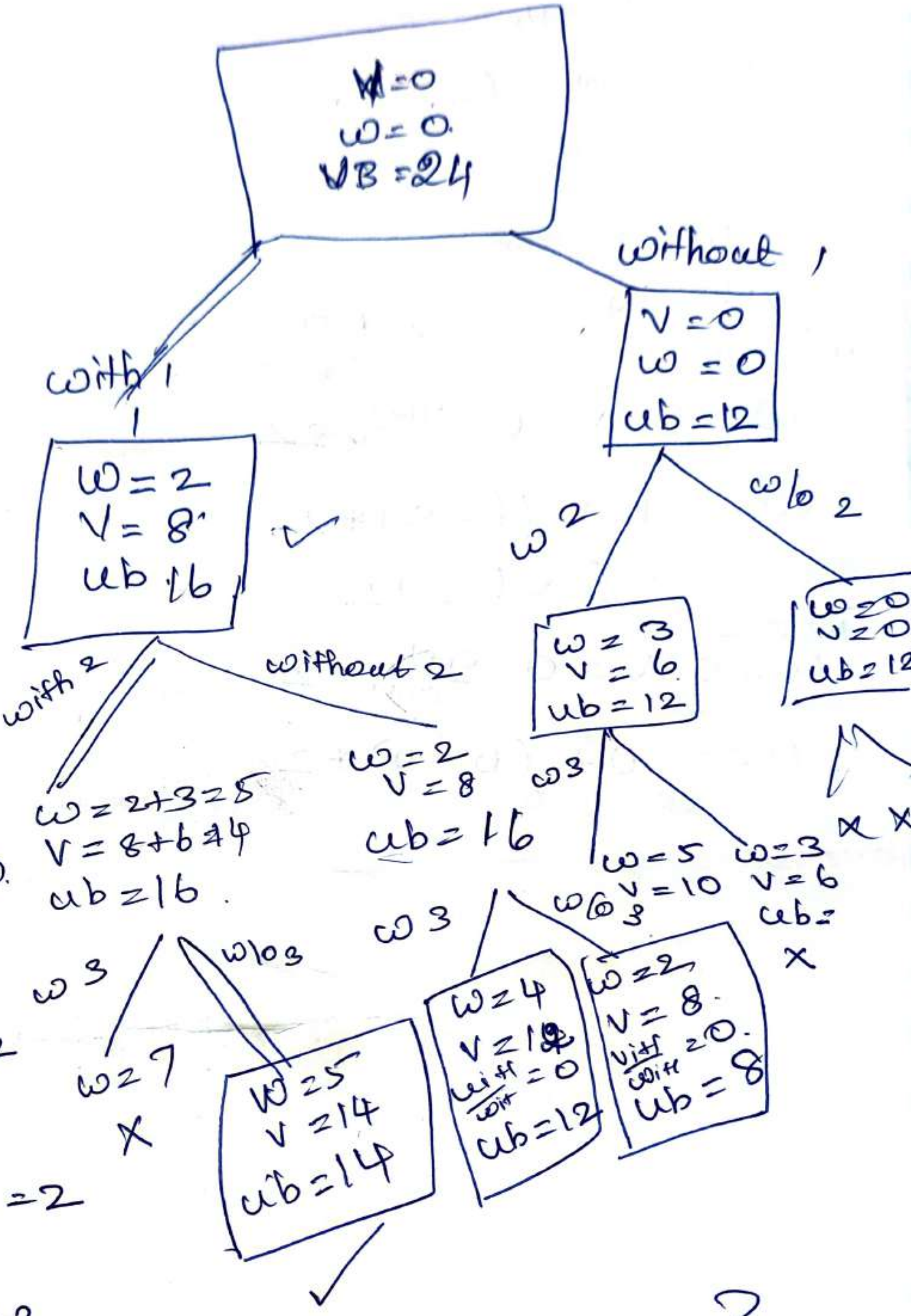
without 1

$ub = V = 0 \quad W = 6 \quad w = 0.$

$v_{i+1}/w_{i+1} = 2.$

$0 + (6 - 0) * 2.$

$0 + 6 * 2 = 12$



with 2

$w = 5, V = 14 \quad v_{i+1}/w_{i+1} = 2$

$ub = 14 + (6 - 5) * 2$

$14 + 2.$

{item 1, item 2}

Optimal solution = 14.

without 2

$w = 2, V = 8 \quad v_{i+1}/w_{i+1} = 2.$

$ub = 8 + (6 - 2) * 2$

$8 + 4 * 2 = 16.$

without 3

$w = 5, V = 14, \frac{v_{i+1}}{w_{i+1}} = 0$

$ub = 14 - (6 - 1) * 0$

$14 - 0$

(2)

item	weight	values.	v_i/w_i
1	3	21	$21/3 = 7$
2	2	16	$16/2 = 8$.
3	4	24	$24/4 = 6$.

$W = 12$

Values Arrange in descending order.

item	weight	Values	v_{i+1}/w_{i+1}
2	2	16.	8.
1	3	21	7
3	4	24	6.

$$ub = V + (W - w) * \frac{v_{i+1}}{w_{i+1}}$$

$$= 0 + (12 - 0) * 8. = 96.$$

$ub = 96.$

with item 1

$w = 2$

$V = 16.$

$v_{i+1}/w_{i+1} = 7$

$ub = 16 + (12 - 2) * 7$

$= 16 + 70 = 86.$

without item 1

$w = 0$

$V = 0$

$v_{i+1}/w_{i+1} = 7$

$\frac{12}{\frac{7}{84}}$

$ub = 0 + (12 - 0) * 7$

$ub = 84$

with item 2

$w = 2 + 3 = 5$

$V = 16 + 21 = 37$

$\frac{v_{i+1}}{w_{i+1}} = 6$

$ub = 37 + (12 - 5) * 6.$

$37 + 42$

79

without item 2

$w = 2$ $V = 16$ $\frac{v_{i+1}}{w_{i+1}} = 6.$

$ub = 16 + (12 - 2) * 6.$ $\frac{12}{\frac{6}{72}}$

$ub = 16 + 60 = 76$

with item 3

$w = 5 + 4 = 9$

$V = 37 + 24 = 61$

$v_{i+1}/w_{i+1} = 0$

$ub = 61 + (12 - 9) * 0.$

~~Lab~~ Assignment

7-42

without item 3
$w = 5$ $v = 37$ $\frac{v_{i+1}}{w_{i+1}} = 0.$

$ub = 37 + (12 - 5) * 0.$

item 3
$w = 4 + 2 = 6$
$V = 24 + 16$ 40
$\frac{v_{i+1}}{w_{i+1}} = 0.$

$ub = 40 - (12 - 6) * 0.$
$ub = 40.$

w/o 3
$w = 2$ $V = 16.$ $v_{i+1}/w_{i+1} = 0$

$ub = 16 + (12 - 2) * 0$
16.

with item 2
$w = 2$ $V = 21$ $\frac{v_{i+1}}{w_{i+1}} = 6.$

$ub = 21 + (12 - 2) * 6.$
$ub = \cancel{76}21 + 60$

without 2
$w = 0$ $V = 0$ $\frac{v_{i+1}}{w_{i+1}} = 6.$

$\frac{12}{\frac{6}{72}}$

$ub = 0 + (12 - 0) * 6.$
$ub = 72$

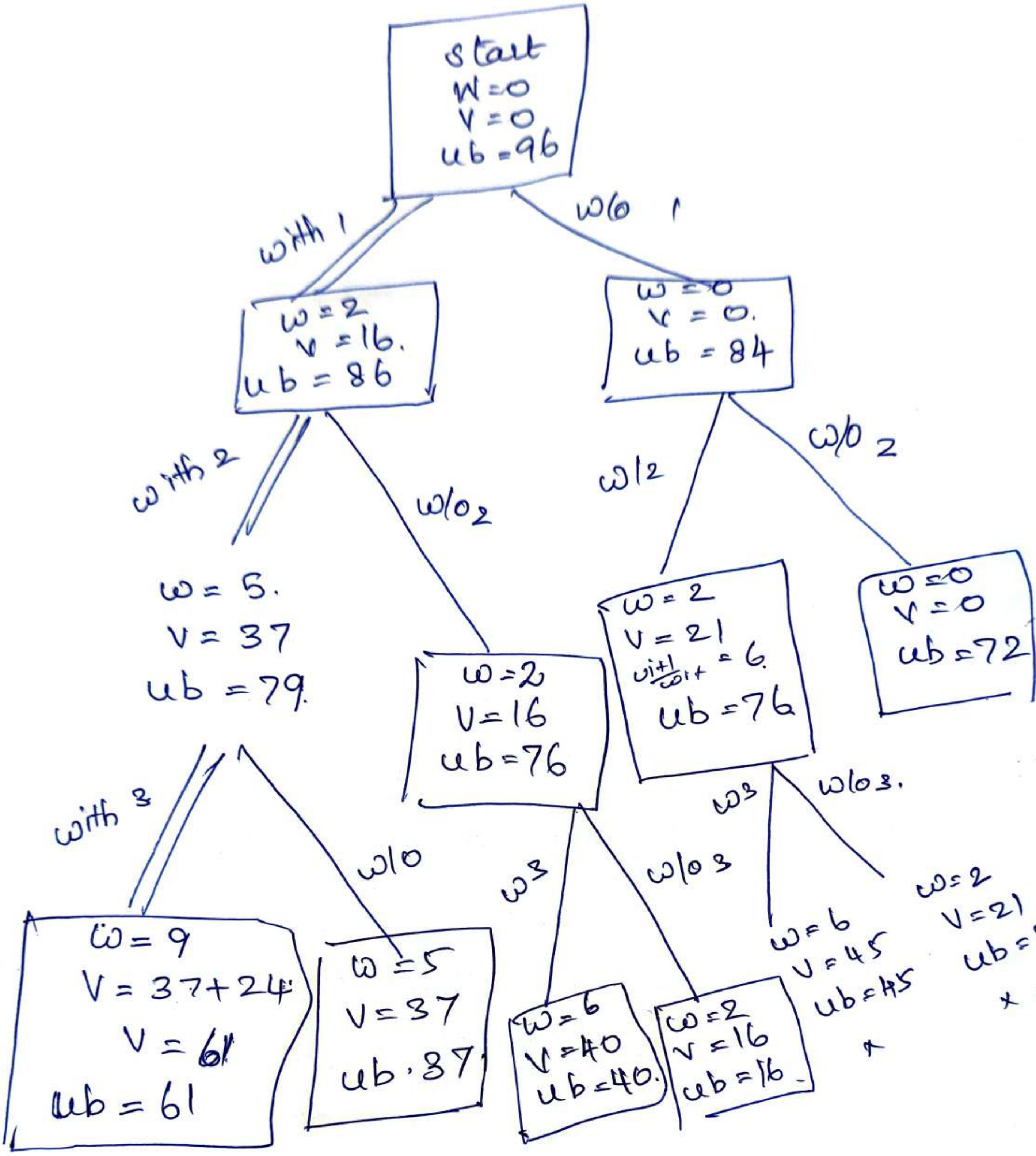
with item 3

$w = 2 + 4 = 6$
$V = 21 + 24 = 45$
$\frac{v_{i+1}}{w_{i+1}} = 0$

$ub = 45 + (12 - 6) * 0.$
$= 45 + 0.$
$ub = 45$

without item 3
$w = 2$
$V = 21$

$ub = 21 + (12 - 2) * 0$
$ub = 21$



Items = { item 1, item 2, item 3 }.

optimal solution = 61

Ex: 3.

item	weight	Value	vi/wi
1	4	10	10/4 = 2.5
2	6	15	15/6 = 2.5
3	3	6	6/3 = 2
4	4	8	8/4 = 2
5	2	4	4/2 = 2

10/4 2.5 15/6 2. W = 12.

1	4	16	16/4 = 4
2.	5	15	15/5 = 3.
3	3	6	6/3 = 2.
4.	4.	20	20/4 = 5
5	2	4	4/2 = 2.

Reorder.

item	w	Value.	$\frac{vi+1}{wi+1}$
4	4	20	20/4 = 5
1	4	16.	16/4 = 4.
2	5	15	15/5 = 3
3	3	6	6/3 = 2.
5	2	4	4/2 = 2.

W = 12.

$$ub = V + (W - w) * \frac{vi+1}{wi+1}$$

$$= 0 + (12 - 0) * 5.$$

$$12 * 5$$

$$ub = 60.$$

With 4^{th} item

$w = 4 \quad V = 20. \quad \frac{v_{i+1}}{w_{i+1}} = 4$

$ub = 20 + (12 - 4) * 20.$

$= 20 + 8 * 20.$

$20 + 160$

$ub = 180$

without 4^{th} item

$w = 0; \; V = 0, \quad \frac{v_{i+1}}{w_{i+1}} = 4.$

$ub = 0 + (\underset{4}{12} - 0) * 4$

$ub = 48$

with 1^{st}

$w = 8, \; V = 36 \quad v_i = 9$

$ub = 36 + (12 - 8) * 9$

$4 * 9$

$36 + 32$

$ub = 68$

w/o 1^{st}

$w = 4, \; V = 20. \quad \frac{v_{i+1}}{w_{i+1}} = 9.$

$20 + (12 - 4) * 9$

$80 + 72$

152

9	8
18	16
27	24
36	32
45	50
54	
63	
72	
81	

With 4th item

$w = 4 \quad v = 20 \quad \frac{v_{i+1}}{} = 4$

2