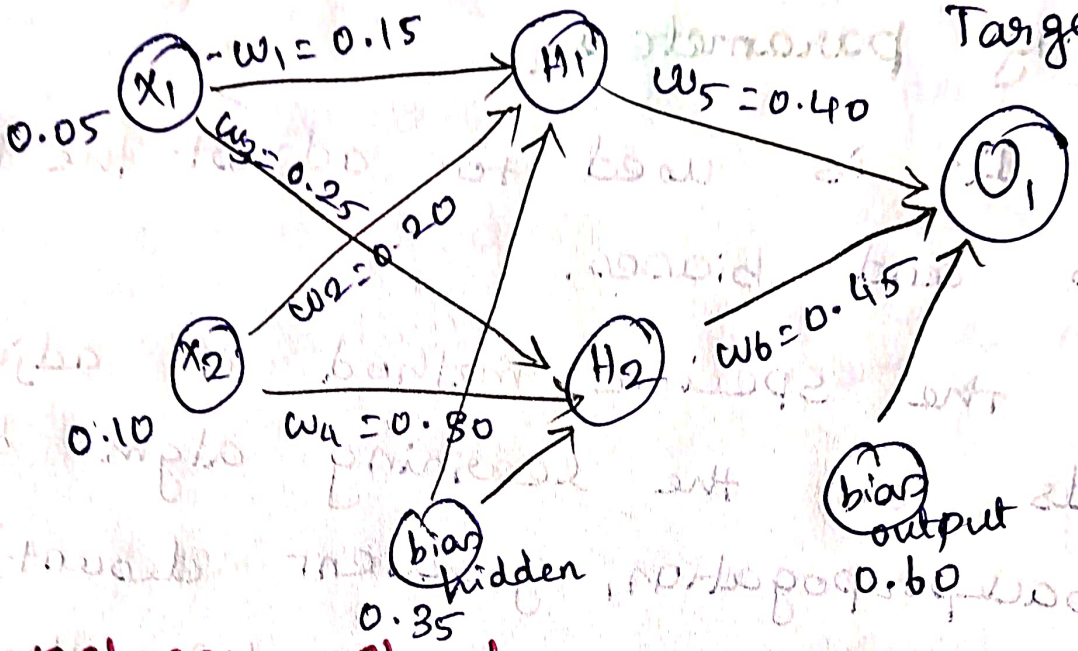
Backpropagation Algorithms

learning rate = 0.5

Target = 0.01



Neural network Structure

* Input layer: 2 neurons ($x1, x2$)
* Hidden layer: 2 neurons (H_1, H_2)
* Output layer: 1 neuron (O_1)
* Activation function: Sigmoid(x) = $\frac{1}{1+e^{-x}}$
* Learning Rate (η): 0.5.

Step 1: Input and Initial weights.

Input:

$x_1 = 0.05, x_2 = 0.10$

Target output

$T = 0.01$

Initial weights

from Input to Hidden

$w_1 = 0.15 \quad w_2 = 0.20, \quad w_3 = 0.25 \quad w_4 = 0.30$

From Hidden to output:

$w_5 = 0.40, \quad w_6 = 0.45$

Biases:

Hidden = 0.35, Output = 0.60.

Step 2: Forward pass.

Hidden layer

$net\_h1 = (x_1 \cdot w_1) + (x_2 \cdot w_2) + bias\ hidden$

$= (0.05 * 0.15) + (0.10 * 0.20) + 0.35$

$= 0.3775$

$$out_{h_1} = \text{sigmoid}(0.3775)$$

$$= \frac{1}{1+e^{-0.3775}}$$

$$= 0.5933$$

$$net_{h_2} = (x_1 \cdot w_3) + (x_2 \cdot w_4) + \text{bias hidden}$$

$$= (0.05 * 0.25) + (0.10 * 0.30) + 0.35$$

$$= 0.3925$$

$$out_{h_2} = \text{sigmoid}(0.3925)$$

$$= \frac{1}{1+e^{-0.3925}}$$

$$= 0.5969$$

Output layer

$$net_{y_1} = (out_{h_1} \cdot w_5) + (out_{h_2} \cdot w_6) + \text{bias output}$$

$$= (0.5933 * 0.40) + (0.5969 * 0.45) + 0.60$$

$$= 1.1059$$

$$out_{y_1} = \text{sigmoid}(1.1059)$$

$$= \frac{1}{1+e^{-1.1059}} = 0.7514.$$

Step 3 : Calculate Error

$$E_{total} = \sum \frac{1}{2} * (Target - output)^2$$

$$= \frac{1}{2} * (0.01 - 0.7514)^2$$

$$= 0.2748.$$

Step 4 : Backpropogation

Updating the neuron values (step 4)

Output layer:

$$\delta_y = (out\ y - Target) \times \boxed{\sigma'(out\ y)}$$

$$\hookrightarrow (out\ y)(1 - out\ y)$$

$$\sigma'(x) = x(1-x)$$

Hidden layer:

$$\delta_H = \delta_y \times w \times \sigma'(out\ H)$$

O/p layer:

$$\delta_y = (out\ y - Target) \times \sigma'(out\ y)$$

$$= (0.7514 - 0.01) \times (0.7514(1 - 0.7514))$$

$$\delta_{01} = 0.1385$$

Hidden layer

$$\delta_H = \delta_y \times w \times \sigma'(out_H)$$

$$\delta_{h_1} = \delta_{o_1} \times w_5 \times \sigma'(out\ h_1) \rightarrow out\ h_1(1-out h_1)$$

$$= 0.1385 \times 0.40 \times 0.5933\ (1-0.5933)$$

$$= 0.01337$$

$$\delta_{h_2} = \delta_{o_1} \times w_6 \times \sigma'(out\ h_2)$$

$$\hookrightarrow out h_2(1-out h_2)$$

$$= 0.1385 \times 0.45 \times 0.5969\ (1-0.5969)$$

$$= 0.01499$$

Step 5: weight & bias Update (Hidden to output layer)

$$w_{new} = w_{old} - \eta \times \delta_y \times out_H$$

$$bias_{output\ new} = bias_{output\ old} - \eta \times \delta_y$$

$$w_{5new} = w_{5old} - \eta \times \delta_{o_1} \times out_{h_1}$$

$$= 0.40 - 0.5 \times 0.1385 \times 0.5933$$

$$= 0.3589$$

$w_{b\,new} = w_{b\,old} - \eta \times \delta_{o_1} \times out_{h_2}$

$= 0.45 + 0.5 \times 0.1385 \times 0.5969$

$= 0.40866$

$bias\,o/p_{new} = 0.60 - 0.5 \times 0.1385$

$= 0.5307$

Input to hidden layer.

$w_{new} = w_{old} - \eta \times (x)$

$bias_{hidden\,new} = bias_{hidden\,old} - \eta \times \Sigma(\delta_H)$

$w_{1\,new} = w_{1\,old} - \eta \times \delta h_1 \times x_1$

$= 0.15 - 0.5 \times 0.01337 \times 0.05$

$= 0.1496$

$w_{2\,new} = w_{2\,old} - \eta \times \delta h_1 \times x_2$

$= 0.20 - 0.5 \times 0.9337 \times 0.10$

$= 0.19933$

$w_{3\,new} = w_{3\,old} - \eta \times \delta h_2 \times x_1$

$= 0.25 - 0.5 \times 0.01499 \times 0.05$

$= 0.2496$

$$w_4 \text{ new} = w_4 \text{ old} - \eta \times \delta h_2 \times x_2$$

$$= 0.30 - 0.5 \times 0.01499 \times 0.10$$

$$= 0.2992$$

$$\text{biashidden}_{\text{new}} = \text{biashidden}_{\text{old}} - \eta \times \Sigma(S_H)$$

$$= 0.35 - 0.5 \times (0.01337 + 0.01499)$$

$$= 0.33582$$

Step 6: Summary Table & updated diagram.

weight/bias	updated value
w_1 new	0.1496
w_2 new	0.1993
w_3 new	0.2496
w_4 new	0.2992
biashidden new	0.3358
w_5 new	0.3589
w_6 new	0.4086
$\text{bias}_{O/P}$ new	0.5307