Karatsuba algorithm using divide & conquer

$a = x_h y_h$

$d = x_l y_l$

$e = (x_h + x_L) * (y_h \times y_L) - a - d$

$xy = ab^n + eb^{n/2} + d$

ex 1234×4321

$a1 = 12 \times 43$ $\quad d1 = 34 \times 21$

$e1 = 12 + 34 \times 43 + 21 - a1 - d1$

solving for a1

$a_1 = 12 \times 43$

$a_2 = 1 \times 4 = 4$

$d_2 = 2 \times 3 = 6$

$e_2 = (1+2)(4+3) - a_2 - d_2$

$= 11$

$xy = ar^n + eb^{n/2} + d$

$= 12 \times 43$

$= 4 \times 10^2 + 11 \times 10^1 + 6 = 516$

solving for d_1

$d_1 = 34 \times 21$

$a_2 = 3 \times 2 = 6$

$d_2 = 4 \times 1 = 4$

$e_2 = (3+4)(2+1) - a_2 - d_2 = 11$

$xy = 6 \times 10^2 + 11 \times 10 + 4 = 714$

solving for $e1 = (46 \times 64) - a_1 - d_1$

$a_2 = 4 \times 6 = 24$

$d_2 = 6 \times 4 = 24$

$e_2 = (4+6)(6+4) - a_2 - d_2 = 52$

$xy = 24 \times 10^2 + 52 \times 10^1 + 24 - 714 - 516 = 1714$

$a_1 = 516 \quad d_1 = 714 \quad e_1 = 1714$

Plugging into xy

$$ar^n + er^{n/2} + d.$$

$$xy = (516)10^4 + (1714)10^2 + 714$$

$$xy = 5,332,114.$$

$$T(n) = 3T(n/2) + O(n)$$

$$= O(n^{\log_2 3})$$

$$\boxed{T(n) = O(n^{\log_2 3})}$$

Algorithm karatsuba (a,b):
  // input a,b
  // output solution
  If a or b has one digit then
      return a * b.
  else :
    let n be number of digit in max{a,b}.
    let aL and aR be left & right halves of a.
    Let bL and bR be left & right halves of b.
    let x_1 hold karatsuba(a_L, b_L)
    let x_2 hold karatsuba(a_L+aR, b_L+bR)
    let x_3 hold karatsuba(aR, bR)
    return $x_1 * 10^n + (x_2 - x_1 - x_3) * 10^{n/2} + x_3$.
  end if
end