

Algorithm Dijkstra ( $G, s$ ).

// input: A weighted graph  $G = (V, E)$ , vertex  $s$ .

// output: The length  $d_v$  of the shortest

Path from  $s$  to  $v$  and its

Perultimate vertex  $P_v$  for every vertex  
 $v$  in  $V$

Initialize ( $Q$ )

for every vertex  $v$  in  $V$

$d_v \leftarrow \infty$ ,  $P_v \leftarrow null$

insert ( $Q, v, d_v$ )

$d_s \leftarrow 0$ ; Decrease ( $Q, s, d_s$ ) // update priority of  $s$  with  $d_s$

$V_T \leftarrow \emptyset$

for  $i \leftarrow 0$  to  $|V| - 1$  do

$u^* \leftarrow \text{DeleteMin}(Q)$  // delete the minimum  
Priority element.

$V_T \leftarrow V_T \cup \{u^*\}$

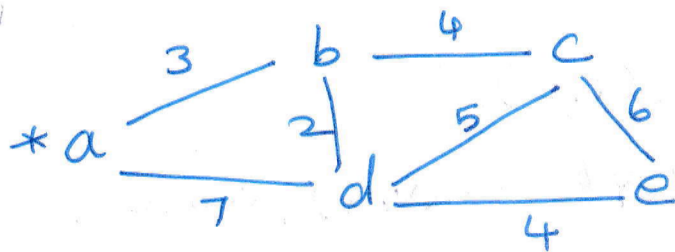
for every vertex  $u$  in  $V - V_T$  that is  
adjacent to  $u^*$  do

if  $d_{u^*} + w(u^*, u) < d_u$

$d_u \leftarrow d_{u^*} + w(u^*, u)$

$P_u \leftarrow u^*$

Decrease ( $Q, u, d_u$ )



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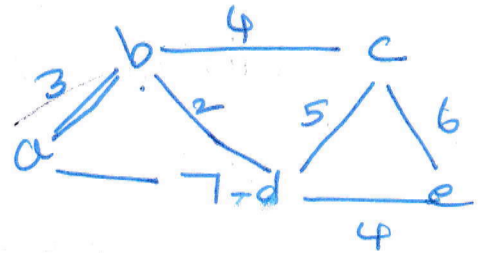
Tree  
Vertices

Remaining  
Vertices

Illustration

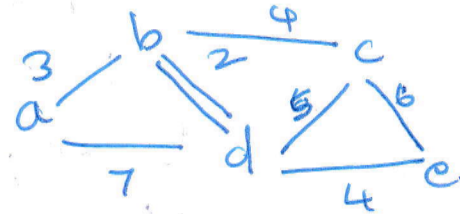
$a(-, 0)$

$b(a, 3)$   $c(-\infty)$   
 $d(a, 7)$   $e(-\infty)$



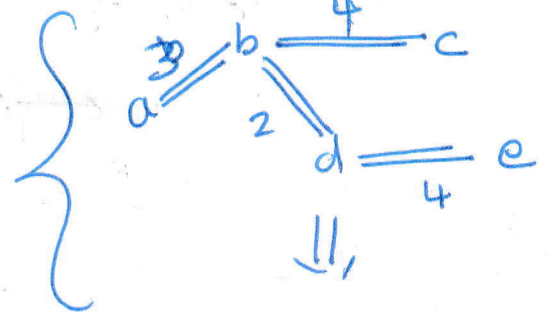
$b(a, 3)$

$c(b, 3+4)$   $d(b, 3+2)$ ,  
 $e(-\infty)$



$d(b, 5)$

$c(b, 4)$   $e(d, 5+4)$



$c(b, 4)$

$e(d, 9)$

length.  
from  $a-b$  3  
from  $a$  to  $d$ :  $a-b-d$  5  
from  $a$  to  $c$ :  $a-b-c$  7  
from  $a$  to  $e$ :  $a-b-d-e$  9

$$T(n) = O(\sqrt{n})$$

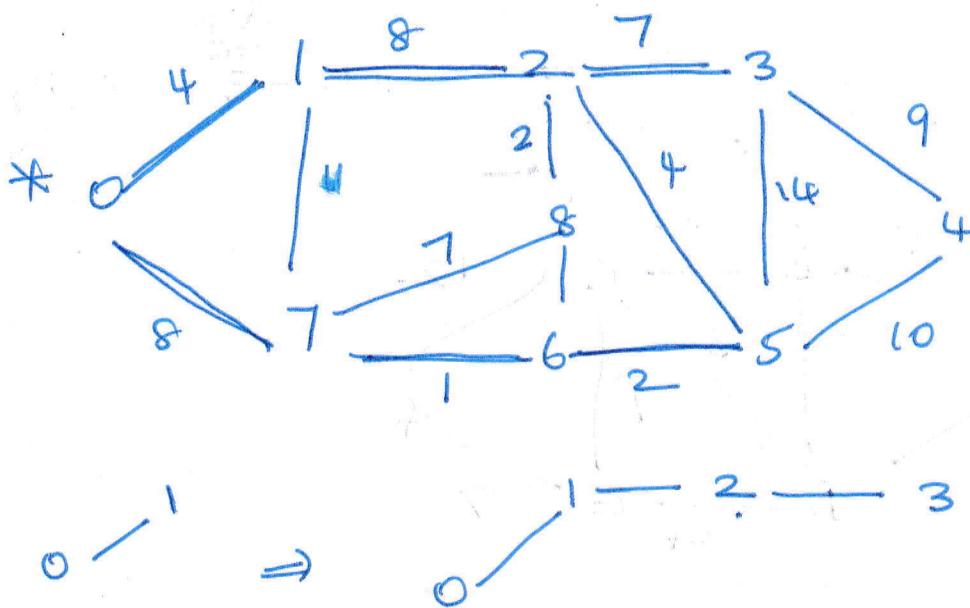
Two operation.

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\* move  $u^*$  from the fringe to the set of tree vertices.

\* for each remaining fringe vertex  $u$  that is connected to  $u^*$  by an edge of weight  $w(u^*, u)$  such that  $d_{u^*} + w(u^*, u) < d_u$  update the labels of  $u$  by  $u^*$  and  $d_{u^*} + w(u^*, u)$

Example Problems :



distance from 0 to 1 is 4

0 to 2 is  $0-1-2 = 12$

0 to 3 is  $0-1-2-3 = 19$

0 to 4 is  $0-7-6-5-4 = 2$

0 to 5 is  $0-7-6-5 = 1$

6 is  $0-7-6 = 9$

7 is  $0-7 = 8$

8 is  $0-1-2-8 = 14$