



## Experiment Number:2

Roll Number	Class	Date of Performance	Date of Submission	Signature
42428	BE 8	20/05/2021		

**Aim:** Write a program to elaborate Lost call system/ delay system used in the analysis of voice/data Traffic.

**Apparatus:** Algorithm, Traffic Table, Machine with MATLAB and Printer.

### Theory:

#### Traffic Engineering:

The telecommunication system has to service the voice traffic and data traffic. The traffic is defined as the occupancy of the server. The basic purpose of the traffic engineering is to determine the conditions under which adequate service is provided to subscribers while making economical use of the resources providing the service. The functions performed by the telecommunication network depends on the applications it handles. Some major functions are switching, routing, flow control, security, failure monitoring, traffic monitoring, accountability internetworking and network management.

#### Traffic Statistics:

The following statistical information provides answer for the requirement of trunk circuits for a given volume of offered traffic and grade of service to interconnect the end offices. The statistical descriptions of a traffic is important for the analysis and design of any switching network.

**1. Calling rate.** This is the average number of requests for connection that are made per unit time. If the instant in time that a call request arises is a random variable, the calling



rate may be stated as the probability that a call request will occur in a certain short interval of time.

If 'n' is the average number of calls to and from a terminal during a period T seconds, the calling rate is defined as:  $\lambda = n / T$ ,

In telecommunication system, voice traffic and data traffic are the two types of traffic. The calling rate ( $\lambda$ ) is also referred as average arrival rate. The average calling rate is measured in calls per hour.

**2. Holding time.** The average holding time or service time 'h' is the average duration of occupancy of a traffic path by a call. For voice traffic, it is the average holding time per call in hours or 100 seconds and for data traffic, average transmission per message in seconds. The reciprocal of the average holding time referred to as service rate ( $\mu$ ) in calls per hour is given as

$$\mu = 1 / h$$

**3. Distribution of destinations:** Number of calls receiving at a exchange may be destined to its own exchange or remotest exchange or a foreign exchange. The destination distribution is described as the probability of a call request being for particular destination. As the hierarchical structure of telecommunication network includes many intermediate exchanges, the knowledge of this parameter helps in determining the number of trunks needed between individual centres.

**4. User behavior:** The statistical properties of the switching system are a function of the behavior of users who encounter call blocking. The system behaves differently for different users. The user may abandon the request if his first attempt to make a call is failed. The user may make repeated attempts to setup a call. Otherwise the user may wait some times to make next attempt to setup a call. These behavior varies person to person and also depends on the situation.

**5. Average occupancy:** If the average number of calls to and from a terminal during a period T seconds is 'n' and the average holding time is 'h' seconds, the average occupancy of the terminal is given by

$$A = nh / T = \lambda h = \lambda / \mu$$

Thus, average occupancy is the ratio of average arrival rate to the average service rate. It is measured in Erlangs. Average occupancy is also referred as traffic flow or traffic intensity or carried traffic.

### **Various parameters related to traffic pattern:**

**Busy hour.** Traditionally, a telecommunication facility is engineered on the intensity of traffic during the busy hour in the busy session. The busy hour vary from exchange to exchange, month to month and day to day and even season to season. The busy hour can be defined in a variety of ways. In general, the busy hour is defined as the 60 minutes interval in a day, in which the traffic is the highest. Taking into account the fluctuations in traffic, CCITT



in its recommendations E.600 defined the busy hour as follows.

1. **Busy hour :** Continuous 60 minutes interval for which the traffic volume or the number of call attempts is greatest.
2. **Peak busy hour :** It is the busy hour each day varies from day to day, over a number of days.
3. **Time consistent busy hour :** The 1 hour period starting at the same time each day for which the average traffic volume or the number of call attempts is greatest over the days under consideration.

In order to simplify the traffic measurement, the busy hour always commences on the hour, half hour, or quarter hour and is the busiest of such hours. The busy hour can also be expressed as a percentage (usually between 10 and 15%) of the traffic occurring in a 24 hour period.

**Call completion rate (CCR):** Based on the status of the called subscriber or the design of switching system the call attempted may be successful or not. The call completion rate is defined as the ratio of the number of successful calls to the number of call attempts. A CCR value of 0.75 is considered excellent and 0.70 is usually expected.

**Busy hour call attempts:** It is an important parameter in deciding the processing capacity of an exchange. It is defined as the number of call attempts in a busy hour.

**Busy hour calling rate:** It is a useful parameter in designing a local office to handle the peak hour traffic. It is defined as the average number of calls originated by a subscriber during the busy hour.

**Day-to-day hour traffic ratio:** It is defined as the ratio of busy hour calling rate to the average calling rate for that day. It is normally 6 or 7 for rural areas and over 20 for city exchanges.

**Units of Telephone Traffic :-** Traffic intensity is measured in two ways. They are (a) Erlangs and (b) Cent call seconds (CCS).

**Erlangs:** The international unit of traffic is the Erlangs. It is named after the Danish Mathematician, Agner Krarup Erlang, who laid the foundation to traffic theory in the work he did for the Copenhagen telephone company starting 1908. A server is said to have 1 erlang of traffic if it is occupied for the entire period of observation. More simply, one erlang represents one circuit occupied for one hour.

The maximum capacity of a single server (or channel) is 1 erlang (server is always busy). Thus the maximum capacity in erlangs of a group of servers is merely equal to the number of servers.

Thus, the traffic intensity which is the ratio of the period for which the server is occupied to the total period of observation is measured in erlangs.

**Cent call seconds (CCS).** It is also referred as hundred call seconds. CCS as a measure of traffic intensity is valid only in telephone circuits. CCS represents a call time product. This is used as a measure of the amount of traffic expressed in units of 100 seconds. Sometimes call



seconds (CS) and call minutes (CM) are also used as a measure of traffic intensity. The relation between erlang and CCS is given by ,

$$1E = 36 CCS = 3600 CS = 60 CM$$

**Grade of Service (GOS):** For non-blocking service of an exchange, it is necessary to provide as many lines as there are subscribers. But it is not economical. So, some calls have to be rejected and retried when the lines are being used by other subscribers. The grade of service refers to the proportion of unsuccessful calls relative to the total number of calls.

GOS is defined as the ratio of lost traffic to offered traffic.

$$GOS = \text{Blocked Busy Hour calls} / \text{Offered Busy Hour calls}$$

$$GOS = (A - A_0) / A$$

Where,  $A_0$  = carried traffic

$A$  = offered traffic

$A - A_0$  = lost traffic.

The smaller the value of grade of service, the better is the service. The recommended GOS is 0.002, i.e. 2 call per 1000 offered may lost. In a system, with equal no. of servers and subscribers, GOS is equal to zero.

GOS is applied to a terminal to terminal connection. But usually a switching centre is broken into following components ,

- (a) an internal call (subscriber to switching office)
- (b) an outgoing call to the trunk network (switching office to trunk)
- (c) the trunk network (trunk to trunk)
- (d) a terminating call (switching office to subscriber).

The GOS calculated for each component is called component GOS. The overall GOS is in fact approximately the sum of the component grade of service.

There are two possibilities of call blocking. They are (a) Lost system and (b) Waiting system. In lost system, a suitable GOS is a percentage of calls which are lost because no equipment is available at the instant of call request. In waiting system, a GOS objective could be either the percentage of calls which are delayed or the percentage which are delayed more than a certain length of time.

### **LOSS SYSTEMS :**

The service of incoming calls depends on the number of lines. If number of lines equal to the number of subscribers, there is no question of traffic analysis. But it is not only uneconomical but not possible also. So, if the incoming calls finds all available lines busy, the call is said to be blocked. The **blocked** calls can be handled in two ways.

The type of system by which a blocked call is simply refused and is lost is called **loss system**. Most notably, traditional analog telephone systems simply block calls from entering the system, if no line available. Modern telephone networks can statistically multiplex calls or even packetize for lower blocking at the cost of delay. In the case of data networks, if dedicated buffer and lines are not available, they block calls from entering the system.



In the second type remains in the system and waits for a free line. This type of system is known as **delay system**.  
of system, a  
blocked call

These two types differs in network, way of obtaining solution for the problem and GOS.

For loss system, the GOS is probability of blocking. For delay system, GOS is the probability of waiting.

Erlang determined the GOS of loss systems having N trunks, with offered traffic A, with the following assumptions. (a) Pure chance traffic (b) Statistical equilibrium (c) Full availability and (d) Calls which encounter congestion are lost. The first two are explained in previous section. A system with a collection of lines is said to be a fully-accessible system, if all the lines are equally accessible to all in arriving calls. For example, the trunk lines for inter office calls are fully accessible lines. The lost call assumption implies that any attempted call which encounters congestion is immediately cleared from the system. In such a case, the user may try again and it may cause more traffic during busy hour.

The Erlang loss system may be defined by the following specifications:

1. The arrival process of calls is assumed to be Poisson with a rate of  $\lambda$  calls per hour.
2. The holding times are assumed to be mutually independent and identically distributed random variables following an exponential distribution with  $1/\mu$  seconds.
3. Calls are served in the order of arrival.

There are three models of loss systems. They are :

1. Lost calls cleared (LCC)
2. Lost calls returned (LCR)
3. Lost calls held (LCH)

### 1. Lost Calls Cleared (LCC) System:

The LCC model assumes that, the subscriber who does not avail the service, hangs up the call, and tries later. The next attempt is assumed as a new call. Hence, the call is said to be cleared.

This also referred as blocked calls lost assumption. The first person to account fully and accurately for the effect of cleared calls in the calculation of blocking probabilities was A.K.

Erlang in 1917.

$$P(k) = (A^k / k!) / \sum_{k=0}^N (A^k / k!)$$

The probability distribution is called the truncated **Poisson distribution** or **Erlang's loss distribution**.

In particular when  $k = N$ , the probability of loss is given by;

$$P(N) = B(N, A) = A^N / N! \sum_{k=0}^N (A^k / k!) \quad \text{where } A = \lambda/\mu .$$

This result is variously referred to as **Erlang's formula of the first kind, the Erlangs-**



### **B formula or Erlangs loss formula.**

The Erlang B formula gives the time congestion of the system and relates the probability of blocking to the offered traffic and the number of trunk lines.

### **2.Lost Calls Returned (LCR) System :**

In LCC system, it is assumed that unserviceable requests leave the system and never return. This assumption is appropriate where traffic overflow occurs and the other routes are in other calls service. If the repeated calls not exist, LCC system is used. But in many cases, blocked calls return to the system in the form of retries. Some examples are subscriber concentrator systems, corporate tie lines and PBX trunks, calls to busy telephone numbers and access to WATS lines. Including the retried calls, the offered traffic now comprise two components viz., new traffic and retry traffic.

The model used for this analysis is known as lost calls returned(LCR) model. The following assumptions are made to analyse the CLR model.

1. All blocked calls return to the system and eventually get serviced, even if multiple retries are required.
2. Time between call blocking and regeneration is random statistically independent of each other. This assumption avoid complications arising when retries are correlated to each other and tend to cause recurring traffic peaks at a particular waiting time interval.
3. Time between call blocking and retry is somewhat longer than average holding time of a connection. If retries are immediate, congestion may occur or the network operation becomes delay system.

### **3.Lost Calls Held (LCH) System :**

In a lost calls held system, blocked calls are held by the system and serviced when the necessary facilities become available. The total time spend by a call is the sum of waiting time and the service time. Each arrival requires service for a continuous period of time and terminates its request independently of its being serviced or not. If number of calls blocked, a portion of it is lost until a server becomes free to service a call. An example of LCH system is the time assigned speech interpolation (TASI) system.

LCH systems generally arise in real time applications in which the sources are continuously in need of service, whether or not the facilities are available. Normally, telephone network does not operate in a lost call held manner. The LCH analysis produces a conservative design that helps account for retries and day to day variations in the busy horn calling intensities. A TASI system concentrates some number of voice sources onto a smaller number of transmission channels. A source receives service only when it is active. If a source becomes active when all channels are busy, it is blocked and speech clipping occurs. Each speech segment starts and stops independently of whether it is served or not.



Digital circuit multiplication DCM) systems in contrast with original TASI, can delay speech for a small amount of time, when necessary to minimize the clipping.

LCH are easily analysed to determine the probability of the total number of calls in the system at any one time. The number of active calls in the system at any time is identical to the number of active sources in a system capable of carrying all traffic as it arises. Thus the distribution of the number in the system is the poisson distribution. The poisson distribution given as,

$$P(x) = \frac{\mu^x}{x!} e^{-\mu}$$

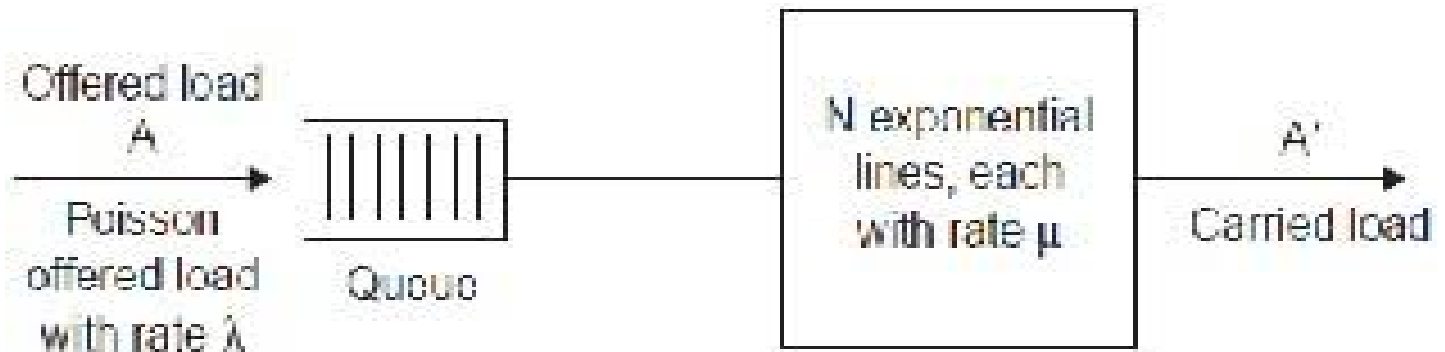
The probability that k sources requesting service are being blocked is simply the probability that k + N sources are active when N is the number of servers.

#### **DELAY SYSTEMS :**

The delay system places the call or message arrivals in a queue if it finds all N servers (or lines) occupied. This system delays non-serviceable requests until the necessary facilities become available. These systems are variously referred to as delay system, waiting-call systems and queueing systems. The delay systems are analysed using queueing theory which is sometimes known as waiting line theory. This delay system have wide applications outside the telecommunications. Some of the more common applications are data processing, supermarket check out counters, aircraft landings, inventory control and various forms of services.

Consider that there are k calls (in service and waiting) in the system and N lines to serve the calls. If  $k \leq N$ , k lines are occupied and no calls are waiting. If  $k > N$ , all N lines are occupied and  $k - N$  calls waiting. Hence a delay operation allows for greater utilization of servers than does a loss system. Even though arrivals to the system are random, the servers see a somewhat regular arrival pattern. A queueing model for the Erlang delay system is shown in Fig.





The basic purpose of the investigation of delay system is to determine the probability distribution of waiting times. From this, the average waiting time  $W$  as random variable can be easily determined. The waiting times are dependent on the following factors :

1. Number of sources
2. Number of servers
3. Intensity and probabilistic nature of the offered traffic
4. Distribution of service times
5. Service discipline of the queue.

In a delay system, there may be a finite number of sources in a physical sense but an infinite number of sources in an operational sense because each source may have an arbitrary number of requests outstanding. If the offered traffic intensity is less than the servers, no statistical limit exists on the arrival of calls in a short period of time. In practice, only finite queue can be realised. There are two service time distributions. They are constant service times and exponential service times. With constant service times, the service time is deterministic and with exponential, it is random. The service discipline of the queue involves two important factors.

1. Waiting calls are selected on of first-come, first served (FCFS) or first-in-first-out (FIFO) service.
2. The second aspect of the service discipline is the length of the queue. Under heavy loads, blocking occurs. The blocking probability or delay probability in the system is based on the queue size in comparison with number of effective sources.

Now, the probability of waiting (the probability of finding all lines occupied) is equal to  $P(W > 0) = C(N, A)$





$$C(N, A) = \frac{A^N}{N!} / \sum_{k=0}^N \frac{A^k}{k!} + \frac{A^N}{N!} \left( \frac{A}{N-A} \right) \text{----- Erlang's second formula}$$

Simplifying

$$\frac{1}{C(N,A)} = \frac{\sum_{k=0}^N \frac{A^k}{k!}}{\frac{A^N}{N!}} + \frac{\frac{A^N}{N!} \left( \frac{A}{N-A} \right)}{\frac{A^N}{N!}}$$

$$\frac{1}{C(N,A)} = \frac{1}{B} + \frac{A}{N-A} \text{----- Erlang's delay formula or Erlang's C formula.}$$

$$\text{Prob. (delay)} = P(> 0) C(N, A) = \frac{BN}{N - A (1 - B)} \text{ ,,}$$

where B = Blocking probability for a LCC system

N = Number of servers

A = Offered load (Erlangs)

### COMBINED LOSS AND DELAY SYSTEM :

In the Erlang loss system, no waiting is allowed, while in the Erlang delay system, if number of sources is greater than the channels, they are placed in queue and no loss occurs. The combined delay and loss system can be modelled by the birth and death process.

### Algorithm :

- Take the value of N
- Declare the output vector P
- Apply the Earlang's first distribution formula for traffic A varying from 1 to N
- Plot P vs A



### Conclusion:

In this experiment we understood how traffic is defined in cellular system and its related parameters. We understood Lost call system and implemented the Erlang's first distribution plot to demonstrate the same. As the traffic increases, the probability of a call getting lost increases for given number of trunks, which can be observed from the graph plotted.

### Code :

```
clear all;
close all;
clc;

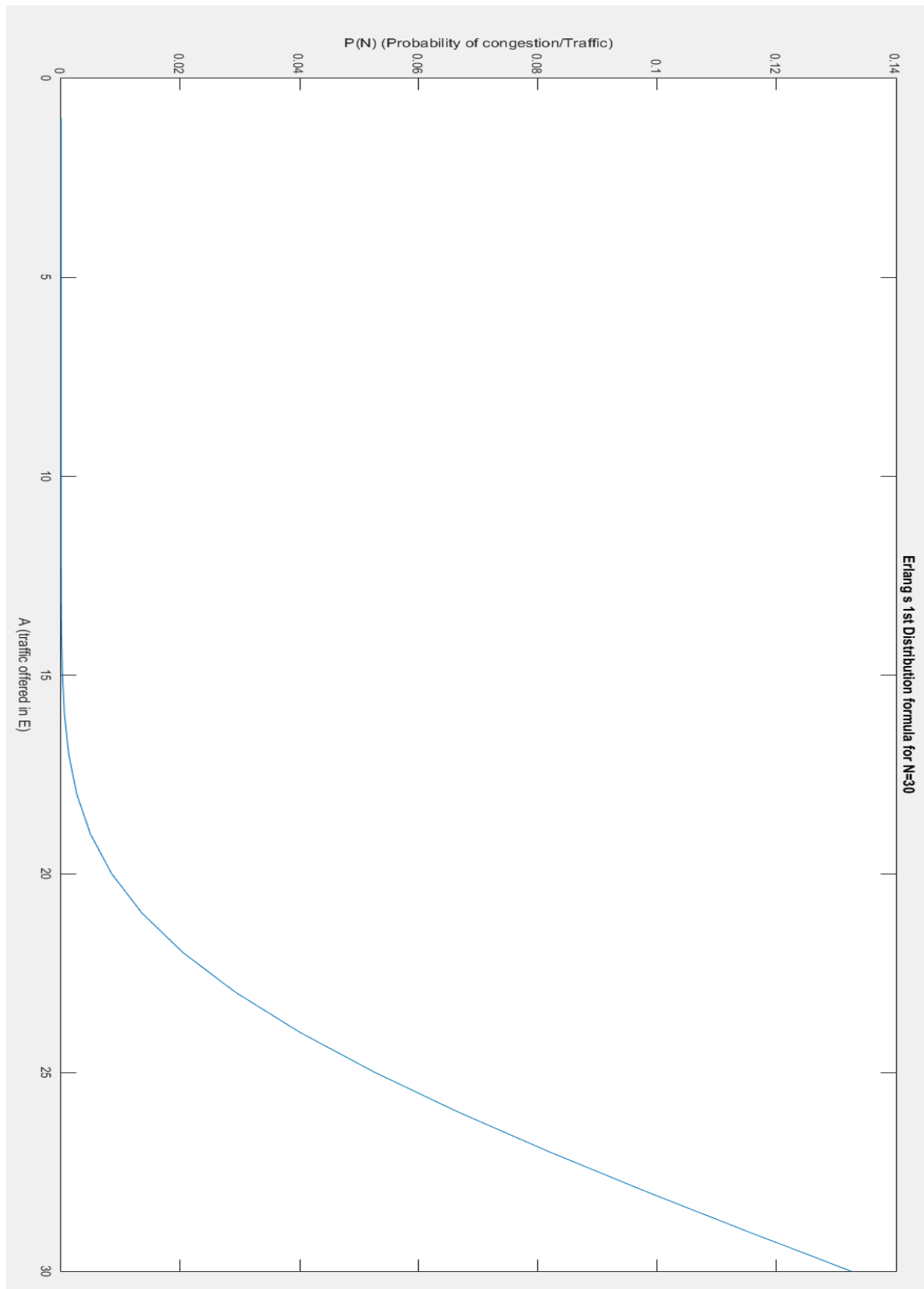
%Erlang's first distribution formula plot
%P(N)=(A^N/N!)/(Sum k from 0 to N A^k/k!)
%N : Number of Lines
%k : Availability

N = 30

P = zeros(1,N)

for A = 1 : N
    num = A^N/factorial(N)
    denominator = 0
    for k = 0 : N
        denominator=denominator + A^k/factorial(k)
    end
    P(A) = num/denominator
end

A = 1 : N
plot(A,P)
xlabel('A (traffic offered in E)')
ylabel('P(N) (Probability of congestion/Traffic)')
title('Erlang s 1st Distribution formula for N=30');
```





**Answer The Following Questions in Separate Sheets:**

**Que 1:** Derive an expression to obtain the Erlang's formula for the first kind of loss system.

Ans.:

x calls are in progress is given as :

$$P(x) = \frac{A^x}{x!} P(0) \text{ for } 0 \leq x \leq N$$

But there cannot be more than N calls nor there can be a negative number of calls. Thus we can conclude that  $0 \leq x \leq N$ .

$$\therefore \sum_{x=0}^N P(x) = 1 = \sum_{x=0}^N \frac{A^x}{x!} P(0)$$

$$\text{Hence, } P(0) = \frac{1}{\sum_{x=0}^N \frac{A^x}{x!}}$$

Substituting in Equation (2.6.1), we get

$$P(x) = \frac{\frac{A^x}{x!}}{\sum_{k=0}^N \frac{A^k}{k!}}$$

This is known as First Erlang Distribution.

$P/N$  is of specific importance, as it is the probability of congestion i.e. the probability of a lost call, which is also the grade of service B.

This is given by symbol  $E_{1,N}(A)$

$$\therefore B = E_{1,N}(A) = \frac{\frac{A^N}{N!}}{\sum_{k=0}^N \frac{A^k}{k!}}$$

$E_{1,N}(A)$  gives the grade of service for loss system for availability group of N trunks offered 'A' Erlangs of traffic.

Most of the engineers remember this equation as Erlangs lost-call formula .

It can be calculated directly or by iterative application of a recurrence relation obtained as shown below :

$$E_{1,N-1} = \frac{A^{N-1}}{(N-1)!} \sum_{k=0}^{N-1} \frac{A^k}{k!}$$

$$\therefore \sum_{k=0}^N \frac{A^k}{k!} = \frac{A^{N-1}}{(N-1)!} + \frac{A^N}{N!}$$

Substituting this in Equation (2.6.3) we get,

$$E_{1,N}(A) = \frac{AE_{1,N-1}(A)}{N + AE_{1,N-1}(A)}$$

**Que 2:** A group of 10 trunks is offered 5E of traffic, find (a) GOS (b) the probability that only one trunk is busy (c) the probability that only one trunk is free and (d) the probability that at least one trunk is free.

**Que 3 :** Derive an expression to obtain the Erlang's second formula of delay system.

**Que 4:** Comment on modelling of traffic.

**Que 5:** A group of 7 trunks is offered 4E of traffic, find (a) the grade of service (b) the probability that only one trunk is busy (c) the probability that only one trunk is free and (d) the probability that at least one trunk is free.

**Que 6:** Consider a trunk group with an offered load 4.5 erlangs and a blocking probability of 0.01. If the offered traffic increased to 13 erlangs, to keep same blocking probability, find the number of trunks needed. Also calculate the trunk occupancies.

**Que 7:** During a busy hour, 1400 calls were offered to a group of trunks and 14 calls were lost. The average call duration has 3 minutes. Find (a) Traffic offered (b) Traffic carried (c) GOS and (d) The total duration of period of congestion.

**Que 8:** Consider a group of 1200 subscribers which generate 600 calls during the busy hour. The average holding time is 2.2 minutes. What is the offered traffic in erlangs, CCS and CM.



**Que 9:** If a group of 20 trunk carries 10 erlangs and the average call duration is 3 minutes, calculate (a) average number of calls in progress (b) total number of calls originating per hour.