

MATRIX PROJECT - Q33

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QUESTION:

A circle passes through

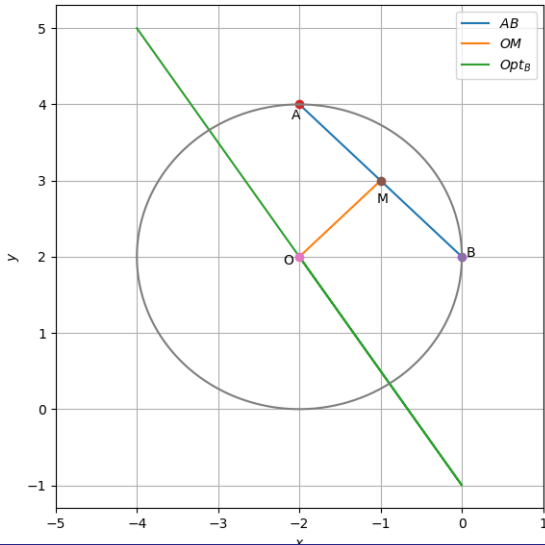
$$A = \begin{pmatrix} -2 \\ 4 \end{pmatrix} \quad (1)$$

and touches the y -axis at

$$B = \begin{pmatrix} 0 \\ 2 \end{pmatrix}. \quad (2)$$

Which one of the following equations can represent a diameter of this circle?

- 1 $\begin{pmatrix} 4 & 5 \end{pmatrix} \vec{x} = 6$
- 2 $\begin{pmatrix} 2 & -3 \end{pmatrix} \vec{x} + 10 = 0$
- 3 $\begin{pmatrix} 3 & 4 \end{pmatrix} \vec{x} = 3$
- 4 $\begin{pmatrix} 5 & 2 \end{pmatrix} \vec{x} + 4 = 0$



SOLUTION :

First, we compute direction vector of line passing through given two points using matrix it comes out to be

$$\text{DirectionVector} = T = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad (3)$$

Now, we compute perpendicular bisector of this line segment **AB** using its mid point and direction vector of $AB = T$ (it is equivalent to normal vector of the perpendicular bisector)

$$\text{MidPoint} = M = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \quad (4)$$

SOLUTION:

Equation of perpendicular bisector:

$$T^T X = p \quad (5)$$

To compute value of p we put X equal to M in the above equation.

$$T^T M = p \quad (6)$$

$$\begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \end{pmatrix} = p \quad (7)$$

$$p = 4 \quad (8)$$

Solution:

Final Equation of perpendicular bisector,

$$\begin{pmatrix} -1 & 1 \end{pmatrix} \vec{x} = 4 \quad (9)$$

As the circle touches the y-axis at (0,2) then circle should lie on the line

$$y = 2 \quad (10)$$

which can be represented in the following way using matrices

$$\begin{pmatrix} 0 & 1 \end{pmatrix} \vec{x} = 2 \quad (11)$$

Solution:

The intersection of these two lines (9) and (11) is the center because both these lines pass through center.

$$\text{Center} = O = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \quad (12)$$

Now whichever line given in options passes through the center **O**, is the answer.

Solution:

$$\mathbf{1:} \quad (4 \quad 5) \vec{x} = 6$$

$$(4 \quad 5) \begin{pmatrix} -2 \\ 2 \end{pmatrix} \neq 6 \quad (13)$$

$$\mathbf{2:} \quad (2 \quad -3) \vec{x} + 10 = 0$$

$$(2 \quad -3) \begin{pmatrix} -2 \\ 2 \end{pmatrix} + 10 = 0 \quad (14)$$

As L.H.S is equal to R.H.S.,this answer is correct.

3: $(3 \ 4) \vec{x} = 3$

$$(3 \ 4) \begin{pmatrix} -2 \\ 2 \end{pmatrix} \neq 3 \quad (15)$$

4: $(5 \ 2) \vec{x} + 4 = 0$

$$(5 \ 2) \begin{pmatrix} -2 \\ 2 \end{pmatrix} + 4 \neq 0 \quad (16)$$

Line represented in 2nd option is one of the diameters of the given circle