Bézier Curves

Definition

A **Bézier curve** is a **parametric curve** used in computer graphics to model **smooth curves**.
It is defined by a **set of control points**, and the curve shape is influenced by these points.

Bézier curves are widely used in:

- Vector graphics (Adobe Illustrator)
- Font design (TrueType fonts)
- Animation paths
- CAD systems

Control Points

- The **first and last control points** lie on the curve.
- The **intermediate control points** do not lie on the curve but control its shape.
- A curve with n + 1 control points is called an **n-th degree Bézier curve**.

Example:

- 2 points → Line (1st degree)
- 3 points → Quadratic Bézier
- 4 points → Cubic Bézier (most common)

Mathematical Representation

A Bézier curve is defined using **Bernstein polynomials**:

$$B(t) = \sum_{i=0}^{n} P_i \cdot B_{i,n}(t), 0 \leq t \leq 1$$

Where:

- P_i= control points
- $B_{i,n}(t)$= Bernstein basis function

$$B_{i,n}(t) = \binom{n}{i} t^i (1-t)^{n-i}$$

Properties of Bézier Curves

1. **Endpoint Interpolation**
   Curve passes through the first and last control points.
2. **Convex Hull Property**
   The curve always lies inside the convex hull of control points.
3. **Affine Invariance**
   Transformations (translate, rotate, scale) applied to control points affect the curve in the same way.
4. **Global Control**
   Moving one control point affects the **entire curve**.
5. **Smoothness**
   The curve is smooth and continuous.

Advantages

- Simple to understand and implement
- Easy to design smooth curves
- Widely supported in graphics software

Disadvantages

- Poor local control (changing one point affects whole curve)
- Not efficient for complex shapes
- Degree increases with number of control points

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2. B-Spline Curves

Definition

A **B-Spline (Basis Spline) curve** is a **generalization of Bézier curves** that provides **better control and flexibility**.

Instead of a single polynomial, B-splines use **piecewise polynomials**.

Components of B-Spline Curve

1. **Control Points**
2. **Degree (k)**
3. **Knot Vector**

Knot Vector

A **knot vector** is a non-decreasing sequence of parameter values:

$$T = \{t_0, t_1, t_2, \ldots, t_m\}$$

Knots determine:

- Where curve segments join
- Continuity of the curve

Mathematical Representation

$$C(t) = \sum_{i=0}^{n} P_i \cdot N_{i,k}(t)$$

Where:

- P_i= control points
- $N_{i,k}(t)$= B-Spline basis functions
- k= degree of curve

Properties of B-Spline Curves

1. **Local Control**
   Moving a control point affects only a **small portion** of the curve.
2. **Continuity**
   B-splines are C^{k-1}continuous.

3. **Convex Hull Property**
   Curve lies inside the convex hull of control points.
4. **Affine Invariance**
5. **Piecewise Polynomial**
   Efficient and flexible.

Types of B-Splines

1. **Uniform B-Spline**
   - Knots are equally spaced
2. **Non-Uniform B-Spline**
   - Knots are not equally spaced
3. **Open B-Spline**
   - Curve passes through first and last control points

Advantages

- Excellent local control
- Suitable for complex shapes
- Lower degree even with many control points
- Efficient for modeling and animation

Disadvantages

- More complex than Bézier curves
- Knot vector management is required
- Harder to implement

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3. Difference Between Bézier and B-Spline Curves

Feature	Bézier Curve	B-Spline Curve
Control	Global	Local

Feature	Bézier Curve	B-Spline Curve
Degree	Depends on number of points	Fixed
Complexity	Simple	More complex
Knot Vector	Not required	Required
Flexibility	Less	More
Shape Editing	Difficult	Easy
Usage	Fonts, logos	CAD, modeling

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4. Applications

Bézier Curves

- Font design
- Vector graphics
- Animation paths
- UI design

B-Spline Curves

- CAD/CAM systems
- 3D modeling
- Automotive and aircraft design
- Animation and motion paths

Projection in Computer Graphics

Definition

Projection is the process of converting a **3D object** into a **2D view** on a display screen or paper.
It shows how a 3D scene appears when viewed from a particular position.

Since computer screens are 2D, projection is necessary to display 3D objects.

Why Projection is Needed

- To represent 3D objects on 2D screens
- To visualize depth, height, and width
- Used in **CAD, games, simulations, animation, and engineering drawings**

Types of Projection

Projections are broadly classified into:

1. **Parallel Projection**
2. **Perspective Projection**

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1. Parallel Projection

Definition

In **parallel projection**, the **projectors (projection lines)** are **parallel to each other**.
The center of projection is assumed to be at **infinity**.

✓ Object size remains the same
❌ No depth perception (no foreshortening)

Characteristics

- Parallel projection lines
- No vanishing point
- Preserves relative dimensions
- Used in engineering and technical drawings

Types of Parallel Projection

(A) Orthographic Projection

Definition

In **orthographic projection**, projection lines are **perpendicular** to the projection plane.

It shows **true shape and size** of an object.

Views in Orthographic Projection

- Front view
- Top view
- Side view

Types of Orthographic Projection

1. Multiview Orthographic Projection

- Shows multiple views (front, top, side)
- Used in engineering drawings

2. Axonometric Projection

Object is rotated, then projected.

Types:

- **Isometric Projection**
  - All axes equally inclined
  - All dimensions equally foreshortened (120° between axes)
- **Dimetric Projection**
  - Two axes equally foreshortened
- **Trimetric Projection**
  - All three axes unequally foreshortened

(B) Oblique Projection

Definition

In **oblique projection**, projection lines are **not perpendicular** to the projection plane.

- One face appears in true shape
- Depth is drawn at an angle

Types of Oblique Projection

1. **Cavalier Projection**
    - Depth drawn at 45°
    - Full scale depth
2. **Cabinet Projection**
    - Depth drawn at 45°
    - Half scale depth (more realistic)
3. **General Oblique Projection**
    - Any angle and scale

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2. Perspective Projection

Definition

In **perspective projection**, projection lines **converge at a single point** called the **Center of Projection (COP)**.

✓ Gives realistic view
✓ Objects appear smaller when farther away

Characteristics

- Has vanishing points
- Shows depth perception
- Mimics human eye vision
- Used in games, animation, movies

Types of Perspective Projection

1. One-Point Perspective

- One vanishing point
- One face parallel to projection plane
- Used for roads, corridors

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2. Two-Point Perspective

- Two vanishing points
- Two faces visible
- Used for buildings

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3. Three-Point Perspective

- Three vanishing points
- Most realistic
- Used for tall structures and dramatic views

Comparison: Parallel vs Perspective Projection

Feature	Parallel Projection	Perspective Projection
Projection lines	Parallel	Converging
Realism	Less	High
Size accuracy	Preserved	Not preserved
Depth effect	No	Yes
Vanishing point	No	Yes
Usage	Engineering drawings	Games, movies

Applications of Projection

- Computer-Aided Design (CAD)

- Engineering drawings
- Video games
- Virtual reality
- Animation
- Architecture
