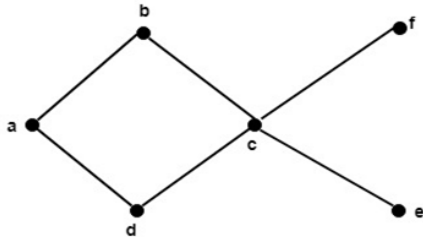


Question Bank
Discrete Mathematics KCA104
UNIT 1

1. What is the cardinality of the set? Find the cardinality of the set $\{1, \{2, \varnothing, \{\varnothing\}\}, \{\varnothing\}\}$.
2. Let the two following functions be defined on set of real numbers be as: $f(x) = 2x+3$ and $g(x) = x^2+1$. Find the $(f \circ g)(x)$ and $(g \circ f)(x)$.
3. In a survey of 60 people, it was found that 25 eat Apple, 26 eat Orange and 26 eat Banana fruit. Also 9 eat both Apple and Banana, 11 eat both Orange and Apple, and 8 eat both Orange and Banana. 8 eat no fruit at all. Then determine
 - i). the number of people who eat all three fruit.
 - ii). the number of people who eat exactly two fruit.
 - iii). the number of people who eat exactly one fruit.
4. State and Prove De Morgan's laws for set theory.
5. Define the Power set.
If $A = \{1,2,3\}$ find $P(A)$ and $n\{P(A)\}$.
6. Define the Cartesian Product of sets.
If $U = \{1,2,3,4,5,6,7,8\}$, $A = \{2,4,6,8\}$, and $B = \{3,5,6,7\}$ then find $A \times B$, $A - B$?
7. Define Equivalence Relation with a suitable examples.
8. Let $A = \{2, 3, 4\}$ $B = \{3, 4, 5\}$.
List the elements of each relation R defined below and the Domain and Range.
 - a) $a \in A$ is related to $b \in B$, that is aRb if and only if $a < b$.
 - b) $a \in A$ is related to $b \in B$, that is aRb if a and b are both odd numbers.
9. Define Bijective function with a suitable example.
10. Explain Symmetric Relation, Antisymmetric Relation and Asymmetric Relation in detail.

UNIT 2

1. Is $(\mathbb{Z}^+, <)$ a POSET?
2. Draw the Hasse diagram of the lattice of $(D_6, |)$.
3. Show that (\mathbb{Z}, \geq) is a Poset.
4. Define Lattice and differentiate between Complete Lattice and Bounded Lattice.
5. Determine all the maximal and minimal elements of the Poset whose Hasse diagram is shown in fig:



6. Simplify the following Boolean expression using K-Map method:
 $A'B'C' + A'B'C + A'BC + A'BC' + AB'C + ABC$.
7. If $S = \{10, 11, 12\}$. Determine the power set of S. Draw the Hasse diagram of Poset $(P(S), \subseteq)$.
8. Simplify the following Boolean expression using K-Map method: $Y = A'B' + A'B + AB$
9. Simplify the expression using K-maps: $F(A, B, C, D) = \sum (1, 3, 5, 6, 7, 11, 13, 14)$.
10. Simplify the expression using K-maps: $F(A, B, C) = \pi(0, 2, 4, 5, 7)$.

UNIT 3

1. Define Tautology and Contradiction.
2. Discuss the truth table of $p \leftrightarrow q$.
3. Prove that conditional proposition and its contrapositive are equivalent, i.e.
 $(p \rightarrow q) \equiv \sim q \rightarrow \sim p$
4. Using the truth table, prove the following logical equivalence:
 $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$.
5. State and Prove De Morgan's laws for propositions using truth table.
6. Construct the truth table of the following- $\sim (P \vee (Q \wedge R)) \Leftrightarrow ((P \vee Q) \wedge (P \vee R))$.
7. Using logical equivalent formulas, show that $\sim (P \vee (\sim P \wedge Q)) \equiv \sim P \wedge \sim Q$.
8. Show that which of the following statements tautology are. $((P \vee \sim Q) \wedge (\sim P \vee \sim Q)) \vee Q$.
9. Explain Modus Ponens Rule in detail.
10. Briefly explain Disjunctive Syllogism with a suitable example.

UNIT 4

1. Define Ring and Field. Give an example of a Ring and a Field.
2. Prove that every cyclic group is abelian.
3. Show that set $Z_6 = (0,1,2,3,4,5)$ forms a group with respect to addition modulo 6.
4. What is the generator of a cyclic group?
5. Find the order of each element in the group $(\{1, -1\}, .)$.
6. $G = \{1, w, w^2\}$ is an abelian group under multiplication. Where $1, w, w^2$ are cube roots of unity.
7. What is the inverse of a , if $(Z, *)$ is a group with $a*b = a+b+1 \forall a, b \in Z$?
8. If every element of a group is its own inverse, then show that the group must be abelian.
9. The set $G = \{1,2,3,4,5,6\}$ is a group with respect to multiplication modulo 7.
10. Let $G = \{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}$ under addition,
 $H = \{\dots -9, -6, -3, 0, 3, 6, 9, \dots\}$ Find left and right cosets.

UNIT 5

1. Find the number of handshakes in party of 12 people, where each two of them shake hands with each other.
2. Discuss the pigeonhole principle?
3. State all PEANO's axioms.
4. State Mathematical Induction. Explain with a suitable example.
5. Explain generating functions to solve the recurrence relation, with an example.
6. Using mathematical Induction Show that: For every positive integer n , $1 + 2 + \dots + n = n(n+1)/2$.
7. Using the principle of mathematical induction, prove that $7^{2n} + 2^{3n-3} \cdot 3^{n-1}$ is divisible by 25 for all $n \in \mathbb{N}$.
8. What is the solution of the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ with $a_0=2$ and $a_1=7$?
9. What is the solution of the recurrence relation $a_n = 2a_{n-1} - a_{n-2} + 2^n$ for $n \geq 2$, with $a_0=1$ and $a_1=2$?
10. Solve the recurrence relation $a_{r+2}-3a_{r+1}+2a_r=0$. By the method of generating functions with the initial conditions $a_0=2$ and $a_1=3$.