{QUANTITATIVE ABILITY - NUMBER SYSTEM}
CONCEPTS

BASIC NUMBERS

Number system is a very important chapter and you will get questions from this area in many placement Exams. We start with classification of numbers.

Types of numbers:

- Natural numbers (N) = 1, 2, 3 . . .
- Whole numbers (W) = 0, 1, 2, 3 . . .
- Integers (Z) = −∞ . . . −2, −1, 0, 1, 2, 3 . .
- Rational numbers (Q) = The numbers of the form p⁄q where q ≠ 0. Ex: 1⁄5, 0.46, 0.333333
- Irrational numbers (I) = The numbers of the form x1⁄n ≠ Integer. Also π and e also irrational numbers.

Other types of numbers:

- Even numbers : Integers which are exactly divisible by 2. These numbers are in the format of 2n.
- Odd numbers: Integers which gives remainder 1 when divided by 2. These numbers are in the format of 2n ± 1.
- Prime numbers: Natural numbers which are divisible by 1 and the number itself are primes. The least prime is 2.
- Composite numbers: Natural numbers which are divisible by more than 2 numbers.

The following rules related to Even and Odd numbers are important:
Odd ± Odd = Even; Even ± Even = Even; Even ± Odd = Odd
Odd × Odd = Odd; Even × Even = Even; Even × Odd = Even.
Odd to the power (any number) = Odd; Even to any power (any number) = Even

FACTORS

Any integer greater than 1 is either prime or product of primes. Writing a number as a product of primes is called prime factorization. For example, 100 can be written as $2^2 \times 5^2$

a. The total number of factors of a number N = $a^p \times b^q \times c^r$ ... = (p+1) × (q+1) × (r+1)...
b. The total number of even factors of a number N = $a^p \times b^q \times c^r$.... = p (q + 1) × (r + 1)...
c. The total number of odd factors of a number N =$a^p \times b^q \times c^r$ ... = (q + 1) × (r + 1)...
d. The sum of factors of a number N=$a^p \times b^q \times c^r$ can be written as $(a^0 + a^1 + \cdots + a^p)(b^0 + b^1 +$ *…+bqc0+c1+…+cr….*
e. The product of factors of a number N = $a^p \times b^q \times c^r$ can be written as$N^{f/2}$, where f is number of factors and should be even.
f. The number of ways of writing a number as a product of two factors = f/2 (if the number is not a perfect square)
g. ThenumberofwaysofwritinganumberNasaproductoftwoco-primefactors = 2^{p-1}wherep=thenumberofprimefactorsofanumber.

HCF & LCM

HCF is the maximum divisor which divides all the given numbers exactly. Let us say for 16, 24 there are several numbers i.e., 1, 2, 4, 8 divide them exactly. Of all these numbers 8 is maximum number so we could call 8 as HCF

LCM is defined as the least number which is divisible by all the given divisors. Take 4,6 as two divisors which divide 12, 24, 36... Perfectly with no remainder. So 12, 24, 36 are called common multiples of 4 and 6. In other words, 4 and 6 are factors of all these number. Of all these common multiples, 12 is the least number. So we can say 12 is Least common multiple of all the given numbers or LCM of 4, 6.

Finding HCF and LCM

Let there are two numbers 60 and 90.

To finding the HCF and LCM we need to do the prime factorization of 60 and 90.

60 = $2^2 \times 3^1 \times 5^1$

90 = $2^1 \times 3^2 \times 5^1$

To find out the HCF take the least powers of the prime numbers and to find out the LCM take the highest power of the prime numbers.

HCF = $2^1 \times 3^1 \times 5^1$

LCM = $2^2 \times 3^2 \times 5^1$

Example: A teacher when distributed certain number of chocolates to 4 children, 5 children, 7 children, he always left with 1 chocolate. Find the least number of chocolates the teacher brought to the class

Solution: N = K (LCM (4, 5, 7) + 1 = 140K + 1. Where K = natural number. When we substitute K = 1, we get the least number satisfies the condition. So minimum chocolates = 141

Example: When certain number of marbles are divided into groups of 4, one marble remained. When the same number of marbles are divided into groups of 7 and 12 then 4, 9 marbles remained respectively. If the total marbles are less than 10,000 then find the maximum possible number of marbles.

Solution: In this case the difference between the remainders and divisors is constant. i.e., 3. So,N = K (LCM (4, 7, 12) - 3 = 84K - 3. Where K = natural number.

But we know that 84K - 3 < 10,000 ⇒⇒ 84 x 119 - 3 < 10,000 ⇒⇒ 9996 - 3 = 9993

Example: Find the greatest number, which will divide 260, 281 and 303, leaving 7, 5 and 4 as remainders respectively.

Solution: We have to find the HCF of (260 - 7, 281 - 5, 303 - 4) = HCF (253, 276, 299) = 23

Example: Find the greatest number by which if we divide 740, 838 and 985, then in each case the remainder is the same.

Solution: Given number is HCF (838 - 740, 985 – 838, 985 - 740) = 49

Note:

If we divide the given numbers with their HCF, the quotients must be co-primes with each other.

The product of two numbers is equal to the product of LCM and HCF of the two given numbers.

Also HCF is always a factor of LCM.

REMAINDERS

Finding remainders is one of important concept in arithmetic. For example, in finding units digit of an expression, H.C.F etc finding remainder is very important.

When 100 is divided by 8, we get 4 as remainder. This can be represented as 100 = 8×k + 4. Here k = 12 and remainder is 4.

In exams, the problems are not straight forward. So learn the following rules and techniques carefully.

The following rules are very important:

- If N=A×B×C..... Then the remainder when N is divided by D is equal to the product of the remainders when A, B, C ... are individually divided by D.

Example:

Find the remainder when 1201 × 1203 ×1205 × 1207 is divided by 6.

Solution:

If you don't know the above rule, this problem is really calculation intensive.

But by applying the above rule, when 1201, 1203, 1205, 1207 divided by 6, leaves remainders 1, 3, 5, 1. The product of these remainders = 15.

When 15 is divided by 6, Remainder is 3.

- If N=A+B+C....... Then the remainder when N is divided by D is equal to the sum of the remainders when A, B, C ... are divided by D.

Example:

Find the remainder when 1! + 2! + 3! + 4! + 5! + ...100! is divided by 24.

Solution:

By applying rule, we divide the terms of the above expression individually, and add them to get the final remainder. But from 4! Onwards all the terms leave a remainder 0 when divided by 24.

So the remainder = 1 + 2 + 6 + 0 + 0....... = 9

Divisibility:

Let us a take a number ABCDEF. In decimal system this number can be written as

100,000A + 10,000B + 1000C + 100D + 10E + F

Divisibilityfor 2:

We can easily observe that from rule 2, if ABCDEF has to be divisible by 2, 2 must divide all the six terms above. It is evident that except F remaining numbers are divisible by 2. So if F is divisible by 2 then the number ABCDEF is divisible by 2.

Divisibility for 5:

Since all the terms except F is divisible by 5, the number is divisible when F is divisible by 5, or F must be 0 or 5.

Divisibility for 4:

We can see that except last two terms 10E and F, the remaining terms are divisible by 4. So, if the last two digits are divisible by 4, the entire number is divisible by 4.

Divisibility for 8:

Except last three terms the remaining terms are divisible by 8. So if the last three digits are divisible by 8 then the number is divisible by 8.

Thumb Rule: for 2, 4, 8, 16... We need to check the last 1, 2, 3, 4 ... digits. Observer there 1, 2, 3, 4 are the powers of the divisor with base 2.

Divisibility for 3, 9:
100,000A + 10,000B + 1000C + 100D + 10E + F = 99999A + 9999B + 999C + 99D + 9E + (A + B + C + D + E + F)
We can see that Except (A + B + C + D + E + F) remaining terms are divisible by 3, 9. If the digit sum is divisible by 3, 9 then the number ABCDEF is divisible by 3, 9. (A + B + C + D + E + F) is called digit sum of a number.

Divisibility for 11:
100,000A + 10,000B + 1000C + 100D + 10E + F = 100,001A + 9,999B + 1,001C + 99D + 11E + (-A + B - C + D - E + F)
From above we know that except (-A + B - C + D - E + F) the remaining digits are divisible by 11. So if the difference between the sum of the digits in the even places and odd places is 0 or multiple of 11 then the number is divisible by 11.

Divisibility for 6, 12 or any composite number:
If a composite divisor can be written as a product of co-primes and each of these co-primes divide the given number exactly, then that number is divisible by the divisor. So if 2, 3 divide the given number exactly then 6 divides that number exactly. Similarly, divisibility for 12 is to check divisibility for 3, 4.

Divisibility for 7, 11 and 13
Triplet Rule.
Difference of Sum of odd triplets and sum of even triplets.

Example:
Find the remainder when 111222333444 when divided by 7, 11 and 13
Solution:
Make triplets from right side and apply the rule
(444+222) – (333+111) = 222
Divide 222 by 7, 11 and 13 we get remainders as 5, 2 and 1 respectively

- Fermat little theorem:

A number $a^{p-1}$is divided by p, then remainder is 1. Here p is prime and a & p must be co-prime.

Example:
What is the remainder when 8^{80} is divisible by 17.
Solution:
As per Fermat theorem we need to make power as multiple of 16.
Hence we can easily say the remainder is 1.

- Wilson's theorem:

If P is a prime number then (P - 1)! + 1 is divided by P the remainder is 0
If P is a prime number then (P - 2)! - 1 is divided by P the remainder is 0.

Example:
Find the remainder when 39! is divided by 41.
Solution:
Substituting P = 41 in the wilson's theorem, we get remainder as -1 or 40.

FACTORIALS:

To find the maximum power of a number which divides a factorial number, we need to consider how many of these numbers contained in the factorial.

Example:
The maximum power of 5 in 60!
Solution:
60! = 1 x 2 x 3 ....60 so every fifth number is a multiple of 5. So there must be 60/5 = 12
In addition to this 25 and 50 contribute another two 5's. So, total number is 12 + 2 = 14
Short cut: [60/5] + [12/5]=12+2=14
Here [ ] Indicates greatest integer function.
Shortcut:
Divide 60 by 5 and write quotient. Omit any remainders. Again divide the quotient by 5. Omit any remainder. Follow the procedure, till the quotient not divisible further. Add all the numbers below the given number. The result is the answer.

Example:
Find the highest power of 12 that divide 49!
Solution:
We should commit to the memory that the above method is applicable only to prime numbers. So we should write 12 in its prime factors. $12 = 2^2 \times 3^1$

We find the maximum power of 2 in 49! = [49/2] + [24/2] + [12/2] + [6/2] + [3/2] = 24 + 12 + 6 + 3 + 1 = 46

So maximum power of 2^2in 49! is 23.

Now we find the maximum power of 3 in 49! = [49/3] + [16/3] + [5/3] = 16 + 5+1=22
Now we take the minimum of 22 and 23. Which is 22

Example:
How many zero's are there at the end of 100!
Solution:
A zero can be formed by the multiplication of 5 and 2. Since 100! Contains more 2's than 5's, we can find the maximum power of 5 contained in 100!

UNIT & TENS PLACE

Number/power	**1**	**2**	**3**	**4**
2	2	4	8	6
3	3	9	7	1
7	7	9	3	1
8	8	4	2	6
4	4	6	4	6
9	9	1	9	1
5	5	5	5	5
6	6	6	6	6

From the table it is clearly visible that for all the numbers whose unit digit in the format of 2b, the unit digits are respectively 2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 6, 2^5 = 2……..

Similarly we can find unit digits for the remaining numbers easily.

Please observe. The cyclicity of the numbers 2, 3, 7, 8 is 4, and for 4, 9 is 2 as the pattern is repeating after power 4. The cyclicity of 0, 1, 5, 6 is 1.

Example: What is the unit digit of the expression317^{171}
Solution: Here we can concentrate only on the unit digit of the base and the power. Unit digit of the base is 7 so from the table its cyclicity is 4.
Let us find the remainder when 171 is divided by 4. For the divisibility rule for the 4 is to find the remainder of the last two digits of 171, so 71 when divided by 4 gives a remainder 3. So from the table unit digit of 73 is 3.

Example: Find the unit digit of the expression$1^{781} + 2^{781} + 3^{781} \ldots \ldots . + 9^{781}$
Solution: We know that 781 when divided by 4 gives a remainder 1. As is visible clearly from the table that for every unit digit after the power 4 the same unit digit repeats.

So unit digit = 1 + 2 + 3 ....9 = 45 so unit digit is 5

Last two digits of an expression:

If we need to find the last two digits of an expression we need to consider the last two digits of the base. We need to consider two cases separately.

Case 1: Numbers which base end with 1.
These numbers are in the format of ...abc1...xyz.
Unit digit of this expression is always 1 as the base ends with 1. For the tenth place digit we need to multiply the digit in the tenth place of the base and unit digit of the power and take its unit digit

Example: The last two digits of 2341^{369}= (4 x 9),1 = 61
Case 2: Numbers which end with 5 as unit digit
The last two digits are always 25 or 75. Let the give number is ...$ab5^{xyz}$ If the product of units digit of the power (i.e., z) and digit left to the 5 in the base (i.e.,b), is even then last two digits of the expression is 25, If the power is odd then it is 75.
Example: Last two digits of 2345^{369} are 25 as the product 4*9 = 36 which is even.

BASE SYSTEM

Suppose for example, we have to convert $(134)_{10}$ to base 7. Then the following process is to be employed.
Divide 134 by 7 we get remainder 1 and quotient 19
Divide 19 by 7 we get remainder 5 and quotient 2
Divide 2 by 7 we get remainder 2 and quotient 0.

So $(134)_{10}$ = $(251)_7$

We can easily convert a number in any base other than 10 to base system 10.
$(251)_7$ = 2 × 7^2+5×7^1+1× 7^0 = $(134)_{10}$

NUMBER SYSTEM – WORKSHEET (BASIC)

H.C.F and L.C.M

Q 1. Find the HCF and LCM of $2^4 \times 3^5$, $2^5 \times 3^2$ and $3^3 \times 5^2$ respectively.

(a) $3^2, 2^5 \times 3^5 \times 5^2$
(b) $3^2, 2^5 \times 3^3 \times 5^2$
(c) $3^3, 2^5 \times 3^5 \times 5^2$
(d) $3^2, 2^4 \times 3^5 \times 5^2$

Q 2. There is a school in which annual sports day is celebrating. 1/12 of the girls and 3/8 of the boys play Hockey, 1/5 of the girls and 1/12 of the boys play Cricket,3/20 of the girls and 1/6 of the boys play football. Remaining students does not play anything. Also the total students of the school is less than 120. Then how many students can be there in the school?

(a) 120 (b) 84 (c) 60 (d) data insufficient

Q 3. In the above question, what is the ratio of number of boys and girls?

(a) 2:5 (b) 3:5 (c) 4:5 (d) data insufficient

Q 4. Three lights change after 48, 72 and 108 seconds. At 9:20:00 they all changed together. At what time will again change together 3rd time?

(a) 9:41:36 (b) 9:41:00 (c) 9:40:36 (d) 9:20:24

Q 5. If the sum of two numbers is 55 and the HCF & LCM of these two numbers are 5 and 120 respectively, then the sum of reciprocals of the numbers will be

(a) 55/601 (b) 601/55 (c) 11/120 (d) 120/11

Q 6.Find the greatest possible length which can be used to measure exactly the lengths 4m 95cm, 9 m and 16 m 65 cm.

(a) 45 cm (b) 44 cm (c) 43 cm (d) 42 cm

Q 7. What is the second smallest number which when increased by 7 is completely divisible by 8, 11 and 24?

(a) 535 (b) 528 (c)521 (d) 257

BASIC NUMBERS

Q 8. How many zeros we get in the multiplication of $55 \times 44 \times 22 \times 25 \times 125$.

(a) 3 (b) 4 (c) 6 (d) 7

Q 9.Linda has 3 types of flowers (Lotus, Tulip, Rose) in her garden. Once she was counting the flowers and noticed that the number of each flowers were different prime numbers. She also observed that if she multiplied the number of Lotus flowers by the sum of Lotus and Tulip flowers, she obtained a number just 120 more than Rose flowers. How many Rose flowers are there in the garden?

(a) 21 (b) 22 (c) 23 (d) 11

Q 10. Let a = 0.xyzxyzxyzxyz……… where x, y and z are non-zero digits. If a is multiplied by a certain natural number N, then the result obtained is also a natural number. Which of the following can be a possible value of N?

(a) 9999 (b) 990 (c) 6993 (d) none of these

Q 11. If p, q and r are odd natural numbers, which of the following is definitely false?

(a) $\frac{p^2q}{r}$ is an odd natural number (b) $\frac{p^q}{r}$ is an odd natural number.

(c) $p^q - r$ is an even natural number. (d) $p^q r$ is an odd natural number.

Q 12. If P is a prime number greater than 3. Find the sum of all numbers which must divide (P - 1)(P + 1).

(a) 30 (b) 40 (c) 50 (d) 60

Q 13. If A and B (A > B) are two prime numbers such that A + B = 2xy345. Find the value of B.

(a) 2 (b) 3 (c) none of the above (d) data inadequate

DIVISIBILITY

Q 14. Find the value of |A – B| if 32A4873B is divisible by 72.

(a) 0 (b) 1 (c) 2 (d) 3

Q 15. How many numbers of the form 34a5b are divisible by 36?

(a) 2 (b) 3 (c) 4 (d)8

Q 16. The director of TPC gives a task to the students of UPES. He told students to write the sum of digits of all natural number starting from 1 to 2200 till single digit. For example 995 = (9+9+5)19 =(1+9) 10 =(1+0) 1. How many of these naturals will have a sum as 9.

(a) 200 (b) 244 (c) 344 (d) data insufficient

Q 17. Find the remainder when 111222333444555666777888999 divided by 11.

(a) 0 (b) 1 (c) 5 (d)2

UNIT and TENS DIGITS

Q 18. Find the unit place of $456^{456} \times 234^{234} \times 567^{567} \times 912^{912}$.

(a) 2 (b) 4 (c) 6 (d) 8

Q 19. Find the unit place of $47^{23} - 23^{47}$

(a) 6 (b) 0 (c) 2 (d)4

Q 20. The unit's digit of $\frac{324^{215} \times 343^{146}}{196^{125} \times 3^{403}}$

(a) 2 (b) 4 (c) 6 (d) 8

Q 21. Find the last two digits of 15 × 37 × 63 × 51 × 97 × 17
(a) 25 (b) 35 (c) 45 (d) 55

Q 22. Find the last two digits of $133^{133!}$.
(a) 01 (b) 03 (c) 07 (d) 09

REMAINDERS

Q 23. Find the remainders when 50^{51}&51^{51}&52^{51} are divided by 7 respectively.
(a) 1, 1, 1 (b) 1, 1, 0 (c) 1, 1, 2 (d) 1, 1, 6

Q 24. What is the remainder when 2^{96} is divided by 96
(a) 2 (b) 32 (c) 64 (d) 95

Q 25. Let N = 1223334444……………999999999101010101010101010101111111111111111111111. What is the remainder when N is divided by 9?
(a) 0 (b) 1 (c) 2 (d) none

FACTORS

Q 26. Find the number of total factors, even factors and odd factors of 1200 respectively.
(a) 30, 24, 6 (b) 30, 6, 24 (c) 24, 12 , 6 (d) 30, 12, 6

Q 27. Find the sum and product of all the factors 180 respectively.
(a) 546, 180^{18} (b) 546, 180^{9} (c) 532, 180^{18} (d) 532, 18^{8}

Q 28. Which one of the following is the least number which can be expressed as a product of two co prime numbers in 8 ways?
(a) 30 (b) 120 (c) 210 (d) 1155

BASE SYSTEM

Q 29. In a certain base system, the following addition operation is true. Find the sum of digits (in base 8) in place of alphabets?

$$\begin{array}{rccccc} & (2 & 3 & b & 5 & c) \\ + & (1 & a & 6 & 4 & 2) \\ \hline & (4 & 2 & 4 & 2 & 3) \\ \hline \end{array}$$

(a) 2 (b) 3 (c) 4 (d) 5

Q 30. In a certain base system b, $(52)_b = 2(25)_b$, then find b?
(a) 7 (b) 8 (c) 9 (d) 11

NUMBER SYSTEM – WORKSHEET (PROGRESSIVE)

Q 1. L.C.M of two numbers $a = 2^5 \times 3^{21}$ and $b$is $2^7 \times 3^{21}$. How many possible values of b exists?
(a) 21 (b) 20 (c) 22 (d) 0

Q 2. If L.C.M of different two numbers is 91. How many pairs of these two numbers exists?
(a) 2 (b) 3 (c) 4 (d) 5

Q 3. At world trade center, a meeting was organized by USA. USA invited 4 countries India, Russia, China and Japan in the meeting. There were 36, 72, 81 and 108 members respectively attended the meeting from the above 4 countries. At the time of dinner the seating arrangement is such that only the members of same country are allowed at one dining table. The number of members in each table should be same. What is the minimum number of tables should be arranged to meet all these conditions.
(a) 32 (b) 33 (c) 40 (d) 432

Q 4. The sum of two numbers is 136 and their HCF is 17. The numbers of pairs of such numbers satisfying the given condition is
(a) 2 (b) 4 (c) 6 (d) 8

Q 5. Find the unit place in $32^{33^{34}}$.
(a) 2 (b) 4 (c) 6 (d) 8

Q 6. Find the last place in the expansion of $1^5 + 2^5 + 3^5 + 4^5 \ldots\ldots\ldots + 29^5$.
(a) 1 (b) 0 (c)5 (d) 9

Q 7. Find the tens place digit in $41^{200} + 42^{200} + 43^{200} + \cdots + 49^{200}$.
(a) 0 (b) 1 (c) 2 (d) 3

Q 8. Find the last two digits of $11^{10} - 9$.
(a) 72 (b) 82 (c) 92 (d) 02

Q 9. The first non-zero digit from the right of the number170^{4443}.
(a) 1 (b) 3 (c) 7 (d) 9

Q 10. If $x^2 - y^2 = 101$ $where$ $x, y \in N$, find the value of$x^2 + y^2$?
(a) 5001 (b) 5101 (c) 5201 (d) none

Q 11. If $x^2 - y^2 = 2345678$. How many positive values of x and y exists?
(a) 0 (b) 1 (c) 2 (d) 3

Q 12. Find the number of zeros in 38!.
(a) 5 (b) 7 (c) 8 (d) 9

Q 13. Find the number of zeros in $43!^{43!} \times 23!^{23!}$.
(a) $9^{43!} \times 4^{23!}$ (b) $9^{43!} + 4^{23!}$ (c) 9(43!) × 4(23!) (d) 9(43!) + 4(23!)

Q 14. Find the number of zeros in 62! - 32!
(a) 15 (b) 14 (c) 7 (d) 6

Q 15. Find the remainder when 1× 1! + 2×2! + 3×3! +…….. +54×54! +1 is divided by 55!
(a) 0 (b) 1 (c) 2 (d) 3

Q 16. (3333333333………………..3) 1000 times find the remainder when divided by 91.
(a) 52 (b) 57 (c) 60 (d) 61

Q 17. Find the remainder 112123123412345123456123456712345678 when divided by 36.
(a) 24 (b) 28 (c) 30 (d) 32

Q 18. How many factors of $2^5 \times 3^5 \times 5^8$ have odd number of factors?
(a) 324 (b) 45 (c) 72 (d) none of these

Q 19. How many factors of the$2^5 \times 5^3 \times 7^4$ are multiple of 10?
(a) 60 (b)75 (c) 90 (d) 105

Q 20.On some planet A, a certain number system is followed. Once there was a concert in a hall in which there were 45 chairs in each row and there 32 rows. Inn all, there were 2133 chairs. If from earth, where decimal number system is followed, 460 persons went to the planet A to attend the concert, then according to planet A, how many people arrived?
(a) 460 (b) 1225 (c) 5221 (c) 225

SOLUTIONS - BASIC

Solution 1:
We take least power of prime numbers to find out the HCF. For LCM we take the highest powers.
So, for$2^4 \times 3^5$, $2^5 \times 3^2$ and $3^3 \times 5^2$
HCF = 3^2 and LCM = $2^5 \times 3^5 \times 5^2$

Solution2 and3:
Number of girls cannot be in fraction. Hence it must be a multiple of 12, 5 and 20. LCM of 12, 5 and 20 is 60. So, number of girls will be 60.
Similarly number of boys cannot be in fraction. Hence it must be a multiple of 8, 12 and 6 which is 24.
Number of boys can be 24 or 48.

Solution4:
LCM of 48, 72 and 108 is 432 seconds.
So the lights changes color after every 432 seconds. So third time it changes color after 432 × 3 =1296 seconds or 21 min 36 seconds.

Solution5:
Product of HCF and LCM is equal to Product of two numbers. Therefore, product of two numbers will be equal to 600. Also sum is 55. We get the two numbers as 40 and 15.
Now, 1/40 + 1/15 = 55/600 = 11/120

Solution6:
First convert all in same units' i.e., 495 cm, 900 cm and 1665 cm.
To the maximum length which measure these three rods, find out the HCF.
HCF of 495, 900 and 1665 is 45.

Solution7:
LCM of 8, 11 and 24 is 264. Now we need to find out the second least multiple, which is 528.
521 is the number which when increased by 7 gives second least multiple.

Solution8:
To find out the number of zeros, we need to find out the min (number of 5, number of 2)
Number of 5 = 6 and number of 2 = 3. Hence number of zeros = 3

Solution9:
Option A and B are eliminated as number of rose flowers is prime number.
Also, Lotus × (Lotus + Tulip) = Rose + 120
Let number of Rose flowers is 11. We get Lotus × (Lotus + Tulip) = 131 which is not possible because product of two numbers will be prime only when if one of them is 1.
Hence only C option will satisfy the condition.

Solution10:
A = 0.xyzxyzxyzxyz………

1000 A = xyz.xyzxyzxyz……
A = xyz/999,
A must be multiple of 999, which is option C

Solution11:
Check options

Solution12:
24 is the highest number which always divide (P-1)(P+1).
Sum of factors of 24 is 60.

Solution13:
Sum of 2 prime numbers is odd only when one of the prime numbers is 2.
Also A > B. So this is possible only when B = 2.

Solution14:
A number is divisible by 72 when it is divisible by 8 and 9 both. Hence 32A4873B is divisible by 8 when B is equal to 6 and 32A48736 is divisible by 9 when A = 3. |A-B| = 3

Solution15:
A number is divisible by 36 when it is divisible by 4 and 9 both. 34a5b will be divisible by 4 when b will be equal to 2 or 6. When b = 2, a = 4 and when b = 6, a can be 0 or 9. So we get 3 numbers.

Solution16:
This is possible only for the multiples of 9. We have 244 multiples of 9 from 1 to 2200.

Solution17:
Use the rule of triplets.

Solution18:
$456^{456} \times 234^{234} \times 567^{567} \times 912^{912}$.
(….6) × (….6) × (…..3) × (…..6) = (……..8)

Solution19:
Last digit in 47^{23} is 3 and in 23^{47} is 7. But$47^{23} is\ smaller\ than\ 23^{47}$. Hence the last digit will be 4.

Solution20:
$\frac{324^{215} \times 343^{146}}{196^{125} \times 3^{403}} = \frac{(.....4)\times(.....9)}{(.....6)\times(.....7)} = \frac{(.....6)}{(.....2)}$. Now last will be either 3 or 8.

Solution21:
Multiply last 2 digits.

Solution22:
We know that any power of 3 which is a multiple of 20 will results last two places as 01. !133 is a multiple of 20 hence the last two digits would be 01.

Solution23:

$\frac{50^{51}}{7} = \frac{1^{51}}{7} = 1$. Hence leaves remainder 1.

$\frac{51^{51}}{7} = \frac{2^{51}}{7} = \frac{8^{17}}{7} = \frac{1^{17}}{7} = 1$. Hence remainder is 1.

$\frac{52^{51}}{7} = \frac{3^{51}}{7} = \frac{27^{17}}{7} = \frac{-1^{17}}{7} = -1$. Hence remainder is 6.

Solution24:

$\frac{2^{96}}{96} = \frac{2^{96}}{32\times3} = \frac{2^{91}}{3} = \frac{-1^{17}}{3} = -1$. Remainder would be 2. But we need to multiply by 32. Hence the remainder would be 64.

Solution25:

Add all the digits $1^2 + 2^2 + 3^2 \ldots \ldots 11^2 = 385$. Divide 506 by 9, we will get remainder as 2.

Solution26:

1200 can be written as$2^4 \times 3^1 \times 5^2$.
Total factors: (4 + 1) (1 + 1) (2 + 1) = 30.
Even factors: (4) (1 + 1) (2 + 1) = 24.
Odd factors: (1 + 1) (2 + 1) = 6

Solution27:

180 = $2^2 \times 3^2 \times 5^1$
Sum of factors = $(2^0 + 2^1 + 2^2) \times (3^0 + 3^1 + 3^2) \times (5^0 + 5^1)$ = 546
Product of factors = 180^9

Solution28:

A number can be expresses as a product of 2 co prime numbers in 2^{p-1},where p is the number of prime numbers in prime factorization.
2^{p-1} = 8. Hence p will be 4.
The smallest number which satisfy the condition would be $2^1 \times 3^1 \times 5^1 \times 7^1$ = 210

Solution29:

	(2	3	b	5	c)
+ (1	a	6	4	2)	
	(4	2	4	2	3)

C = 1. 5 + 4 = 9, hence we can easily conclude the base which is 7.

Solution 30:

$$(52)_b = 2(25)_b$$

5 × b + 2 = 2 (2 × b + 5)
Hence b = 8.

MODERATE

Solution 1:

$a = 2^5 \times 3^{21}$
b = ?
LCM = $2^7 \times 3^{21}$

b must have 2^7 and can have any power of 3 from 0 to 21. Hence 22 values.

Solution 2:
91 = 7 × 13.
(1, 91), (7, 91), (13, 91) and (7, 13). Only 4 pairs exist.

Solution 3:
HCF of 36, 72, 81 and 108 is 9.
USA need 4 tables for INDIA, 8 tables for RUSSIA, 9 tables for CHINA and 12 tables for JAPAN.

Solution 4:
Let the number be 17a and 17b. (As their HCF is 17, a and b must be co-prime)
17a + 17b = 136.
a + b = 8.
Only 2 co-prime pairs exist.

Solution 5:
$32^{33^{34}}$. To find out the unit place divide the power by 4 we get remainder as 1. Hence unit place would be 1.

Solution 6:
$1^5 + 2^5 + 3^5 + 4^5 \ldots \ldots \ldots. + 29^5$. As per the rule last digits would be
1 + 2 + 3 + 4 … …. + 9 = 45

Solution 7:
$41^{200} + 42^{200} + 43^{200} + \cdots + 49^{200}$.
If an odd number has a power of multiple of 20 then the last 2 digits would always be 01(except 5). If an even number has a power of multiple of 20 then the last digit would always be 76.
We get, 01 + 76 + 01 +76 + 25 + 76 + 01 +76 + 01 = 33.

Solution 8:
$11^{10} - 9 =$ 01 – 09 = 92

Solution 9:
We can write 170^{4443}as $17^{4443} \times 10^{4443}$. In 17^{4443} the last digit will be 3 and rest all are zeros.

Solution 10:
We can write this as (x - y)(x + y) = 101. 101 is prime number and the product of two numbers will be prime only when one of the two numbers is 1 and other must be that number itself.

Hence (x – y) = 1 and (x + y) = 101. By solving the equation we get x = 51 and y = 50.

Solution 11:
$x^2 - y^2 = 2345678$. We can write this as
(x - y) (x + y) = 2 × 1172839.
This is not possible for prime values of x and y.

Solution 12:
Count the number of 5's.
[38/5] + [7/5] + [1/5] = 7 + 1 = 8.

Solution 13:
Find out the number of 5's in both the factorials and add them.

Solution 14:
When we subtract a small number from a large number, number of zeros obtained are same as in the smaller number.
Hence we just need to find the number of zeros in 32!
Number of zeros in 32! is 7.

Solution 15:
Using property, when 1 × 1! + 2 × 2! + 3 × 3! +…….. +n × n! is divided by (n+1)!, the remainder obtained is always 1.

Solution 16:
91 = 7 × 13.
Now using triplet rule and finding the remainder individually from 7 and 13.
When we divide (333333333333……….3) 1000 times by 7, we get the remainder as 1.
When we divide (333333333333………..3) 1000 times by 13, we get the remainder as 5.
Hence only 57 satisfies the condition.
Short trick:
When we divide this number by 7, we get remainder 1. Only 57 satisfies the condition. No need to check for 13.

Solution 17:
36 = 4 × 9
To check the divisibility of 4 we divide the last 2 digits by 4 and for 9 we add all the digits.
When we divide 1121231234……..12345678 by 4, we get the remainder as 2.
When we divide 1121231234……..12345678 by 9, we get the remainder as 3.
Hence only option c satisfies the condition.
Short trick:
When we divide this number by 4, we get remainder 2. Only option c satisfies the condition. No need to check for 9.

Solution 18:

Only Perfect squares are the number which have odd number of factors.
We need to find out the factors which are perfect squares.
Total number of perfect squares: 3 × 3 × 5 = 45.

Solution 19:
To get a factor multiple of 10, we need at least one 2 and at least one 5.
Total number of factors which are multiple of 10: 5 × 3 × 5 = 75.