

QUANTITATIVE ABILITY - RATIO & PROPORTION, PARTNERSHIP CONCEPTS

The concept of Ratio and Proportion is very important for various Campus Exams. Besides, this concept is very important for Data Interpretation and Data sufficiency.

A ratio is a comparison of two numbers. We generally separate the two numbers in the ratio with a colon ":". Suppose we want to write the ratio of 8 and 12.

We can write this as 8:12 or as a fraction $\frac{8}{12}$, and we say the ratio is eight to twelve.

Examples

In a certain class, the ratio of passing grades to failing grades is 7 to 5. How many of the 36 students failed the course?

The ratio, "7 to 5" (or 7 : 5 or $\frac{7}{5}$), tells you that, of every 7 + 5 = 12 students, five failed. That is, $\frac{5}{12}$ of the class flunked. Then $(\frac{5}{12})(36) = 15$ students failed.

Express the ratio in simplest form: \$10 to \$45

This means that you should write the ratio as a fraction, and you should then reduce the fraction:
 $\frac{10}{45} = \frac{2}{9}$.

This reduced fraction is the ratio's expression in simplest form. Note that the units "canceled" on the fraction, since the units, "\$", were the same on both values. So there is no unit on the answer.

Express the ratio in simplest form: 240 miles to 8 gallons

Depending on the text (or instructor), you may be supposed to keep the units on a ratio. In this case, you would have $(240 \text{ miles})/(8 \text{ gallons}) = (30 \text{ miles})/(1 \text{ gallon})$, or, in more common language, 30 miles per gallon.

Comparing Ratios

To compare ratios, write them as fractions. The ratios are equal if they are equal when written as fractions.

Example:

Are the ratios 3 to 4 and 6:8 equal?

The ratios are equal if $\frac{3}{4} = \frac{6}{8}$.

These are equal if their cross products are equal; that is, if $3 \times 8 = 4 \times 6$.

Since both of these products equal 24, the answer is yes, the ratios are equal.

Remember to be careful! Order matters!, A ratio of 1:7 is not the same as a ratio of 7:1

Let $x : y$ be a ratio, which can also be written as $\frac{x}{y}$. We will try to find out what will happen when a constant a is added both to the numerator and denominator

Case I: $(\frac{x}{y} < 1)$

If $\frac{x}{y} < 1$, then addition of a constant positive number to numerator and denominator leads to a bigger ratio than the ratio itself, i.e.

$\frac{x}{y} < \frac{(a+x)}{(a+y)}$ for $\frac{x}{y} < 1$

where 'a' is a constant positive number. e.g. $\frac{1}{2}$ is less than 1 and when we add 2 to both numerator and denominator we get $\frac{3}{4}$ and $\frac{3}{4}$ is greater than $\frac{1}{2}$. Similarly, Subtraction leads to a similar ratio, i.e., $\frac{x}{y} > \frac{(x-a)}{(y-a)}$ for $\frac{x}{y} < 1$ Lets consider a fraction $\frac{5}{10}$, if 5 is subtracted from numerator as well as denominator, we get 0 and it is less than $\frac{5}{10}$ (i.e. $\frac{1}{2}$) Thus the rule for the case of subtraction is the reverse of the case of addition, as can be easily seen by the given example.

CASE II: ($x/y > 1$)

The above rule gets totally and exactly reversed for $x/y > 1$. Therefore,
 $x/y > (x+a)/(y+b)$ and $x/y < (x-a)/(y-b)$.

Proportion:

A proportion is an equation with a ratio on each side. It is a statement that two ratios are equal.

$3/4 = 6/8$ is an example of a proportion. When one of the four numbers in a proportion is unknown, cross products may be used to find the unknown number. This is called solving the proportion. Question marks or letters are frequently used in place of the unknown number.

Example: Solve for n: $1/2 = n/4$.

Using cross products we see that $2 \times n = 1 \times 4 = 4$, so $2n = 4$. Dividing both sides by 2, $n = 4 / 2$ so that $n = 2$.

NOTES

1. Compounded Ratio of two ratios a/b and c/d is ac/bd , i.e., $ac : bd$.
2. Duplicate ratio of $a : b$ is $a^2 : b^2$
3. Triplicate ratio of $a : b$ is $a^3 : b^3$
4. Sub-duplicate ratio of $a : b$ is $a : b$
5. Sub-triplicate ratio of $a : b$ is $^3a : ^3b$
6. Reciprocal ratio of $a : b$ is $b : a$
- Proportion. Four (non-zero) quantities of the same kind a, b, c, d are in proportion, written as $a : b :: c : d$ iff $a/b = c/d$
- $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \left(\frac{Ap^n + Br^n + Ct^n}{Aq^n + Bs^n + Cu^n} \right)^{\frac{1}{n}}$
- The non-zero quantities of the same kind a, b, c, d, \dots are in continued proportion if $a/b = b/c = c/d = \dots$
In particular, a, b, c are in continued proportion iff $a/b = c/d$. In this case b is called the mean proportion; $b = ac$; c is called third proportional. If a, b, c, d are in proportion, then d is called fourth proportional.
1. Invertendo. If $a : b :: c : d$ then $b : a :: d : c$
2. Alternendo. If $a : b :: c : d$ then $a : c :: b : d$
3. Componendo. If $a : b :: c : d$ then $(a+b) : b :: (c+d) : d$
4. Dividendo. If $a : b :: c : d$ then $(a-b) : b :: (c-d) : d$

Componendo and dividendo

If $a : b :: c : d$ then $(a+b) : (a-b) :: (c+d) : (c-d)$

i.e., $a/b = c/d \Rightarrow (a+b)/(a-b) = (c+d)/(c-d)$

- If $a/b = c/d = e/f = \dots$, then each ratio $= (a+c+e+\dots)/(b+d+f+\dots)$
Which is same as $(pa+qc+re+\dots)/(pb+qd+rf+\dots)$
- If $x = \frac{2pq}{p+q}$, then $\frac{x+p}{x-p} + \frac{x+q}{x-q} = 2$

RATIO & PROPORTION, PARTENERSHIP – WORSHEET (BASIC)

RATIO & PROPORTION

Q 1: If $18 : x = x : 8$, then x will be equal to

- (a) 144 (b) 72 (c) 26 (d) 12

Q 2: The ratio of two numbers is 4:5. If 15 is added to both the numbers, the ratio becomes 17:20. Find the two numbers?

- (a) 12 and 15 (b) 24 and 30 (c) 36 and 45 (d) 40 and 50

Q 3: Two numbers are respectively 50% and 100% more than a third number. The ratio of the two numbers is:

- (a) 2:3 (b) 3:4 (c) 4:5 (d) 1:2

Q 4: 4 numbers are in the ratio of 1:3:6:8 when added gives 216. Find the biggest number?

- (a) 96 (b) 104 (c) 88 (d) 112

Q 5: Seats in Wipro, TCS and HCL are in the ratio of 2: 3: 5. There is a proposal to increase the seats by 20%, 20% and 20%. What will be the ratio of increased seats?

Q 6: The difference between two positive numbers is 10 and the ratio between them is 5:3. Find the product of the two numbers?

- (a) 375 (b) 175 (c) 275 (d) 125

Q 7: In a bag there are coins of 25 p, 10 p and 5 p in the ratio of 1: 2: 3. If there is Rs. 30 in all, how many 5 p coins are there?

- (a) 50 (b) 100 (c) 150 (d) None of these

Q 8: The fourth proportional to 3, 6, and 10 is:

- (a) 20 (b) 40 (c) 5 (d) 1.8

Q 9: If $\frac{p}{q} = \frac{r}{s} = \frac{t}{u} = \frac{2}{3}$, then $\frac{3p+4r+5t}{3q+4s+5u} =$

- (a) 3/4 (b) 1/2 (c) 4/5 (d) None of these.

Q 10: The incomes of A and B are in the ratio 4:3 and their expenditures are in the ratio 2:1. If both A and B save Rs. 1000 each, their respective incomes are

- (a) Rs. 2000 and Rs. 1500 (b) Rs. 3000 and Rs. 2250
(c) Rs. 4000 and Rs. 3000 (d) None of these

Q 11: Jai spends 90% of his salary and Veer spends 85% of his salary. But savings of both are equal. Find the income of Veer, if sum of their incomes is Rs. 5000.

- (a) 1000 (b) 2000 (c) 3000 (d) 1200

Q 12: If $8x^2 + 6y^2 = 19xy$, find the ratio of x and y?

- (a) 2:1 (b) 3:8 (c) 1:2 (d) Cannot say

Q 13: The incomes of Chan, Khan and Rhan are in the ratio of 8: 9: 13 and their expenses are in the ratio of 4: 5: 6. If Chan saves $\frac{1}{5}$ th of his income, then what is the ratio of their savings?

- (a) 36: 14: 69 (b) 8: 5: 34 (c) 16: 5: 34 (d) None of these

Q 14. A, B and C start a business. A invests 3 times as much as B invests two-third of what C invests. Then the ratio of capitals of A, B and C?

- (a) 3: 9: 2 (b) 6: 10: 15 (c) 5: 3: 2 (d) 6: 2 : 3

Q 15: A certain amount was divided between Joy and Toy in the ratio of 4:3. If Toy's share was Rs. 2400, the amount is:

- (a) Rs. 5600 (b) Rs. 3200 (c) Rs. 9600 (d) None of these

Q 16: The prices of a scooter and a television set are in the ratio 3:2. If a scooter costs Rs. 6000 more than the television set, the price of the television set is:

- (a) Rs. 6000 (b) Rs. 10,000 (c) Rs. 12000 (d) Rs. 18000

Q 17: Three friends divide Rs. 624 among themselves in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$. The share of the third friend is:

- (a) Rs. 288 (b) Rs. 192 (c) Rs. 148 (d) Rs. 144

Q 18: A circle and square have same area. Therefore, the ratio of the side of the square and the radius of the circle is:

- (a) $\sqrt{\pi} : 1$ (b) $1 : \sqrt{\pi}$ (c) $1 : \pi$ (d) $\pi : 1$

Q 19: 8 times the salary of A is equal to 12 times the salary of B is equal to 20 times the salary of C. find the respective ratio of their salary?

Q 20: Divide 1440 in the ratio of 5: 6: 13?

PARTNERSHIP

Q 21: Vishnu and Sidhu enter into a partnership. Vishnu invests 10000 rupees and Sidhu invests 8000 rupees in it for the same time period. Find the ratio of their profit?

- (a) 5: 4 (b) 1:2 (c) 2:3 (d) Indeterminate

Q 22: A invests 400 rupees in business for 5 months whereas B invests 250 rupees in the same business for 10 months. Find the ratio of profit of A and B?

- (a) 1:2 (b) 3:1 (c) 5:8 (d) None of these

Q 23: The ratio of profit of A and B is 4:3. Find the ratio of their investment?

- (a) 4:3 (b) 3:4 (c) 1:1 (d) data insufficient

Q 24: Ram invests 300\$ in business for 7 years whereas Shyam invests 700\$ in the same business for 2 years. Find the ratio of profit of Ram and Shyam?

- (a) 3: 2 (b) 3:1 (c) 5:8 (d) None of these

Q 25: Karan and Arjun enters into a partnership. Karan invests Rupees 24000 for 10 months and Arjun remains in business for 5 months. Out of the total profit, Arjun claims half of the profit. How much money did Arjun contribute?

- (a) 24000 (b) 4800 (c) 48000 (d) 12000

VARIATION

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Q 26. A varies directly as B. If B = 7 and A = 18, find A when B = 21?

- (a) 36 (b) 54 (c) 72 (d) 18

Q 27. Y varies directly as third power of X. What is the percentage change in the value of Y, when X is halved?

- (a) -87.5% (b) -75% (c) -5% (d) 12.5%

Q 28. Price of a diamond is directly proportional to the square of its weight. A piece of diamond breaks into two pieces whose ratio of weights is 3:4. What is the % profit or loss due to this breakage?

- (a) 77% (b) 0% (c) 252% (d) 73.4%

MORE QUESTIONS

Q 29: An engineering college having 600 students collected fund to build a Chemical laboratory. The amount contributed by each boy and girl was equal to their respective total numbers in the college. The UGC grant was equal to the fund collected by the students. Ratio of boys and girls is 7: 3. What amount was available to build the laboratory?

- (a) Rs.208000 (b) Rs.416000 (c) Rs.352000 (d) Data insufficient

Q 30: One-fifth of the boys and three-seventh of the girls in a school participated in fancy dress competition. What fractional part of the students of the students participated for the competition?

- (a) 4/13 (b) 4/12 (c) 1/12 (d) Data insufficient

ANSWER KEY:

Q 1: d	Q 2: c	Q 3: b	Q 4: a	Q 5: 2: 3: 5
Q 6: a	Q 7: c	Q 8: a	Q 9: d	Q 10: a
Q 11: b	Q 12: d	Q 13: d	Q 14: d	Q 15: a
Q 16: c	Q 17: d	Q 18: a	Q 19: 15: 10: 6	Q 20: 300, 360, 780
Q 21: a	Q 22: d	Q 23: d	Q 24: a	Q 25: c
Q 26: b	Q 27: a	Q 28: d	Q 29: b	Q 30: d

RATIO & PROPORTION, PARTENERSHIP – WORSHEET (PROGRESSIVE)

Q 1: The cost of a certain necklace varies directly as the cube of the number of diamond beads in it. Once this diamond broke into three pieces with number of beads in the ratio of 1: 3: 5 respectively. When the three pieces were sold individually, the merchant got Rs. 19,200 less than the price the original necklace would have fetched him. What was the initial cost of the necklace?

- (a) 2.76 lakh (b) 7.29 lakh (c) 2.43 lakh (d) None of these

Q 2: An alloy contains silver, platinum and copper in the ratio 2:3:4. If silver is 10% impure, platinum is 12% impure and copper is 9% impure, then the purity of alloy is

- (a) 91.67% (b) 10.33% (c) 84.66% (d) 89.77%

Q 3: If $\frac{a}{b} = \frac{2}{3}$, $\frac{b}{c} = \frac{3}{1}$, $\frac{c}{d} = \frac{4}{3}$, $\frac{d}{e} = \frac{1}{2}$, $\frac{e}{f} = \frac{3}{4}$. Find the value of $\frac{abc}{def}$
(a) 3/8 (b) 3/4 (c) 1/4 (d) 3/7

Q 4: Solve the equation $\frac{\sqrt{x+1}-\sqrt{x-1}}{\sqrt{x+1}+\sqrt{x-1}} = \frac{4x-1}{2}$
(a) 5/4 (b) 4/5 (c) 1/2 (d) None of these

Q 5: There are two containers: the first contains 800 ml of milk, while the second contains 800 ml of water. Three cups of milk from the first container is taken out and is mixed well in the second container. Then three cups of this mixture is taken out and is mixed in the first container. Let P denote the proportion of water in the first container and Q denote the proportion of milk in the second container. Then

- (a) P is greater than Q (b) Q is greater than P
(b) P is equal to Q (d) Data is incomplete. Depends upon the quantity of cup.

Q 6: BJP, AAP, CONG and SP enter into a yearlong partnership. AAP and SP are sleeping partners who invest sum of Rs.100000 for 5 months and Rs.110000 for 6 months respectively. BJP invests Rs.30000 for 6 months besides working for 11 months. CONG only works for 9 months and does not contribute any money. The working partners receive a share of 1% of the total profit of every month of work, while the remaining profit is distributed amongst the partners in the ratio of their investments. If CONG receives a sum of Rs.3879 as profit at the end of the year, the total profit earned during the year is

- (a) 38,790 (b) 24,560 (c) 43,100 (d) 45,500

Q 7: If $\frac{p}{q} = \frac{r}{s} = \frac{t}{u} = \frac{3}{7}$, then find the value of $\sqrt[13]{\frac{7p^{13}+12r^{13}-97t^{13}}{7q^{13}+12s^{13}-97u^{13}}}$
(a) 7/3 (b) 3/7 (c) 27/343 (d) 343/27

Q 8: If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b} = L$, then L cannot take any value except
(a) 1/2 (b) -1 (c) 1/2 or -1 (d) None of these

Q 9: A, B and C invest Rs.4000, Rs.5000 and Rs.6000 respectively, in a business. A gets 25% of profits for managing the business and the rest of the profit is divided among A,B and C in proportion to their investments. If in a year A gets Rs.100 less than B and C together, what was the profit that year?

- (a) Rs.500 (b) Rs.750 (c) Rs.1000 (d) Rs.12,00

Q 10: The ratio of the incomes of A and B in 3 : 1 and the ratio of their expenditure is 6 : 5. B's savings equals $\frac{1}{7}$ th of his income. What is A's expenditure, if A's savings amount to Rs.1150?

- (a) Rs.166.67 (b) Rs.600 (c) Rs.1000 (d) Rs.2000

Q 11: The cost of a certain necklace varies directly as the cube of the number of diamond beads in it. Once this diamond broke into 3 pieces with number of beads in the ratio 1 : 3 : 5 respectively. When the 3 pieces were sold individually, the merchant got Rs.19200 less than the price the original necklace would have fetched him. Originally the necklace was worth

- (a) Rs.2.76 lakh (b) Rs.7.29 lakh (c) Rs.2.43 lakh (d) None of these

Q 12: Y varies directly as the 4th power of X. What is the percentage change in the value of Y, when X is halved?

- (a) -87.5% (b) -93.75% (c) -6.25% (d) 12.5%

Q 13: A right cylinder and a right circular cone have the same radius and the same volume. The ratio of the height of the cylinder to that of the cone is:

- (a) 3:5 (b) 2:5 (c) 3:1 (d) 1:3

Q 14: In a class, the number of boys is more than the number of girls by 12% of the total strength. The ratio of boys to girls is:

- (a) 11:4 (b) 14:11 (c) 25:28 (d) 28:25

Q 15: A, B and C do a work in 20, 25 and 30 days respectively. They undertook to finish the work together for Rs. 2220, and then the share of A exceeds that of B by:

- (a) Rs. 120 (b) Rs. 180 (c) Rs. 300 (d) Rs. 600

Q 16: Salmaan, Shahrukh and Aamir enter into a partnership and their capitals are in the proportion $\frac{1}{3} : \frac{1}{4} : \frac{1}{5}$. Salmaan withdraws half of his capital at the end of 4 months. Out of the total annual profit of Rs.847, Salmaan shares would be

- (a) Rs.252 (b) Rs.280 (c) Rs.315 (d) Rs.412

Q 17: 4 director rented some spot boys, Subash Ghai took 18 spot boys for 4 months, Yash Chopra took 25 spot boys for 2 months, Bhansali took 28 spot boys for 5 months and Rakesh Roshan took 21 spot boys for 3 months. If Subash Ghai's share of rent is 360\$, the total rent of the spot boys is:

- (a) 1500\$ (b) 1600\$ (c) 1625\$ (d) 1650\$

Q 18: A and B are two alloys of Copper and Silver prepared by mixing metals in proportion 7: 2 and 7: 11 respectively. If equal quantities of the alloys are melted to form a third alloy C, the proportion of copper and silver in C will be:

- (a) 5: 9 (b) 5: 7 (c) 7: 5 (d) 9: 5

Q 19: If $x = \frac{2\sqrt{6}}{\sqrt{3}+\sqrt{2}}$, find the value of $\frac{x+\sqrt{3}}{x-\sqrt{3}} + \frac{x+\sqrt{2}}{x-\sqrt{2}}$

- (a) 1 (b) 2 (c) -2 (d) 3

Q 20: Chavanni and Athanni start a business and each invests Rs.8000. After 6 months, Chavanni withdraws half of his capital and Athanni invests the amount withdrawn by Chavanni. If at the end of the year Chavanni gets Rs.7822 as his share of profits, find Athanni share?

- (a) Rs.13036 (b) Rs.4693 (c) Rs.12099 (d) Rs.11099

ANSWER KEY:

Q 1: d	Q 2: d	Q 3: a	Q 4: a	Q 5: c
Q 6: c	Q 7: c	Q 8: c	Q 9: c	Q 10: b
Q 11: d	Q 12: b	Q 13: d	Q 14: b	Q 15: b
Q 16: b	Q 17: c	Q 18: c	Q 19: b	Q 20: a