SET

SET :- Set is well defined collection of distinct object. Set can be finite or infinite.

Represention of Set :-

→ Roaster form
→ Set - Builder form

Roaster form :- Roaster ~~f~~ notation of a Set is a simple mathematical reepresentation of the in mathematical ~~reepresentat~~ form. Every two element are seperated by comma.

eg :- A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

Builder Notation :- The method of defining a set by describing its properties rather than listing its element is known as set Builder notation.

eg :- x = {x : y is a letter in the word dictionary}

Types of Set :-

1. Empty Set / Null Set :- A Set that does not contain any element termed as null set. The cardinality set is zero. It is expressed by the ϕ. cardinality = Modules of set.

2. Singleton set :- A set that has only one element is termed as singleton set.
eg :- $A = \{10\}$

3. Finite Set :- A set that contains a finite number of elements is named as a finite set. Empty set is also termed as finite set.
eg: -
$M = \{x : x \in M, x < 8\}$
$Q = \{3, 5, 7, 11, 13, \ldots, 113\}$

4. Subset :- Consider two set A and B. If each element of A is persent in set B or we can say the if element of set A belongs to set B. Then A is subset of B.
$A \subseteq B$
Every given set is subset of itself.

eg :- $A = \{p, q, r, s, t, u\}$
$B = \{m, n, o, q, r, s, t, u, p\}$
then
$A \subseteq B$.
$A = \{3, 4, 5, 7, 6, 8\}$
$B = \{4, 5\}$
~~$A \subseteq B$~~. $B \subseteq A$

5. Power Set :- Let A be a set, then the set of all possible subset of A is called power set of A and is denoted by P(A)

The number of component of the power set is given by 2^n elements.

eg - Set $\{x, y, z\}$
Component of power set :-
$P(A) = \{\{y\}, \{x\}, \{z\}, \{\ \}, \{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}\}$.

No. of element persent in power set $= 2^n$
$n = 3$,
$2^3 = 8$

5. Complement of set :- The complement of set A is defined as a set that contain the element persent in universal set but not in A.
eg :- $U = \{2, 4, 6, 8, 10, 12\}$
$A = \{4, 6, 8\}$

Complement of set A, $A' = \{2, 10, 12\}$
means $U - A = A'$

6. Proper set :- Consider A and B to be two sets. Then A is declared to be proper set of B if A is subset of B and A is not equivalent to B.
eg :- $A = \{2, 3, 4, 7, 8\}$
$n(A) = 5$
$B = \{1, 2, 3, 4, 7, 8, 10\}$
$n(B) = 7$
$A \subset B$

Difference between Subset and Proper Subset:-

Subset :- $A = \{2, 3, 5, 6\}$
$B = \{2, 3, 5, 6\}$
$A \subseteq B$.

Proper Subset :- $A = \{2, 3, 5, 6\}$
$B = \{2, 3, 5, 6, 7\}$
$A \subset B$.
B has 1 one element that is not in A.

7. Universal Set :- The basic set is called universal set. The universal set is normally indicated by U, and all its subset by letter A, B, C etc. This is the set that is the foundation of every other set developed.

eg :- $A = \{2, 3, 4\}$ $B = \{4, 5, 6, 7\}$
$C = \{6, 7, 8, 9, 10\}$
then,
$U = \{2, 3, 4, 5, 6, 7, 8, 9, 10\}$
All the set are subset of the universal set.

9. Equivalent Set :- Any two sets are stated to be equivalent set if their cardinality is the same.
A and B be two set then if $n(A) = n(B)$ then A and B are equivalent set.

10. Equal Set :- Any two set are declared to be equal set if they hold same element. Each element of P is an element of Q and every element of Q is an element of P.

11. Superset :- whenever a given set P is a subset of set Q, we say the Q is a superset of P and we address it as $Q \supseteq P$. Then symbol $\supseteq$ is applied to denote super set of.

12. Null Set :- Null set is a subset of every set. Improper subset is synonym of null set. Every set A is a subset of itself ϕ is a subset of every set.

* if $|A| = n$
Then no. of subset of cardinality $r = {}^nC_r = \dfrac{n!}{r!\,(n-r)!}$

eg :- $\{a, b, c\}$
find no. of subset if cardinality is 2
$${}^nC_r = {}^3C_2 \Rightarrow \frac{3!}{(3-2)!\,2!} = \frac{3 \times 2 \times 1}{2 \times 1 \times 1} \Rightarrow 3.$$

Subset with even cardinality + Subset with odd cardinality = Total no. of subset.

Set Operation :-

1. Set UNION :- The union of set A and B (denoted by $A \cup B$) is the set of element that are in A, in B or in both A and B.
Hence, $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
eg:-
$A = \{10, 11, 12, 13\}$
$B = \{13, 14, 15\}$
$A \cup B = \{10, 11, 12, 13, 14, 15\}$

2. Set Intersection :- The intersection of set A and B (denoted by $A \cap B$) is the of element which are both in A and B.
Hence $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
eg:- $A = \{11, 12, 13\}$
$B = \{13, 14, 15\}$
then
$A \cap B = \{13\}$

3. Cartisian product / Cross Product :-
The Cartesian product of n number of sets $A_1, A_2, \ldots A_n$ denoted as $A_1 \times A_2 \times A_n$ can be defined as all possible ordered pairs $(x_1, x_2, \ldots x_n)$ where $x_1 \in A_2 \ldots x_n \in A_n$.
$A \times B \neq B \times A$
eg:- $A = \{a, b\}$
$B = \{1, 2\}$
$A \times B = \{(a,1), (a,2), (b,1), (b,2)\}$

4. Symmetric Difference :-
→ Let A and B are two sets. The symmetric difference between both A and B is the set that contain the element that are present in both sets except the common elements.
$A \Delta B$ or $A ? B = (A - B) \cup (B - A)$
eg:- $A = \{1, 2, 3, 4, 5\}$
$B = \{3, 5\}$
$A \Delta B = \{1, 2, 4\}$

5. Disjoint Set :- In a mathematics, two sets are said to be disjoint sets if they have no element in common.
eg:- $A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$
A and B are Disjoint set.
$C = \{1, 2, 3\}$ $D = \{3, 4, 5\}$
C and D are disjoint set as 3 is common.

PROPERTIES OF SET :-

1. If $A \subseteq B$ then $|A| \leq |B|$
2. If $A \subseteq A \cup B$ then $|A| \leq |A \cup B|$
3. If $A \cap B \subset B$ then $|A \cap B| \leq |B|$
4. $(A \cup B)^c = A^c \cap B^c$ → De morgon's law.

* Law of Set theory :-

1. Identity :- $A \cup \phi = A$
$A \cap U = A$

eg :-

$A = \{1, 3, 5\}$

$\phi = \{\ \}$

$A \cup \phi = \{1, 3, 5\}$

2. Domination :- $A \cap \phi = \phi$

$A \cup U = U$

3. Idempotent law :- $A \cup A = A$

$A \cap A = A$

4. Commutative law :- $A \cup B = B \cup A$

$A \cap B = B \cap A$

5. Distributive law :- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap B)$

6. Assosiative Law :- $A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

7. Absorption law :- $A \cap (A \cup B) = A$

$A = \{3, 7, 9\}$

$B = \{3, 13, 17\}$

$A \cup B = \{3, 7, 9, 13, 17\}$

$A \cap (A \cup B) = \{3, 7, 9\} \cap \{3, 7, 9, 13, 17\}$

$= \{3, 7, 9\} = A$

if $A \cup B = A \cap B$

then $A = B$

Suppose $A = \{\ \}$, $B = \{1, 2, 3\}$ then what is $A \times B$?

$\Rightarrow \{\ \} \times \{1, 2, 3\} = 0$

product of null set and ~~only~~ any other set is always empty set.

So,

$A \times B = \phi$ or null set

Q. which of the following statements are true about $B = \{D, \{A\}\}$.

(a) $A \in B$

(b) $\{A\} \in B$

(c) $\{A\} \subseteq B$

(d) $\{D, A\} \in pow(B)$

* Properties of Cartesian product :-

1. $A \times B \neq B \times A$
2. $A \times (B \cup C) = (A \times B) \cup (A \times C)$
3. $A \times (B - C) = (A \times B) - (A \times C)$
4. $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

* Set theory : more result :-

$A - B = A \cap B^c = A - (A \cap B)$

$A - B = B^c - A^c$

$A \subseteq B \Leftrightarrow B^c \subseteq A^c$

$A \subset B$ and $C \subset D \Rightarrow A \times B \subset B \times D$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n($

$+ n(A \cap B \cap C)$

Q. How many positive integer not exceeding 100 are divisible either by 4 or 6

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A) = \frac{100}{4} = 25$

$n(B) = \frac{100}{6} = 16.6 = 16$

$n(A \cap B) = 2 \,|\, 4, 6$; $2, 3$ $= 2 \times 2 \times 3 = 12$

$n(A \cup B) = 25 + 16 - 12$
$= 41 - 12$
$= 29$

Q. How many bit string of length 7 either begin with two zero and end with three once.

Soln

→ $A = \boxed{0}\boxed{0}\boxed{1/0}\boxed{1/0}\boxed{1/0}\boxed{1/0}\boxed{1/0}$ $2 \times 2 \times 2 \times 2 \times 2 = 16$

$B = \boxed{0}\boxed{0}\boxed{1/0}\boxed{1/0}\boxed{1}\boxed{1}\boxed{1}$ $2 \times 2 \times 2 \times 2 \times 2 = 16$

$A \cap B = \boxed{0}\boxed{0}\boxed{1/0}\boxed{1/0}\boxed{1}\boxed{1}\boxed{1}$ $2 \times 2 = 4$

$= 32 + 16 - 4$
$= 48 - 4 = 44$

Q. If A and B subset of universal set U then show that $\bar{A} + \bar{B} = A + B$.

$\bar{A} + \bar{B} = A + B$

$(\bar{A} \cup \bar{B}) - (\bar{A} \cap \bar{B}) = (\overline{A \cap B}) - (\overline{A \cup B})$

$= U - (A \cap B) - [U - (A \cup B)]$

$(A \cup B) - (A \cap B) = A \oplus B$

RELATION

→ Let A and B be two non-empty set then any subset R of the Cartesian product $A \times B$ is called a relation from A to B.

eg:-

$A = \{3, 6, 9\}$

$B = \{4, 8, 12\}$

$A \times B = \{(3,4), (3,8), (3,12), (6,4), (6,8), (6,12), (9,4), (9,8), (9,12)\}$

then,

$R = \{(3,4), (3,8), (6,12)\}$

If $(a,b) \in R$, we offen write aRb and state 'a' is related to 'b'

if $R \subseteq A \times A$ then R is relation from A to A and R is called a relation in A.

eg:-

$A = \{1, 2, 3\}$

$R = \{(1,2), (3,2), (2,3)\}$ is a relation.

* Represention of Relation :-

Relation can be representationed by vario method these are :-

→ Set Builder
→ Listing
→ Matrix method
→ Arrow diagram
→ Graphical method.

1. Set Builder :-

$R = \{(x,y) \mid x<y\}$

$A = \{1,2,3\}$

$R = \{(1,2),(1,3),(2,3)\}$

2. Listing :- $R = \{(1,2),(1,3),(2,3)\}$

$R = \{(x,y) \mid x<y\}$.

3. Matrix method :-

$A = \{1,2,3\}$

$R = \{(x,y) \mid x<y\}$

$$R = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 1 & 1 \\ 2 & 0 & 0 & 1 \\ 3 & 0 & 0 & 0 \end{array}$$

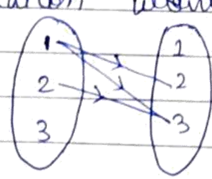
If $(x,y) \in R$ then there will be 1 in the position corresponding to (Row representing element x, column representing element y).

So.

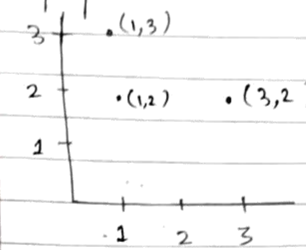
if $a_{ij} = 1$ if there is a relation between i of A and j of B otherwise 0.

4. Arrow diagram :-

Arrow diagram representation of same relation would be



5. Graphical Method :-

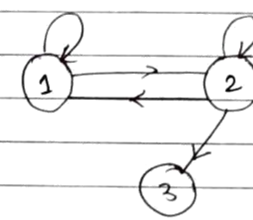


6. Digraph :- A diagraph (directed graph) representation is Suitable only if the relation is between a set A and itself

eg : $A = \{1,2,3,4\}$

then ~~asie~~ consider a relation on $A \times A$ given as follows.

$R = \{(1,1),(1,2),(2,2),(2,1),(2,3)\}$



* Operation on Relation :-

$R = \{(1,1),(1,2),(2,3)\}$

$S = \{(1,2),(2,3),(3,3)\}$

→ Union :- $R \cup S = \{(1,1),(1,2),(2,3)\}$

→ Intersection :- $R \cap S = \{(1,2),(2,3)\}$

→ Compliment :- $A^c = U - A$

$A^c = (A \times A) - A$

eg :- $\{1,2,3\}$

$U = \{ (1,1), (2,2), (3,3), (1,2), (1,3)\ (2,3),$
$\{1, \cancel{2}, \cancel{3}\}, \{3,1)\ (3,2)\ \}$

Properties of relation :-

1. Reflexive :- R is reflexive if aRa holds for all $a \in A$ such that if $(a,a) \in R$ $\forall\ a \in A$.

eg:
let $A = \{a, b, c\}$
$R = \{(a,a), (b,b), (c,c)\}$
Smallest reflexive relation
$= \{(a,a), (b,b), (c,c)\}$

Largest reflexive relation = All the subset of relation R.

2. Symmetric :- R is Symmetric if bRa holds whenever aRb (a is related to b)
eg: let R be a relation "is perpendicular to" in a set of all straight lines".

3. Transitive Relation :- A Relation R in a set A is said to be transitive if $(a,b) \in R, (b,c) \in R$ $\Rightarrow (a,c) \in R$.
eg:- let R be a relation "is parallel to" in a set of all straight lines.

let $A = \{1, 2, 3\}$
$R = \{(1,1), (2,2), (3,3), (2,3)(3,2)\}$

6/12/22

Q1) Let $A = \{1,2,4\}$, $B = \{0,2\}$ find $A \times B$

= $A \times B = \{(1,0), (1,2), (2,0), (2,2), (4,0), (4,2)\}$

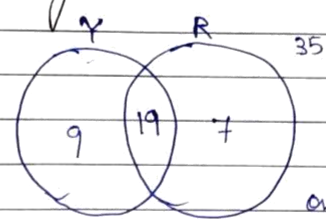
Q2) $A = \{a,b\}$, $B = \{2,3\}$, & $C = \{3,4\}$
find
(i) $A \times (B \cup C)$
=) $B \cup C = \{2,3,4\}$
$A \times (B \cup C) = \{(a,2), (a,3), (a,4), (b,2), (b,3), (b,4)\}$

(ii) $(A \times B) \cup (A \times C)$
=) $A \times B = \{(a,2), (a,3), (b,2) (b,3)\}$
$A \times C = \{(a,3), (a,4) (b,3), (b,4)\}$
$(A \times B) \cup (A \times C) = \{(a,2), (a,3), (b,2), (b,3), (b,4)\}$

(iii) Find $A \times (B \cap C)$
=) $B \cap C = \{3\}$
=) $A \times (B \cap C) = \{(a,3), (b,3)\}$

Q3. 35 children of a class draw a map. 26 uses Red color same uses yellow color. If 19 uses both the color then find the no. of children who use the yellow color.

=)



$n(\text{Total}) = 35$
$n(R) = 26$
$n(Y \cap R) = 19$
only (Red) $= \cancel{35} - 26 - 19 = 7$
only yellow $= 35 - (19 + 7)$
=) $35 - 26 \Rightarrow 9$

The no. of children who uses the yellow color $= 9 + 1$[illegible]
$= 28$

" " " " " only yellow color $= 9$

4) $A = \{3, 7\}$

(i) find $P(A)$ = $n = 2$.

$P(A) = 2^2 = 2^n = 2^2 = 4$

(ii) What is $|A|$? = no. of elements = 2

(iii) What is $|P(A)|$? ⇒

$P(A) = \{(\phi), \{3\}, \{7\}, \{3,7\}\}$

$|P(A)| = 4$

Q5) List all the subsets of $\{a, b\}$

$= \{\phi, (a), (b), (a, b) ~~, (a, b, \phi)~~ \}$

Irreflexive :-

A relation R on set A is irreflexive if and only if $a \not R a$ for every $a \in A$.

ex - Let $A = \{1,2,3\}$

$R = \{(1,2), (1,3), (3,1), (2,1)\}$

ex - Let S be the set of all straight lines the relation R defined by "x is ‖ to y" is irreflexive, since no line is perpendicular to itself.

Equivelance :- A relation R in a set A is set to be an equivalance relation if R is Reflexive, Symmetric and transitive.

$A = \{a, b, c\}$ and

$R = \{(a,a), (b,b), (c,c), (a,b), (b,a), (b,c), (c,a), (c,b), (a,c)\}$

(Symmetric difference ⇒ Exclude intersection)

Asymmetric Relation :-

A relation R on A is an asymetric relation if and only if $(x,y) \in R \Rightarrow (y,x) \notin R$

Supose, This is similar to anti-symmetric property in that all relations are uni directional, except that in anti-symmetric the self loop are allow but here in asymmetric even self loop are not allowed.

ex - $R = \{(x,y) : x$ is father of $y\}$

Let xRy ⇒ x is father of y

further y is not father of x

Such that $xRy \Rightarrow y \not R x$

no. of subsets in $A = \{1,2,3\}$

$= \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$

$\{1\} \subseteq \{1,2,3\}$ $B \subseteq A$

B A

no. of subsets in of $A = 2^n = 2^3 = 8$ Cardinality of A

Total no. of proper Subset $= 2^n - 1$

$= \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{3,1\}\}$.

Power Set $P(A) = 2^n = 8$

Antisymmetric :- A relation R on A is called antisymmetric if and only if $xRy \Rightarrow y \not R x$ unless $x = y$

in other words R is antisymmetric iff $(x,y) \in R$ and $(y,x) \in R \Rightarrow x = y$

(a) antisymmetric property basically means that all relations are one way (except for self loop) which are always two ways.

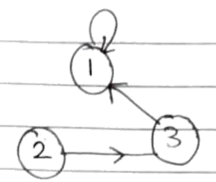
(b) To check for antisymmetric, check the 1's in off diagonal and see if a '0' is there in corresponding mirror image position.

Ignor diagonal is in this check

(c) To check a diagraph for antisymmetric ignore self loops and check that for arrow going from a to b (a and b are distinct) there is no arrow from b to a.

$A = \{1,2,3\}$

$R = \{(1,1), (2,3), (1,3)\}$



$$M_R = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 0 & 1 \\ 2 & 0 & 0 & 1 \\ 3 & 0 & 0 & 0 \end{array}$$

diagonal

diagram representation   matrix representation

(no. of self loop is considered in asymmetric)

(d) To check a set builder relation let xRy & yRx then Solve if the only Solution is $x=y$ then R is Antisymmetric.

ex- $R = \{(x,y) \mid x \text{ divides } y\}$ $x,y \in N$

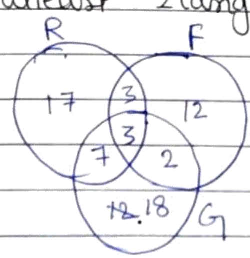
$xRy = yRx$ result $[x=y]$

addition function is not an antisymmetric (exceptional)

8-12-22

Q1) In a class room of 100 Students taking a III langu
30 Study russian, 30 Study german and 20 Study french. 10 Study german & Russian 5 Study german & french. 6 Study russian and french. 3 Study all 3 language. 6 Study

(a) How many students Study none of the language?
(b) Exactly one language
(c) atleast 2 langauge.



$n(R) = 30$, $n(G) = 30$, $n(F) = 20$

$n(R \cap G) = 10$, $n(G \cap F) = 5$, $n(R \cap F) =$

$n(R \cap G \cap F) = 3$

Ans a Total - no. of student study at least one subject
$= 100 - 62 = 38$

Ans b Exactly one lang $= 17+12+18 = 47$

Ans c atleast 2 lang $= 7+3+3+2 = 15$

Q2. Each of the following define a relation on the set N of positive integers. determine which of the following relations are reflexive / symmetric.

(a) R : x is greater than y
(b) T : $x + 4y = 10$
(c) S : $x + y = 10$ → Symmetric
(d) None → non is Reflexive

Q3 which of the following relation is transitive

(a) R : $A \subseteq B$ → Transitive
(b) S : A is disjoint from B → $A \cap B = \phi$ Transitive
(c) T : $A \cup B$ = Universal set → non Transitive
(d) None → $U = \{1,2,3,4,5,6\}$

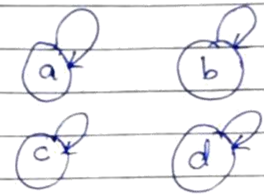
$A = \{1,2\}$ $B = \{3,4,5,6\}$ $C = \{1,2\}$
$A \cup B = U$
$B \cup C = U$
$A \cup C \neq U$.

13-12-22

Identity Relation :- Let A be a non empty set then the relation.

$\{(x,y) \mid x, y \in A \text{ and } x = y\}$ is called identity relation.

ex - Let $A = \{a,b,c,d\}$
then Identity relation $I_A = \{(a,a),(b,b),(c,c),(d,d)\}$



Operations on relation :-

Since relations are sets, all set operations can be performed on relations also such that if R and S are two relations than the following are define.

$R \cup S$, $R \cap S$, R^c, S^c, $R-S$, $S-R$, $R \oplus S$

example :-
$R = \{(1,1),(1,2),(2,3)\}$
$S = \{(1,2),(2,3),(3,3)\}$ on $A \times A$.
$A = \{1,2,3\}$
$A \times A$ = is universal set here $= U$
$A \times A = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$

→ $R \cup S = \{(1,1),(1,2),(2,3),(3,3)\}$
→ $R \cap S = \{(1,2),(2,3)\}$
→ $R^c = U - R = (A \times A) - R = \{(1,3),(2,1),(2,2),(3,1),(3,2),$
$(3,3)\}$

→ $S^c = U - S = (A \times A) - S$
$= \{(1,1), (1,3), (2,1), (2,2), (3,1), (3,2)\}$

→ $R - S = \{(1,1)\}$
→ $S - R = \{(3,3)\}$
→ $R \oplus S = (R-S) \cup (S-R)$
$= \{(1,1), (3,3)\}$

In addition to the above operation, the following sepical operations are also defined on R and S such that

R^{-1}, S^{-1}, $S_\circ R$, $R_\circ S$ → composition operator at Relation S & R

R operator S

Defination :-

$R = \{(1,1), (1,2), (2,3)\}$
$S = \{(1,2), (2,3), (3,3)\}$

$R^{-1} = \{(y,x) \mid (x,y) \in R\}$. in above example

$R^{-1} = \{(1,1), (2,1), (3,2)\}$

$S^{-1} = \{(2,1), (3,2), (3,3)\}$

Note that if R relates x to y then R^{-1} relates y to x.

Composition of Relation :-

Let R be a relation from A to B and S be a relation from B to C.

R ⇒ A to B
S ⇒ B to C

Then we can define a relation, the composition of R and S written as $S_\circ R$. The relation $S_\circ R$ is a relation from the set A to the set C and is deffined as follows

if $a \in A$ and $c \in A$, then $(a,c) \in S_\circ R$ if and only if for some $b \in B$, we have $(a,b) \in R$ and $(b,c) \in S$.

example :- Let $A = \{1,2,3,4\}$ and
$R = \{(1,2), (1,3), (2,4), (3,2)\}$
$S = \{(1,4), (4,3), (2,3), (3,1)\}$

find $S_\circ R$ and $R_\circ S$?

$S_\circ R$ = $(1,2) \in R$
$(2,3) \in S$
$(1,3) \in S_\circ R$.

$(1,3) \in R$
$(3,1) \in S$
$(1,1) \in S_\circ R$

$(3,2) \in R$
$(2,3) \in S$
$(3,3) \in S_\circ R$

$(2,4) \in R$
$(4,3) \in S$
$(2,3) \in S_\circ R$

S∘R find karne ke liye pehle R ke ek set ko lenge aur uske last no. se start hote hue R ke set me [illegible] aur common [illegible]

$S \circ R = \{(1,3), (1,1), (3,3), (2,3)\}$ → Ans

→ $R \circ S$:

$(1,2) \in S \rightarrow (2,3) \in R \rightarrow (1,3) \in R \circ S$

$(1,4) \in S \rightarrow 4$ is not in R

$(4,3) \in S \rightarrow (3,2) \in R \rightarrow (4,2) \in R \circ S$

$(2,3) \in S \rightarrow (3,2) \in R \rightarrow (2,2) \in R \circ S$

$(3,1) \in S \rightarrow (1,2) \in R \rightarrow (3,2) \in R \circ S$

$(3,1) \in S \rightarrow (1,3) \in R \rightarrow (3,3) \in R \circ S$

$R \circ S = \{(4,2), (2,2), (3,2), (3,3)\}$

$\boxed{R^2 = R \circ R}$ $\boxed{R^3 = (R \circ R) \circ R}$

Q find $R \circ R = R^2$ where R is $\{(1,2), (1,3), (2,4), (3,2)\}$

$R \circ R$ =

$(1,2) \in R \rightarrow (2,4) \in R \rightarrow (1,4) \in R \circ R$

$(1,3) \in R \rightarrow (3,2) \in R \rightarrow (1,2) \in R \circ R$

$(2,4) \in R \rightarrow 4 \notin R$

$(3,2) \in R \rightarrow (2,4) \in R \rightarrow (3,4) \in R \circ R$

$R^2 = R \circ R = \{(1,4), (1,2), (3,4)\}$ Ans

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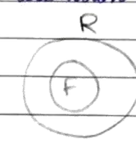
FUNCTION

function :-

Let A and B be any two sets. A relation f from A to B is called function if for every $a \in A$ there is unique element $b \in B$ such that order pair $(a,b) \in f$. In other words, a function is a unique value relation such that every element of a is mapped to only one element of B. however, elements of B may be related to more than one element of a.

Note that every function is a relation, but a relation may or may not be a function.

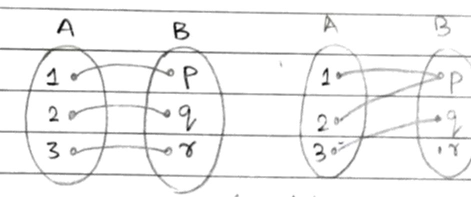


example :- Let $A = \{1,2,3\}$
$B = \{p, q, r\}$

and

$f = \{(1,p), (2,q), (3,r)\}$ → function & Relation

$f_1 = \{(1,p), (1,q), (2,q), (3,r)\}$ → relation but not function.

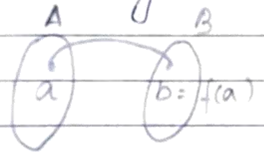


function

then $f(1) = \{p\} = p$
$f(2) = \{q\} = q$
$f(3) = \{r\} = r$

clearly f is a function from A to B given any function $f: A$ to B or $f: A \to B$ the notion $f(a) = b$ means $(a, b) \in f$

it is customery to write $b = f(a)$ the element $a \in A$ is called an argument of the function f, and $f(a)$ is called the value of the function for the argument a or the image of a under f.



example :- $f = \{(1,1), (2,3), (3,3)\}$ is a function on $A \times A$ where $A = \{1,2,3\}$

here
$f(1) = \{1\} = 1$
$f(2) = \{3\} = 3$
$f(3) = \{3\} = 3$. → value

argument

where as $R = \{(1,1), (2,3), (2,4), (3,3)\}$ is not a function since.
$R(1) = \{1\} = 1$
$R(2) = \{3,4\}$ → argument cannot be connected to two value
$R(3) = \{3\}$

Cardinality will be 1 or zero.

Here $R(2)$ has two values 3 & 4 hence R is not a function.

One-One function :-

A function $f: A \to B$ is said to be one-one if different elements of A have different f-images in B such that $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ or equivalently $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$.

To check if a function is one-one, let $f(x_1) = f(x_2)$ and see if this leads to a single solution i.e., $x_1 = x_2$ if so, f is one-one. else it is many-one.

example :- $S = \{(x,y) \mid y = 3x+1\}$ on $R \times R$

(i) $\forall\ x \in R$ $S(x) = 3x+1 \in R$ on $R \times R$

(ii) $3x+1$ has a single values for any real values x, so S is a function.

(iii) Now to check one-one we set $S(x_1) = S(x_2)$

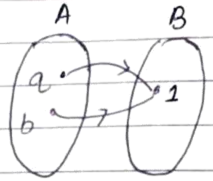
$y_1 = y = 3x_1+1 = 3x_2+1 \Rightarrow 3x_1 = 3x_2$
$\Rightarrow x_1 = x_2$

So S is one-one function

many-one function :-

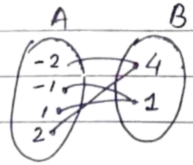
A function $f: A \to B$ is said to be many-one, if and only if two or more different elements in A have the same f-image in B.

A function which is not one-one will be many-one.



example :- $T = \{(x, y) \mid y = x^2\}$ on $R \times R$

T is a function.



now let $T(x_1) = T(x_2) = x_1^2 = x_2^2$

$\Rightarrow y_1 = y_2$

now $x_1^2 = x_2^2$ has two solution

$x_1 = x_2$ or $x_1 = -x_2$.

So we say $x_1^2 = x_2^2 \neq x_1 = x_2$ this means T is not one-one, such that it is manager.

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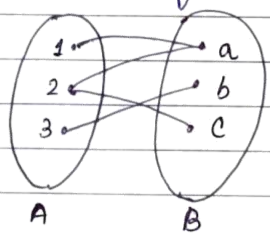
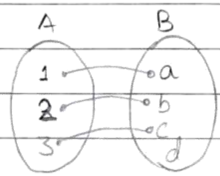
Onto function :- A function f from set A to Set B is onto if each element of B is match to atleast one element of A.





not an onto function

Bijection function :- A mapping $f: A \to B$ is called one-to-one, onto if it is both one-to-one and onto then it is Bijective function.

If the function is both one-one and onto then it is bijection function.

Operation or Composition of function :-

Q $f(x) = x + 1$

$g(x) = 2x^2 + 3x + 1$

(i) $(f+g)(x) = f(x) + g(x)$

$= x + 1 + 2x^2 + 3x + 1$

$= 2x^2 + 4x + 2$ - Ans

(ii) $(f-g)(x) = f(x) - g(x)$

$= x + 1 - (2x^2 + 3x + 1)$

$= x + 1 - 2x^2 - 3x - 1$

$\Rightarrow -2x^2 - 2x$ - Ans

Q. $f(x) = 3x^2 + 7x$

$g(x) = 2x^2 - x - 1$

(i) $gof(x) = 2(3x^2 + 7x)^2 - (3x^2 + 7x) - 1$

$= 2(9x^4 + 42x^3 + 49x^2) - 3x^2 - 7x - 1$

$= 18x^4 + 84x^3 + 98x^2 - 3x^2 - 7x - 1$

$= 18x^4 + 84x^3 + 95x^2 - 7x - 1$ - Ans

(ii) $(f+g)(x) \Rightarrow 3x^2 + 7x + 2x^2 - x - 1$

$\Rightarrow 5x^2 + 6x - 1$ Ans

(iii) $(f-g)(x) \Rightarrow 3x^2 + 7x - 2x^2 + x + 1$

$\Rightarrow x^2 + 8x + 1$ Ans

(iv) $(f * g)(x) = (3x^2 + 7x)(2x^2 - x - 1)$

$= (6x^4 - 3x^3 - 3x^2 + 14x^3 - 7x^2 - 7x)$

$= 6x^4 + 11x^3 - 10x^2 - 7x$ Ans

(v) $(f \circ g)(x)$:- $3(2x^2 - x - 1)^2 + 7(2x^2 - x - 1)$

$\Rightarrow 3(4x^4 - 2x^3 - 5x^2 + 2x + 1) + 7(2x^2 - x - 1)$

$\Rightarrow 12x^4 - 6x^3 - 15x^2 + 6x + 3 + 14x^2 - 7x - 1$

$\Rightarrow 12x^4 - 6x^3 - x^2 - x + 2$ Ans.

Constant function :- Let function A to B is said to be a constant function if every element of A is mapped onto the same element of B.

for-example :- $f : R \rightarrow R$
$f(x) = 5 \quad \forall x \in R$

Set $A = \{1,2,3,4,5,6,7,8\}$
$B = \{2,4,6,8\}$
$B \subseteq A$

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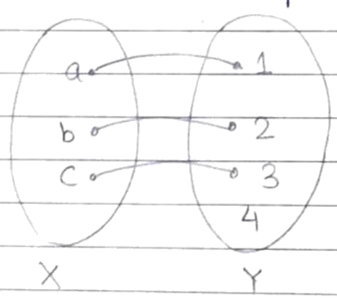
Into function :- A function in which there must be an element of co-domain. Y does not have a pre-image in X.



[Injective - one-one
Surjective - onto
bijective - one-one onto]

Recursively define function :-

It is a function that its value at any point can be calculated from the value of function at some previous point.

Recursive function is a function that calls itself.

ex - fibbonacci Series, factorial of a no.

$f : N \rightarrow N$

$$F(n) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F(n-1) + F(n-2) & \text{if } n > 1 \end{cases}$$

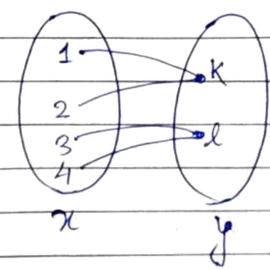
Recursively define function of fibbonacci Series.

factorial $f(n) = \begin{cases} 1 & \text{if } n = 0 \;\&\; n = 1 \\ n * f(n-1) & \text{if } n > 1 \end{cases}$

Q $x = \{1,2,3,4\}$ $\quad y = \{k, l\}$

$f = \{(1,k), (2,k), (3,l), (4,l)\}$

=)



many - one and onto functio

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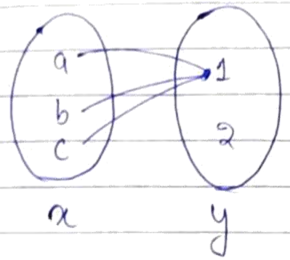
Q. $x = \{a, b, c\}$ $y = \{1, 2\}$
$f = \{(a,1), (b,1), (c,1)\}$

→ 

many-one & Into.

Q $f(x) = 3x^2$ find fof

⇒ $fof(x) = 3(3x^2)^2 \Rightarrow 3 \times 9 \times x^4$
$= 27x^4$ Ans.

Q $f(x) = 2x + 1$
$g(x) = x^2 + 1$

$gof(x) = (2x+1)^2 + 1$
⇒ $4x^2 + 1 + 4x + 1$
⇒ $4x^2 + 4x + 2$ Ans

$fog(x) = 2(x^2+1) + 1$
⇒ $2x^2 + 2 + 1$
$= 2x^2 + 3$ Ans

POSET :- Partial Ordered Set

Partial Ordering Relation - A relation 'R' is said to be a partial order relation if R is reflexive, anti-symmetric, transitive.

A Set 'A' with partial Ordering Relation 'R' defined on 'A' is called Poset. Poset is denoted by [A,R].

Reflexive $aRa \ \forall a \in R$
Anti-Symmetric aRb $b \not R a \ \forall a,b \in R$
Transitive aRb, bRc & $aRc \ \forall a,b,c \in R$

Example :- $A = \{1,2,3\}$
$R = \{(1,1), (2,2), (3,3), (1,2), (2,1)\}$

Reflexive $= \{(1,1), (2,2), (3,3)\}$
Anti-Symmetric $= \{(1,2) \ (2,1)\}$ → not anti Symmetric
Transitive → $\{(1,2), (2,1) \ (1,1)\}$ → Transitive.

So it is not Poset

$\boxed{\text{Equivalance \& Poset}}$ → important

Q $\leq$ relation on natural no.
$A, \leq$
poset

⇒ $A = \{1, 2, 3\}$
$R = \{(1,1), (2,2), (3,3), (1,2), (1,3) \ (2,3)\}$

Relflexive = {(1,1), (2,2), (3,3)}
Anti-Symmetric = {(1,2)} but (2,1) not present so it is anti-Symmetric
Transitive :- {(1,2) (2,3) (1,3)}

So it satisfies all the condition of Poset
So it is Poset relation.

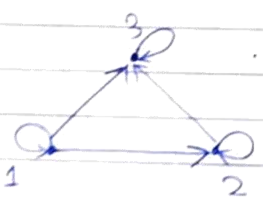
Q. A = {1,2,3}
R = {(1,1),(2,2),(3,3),(1,2),(2,3),(1,3)}

⇒ Reflexive = {(1,1),(2,2),(3,3)}
Anti-Symmetric = {(1,2)} (2,1) ×.
Transitive = {(1,2),(2,3),(1,3)}

it full fills the condition of Poset so this relation is Poset.

Q. ~~A~~ = {1,3,5,7} Relation = (a+b) = even

⇒ R = {(1,1),(3,3),(5,5),(7,7)



diagrametical representation.

no. of vertex = no. of elements in Set

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Hasse Diagrame :- Hasse Diagrame is a pictorial representation of a finite partial order on set 'A' on this representation the objects i.e elements are shown as vertices (or dots)

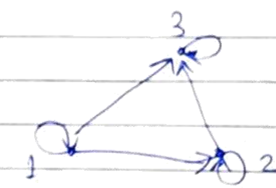
A = {1,2,3} [A, ≤] is a poset or not?

R = {(1,1),(2,2),(3,3),(1,2),(2,3),(1,3)}.

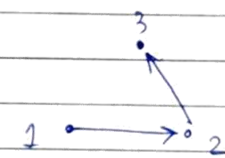
Reflexive = {(1,1),(2,2),(3,3)}
Anti-Symmetric = {(1,2)} but (2,1) not present
Transitive = {(1,2) (2,3) (1,3)}

it satisfies all the condition of POSET

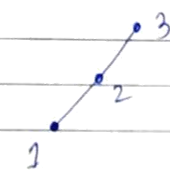
$v_i \to v_j$



Now remove the self-loop if the relation is Poset then self loop automatically exist so no need to show



→ 1 can pair with 3 by going through 2



it can be like this also.

Q. $A = \{1, 2, 3, 4, 6, 9\}$

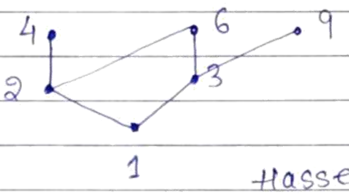
$R = [A, /]$

=) $R = \{(1,1), (2,2), (3,3), (4,4), (6,6), (9,9), (1,2), (1,3), (1,4), (1,6), (1,9), (2,4), (2,6), (3,6), (3,9)\}$

Reflexive = $\{(1,1), (2,2), (3,3), (4,4), (6,6), (9,9)\}$

Anti Symmetric = $\{(1,3)\}$ but $(3,1)$ does not exist
Transitive = $\{(1,2)\ (1,3), (1,3)\}$

So it satisfies all the condition of being reflexive, anti-Symmetric and Transitive
So this is a poset.



Hasse diagrame.

Q. $A = \{2, 3, 6, 12, 24, 36\}$
$R = [A, /]$

$R = \{(2,2), (3,3), (6,6), (12,12), (24,24), (36,36), (2,6), (2,12), (2,24), (2,36), (3,6), (3,12), (3,24), (3,36), (6,12), (6,24), (6,36), (12,24), (12,36)\}$

Reflexive :- $\{(2,2), (3,3), (6,6), (12,12), (24,24), (36,36)\}$
Anti Symmetric = $\{(2,12)\}$ but $(12,2)$ not present

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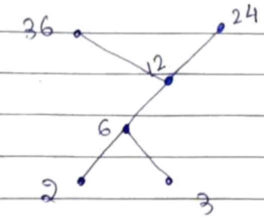
Transitive :- $\{(2,6), (6,12), (2,12)\}$

So it is Poset

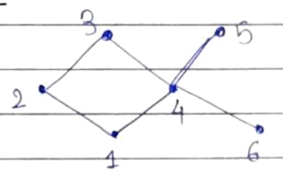


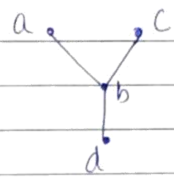
Hasse Diagram

Maximal & Minimal Elements :-

Minimal Elements :- An element $a \in A$ is called minimal ~~mebe~~ member of a relative to $\leq$ if for no. $x \in A$ is $x < a$.

example :-



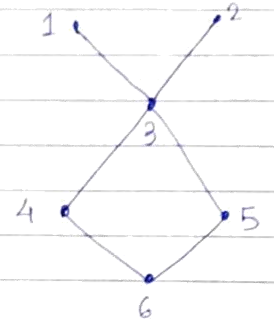


maximal = $\{3, 5\}$
minimal = $\{1, 6\}$

no. of element is related an element in POSET.

Upper bound and Lower bound :-

A = {1,2,3,4,5,6}
B = {4,5}

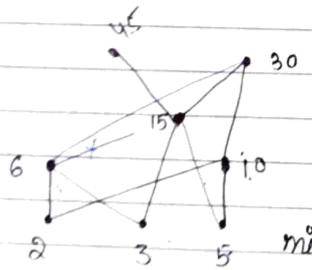


Lower bound = {6}
Upper bound = {1,2}

all element should relate with 4 & 5.

Q. A = {2,3,5,6,10,15,30,45}
[A, /]

R = {(2,2)(3,3),(5,5),(6,6),(10,10),(15,15),(30,30),(45,45),(2,6),(2,10),(2,30),(3,6),(3,15),(3,30),(3,45),(5,10),(5,15),(5,30),(5,45),(6,30),(10,30),(15,30),(15,45)}



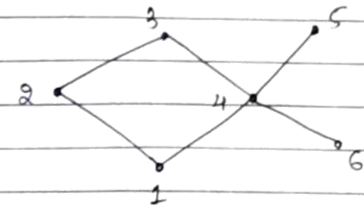
Hasse diagram

minimal = {2,3,5}
maximual = {45,30}

B = {6,10}
Lower Bond = {2}
Upper Bond = {30}

Upper Bound (UB): Let (A, ≤) be partially ordered set & AB ≤ A, any element m∈A is called an upper bound for B if for all x∈A, x≤m

Lower bound (LB): An element l∈A is called a lower bound for B if for all x∈A, l≤x

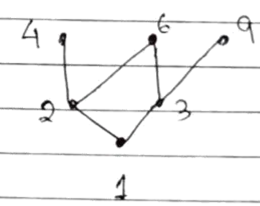


max = {3,5}
mini = {1,6}

Q. A = {1,2,3,4,6,9} [A, /]

R = {(1,1),(2,2),(3,3),(4,4),(6,6),(9,9),(1,2),(1,3),(1,4),(1,6),(1,9),(2,4),(2,6),(3,6),(3,9)}

Reflexive :- {(1,1),(2,2),(3,3),(4,4),(6,6),(9,9)} ✓
Anti-Symmetric = {(1,2), but (2,1) not present} ✓
Transitive = { ~~(1,6)~~ ~~(3,6)~~ ((1,2) (2,4), (1,4)) ✓

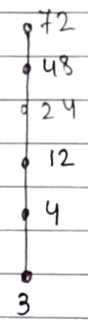


maximal → {4,6,9}
minimal = {1}

Q2. $A = \{3, 4, 12, 24, 48, 72\}$ $[A, \leq]$

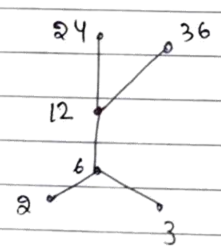
$R = \{(3,3), (4,4), (12,12), (24,24), (48,48), (72,72), (3,4), (3,12), (3,24), (3,48), (3,78), (4,12), (4,24), (4,48), (4,72), (12,24), (12,48), (12,72), (24,48), (24,72), (48,72)\}$

It is a Poset



Q3. $A = \{2, 3, 6, 12, 24, 36\}$ $[A, /]$

$R = \{(2,6), (2,12), (2,24), (2,36), (3,6), (3,12), (3,24), (3,36), (6,12), (6,24), (3,36), (12,24), (12,36)\}$



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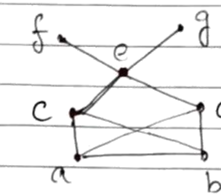
Q. $A = \{a, b, c, d, e, f, g\}$



$B = \{c, e, d\}$

Upper Bond $= \{f, g\}$
Lower Bond $= \{a, b\}$

Q.# Maximal Element :- An element $b \in A$ is called a maximal element of A relative to the partial ordering $\leq$ if for no $x \in A$ is $b < x$.

Greatest Lower bound (GLB) :- Let A be a Poset & B denote a Subset of A. An element L is called Greatest Lower Bound of B if L is a lower of B and ~~L's~~ $L' \leq L$ whenever L' in a lower bound of B. (also called Infimum ($\wedge$) it is meet operation).

Least Upper Bound (LUB) :- Set A be a partialy ordered Set and B is a subset of A. An element M is called Least Upper Bound ($M \in A$) of B if M is an upper bound of B and $M \leq M'$ whenever M' in a upper bound of B. (also called Supremum ($\vee$) it is join operation)

Totaly Ordered Set :- (Toset) Let $(A, \leq)$ be a partially Ordered Set if for every $a, b \in A$ we have $a \leq b$ or $b \leq a$ then $\leq$ is called a Simple ordering or linear ordering on A and the set is called a Totally ordered set (Toset) or chain.

→ If A is Toset then every pair of elements of A

are comparable.

ex ⇒ A = {set of positive integer}

A = {1, 2, 3, 4, 5}

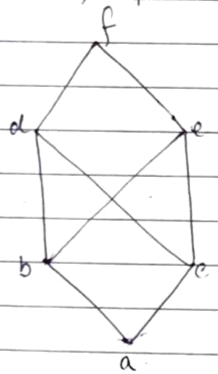
$1 \leq 2 \leq 3 \leq 4 \leq 5$. So every element is comparable then it is Toset.

ex = [A, /]

A = {3, 5, 10, 7, 21}

(3, 21) (5, 10), (7, 21) not every pair is comparable So it is not a Toset.

Q. find Lower bound, Upper bound, Least upper bound, Greatest Lower bound.



B = {d, e}
UB = {f}
LB = {a, b, c}
LUB = {f}
GLB = {ϕ}
[because (b, c) compare in for skte to ϕ]

B = {b, c}
UB = {d, e, f}
LB = {a}
LUB = {ϕ}
GLB = {a}

Lattice :- Lattice is a Poset $(L, \leq)$ in which every pair of elements $a, b \in L$ has a Greatest Lower Bound and Least upper Bound.

→ Propertises of Lattice :-

Let $(L, \leq)$ be a lattice '·' and '+' denotes operation of meet & join respectively on $(L, \leq)$ then for $a, b, c \in L$ we have

1. Idempotent Law
2. Commutative law
3. Associative law
4. Absorption law

1. Idempotent Law = $a \cdot a = a$, $a + a = a$
2. Commutative Law = $a \cdot b = b \cdot a$, $a + b = b + a$
3. Associative Law = $a \cdot (b \cdot c) = (a \cdot b) \cdot c$, $a + (b + c) = (a + b) + c$
4. Absorption law = $a \cdot (a + b) = a$
   $a + (a \cdot b) = a$

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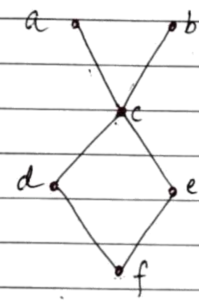
meet / ∧ / GLB — Infimum

join / ∨ / LUB — Supremum

[Lattice — Single top, Single bottom]



→ this is meet Semi lattice
meet / GLB of every pair

{d, e} - GLB = {f} - meet
LUB = {c} - join

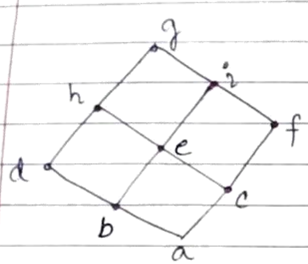
{a, b} - GLB = {c} - meet
LUB = {ϕ} - join nei h } semi Lattice

(d, c) = GLB = {ϕ} {e} } meet Semi Lattice
LUB = {ϕ}

(This is not lattice)

$\{d, c\}$

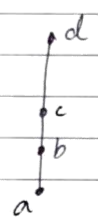
Q.



→ as it have single top and single bottom so meet & join exist so it can be lattice. as every pair ~~have~~ has satisfied the relation of meet (GLB) & join (LUB) So it is lattice.

$\{i, f\}$ = GLB = $\{f\}$
LUB = $\{i\}$

Q.



→ [every straight line is a lattice]

Idempotent law :-

meet $(\cdot)(\wedge)$ join $(+)(\vee)$

$a \cdot a = a = a \wedge a = a$

$a + a = a = \cancel{a+}\; a \vee a = a$

Commutative law.

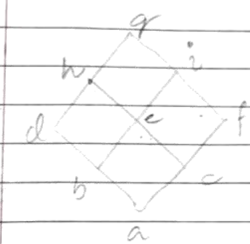
$a \cdot b = b \cdot a = a \wedge b = b \wedge a$

$a + b = b + a = a \vee b = b \vee a$

Associative law:-

$a \cdot (b \cdot c) = (a \cdot b) \cdot c = a \wedge (b \wedge c) = (a \wedge b) \wedge c$

$a + (b + c) = (a + b) + c = a \vee (b \vee c) = (a \vee b) \vee c$



$d \cdot h$

$d \cdot (h \cdot g) = d$

$(d \cdot h) \cdot g = d \cdot g = d$

$h \wedge (e \wedge c) = (h \wedge e) \wedge c$

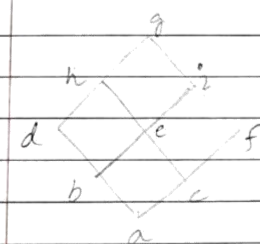
$h \wedge (e \wedge c) = h \wedge c = c$ } = proved

$(h \wedge e) \wedge c = e \wedge c = c$

Absorption Law :-

$a \cdot (a + b) = a = a \wedge (a \vee b) = a$

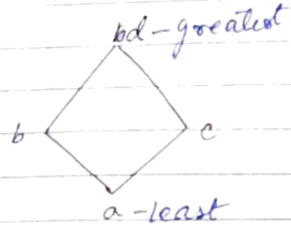
$a + (a \cdot b) = a = a \vee (a \wedge b) = a$



$\{e, f\}$

$e \wedge (e \vee f) = e \wedge i = e$ } hence proved

$e \vee (e \wedge f) = e \vee c = e$



Compliment

$a^c = d$

$d^c = a$

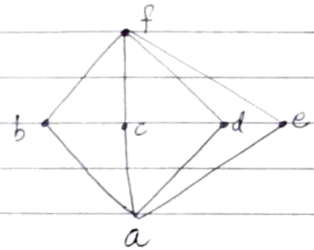
$b^c = c$

$c^c = b$

$b \vee c = \{d\}$

$b \wedge c = \{a\}$

Q.



$b^c = \{\cancel{a}c, d, e\}.$

$a^c = \{f\}$

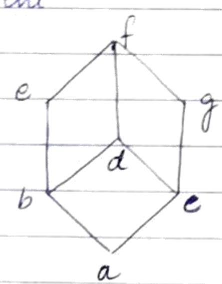
$f^c = a$

$c^c = \{b, d, e\}$

$d^c = \{b, c, e\}$

$e^c = \{b, c, d\}$ So this is complimented Lattice.

Complemented Lattice :- A Lattice L is said to be complimented if every element in lattice has a complement / atleast one compliment



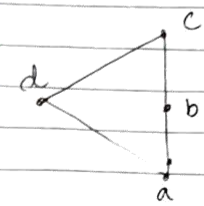
$b^c = g$ $c^c = e$

$g^c = b$ $e^c = c$

$a^c = f$

$f^c = a$

Q.



$a^c = c^c$

$c^c = a^c$

$d^c = b^c$

$b^c - d^c$

(i) a is complement of c

$a \vee c = c$

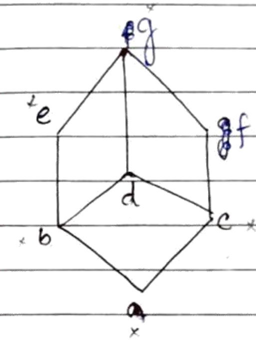
$a \wedge c = a$

(ii) b is complement of d

$b \vee d = d$

$b \wedge d = b$

Q.



(i) a is complement of g

$a \vee g = g$

$a \wedge g = a$

$a^c = g$

$g^c = a$

(ii) e is complement of c

$e \vee c = g$ — greatest

$e \wedge c = a$ - least

$e^c = c$

$c^c = e$

(iii) b is complement of c

$b \vee c = d$ → but this is not greatest

$b \wedge c = a$ → least

$b^c \neq c$ So b is not complement of c

$c^c = b$

(iv) is b compliment of f.

$b \vee f = g$ → greatest

$b \wedge f = a$ → least.

$b^c = f$

$f^c = b$.

as d is does not have any complemen
So this is not a complimented lattice.

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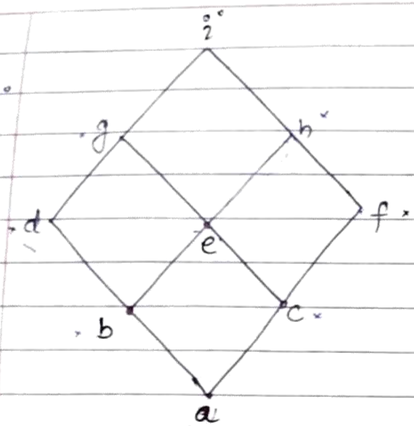
Q.



a = least bound
i = greatest bound

(i) is a a Compliment of i
$a \vee i = i$ - greatest
$a \wedge i = a$ - least
$a^c = i$
$i^c = a$

(ii) is d complement of f
$d \vee f = i$ - greatest
$d \wedge f = a$ - least
$d^c = f$
$f^c = d$

(iii) is g is complement of c
$g \vee c = i$ = greatest
$g \wedge c = a$ = least

(iv) is b complement of h
$\Rightarrow$ $b \vee h = i$ = greatest
$b \wedge h = a$ - least

(iv) is e a
$e \vee f = h$ ×
$e \wedge f = c$ ×

$e \vee d = g$ ×
$e \wedge d = b$ ×

$e \vee a = e$ ×
$a \wedge a = a$ ✓

$\left[\frac{n-1}{m}\right] + 1$ $\quad m < n$

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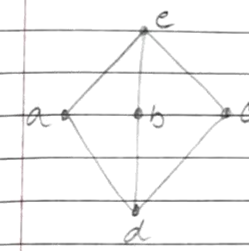
Practice

Distributive lattice :- A lattice is distributive lattice if $\forall a, b, c \in L$

(i) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$
(ii) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

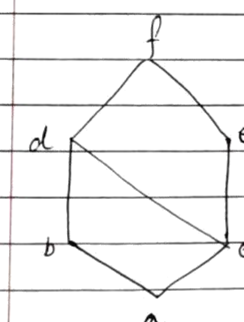


$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$
$a \vee d = e \wedge e$
$a \neq e$ so this is not an distributive lattices.

→ A lattice is said to be distributive lattice if every element in lattice L has atmost one compleme[nt]

$d^c = e$
$e^c = d$
$a^c = b, c$, → 1 se jyada compliment hai to ye distributive nhi hoga.

Q.



$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$
$= a \vee a = b \wedge c$
$= a = a$ this is distributive lattice

$a^c = f$, $f^c = a$
$b^c = e$, $e^c = b$
$d^c = c$ × d does not have its complement. atmost 1 complement so this is distributive lattice.

$(D_6, /) \rightarrow$ Relation of division or division of 6.
$\{1, 2, 3, 6\}$

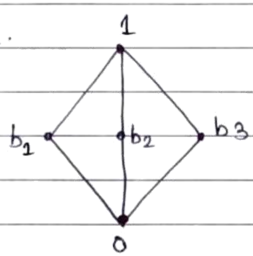
$\subseteq \{(1), (2), (3), (6), (1,2), (1,3), (1,6), (2,3), (2,6), (3,6),$
$(1,2,3), (1,2,6), (2,3,6), (1,2,3,6), (\phi)\}$

Modular Lattice :- A modular lattice if

(i) $a \vee (b \wedge c) = (a \vee b) \wedge c$
whenever $a \leq c \;\forall\; a, b, c, \in L$

A lattice is said to be modular lattice if
$a \vee (b \wedge c) = (a \vee b) \wedge c$ whenever $a \leq c \;\forall\; a, b, c \in L$

ex.



(i) $(0, b_1, b_2)$
(ii) $(b_2, b_3, 1)$
(iii) $(0, b_2, 1)$

(i) $(0, b_1, b_2)$
$0 \vee (b_1 \wedge b_2) = (0 \vee b_1) \wedge b_2$
$\Rightarrow 0 \vee 0 = b_1 \wedge b_2$
$= 0 = 0$

(ii) $(b_2, b_3, 1)$
$\Rightarrow b_2 \vee (b_3 \wedge 1) = (b_2 \vee b_3) \wedge 1$
$\Rightarrow b_2 \vee b_3 = 1 \wedge 1$
$1 = 1$

(iii) $(0, b_2, 1)$
$\Rightarrow 0 \vee (b_2 \wedge 1) = (0 \vee b_2) \wedge 1$
$\Rightarrow 0 \vee b_2 = b_2 \wedge 1$
$\Rightarrow b_2 = b_2$

Complete Lattice :- A lattice is said to be complete lattice if each of its non-empty subsets has a greatest lower bound and least upper bound.

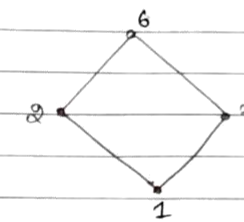
$(D_6, /) = \{1, 2, 3, 6\}$

$\Rightarrow \{(1,6), (1,2), (1,3), (2,6), (3,6), (1,2,6), (1,3,6)\}$

$(1,2) \rightarrow$ GLB = 1, LUB = 2

$(1,2,6) \rightarrow$ GLB = 1, LUB = 6

$(1,5,6) =$ GLB = 1, LUB = 6



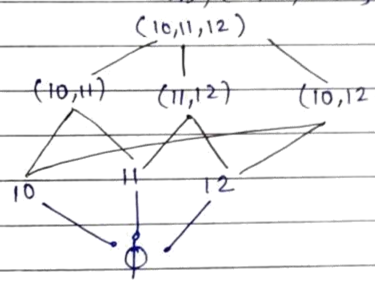
So this complete lattice

Dual

$\leq \quad \geq$

$+ \quad -$

$\{10, 11, 12\}$
$\Rightarrow \{(10), (11), (12), (10,11), (11,12), (10,12)$
$(\phi), (10,11,12)\}$



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BOOLEAN ALGEBRA

→ Boolean Algebra is a distributive, complemented lattice having atleast two element as well as zero and one.

→ A Boolean Algebra is generally denoted by 6-tuple $(B, +, \circ, ', 0, 1)$

where $(B, +, \cdot)$ is a lattice with two binary operations + and • called join & meet respectively and (′) is a unary operation in B.

The elements 0 and 1 are the least and greatest elements of lattics B.

There are three term :-

Boolean Expression → A boolean expression in n variables $(x_1, x_2, x_3 \ldots x_n)$ is any finite string of symbols formed as given below:

(1) 0 and 1 are the boolean expression.
(2) $(x_1, x_2, x_3 \ldots x_n)$ boolean expression.
(3) $x_1 x_2' + x_1' x_2'$

Boolean function → A function $f: X^n \to X$ which is associated with a boolean expression in n variables is called boolean function.

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→ Axioms of boolean algebra :- $a, b \in B$

(i) Communitative : $a+b = b+a$
$a \circ b = b \cdot a$

(ii) Distributive : $a+(b \circ c) = (a+b)\circ(a+c)$
$a \circ (b+c) = (a \circ b) + (a \circ c)$

(iii) Idempotent Law : $a+a = a$
$a \cdot a = a$

(iv) Boundedness Law : $a+1 = 1$ $\quad a \cdot 0 = 0$
$a \circ 1 = a$ $\quad a+0 = a$

(v) Absorption law : $a + (a \circ b) = a$

(vi) Complement Law : $0' = 1$
$1' = 0$

(vii) Involution Law : $(\bar{\bar{A}}) = A$

(viii) Associative law : $(a+b)+c = a+(b+c)$
$(a \circ b) \circ c = a \circ (b \circ c)$

(ix) De morgan's law : $\overline{a+b} = \bar{a} \circ \bar{b}$
$\overline{a \circ b} = \bar{a} + \bar{b}$

(x) Uniqueness of complement : $a + \bar{a} = 1$
$a \circ \bar{a} = 0$

Gates: AND, OR, NOT, NAND, NOR, Ex-OR, Ex-NOR

Boolean Expression can be graphically represented by using logic circuits these circuits can be constructed by using Solid State devices called gates.

AND, OR & NOT are the basic gates.
NAND, NOR, are called universal gates because these are used to implemented any digital circuit without using any other gate.

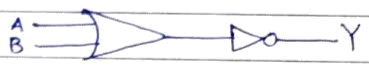
→ NAND – AND + NOT

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

$Y = (A \cdot B)' = \overline{A \cdot B}$

if both the inputs are 1 then output will be zero.

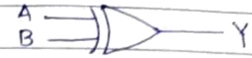
→ NOR – OR + NOT



A	B	Y
0	0	~~0~~1
0	1	0
1	0	0
1	1	0

if ~~a~~ both the inputs are zero than output will be one.

→ XOR (Ex-OR)



A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

$Y = A \oplus B = \bar{A}B + A\bar{B}$

if both the inputs are different then output will be one.

→ Ex NOR

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

$Y = A \odot B = \bar{A}\bar{B} + AB$

if the inputs are same then output will be 1

Karnaugh Maps (K-maps): K-maps are used to simplify the boolean expressions there are two type of terms that are:

SOP (minterm) (Σ): Sum of Product [$0 = \bar{A}$, $1 = A$] [

POS (maxterm) (π): Product of Sum [$0 = A$, $1 = \bar{A}$] [

$0 = A,\ 1 = \bar{A}$ $\quad$ $0 = \bar{A},\ 1 = A$

A	B	C	M (POS)	m (SOP)
0	0	0	$M_0 = A+B+C$	$m_0 = \bar{A}\bar{B}\bar{C}$
0	0	1	$M_1 = A+B+\bar{C}$	$m_1 = \bar{A}\bar{B}C$
0	1	0	$M_2 = A+\bar{B}+C$	$m_2 = \bar{A}B\bar{C}$
0	1	1	$M_3 = A+\bar{B}+\bar{C}$	$m_3 = \bar{A}BC$
1	0	0	$M_4 = \bar{A}+B+C$	$m_4 = A\bar{B}\bar{C}$
1	0	1	$M_5 = \bar{A}+B+\bar{C}$	$m_5 = A\bar{B}C$
1	1	0	$M_6 = \bar{A}+\bar{B}+C$	$m_6 = AB\bar{C}$
1	1	1	$M_7 = \bar{A}+\bar{B}+\bar{C}$	$m_7 = ABC$

Octal (8)
Quad four(4)
Pair (2)

3-variables

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Example: $F(A,B,C) = \Sigma(1,3,5,6,7)$

$= m_1 + m_3 + m_5 + m_6 + m_7$ $\Rightarrow AB + C$

A \ BC	00	01	11	01
0	0	1 (1)	1 (3)	2
1	4	1 (5)	1 (7)	1 (6)

Q. $F(P,Q,R,S) = \Sigma(0,2,5,7,8,10,13,15)$

$= m_0 + m_2 + m_5 + m_7 + m_8 + m_{10} + m_{13} + m_{15}$

PQ \ RS	00	01	11	01
00	1 (0)	1	3	1 (2)
01	4	1 (5)	1 (7)	6
11	12	1 (13)	1 (15)	14
10	1 (8)	9	11	1 (10)

$= QS + \bar{Q}\bar{S}$

Q1.

Q2. $R = \{(x,y) \mid x \parallel y\}$ on a straight line on plane?

Let $R = \{(x,y)$: line x is parallel to line y, $x, y \in$ set of straight line$\}$

→ Every line is parallel to itself So x,x if $x \in S$ then $(x,x) \in R$

$\Rightarrow$ R is reflexive.

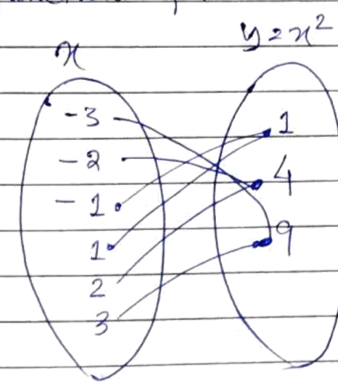
→ if $(x,y) \in R \Rightarrow x \parallel y \Rightarrow y \parallel x \Rightarrow (y,x) \in R$.
is So R is Symmetric

→ $(x,y) \in R$ & $(y,z) \in R \Rightarrow x \parallel y,\ y \parallel z \therefore x \parallel z$
(x,y) $\therefore (x,z) \in R$.

→ R is transitive.

R being reflexive, Symmetric & transitive it is an equivalance relation and Straight line are always equivalance.

Q5. $f = \{(x,y) \mid y = x^2\}$ on $R \times R$ State type of function f.



function type
= many-to-one

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Q6. Find two incomparable elements in poset $(\{1,2,4,6,8\}, /)$.

⇒ $R = \{(1,2),(1,4),(1,6),(1,8),(2,4)(2,8)(2,6)(4,8)\}$.

but the pair (4,6) & (6,8) is not comparable in divisor relation.

Q8. What is composition of functions? Also prove that $f^{-1} \circ g^{-1} = (g \circ f)^{-1}$.

where $f: Q \to Q$ Such that $f(x) = 4x$
$f \to Q \to Q$ Such that $g(x) = x+4$
are two function?

→ Composition of function is a unique operation between functions is the ~~unique~~ composition of functions. when the output of one function is input of second function, we will need to use a composition of two functions to solve.

$f(x) = 4x$ $\quad$ $g(x) = x+4$

$4x = y$ $\quad$ $x+4 = y$

$x = y/4$ $\quad$ $x = y-4$

$f^{-1}(x) = \frac{x}{4}$ $\quad$ $g^{-1}(x) = x-4$

$(f^{-1} \circ g^{-1})(x) = \frac{x-4}{4} \Rightarrow \frac{x}{4} - 1$

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$(g \circ f)(x) = 4x+4 \Rightarrow 4(x+1)$

$(g \circ f)^{-1} = 4(x+1)$

$4(x+1) = y$

$x+1 = \frac{y}{4}$

$x = \frac{y}{4} - 1$

$(g \circ f)^{-1} = \frac{x}{4} - 1 = (f^{-1} \circ g^{-1})x$.

∴ $f^{-1} \circ g^{-1} = (g \circ f)^{-1}$ proved

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Q. SOP (minterm)
K-map
$F(A,B,C,D) = \Sigma_m(0,2,3,6,7,12,13,14) + \Sigma d(1,4,11,15)$

AB \ CD	00	01	11	10
00	1 (0)	X (1)	1 (3)	1 (2)
01	X (4)	(5)	1 (7)	1 (6)
11	1 (12)	1 (13)	X (15)	1 (14)
10	(8)	(9)	X (11)	(10)

$\bar{A}\bar{B}$; $\bar{A}C$; AB

$\Rightarrow \bar{A}\bar{B} + AB + \bar{A}C$

Q. Function is 4 variable A,B,C,D.
POS $F(A,B,C,D) = \pi M($

Ex ⇒ $A\bar{B}C + \bar{A}BC\bar{D} + AB\bar{C}D + ABC$

$\Rightarrow A\bar{B}CD + A\bar{B}C\bar{D} + \bar{A}BC\bar{D} + AB\bar{C}D + ABCD + ABC\bar{D}$
1011 1010 0110 1101 1111 1110

$\Rightarrow \Sigma(11, 10, 6, 13, 15, 14)$
$\Rightarrow \pi = (0,1,2,3,4,5,7,8,9,12)$

A+B \ C+D	00	01	11	10
00	0 (0)	0 (1)	0 (3)	0 (2)
01	0 (4)	0 (5)	0 (7)	1 (6)
11	0 (12)	1 (13)	1 (15)	1 (14)
10	0 (8)	0 (9)	1 (11)	1 (10)

$(A+\bar{D})$; $(A+B)$; $(BC\bar{D})$; $(AB\bar{D})$; $(C+D)$; $(B+C)$; (AC)

$\Rightarrow (C+D)\cdot(A+B)\cdot(B+C)\cdot(A+\bar{D})$

$\Rightarrow (AC + BC\bar{D} + AB\bar{D}$

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UNIT-3

PROPOSITION

A Proposition is a declarative sentence or a statment that has either a true value or false

→ Compound Proposition – when one or more proposition are connected through various connectives is called compound proposition.

→ example of proposition :– 1+1 = 2 True
1 ≤ 2 false

example of compound proposition :–

I am in the class and i am disturbing the class → ~~pqe~~

→ Primitive Proposition :– A proposition is said to be primitive if it can not be broken down into simpler proposition. (it is also ~~An~~ Atomic proposition)
Example :– Roses are Red

→ Truth Tables :– A table showing the truth values of a statment formula is called truth table.

→ Basic Operation :– Conjunction ($\wedge$) – AND
Disjunction ($\vee$) – OR
Negation – ($\sim$) – NOT
(Tilde)

Example :-

A	B	A	B	$A \wedge B$	$A \vee B$	$\tilde{A}$	$\tilde{B}$
0	0	F	F	F	F	T	T
0	1	F	T	F	T	T	F
1	0	T	F.	F	T	F	T
1	1	T	T	T	T	F	F

If then Implication $A \rightarrow B$

A	B	$A \rightarrow B$
F	F	T
F	T	T
T	F	F
T	T	T

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Q. p: It is cold } statments that are either true
q: It is raining   or false

~p : It is not cold
~q : It is not raining.

$\sim p \wedge \sim q$: It is not cold ~~not~~ and it is ~~not~~ raining

$\sim p \wedge \sim q$ $\sim$~~p~~$\wedge$q : It is neither cold ~~not~~ nor raining

Q. p : Agra is in England
q : ~~One~~ 1+9 = 8

~p : Agra is not in England
~q : 1+9 is not equal to 8

Q. Tautology

p	q	~p	$\sim p \vee q = \sim p \vee p$
0	0	1	1
0	1	1	1
1	0	0	1
1	1	0	1

↳ Tautology.

A Statment formula that is true for all possible values of its propositional variables is called Tautology.

$\boxed{(p \vee q) \longleftrightarrow (q \vee p)}$

Q. Contradiction :- A Statment formula that is false for all possible values of its propositional variable is called Contradiction.

p	q	~p	$\sim p \wedge q = \sim p \wedge p$
0	0	1	0
0	1	1	0
1	0	0	0
1	1	0	0

Q. contingency :- A statment formula that can be either true or false depending upon the truth values of its propositional variables is called contingency.

$(p \rightarrow q) \wedge (p \wedge q)$

p	q	$p \to q$		$p \wedge q$	$(p\to q)\wedge(p\wedge q)$
0	0	T	1	0	0
0	1	T	1	0	0
1	0	F	0	0	0
1	1	T	1	1	1

1. Conditional Statments :- $p \to q$ (conditional)

$p \leftrightarrow q$ (Bi- conditional)

(Converse, inverse, contrapositive)

__Conditional statments__ : If p and q are the two statments then the statment $p \to q$ which is read as "if $p \to$ then q"

Truth table.

p	q	$p \to q$	p	q	$p \to q$
0	0	1	F	F	T
0	1	1	F	T	T
1	0	0	T	F	F
1	1	1	T	T	T

2. __Biconditional__ : A statment of the form "p if and only if q" $p \leftrightarrow q$ is called a biconditional statment. it is denoted as $p \leftrightarrow q$ or $p \rightleftarrows q$

Truth table

p	q	$p \leftrightarrow q$	p	q	$p \leftrightarrow q$
0	0	1	F	F	T
0	1	0	F	T	F
1	0	0	T	F	F
1	1	1	T	T	T

imp

$p \to q = \sim p \vee q$

p	q	$p \to q$	$\sim p$	$\sim p \vee q$
0	0	1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	1	0	1

$(p \vee q) \leftrightarrow (q \vee p)$

p	q	$p \vee q$	$q \vee p$	$(p\vee q) \leftrightarrow (q \vee p)$
0	0	0	0	1
0	1	1	1	1
1	0	1	1	1
1	1	1	1	1

↳ Tautology.

If they have identical truth value
$\sim(p \wedge q) = \sim p \vee \sim q$.

Well formed formula (wff) :- A Statment formula is called wff if it can be generated by the following rules.

1. If p is wff then negation of p is wff
2. If p is a propositional variable then it is wff
3. If p and q are wff then $(p \vee q)$, $(p \wedge q)$, $(p \rightarrow q)$ & $(p \leftrightarrow q)$ are wff
4. A String of symbols is a wff if and only if it is obtained by finitly many applications of rules 1, 2 & 3

Statments → premises and conclusion.

#1. Premises / Evidenses / assumption / anticidents :-

It is a ~~g.~~ set of ~~S'~~ given Statments

2) Concalusion :- It is a preposition getting by the given set of premises.

[If premises then conclusion]

3. Argument :- It is a set of one or more premises and a conclusion

Valid Argument :- $(p_1, p_2, \cdots p_n) \rightarrow Q$ or $\vdash$ (detachment symbol)

An argument is valid argument were where Q is true when all $(p_1, p_2, \cdots p_n)$ are true.

An argument is valid if and only if it is not possible to make all premises true and conclusion false.

If it is ~~true~~ tautology then it is valid argument

example :- (p: if "i love cat" then "i love dog")

~~conclusion: p~~ "I love cat")

therefore

Conclusion :- q: ("I love dog")

Solution :- Let p: I love cat

q: I love dog

$$p \rightarrow q$$
$$\underline{p}$$
$$\therefore q$$

$\Rightarrow \boxed{((p \rightarrow q) \wedge p) \rightarrow q}$

Q.1 (If it rain then Ram will be sick

It did not rain) therefore Ram was not sick

Q2. Premises → "Ram works hard", "if ram works hard then he is a dull boy" and

"If ram is a dull boy then he will not get the job".

Conclusion : Ram will not get the job.

$(p \rightarrow q) \rightarrow$

Q1. p: if it Rain

q: Ram will be Sick

premises

~~p: It~~ $\sim p$: if did not rain

Conclusion : $\sim q$: Ram was not sick

$$((p \rightarrow q) \wedge \sim p) \rightarrow \sim q.)$$

Q2. p: Ram was

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$(p \rightarrow q) \wedge \sim p$	$(p \rightarrow q) \wedge \sim p \rightarrow \sim q$
F	F	T	T	T	T	T
F	T	T	F	T	T	F
T	F	F	T	F	F	T
T	T	F	F	T	F	T

↓

Hence it is not valid

p	q	p→q	(p→q)^p	(p→q)^p → q
F	F	T	F	T
F	T	T	F	T
T	F	F	F	T
T	T	T	T	T

↳ valid argument

Q2. p: Ram works hard
q: he is a dull boy

~r: he will not get job

~~$(p\to q)\leftrightarrow r$~~

$p\wedge(p\to q)\wedge(q\to r)\to \sim r$

p	q	~r	p→q	q→r	(p→q)^(q→r)	(p→q)^(q→r)→~r
F	F	F	T	F	F	T
F	F	T	T	T	T	T
F	T	F	T	F	F	T
F	T	T	T	T	T	T
T	F	F	F	T	F	T
T	F	T	F	T	T	T
T	T	F	T	F	F	T
T	T	T	T	T	T	T

$p\wedge(p\to q)\wedge(q\to \sim r)\to \sim r$

p	q	r	~r	p→q	q→~r	p^(p→q)^(q→~r)	
F	F	F	T	T	T	F	T
F	F	T	F	T	T	F	T
F	T	F	T	T	T	F	T
F	T	T	F	T	F	F	T
T	F	F	T	F	T	F	T
T	F	T	F	F	T	F	T
T	T	F	T	T	T	T	T
T	T	T	F	T	F	F	T

$\overset{T}{p}$

T $p\to \overset{T}{q}$

T $q\to \overset{T}{\sim r}$ ⇒ T

~r

T

Rules of Inference :- Rules of inferences (of propositional logic) specifies which conclusion may be infere from ~~Know~~ known, assum or given Premises

1. $p \to q$
   p
   $\therefore q$ — modus Ponens

2. $p \to q$
   $\sim q$
   $\therefore \sim p$ — modus tollens

3. $p \to q$
   $q \to r$
   $\therefore p \to r$ — Hypothetical Syllogism

4. $p \vee q$
   $\sim p$
   $\therefore q$ — Disjunction

5. p
   $p \vee q$ — Addition

6. p
   q
   $p \wedge q$ — conjuction ✓

7. $p \wedge q$
   $\therefore p$ — simplication

8. $(p \to q) \wedge (r \to s)$
   $p \vee r$
   $\therefore q \vee s$ — Constructive Dilemma

9. $(p \to q) \wedge (r \to s)$
   $\sim q \vee \sim s$
   $\sim p \vee \sim r$ — Distructive Dilemma

Predicates :- It is a part of declarative sentence describing the properties of an object.

Predicates are the statments involving variables which are neither true nor false.

example :-
1. x is an animal
2. x is $\underbrace{\text{greater than } 3}$

Subject (about what we are talking) — Predicate (are the property that Subject can have)

Quantifiers :-
$\forall$ universal quantifiers
$\exists$ existential quantifiers (there exist)

These are used express the quantities such as all, some, few etc

example of universal Quantifiers $\forall$:-
Let $P(x)$ be a statment where $x+1 > x$
$\forall x \; P(x)$ it means
$P(x)$ is true for all +ve integers x

$P(1)$ $1+1 > 1$ (t)
$P(2)$ $2+1 > 2$ (t)
⋮
$P(x)$ $x+1 > x$ (t)

UoD → Universe of Discourse

8. valid argument
Q. Tautology vs contradiction
Q. Contigency.

FOPL → First order Predicate Logic

Example of $\exists$ existential :-

$x^2 = 4$

$x \in \{-2, 2\}$ there exist some x for which it is true

$\exists$ -2, 2

$\exists x\, P(x)$

Statment : Every Student is clever

(This is not a proposition because we do not know the exact student name)

Student ⇒ universe of discord

$M(x)$: x is Student
$N(x)$: x is clever.

$\forall x\, (M(x) \longrightarrow N(x))$

Normal form / Standard form :-

→ (elementary Sum form) (POS) (DNF) $((\wedge) \vee (\wedge))$

1. Disjunctive normal form $(\vee)$ OR $[P_1 \vee P_2 \vee P_3 \ldots \vee P_n]$
2. Conjunctive normal form $(\wedge)$ AND $[P_1 \wedge P_2 \wedge P_3 \ldots \wedge P_n]$

↳ (elementary Product form) (SOP)

$(p \vee q) \wedge (p \vee q)$ (CNF) $((v) \wedge (v))$

$p \to q = (\sim p \vee q)$ $p \leftrightarrow q = [(\sim p \vee q) \wedge (\sim q \vee p)]$

Q The $p \wedge (p \to q)$ (conjuctive normal form) (CNF)

Step 1: $p \to q = \sim p \vee q$

$p \wedge (\underbrace{\sim p \vee q}_{p \to q})$

Step 2: use distributive Law.

$p \wedge (\sim p \vee q)$

$(p \wedge \sim p) \vee (p \wedge q)$ — DNF

Q. $(p \to q) \wedge (q \vee (p \wedge r))$

Step 1:- $(\sim p \vee q) \wedge ((q \vee p) \wedge (q \vee r))$

$(\sim p \vee q) \wedge (q \vee (p \wedge r))$ → CNF

Step 2:- using distributive

$(\sim p \vee q) \wedge ((q \vee p) \wedge (q \vee r))$

$(\sim p \vee q) \wedge (q \vee p) \wedge (q \vee r)$ → ~~DNF~~ CNF

$(v) \wedge (v) \wedge (v)$

Rules of Inferences of predicate logic :-

1. Universal instantiation : This rule is used to conclude that $P(c)$ is true when $\forall x\, P(x)$ is true where you can say c is the element of universe. This rule is use to conclude that $P(c)$ is true

$\forall x\, P(x)$
$\therefore P(c)$

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2. Universal Instantiation or Generalization :-

$$\frac{P(c)}{\therefore \forall x\, P(x)}$$

1. Existential Instantiation :-

$$\frac{\exists x\, P(x)}{\therefore P(c)}$$

3. Existential Generalization :-

$$\frac{P(c)}{\therefore \exists x\, P(x)}$$

Example :-

Given Premises $p \to q$ -(1)

$\sim p \to r$ -(2)

$r \to s$ -(3)

conclusion :- $\sim q \to s$

1. $p \to q$ premise (1)
2. $\sim q \to \sim p$ contrapositive of (1)
3. $\sim p \to r$ premise (2)
4. $\sim q \to r$ by Hypothetical Syll (3)
5. $r \to s$ premise (3)
6. $\sim q \to s$ Hypothetical Syll (4 & 5)

premises lead to the conclusion hence it is a valid argument proved without using truth table

Natural deduction :- When an inference rule is used as a part of proof meta variables are replaced in a consistent way. Most rules are comes into two following ways :-

1) Introduction
2) Ellimination

Introduction :- Introduces the use of logical operation.

Ellimination :- Elliminates the use of logical operation

Rule of Introduction :- $\frac{p \quad q}{p \wedge q}$ $\wedge I$ (introduce and ($\wedge$))

Rule of Ellimination :- (1) $\frac{p \wedge q}{p}$ $\wedge E_1$ (eliminate and)

(2) $\frac{p \wedge q}{q}$ $\wedge E_2$ (eliminate and)

Rule of double negation :- $\frac{\sim\sim p}{p}$ $\sim\sim E$ (eliminate negation)

$\frac{p}{\sim\sim p}$ $\sim\sim I$ (introduce negation)

example :-
Statment :-
Either Ram will cook or Hari will practice Karatte
If hari Practices karate then Ram Studies.

Ram does not Study.

therefore Hari will practise karatte

p :- Ram will cook
q :- Hari will practise Karatte
r :- Ram Studies
$\sim r$:- Rama does not Study

$$(p \vee q) \wedge (q \rightarrow r) \wedge \sim r \rightarrow q$$

$$\begin{array}{ll} p \vee q & -(1) \\ q \rightarrow r & -(2) \\ \sim r & -(3) \\ \hline \therefore\ q & \end{array}$$

	$p \vee q$	premises (1)
①	$q \rightarrow r$	premise (2)
②	$\sim r$	premise (3)
③	$\sim q$	by tollens (2 & 3)
	~~q~~	~~premise~~
	~~q~~	~~by disjunction (3 & 4)~~
④	$\sim q$	(by tollens (2 & 3)
⑤	$p \vee q$	
	p	by disjunction (4 & 5)

not valid argument.

Q. write the converse and inverse
"If $x+3=8$ then $x=6$"

p : $x+3=8$
q : $x=6$
if $p \rightarrow q$.
Converse $q \rightarrow p$
inverse $\sim p \rightarrow \sim q$

Converse - if $x=6$ then $x+3=8$

inverse - if $x+3 \neq 8$ then $x \neq 6$
Contrapositive $\sim q \rightarrow \sim p$.

Q. Direct Proof, Indirect Proof, Proof by counter example, & Proof by Cases

1. Direct Proof :- if a is a no. such that ~~than~~ $a^2 - 7a + 12 = 0$ than show that $a = 3$ and $a = 4$ by direct Proof

$$a^2 - 7a + 12 = 0$$
$$a^2 - 4a - 3a + 12 = 0$$
$$a(a-4) - 3(a-4) = 0$$
$$(a-4)(a-3) = 0$$

$a = 4$, $a = 3$ hence proved by direct method.

2. Proof by Counter example :-

$x^2 = y^2$
$x = \pm y$
$x = 3, \quad y = 3, -3$

$x = y$ $\quad\quad x \neq y$
$3 = 3$ $\quad\quad 3 \neq -3$
$x^2 = y^2$ $\quad\quad (3)^2 = (-3)^2$
$(3)^2 = (3)^2$

$\forall x (x > 0 \vee x < 0) \rightarrow$ Counter example $x = 0$.

$\forall x \ (x = 1) \rightarrow$ counter example $x \neq 1$

→ Since 0 ~~is~~ $0 \nless$ 0 is not less than 0 or greater than 0, so ~~it~~ therefore it is counter example.

UNIT - 4

Algebric structure :-

If a set ~~a~~ A with respect to operator (*) satisfies the closer property then it is called algebric structure (A, *)
↳ any mathematical operation.

$\{1, 2, 3, 4, 5\}$

$1 \times 3 = 3$

Group (G, *) :- A Group (G, *) is an algebric structure in which the binary operation (*) on G satisfies the following condition

1. Closer Property —
2. ~~Identity~~ Associative property
3. Identity
4. Inverse
5. Communitative

Group
- Closer Property :- If $\forall a, b \in A$ then $a * b \in A$
- {semigroup
- Associative Property :- $(a*b)*c = a*(b*c)$
- monoide
- Identity :- $a * e = a = e * a$ [0 is additive, 1 is multiplicative]
- Inverse :- $a * b = b * a = e$
  where $b = a^{-1}, a = b^{-1}$

$5 \times \frac{1}{5} = 1$, $5 + (-5) = 0$

5 Communitative :- $a * b = b * a$

example :- $(\mathbb{Z}, +)$ prove it is a group or not.
$(-\infty, \ldots -3, -2, -1, 0, 1, 2, 3, \ldots \infty)$.

$1 + 2 = 3$ → closer

When all the property Satisfy it is Group

when only closer property Satisfies it is Algebric Structure

when only 1 & 2 Satisfies Semi group

when 1, 2 & 3 satisfies monoid.

when communitative Satisfies it is Abelian

example.

1. $(N, -)$

$N = \{1, 2, 3, 4, 5, \dots \infty\}$ $\quad a = 4, \; b = 1$

Closer Property $(4 - 1 = 3)$

2. $(z, -)$, $z = \{-\infty, \dots -3, -2, -1, 0, 1, 2, 3, \dots \infty\}$

Closer Property: $(-3 - (-2) = -1)$ $\quad -1 \in z$

Associative Property $(-3 - (-2)) - 1 = -3 - ((-2) - 1)$

$\Rightarrow (-3 + 2) - 1 = -3 - (-2 - 1)$

$\Rightarrow -2 = -3 - 3$

$a = 2, \; b = 3, \; c = 4.$

$(2 - 3) - 4 = 2 - (3 - 4)$

$-1 - 4 = 2 - (-1)$

$-5 = 3$

finite Group :- A group G is said to be finite group if the set G is a finite set.

for example :- $G = \{-1, 1\}$ is a finite Set

Order = 2

Infinite Group :- A Group G is said to be infinite group which is not finite forexample

$G = \{\text{Set of integers } (z)\}$

Order of a finite Group :- The order of finite group is the total no. of distinct elements of a group.

It is denoted by $o(G)$ or $|G|$

$G = \{-1, 1\}$

$o(G) = 2$

$|G| = 2$

example :- $G = \{1, -1, i, -i\}$

$(G, *)$ $\quad (G, \times)$

↳ operation.

$1 \times (-1) = -1$ $\quad$ it is so it satisfies closer property

Composition Tabel :-

$\times$	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

So it is closer property

Associative :- $1, -1, i$

$$(1 \times -1) \times i = 1 \times (-1 \times i)$$
$$-i = -i \quad \text{proved}$$

Identity :- i $\quad e = 1$

$$a \times e = a = e \times a$$
$$i \times 1 = i = 1 \times i = i \quad \text{proved}$$

Inverse :- $a \times b = b \times a = e$ $\quad e = 1.$

$$1, \cancel{-1}$$
$$1 \times \cancel{(-1)} = \cancel{(-1) \times 1} \neq 1$$
$$-1 = \cancel{-1} = \cancel{1}.$$

$$a \times a^{-1} = e$$
$$(-1) \times (-1) = 1$$

So it Satisfies all the four property So it is Group.

Example :- $G = \{1, \omega, \omega^2\}$ $\quad \omega^3 = 1$

$(G, \times)$ $\quad \omega + \omega^2 + 1 = 0$

closure property :-

$\times$	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	$\omega^3 = 1$
ω^2	ω^2	$\omega^3 = 1$	$\omega^4 = \omega$.

So it is Satisfies the closure property.

Associative $1, \omega, \omega^2$

$$(1 \times \omega) \times \omega^2 = 1 \times (\omega \times \omega^2)$$
$$1 = 1 \quad \text{proved}$$

Identity = $e = 1$ $\quad \omega$

$$a \times e = a = e \times a$$
$$\omega \times 1 = \omega = 1 \times \omega$$

Inverse = $e = 1$ $\quad \omega \times \omega^2 = 1$

Addition Modulo 5 $(+_5)$:- $\{0, 1, 2, 3, 4\}$

Multiplicative Modulo 5 $(\times 5)$:- $\{1, 2, 3, 4\}$

modulo = remainder.

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	5-5
2	2	3	4	5-5	6-5
3	3	4	5-5	6-5	7-5
4	4	5	6-5	7-5	8-5.

$\Downarrow$ 5-5.

$\Rightarrow$

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	$5/5$ remainder $= 0$
2	2	3	4	$\frac{5}{5} = 0$	$\frac{6}{5} = 1$
3	3	4	$\frac{5}{5} = 0$	$\frac{6}{5} = 1$	$\frac{7}{5} = 2$
4	4	$\frac{5}{5} = 0$	$\frac{6}{5} = 1$	$\frac{7}{5} = 2$	$\frac{8}{5} = 3$

$\therefore$ closure property Satisfy.

Associative :- $(a+\cancel{b}b)+c) = a+(b+c)$

$$(a +_5 b) +_5 c = a +_5 (b +_5 c)$$

$$(1 +_5 2) +_5 3 = 1 +_5 (2 +_5 3)$$
$$3 +_5 3 = 1 +_5 0$$
$$1 = 1$$

agar har row main ek baar bhi zero ayega to wo identity property satisfy karega.

Than Jha pe zero ara hai wo uska inverse ho jayega.

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

inverse of 1 = 4.

inverse of 4 = 1.

inverse of 3 = 2.

All the property satisfies hence it is group.

Multiplicative modulo $*_5 = \{1, 2, 3, 4\}$

*	1	2	3	4
1	1	2	3	4
2	2	4	$\frac{6}{5} = 1$	$\frac{8}{5} = 3$
3	3	$\frac{6}{5} = 1$	$\frac{9}{5} = 4$	$\frac{12}{5} = 2$
4	4	$\frac{8}{5} = 3$	$\frac{12}{5} = 2$	$\frac{16}{5} = 1$

*	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

Sub Group :- Let $(G,*)$ be a group and H, be a non-empty subset of G. If $(H,*)$ is itself is a group then $(H,*)$ is called Sub-group of $(G,*)$.

Let $G = \{1, -1, i, -i\}$ & $H = \{1, -1\}$

G and H are groups with respect to the binary operation, multiplication. H is a subset of G, therefore $(H,*)$ is a Sub-group $(G,*)$

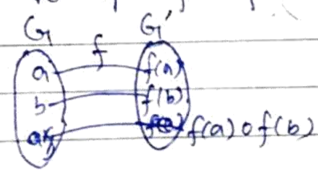
Cyclic Group :- $G = \{1, -1, i, -i\}$

Let $(G,*)$ be a group if there exists an elements $a \in G$ such that $G = \{a^m : m \text{ is an integer}\}$.

$i^1 = i$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$

Homo morphism Group :-

Let $(G,*)$ & $(G',0)$ be any two group. A mapping $f : G \to G'$ is called homomorphism of G to G' if $f(a*b) = f(a) \, 0 \, f(b) \; \forall a, b \in$



"Isomorphism" : Homo + one-to-one + onto

"Homomorphism + bijective"

Q.# Show that addition modulo $+_5$ is a cyclic group = {0, 1, 2, 3, 4}.

+	0	1	2	3	4
0					
1					
2					
3					
4					

Permutation Group :- Set S be a finite set having n distinct elements then one - one mapping of S onto itself is called a permutation of degree n

$f : S \to S$ is said to be permutation of S if

(i) f is one - one

(ii) f is onto

Date 3 2 23

Q (A, *)

Ring :- An algebraic system (R, +, ·) is called a ring if the binary operations '+' and '·' R Satisfy the following properties:-

1. (R, +) is an abelian Group
2. (R, ·) is a Semi - Group
3. The operation '·' is distributive over +, that is for any a, b, c ∈ R.

$a \cdot (b+c) = a \cdot b + a \cdot c$ and

$(b+c) \cdot a = b \cdot a + c \cdot a$.

Special types of Rings :-

1. Commutative Ring or Abelian Ring :- A ring R is Said to be a communitative ring or an abelian ring if it satisfies the communitative law, $\forall$ a, b ∈ R . $a \cdot b = b \cdot a$.

2. Ring with Unity :- A ring R which contains the multiplicative identity (Called unity) is called a ring with unity.
Thus if 1 ∈ R such that $a \cdot 1 = a = 1 \cdot a$ $\forall$ a ∈ R then the ring is called a ring with unity.

3. Ring without unity :- A ring R, which does not contain multiplicative identity is called a ring without unity.

Finite and Infite Ring :- If the number of elements in the ring R. Is finite, then $\langle R, +, \cdot \rangle$ is called a finite ring, otherwise. It is called an infinite ring.

Order of Ring :- The number of elements in a finite ring R is called the order of ring R. This is denoted by $|R|$.

Ring with zero divisors :-

$R = \{0, 1, 2, 3, 4, 5\}$ $\qquad$ if $a \neq 0$, $b \neq 0$ then $ab = 0$

$2 \cdot_6 3 \Rightarrow \frac{6}{6} = 0$

A ring $(R, +, \cdot)$ is called a ring with zero divisions if $a \neq 0$, $b \neq 0$ then $ab = 0$

Ring without zero divisors :-

A ring $(R, +, \cdot)$ is called a ring without zero divisors if $a \neq 0$, $b \neq 0$, then $ab \neq 0$.

Fields :- If a commutative ring $(R, +, \cdot)$ is called a field. $\rightarrow$ if every element has multiplicative inverse.

Integral Domain :- A commutative ring $(R, +, \cdot)$ is called Integral Domain if it is without zero divisions.

Sub-Rings :- Let $(R, +, \cdot)$ be a ring and S be a non-empty subset of R. If $(S, +, \cdot)$ is a ring then $(S, +, \cdot)$ is called a Sub-ring of R.

example :- Let E denote the set of even integers $(E, +, \cdot)$ is a Sub-ring of $(Z, +, \cdot)$ where Z denotes the set of integers.

Every ring $(R, +, \cdot)$ has two trivial sub-rings $(\{0\}, +, \cdot)$ and $(R, +, \cdot)$ where 0 is the additive identity of $(R, +, \cdot)$.

7/2/25.

Q. $\sim r$ valid?

$p \to \sim q$ — 1

$r \to p$ — 2

q — 3

$\sim r$

$((p \to \sim q) \wedge (r \to p) \wedge q) \to \sim r$

$r \to p$ contra positive

$\sim p \to \sim r$

$p \to \sim q$ — 1

q

$\sim p$ → modus tollens (1 – 3)

$\cancel{p \to r}$ → modus tollen (

$\cancel{\sim r}$

$r \to p$

$p \to \sim q$

q modus tollens (1 & 3)

$\sim p$ — 4

$r \to p$ modus tollens. (4 & 2)

$\sim r$ – valid.

p	q	r	$\sim q$	$\sim r$	$p \to \sim q$	$r \to p$	$(p \to \sim q) \wedge (r \to p) \wedge q$	$\to \sim r$
F	F	F	T	T	T	T	F	T
F	F	T	T	F	T	F	F	T
F	T	F	F	T	T	T	T	T
F	T	T	F	F	T	F	F	T
T	F	F	T	T	T	T	F	T
T	F	T	T	F	T	T	F	T
T	T	F	F	T	F	T	F	T
T	T	T	F	F	F	T	F	T

Valid argument

$p \leftrightarrow q \;[(\sim p \vee q) \wedge (\sim q \vee p)]$

Q. If a man is not a fisherman he is not a swimmer.

p : A man is fisherman

$\sim p$: A man is not a fisherman.

q : He is a swimmer

$\sim q$: He is not a swimmer.

p · $\sim p \to \sim q$ → inverse

Converse $q \to p$

If he is a swimmer than a man is fisher-man.

Q. DNF – $(\sim p \to r) \wedge (p \leftrightarrow q)$

~~$\sim p \to r \equiv p \vee r$.~~

~~$p \leftrightarrow q =$~~

$\sim p \to r = \sim(\sim p) \vee r$

$\Rightarrow p \vee r$

$p \to q$

$\sim p \vee q$.

$p \leftrightarrow q$

$(\wedge) \vee (\wedge)$

$(\vee) \wedge (\vee) \to$ CNF

$(\sim p \to r) \wedge (p \leftrightarrow q)$

$(\sim(\sim p) \vee r) \wedge [(\sim p \vee q) \wedge (\sim q \vee p)]$

$(p \vee r) \wedge [(\sim p \vee q) \wedge (\sim q \vee p)]$ – CNF

Q. $p \wedge (p \to q)$ — CNF

~~$p \wedge (\sim p \to$~~

$(\wedge) \vee (\wedge)$

$p \wedge (p \to q)$

$p \wedge (\sim p \vee q)$

$(p \wedge \sim p) \vee (p \wedge q)$

$(p \wedge \sim p) \vee (p \wedge q)$ — distributive law.

↳ DNF

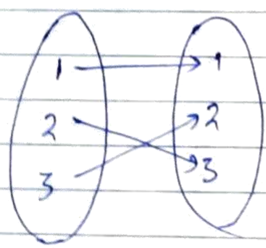
Q. $(p \wedge q) \vee (\sim p \wedge r)$ — DNF.

$((p \wedge q) \vee \sim p) \wedge ((p \wedge q) \vee r)$ → By distributive Law

~~$(p \vee \sim p) \wedge (q \vee \sim p) \wedge ((p \vee r) \wedge (q \vee r))$~~

↳ CNF.

~~$p \vee q$~~



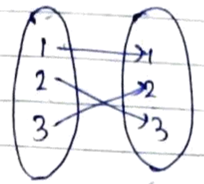
→ Permutation Group

$$\sigma = P_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

$\lfloor 3 = 6$

$(p \vee \sim p) \wedge (q \vee \sim p) \wedge (p \vee r) \wedge (q \vee r)$ — CNF

Permutation Group



$$\sigma = P_1 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

$\lfloor 3 = 6$ possible combination

$$P_2 = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \quad P_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$$

$$P_4 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \quad P_5 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

$$P_6 = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

Q. $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix} \quad g = \begin{pmatrix} 4 & 1 & 3 & 2 \\ 4 & 3 & 2 & 1 \end{pmatrix}$

$S = \{1, 2, 3, 4\}$

$f(1) = g(1)$

$f(2) = g(2)$

This is called equal permutation (pg-382)

Equal Permutation Let S be a non-empty set the permutation f and g define on S are said to be equal if $f(a) = g(a)$ $\forall\, a \in S$.

Identity Permutation :- $\begin{pmatrix}1&2&3&4\\1&2&3&4\end{pmatrix}$

2022

$p \leftrightarrow q \cdot [(\sim p \vee q) \wedge (\sim q \vee p)]$

Q. ~~$(p \to q) \approx p$~~

$(p \to q) \to q = p \vee q$.

p	q	$p \to q$	$(p \to q) \to q$	$p \vee q$
F	F	T	F	F
F	T	T	T	T
T	F	F	T	T
T	T	T	T	T

5 a(ii) $(\sim p \vee \sim q) \to q$ $\quad p \to q$

p	~p	q	~q	$p \to q$	$\sim p \vee \sim q$	$(\sim p \vee \sim q) \to q$
0	1	0	1	1	1	0
0	1	1	0	1	1	1
1	0	0	1	10	1	0
1	0	1	0	1	0	1

(b) $\overline{A+B} = \bar{A} \cdot \bar{B}$

A	B	$\bar{A}$	$\bar{B}$	A+B	$\overline{A+B}$	$\bar{A} \cdot \bar{B}$
0	0	1	1	0	1	0 1
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	1	0	0	1	0	0

O(G) generator Cyclic Group :- A generator is one element through which we can generate all the element

$G = \{1, -1, i, -i\}$.
$O(G) = 4$

·	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	i	-i	-1	1
-i	-i	i	1	-1

e = 1

$(1)^1 = 1$ order = 1
$(-1)^2 = 1$ order = 2
$(i)^4 = 1$ order = 4
$(-i)$

2.(d) $+_6$ $\{0, 1, 2, 3, 4, 5\}$.

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

$[a * b = a + b + 1]$

Q.

9/10/23

"Mathematicals Induction"

→ It is a method of proving a proposition or a statement by
→ Inductive proving approach.

Step 1 : we have to prove that P(n) is true for some initial value $n = n_0$

Step 2 : let us assume that P(n) is true for some k [n = k]

Step 3 :- we have to prove that P(n) is true for [n = k+1]

Bee Is true for every natural number
Then we can say that P(n) is true for all $n >= n_0$

Q. $1^2+2^2+3^2+ \dots +n^2 = \frac{n(n+1)(2n+1)}{6}$; $n \geq 1$

Put n = 1 , $1^2 = \frac{1(1+1)(2\times1+1)}{6}$

$1 = \frac{1\times2\times3}{6} = 1$ Hence proved LHS = RHS

it is true for n = 1.

Let us assume that P(n) = k :

$1^2+2^2+3^2+ \dots +k^2 = \frac{k(k+1)(2k+1)}{6}$

n = k+1 ; $1^2+2^2+3^2+\dots+k^2+(k+1)^2 = \frac{(k+1)(k+2)(2(k+1)+1)}{6}$

$\frac{k(k+1)(2k+1)}{6} + (k+1)^2 = \frac{(k+1)(k+2)(2(k+1)+1)}{6}$

$\frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$

$\Rightarrow \frac{(k+1)(k(2k+1)+6(k+1))}{6} = \frac{(k+1)[2k^2+k+6k+6]}{6}$

$\Rightarrow \frac{(k+1)[2k^2+4k+3k+6]}{6} = \frac{(k+1)[2k(k+2)+3(k+2)]}{6}$

$\Rightarrow \frac{(k+1)(2k+3)(k+2)}{6}$ = R.H.S.

10/10/23

"Peanos Axioms"

→ Peanos Axioms or the Peanos Postulats are the axioms for the natural numbers presented by the 19th century italian Mathematicians "G. Peano".

Properties of natural numbers

1. $1 \in N$ (one belongs to N so 1 is a natural number)
$N \neq \phi$ (it means natural no. can not be empty)

2. For each natural no. there exists a unique natural no. n^* called the successor of n.

$n^* = n+1$

Natural no. are infinite.

3. 1 is not the successor of any no. (1 is the least natural no.)

4. If $m, n \in N$ and $m^* = n^*$ then $m = n$

5. (Mathematical Induction) & Principle of finite induction

STRONG INDUCTION :- Strong Induction is another form of mathematical induction. Through this induction technique, we can prove that a propositional function, P(n) is true for all positive integers, n, using the following steps -

Step 1 (Base Step) - It proves that the initial proposition P(1) true.

Step 2 (Inductive step) - It proves that the conditional statement $[P(1) \wedge P(2) \wedge P(3) \wedge \ldots \wedge P(K)]$ is $\rightarrow P(K+1)$ true for positive integers K.

Q. mathematical induction:-

$3^n - 1$ prove is a multiple of 2 for $n = 1, 2, 3 \ldots$

Put $n = 1 \Rightarrow 3^n - 1 = 3^1 - 1 = 3 - 1 = 2$ it is true for $n = 1$

Put $n = k$ $\quad 3^k - 1$

Put $n = k+1$ $\quad 3^{k+1} - 1 = 3^k \cdot 3 - 1$

$\Rightarrow (2+1)(3^k - 1)$

$\Rightarrow 2(3^k - 1) + 1(3^k - 1)$

$\Rightarrow$ multiple of 2

hence it is multiple of 2 for $n = k+1$

Cosets :- Let $(H, *)$ be a sub group of $(G, *)$ and $a \in G$ then the subset

$a * H = \{a * h, h \in H\}$

is called left coset of H in G

$H * a = \{h * a, h \in H\}$

is called right coset of H in G

if $\boxed{a * H = H * a} \rightarrow$ abelian Group

$G = \{1, -1, i, -i\}$, $\times$ - multiply operation

$H = \{1, -1\}$

$a * H$

$a \times H$

$1 \times H$

$\left.\begin{array}{l} 1 \times 1 = 1 \\ 1 \times -1 = -1 \end{array}\right\}$ belong to the set so it is left coset.

$H \times a$

$\left.\begin{array}{l} 1 \times 1 = 1 \\ -1 \times 1 = -1 \end{array}\right\}$ right coset

14/2/23.

$T = a + a - a \cdot a$

Q $a * b = a + b - ab$

$a * e = a = e * a$

$\downarrow$

$a + e - ae = a$

$\cancel{a} + e(1-a) = \cancel{a}$

$e(1-a) = 0$

$e = 0, \quad 1 - a = 0$

$a = 1$ but $a \neq 1$

so $e = 0$.

$+ = 0$

$m = 1$

$aa^{-1} = 1$

Ex-9.1

Q4 $a * a^{-1} = e = a^{-1} * a$

$\downarrow$

$a * a^{-1} = e$

$a + a^{-1} - aa^{-1} = e$

$a + a^{-1}(1-a) = 0.$

$a^{-1}(1-a) = -a$

$a^{-1} = \dfrac{-a}{(1-a)}$

$a^{-1} = \dfrac{a}{a-1}$ hence proved.

$\{a, b, c, d\}$

$a * b = e$

closer.

Assosiative property

$a * (b * c) = (a * b) * c$

$a + (b * c) - a(b * c)$

$a +$

~~Recurrence relation~~

Associative.

$(a * b) * c = a * (b * c)$

$\Rightarrow (a + b - ab) * c$

$\Rightarrow (a + b - ab) + c - c(a + b - ab)$

$\Rightarrow a + b - ab + c - ac - bc + abc.$

$a * (b * c)$

$a * (b + c - bc)$

$a + (b + c - ab) - (b + c - ab)\, a.$

$\Rightarrow a + b + c - ab - ab - ac + abc.$

$\Rightarrow$ hence proved associative.

Recurrence Relation :- It is an equation that recursively defines a sequence where the next term is a function of the previous terms.

for example :- fabonacci series, factorial

$a_n = a_{n-1} + a_{n-2}$

→ Order of Recurrence Relation :- The difference betw the highest and the lowest power

$n - (n-2) = 2$

→ Degree of Recurrence Relation :- The highest pow of a_n is called ~~go~~ degree.

Linear Recurrance Relation :- with constant co-efficient

order = 1, power only 1

only sum, no product

$C_0 a_n + C_1 a_{n-1} + C_2 a_{n-2} + \dots + C_K a_{n-K} = f(n) \to$ Linear Recurrance Relation with constant co-efficient.

① $f(n) = 0$ this is called linear ~~two~~ homogeneous equation

② $f(n) \neq 0$ this is called linear non-homogenious equation.

① $a_n + 3a_{n-1} + 2a_{n-2} = 0$

Step 1:- axiluary equation

$a_n = x^n$

$x^n + 3x^{n-1} + 2x^{n-2} = 0$

$x^{n-2}(x^2 + 3x + 2) = 0$

$x^{n-2} \neq 0$, $x^2 + 3x + 2 = 0$

$x^2 + 2x + x + 2 = 0$

$x(x+2) + 1(x+2) = 0$

$(x+2)(x+1) = 0$

$x = -1, -2$

different roots :- $y_n = a_1(-1)^n + b_1(-2)^n$

Same roots $= y_n = (a_1 + nb_1)(r)^n$ ↳ roots

$y_n = (a_1 + nb_1 + n^2c_1)(r)^n \to$ jitni roots hai utna expand hoga

Q. $a_n = a_{n-1} + 2a_{n-2}$

put $a_n = x^n$.

$x^n = x^{n-1} + 2x^{n-2}$

$x^n - x^{n-1} - 2x^{n-2} = 0$

$x^{n-2}(x^2 - x - 2) = 0$

$x^{n-2} \neq 0$ $x^2 - x - 2 = 0$

$x^2 + x - 2x - 2$

$x(x+1) - 2(x+1) = 0$

$(x+1)(x-2) = 0$

$x = -1, 2$

$y_n = a_1(-1)^n + b_1(2)^n$ Ans

Q. $a_n - a_{n-1} - 2a_{n-2} = 0$ with $a_0 = 1$, $a_1 = 1$

$x^2 - x - 2 = 0$

$x = 2, -1$

CF = $y_n = a_1(2)^n + a_2(-1)^n$

put $n = 0$.

$a_0 = a_1(2)^0 + a_2(-1)^0$

$a_0 = a_1 + a_2$

$a_1 + a_2 = 1$. —(i)

put $n = 1$.

$a_1 = a_1(2)^1 + a_2(-1)^1$

$1 = 2a_1 - a_2$ —(ii)

$a_1 + a_2 = 1$ C.F

$2a_1 - a_2 = 1$

$3a_1 = 2$

$a_1 = \frac{2}{3}$.

$a_2 = 1 - \frac{2}{3} \Rightarrow \frac{1}{3}$.

$a_n = a_1(2)^n + a_2(-1)^n$

$a_n = \frac{2}{3}(2)^n + \frac{1}{3}(-1)^n$

$f_n \neq 0$ Linear non-homogeneous equation.

Rules:-
1. $f(n) = n$; $P.I = P_1 n + P_2$ then P.I = Particular Integral
2. $f(n) = n^2$; $P.I = P_1 n^2 + P_2 n + P_3$
3. $f(n) = a^n$; $P.I = Pa^n$ or Pn.
4. $f(n) = n \cdot a^n$; $P.I = (P_1 n + P_2)a^n$

Q. $a_n + 4a_{n-1} + 4a_{n-2} = n + 1$

$= x^2 + 4x + 4 = 0$

$x = -2, -2$

$y_n = (a_1 + nb_1)(-2)^n$

$a_n = (a_1 + na_2)(-2)^n$

P.I $\boxed{a_n = (P_1 n + P_2)}$ put in equ

$(P_1 n + P_2) + 4(P_1(n-1) + P_2) + 4(P_1(n-2) + P_2) = n + 1$

$(P_1 + 4P_1 + 4P_1)n + (P_2 - 4P_1 + 4P_2 - 8P_1 + 4P_2) = n + 1$

$9P_1 n + (9P_2 - 12P_1) = n + 1$

Working Rule

Step 1:- Find auxillary equation

Step 2:- (i) when roots are real & distinct

$a_n = a_1(r_1)^n + a_2(r_2)^n + \dots a_k(r_k)^k$

(ii) when roots are co-incident or repeated

$a_n = (a_1 + na_2 + n^2 a_3 + n^3 a_4 + \dots + n^{m-1} a_m)(r)^n$

(iii) If the roots are complexed non repeated

$\alpha \pm i\beta$

$a_n = r^n(A_1 \cos n\theta + A_2 \sin n\theta)$

$r = \sqrt{\alpha^2 + \beta^2}$

$\theta = \tan^{-1}\frac{\beta}{\alpha}$

put $9P_1 n = n$

$P_1 = \frac{1}{9}$

$9P_2 - 12P_1 = 1$

$9P_2 - 12 \times \frac{1}{9} = 1$

$9P_2 = 1 + \frac{12}{9} \Rightarrow \frac{7}{3} = 9P_2$

$P_2 \Rightarrow \frac{7}{3} \times \frac{1}{9}$

$P_2 = \frac{7}{27}$

$P_2 = \frac{7}{27}$

$x^{n+2} - 5x^{n+1} + 6x^n = 0$

Q. $a_{n+2} - 5a_{n+1} + 6a_n = 2$

$a_0 = 1$

$a_1 = 1$

$x^2 - 5x + 6 = 0$

$x = 2, 3.$

$a_n = a_1(2)^n + a_2(3)^n$

put $a_n = A$

$A - 5A + 6A = 2$

$2A = 2$

$\boxed{A = 1}$

$a_n = A = 1.$

append ho jayega final mai

$$\boxed{a_n = a_1(2)^n + a_2(3)^n + 1.}$$

$a_0 = a_1(2)^0 + a_2(3)^0 + 1 \Rightarrow 1 = a_1 + a_2 + 1 \Rightarrow 3(a_1 + a_2 = 0.)$

$a_1 = a_1(2)^1 + a_2(3)^1 + 1 \Rightarrow 1 = 2a_1 + 3a_2 + 1 \Rightarrow 2a_1 + 3a_2 = 0$

$3a_1 + 3a_2 = 0$

$2a_1 + 3a_2 = 0$

$\boxed{a_1 = 0}$

$\boxed{a_2 = 0}$