QMM-Assignment 06

Chandima Attanayake

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Assignment Instructions: Module 11 – Integer Programming

Directions

AP is a shipping service that guarantees overnight delivery of packages in the continental US.The company has various hubs at major cities and airports across the country. Packages are received at hubs, and then shipped to intermediate hubs or to their destination. The manager of the AP hub in Cleveland is concerned about labor costs and is interested in determining the most effective way to schedule workers. The hub operates seven days a week, and the number of packages it handles varies from one day to another. The table below provides an estimate of the number of workers needed each day of the week.

```
days1 <- c("Sunday", "Monday", "Tuesday", "Wednesday", "Thursday", "Friday", "Saturday")
Wokers_required1 <- c(20, 25, 22, 28, 25, 22, 20)

# Create a data frame
data1 <- data.frame(Day = days1, workers_required = Wokers_required1)

# Display the table with column and row lines
data1 %>%
  kable() %>%
  kable_classic() %>%
  column_spec(2,border_left = TRUE) %>%
  column_spec(1,border_left = TRUE) %>%
  column_spec(2,border_right = TRUE)
```

Day	workers_required
Sunday	20
Monday	25
Tuesday	22
Wednesday	28
Thursday	25
Friday	22
Saturday	20

Package handlers at AP are guaranteed a five-day work week with two consecutive days off. The base wage for the handlers is $750 per week. Workers working on Saturday or Sunday receive an additional $20 per day. The possible shifts and salaries for package handlers are:

```r
# Required data
Shift <- c(1,2,3,4,5,6,7)
Days_off <- c("Sunday and Monday", "Monday and Tuesday", "Tuesday and Wednessday", "Wednesday and Thurs
values <- c(770, 790, 790, 790, 790, 770, 750)

# Create a data frame
data <- data.frame(Shift = Shift, Days_off = Days_off, Values = values)

# Display the table with column and row lines
data %>%
  kable() %>%
  kable_classic() %>%
  column_spec(2,border_left = TRUE) %>%
  column_spec(1,border_left = TRUE) %>%
  column_spec(2,border_right = TRUE)%>%
  column_spec(3,border_right = TRUE)
```

Shift	Days_off	Values
1	Sunday and Monday	770
2	Monday and Tuesday	790
3	Tuesday and Wednessday	790
4	Wednesday and Thursday	790
5	Thursday and Friday	790
6	Friday and Saturday	770
7	Saturday and Sunday	750

1. Formulate the problem.

Objective

Minimize Z = 770x1+790x2+790x3+790x4+790x5+770x6+750x7

Constraints

Sunday - x2+x3+x4+x5+x6 >= 20 Monday - x3+x4+x5+x6+x7 >= 25 Tuesday - x1+x4+x5+x6+x7 >= 22 Wednesday - x1+x2+x5+x6+x7 >= 28 Thursday - x1+x2+x3+x6+x7 >= 25 Friday - x1+x2+x3+x4+x7 >= 22 Saturday - x1+x2+x3+x4+x5 >= 20

Non-negativity and Integer Constraint

xi >= 0 and

i=1,2,3,4,5,6,7.

2. Solve the problem in R markdown.

```r
library(lpSolveAPI)

# Initialize model with 7 decision variables
AP2 <- make.lp(0, 7)
```

```r
# Set objective function coefficients (wages per shift)
set.objfn(AP2, c(770, 790, 790, 790, 790, 770, 750))

# Add constraints for each day
add.constraint(AP2, c(0, 1, 1, 1, 1, 1, 0), ">=", 20) # Sunday
add.constraint(AP2, c(0, 0, 1, 1, 1, 1, 1), ">=", 25) # Monday
add.constraint(AP2, c(1, 0, 0, 1, 1, 1, 1), ">=", 22) # Tuesday
add.constraint(AP2, c(1, 1, 0, 0, 1, 1, 1), ">=", 28) # Wednesday
add.constraint(AP2, c(1, 1, 1, 0, 0, 1, 1), ">=", 25) # Thursday
add.constraint(AP2, c(1, 1, 1, 1, 0, 0, 1), ">=", 22) # Friday
add.constraint(AP2, c(1, 1, 1, 1, 1, 0, 0), ">=", 20) # Saturday

# Set variables as integers
set.type(AP2, 1:7, "integer")

# Solve the problem
solve(AP2)
```

```
## [1] 0
```

```r
# Display the minimum cost
total_cost <- get.objective(AP2)
total_cost
```

```
## [1] 25550
```

```r
# Get the optimal number of workers assigned to each shift
optimal_workers <- get.variables(AP2)
optimal_workers
```

```
## [1]  2  6  4  0  8  2 11
```

```r
# Calculate the number of workers available each day
daily_workers <- get.constraints(AP2)
daily_workers
```

```
## [1] 20 25 23 29 25 23 20
```

3. Total cost and the number of workers available each day.

The total cost

According to the above answer, the total cost of the salaries per week which is minimize the wage cost is $25,550.The cost of $ 25,550 is further explained as below accordong to the above answer.

shift 1 - there are 2 employees, paid $770 each (Total:770*2= $1,540) shift 2 - there are 6 employees, paid $790 each (Total:790*6= $4,740) Shift 3 - there are 4 employees, paid $790 each (Total:790*4= $1,540) shift 4 - there are 0 employees, paid $790 each (Total:790*0= $0) shift 5 - there are 8 employees, paid $790 each (Total:790*8= $6,320) shift 6 - there are 2 employees, paid $770 each (Total:770*2= $1,540) shift 7 - there are 11 employees, paid $750 each (Total:750*11= $8,250)

Accordingly, the total cost is (1540+4740+1540+6320+1540+8250=$25,550) $ 25,550.

The Number of workers availble on each day

According to the above calulcation, employees are availble on each day as below. Also, when it comapred with the minimum number of required employees, it is satisfied the requirement. Sunday - Required 20 and 20 available Monday - Required 25 and 25 available Tuesday - Required 22 and 23 available Wednesday - Required 28 and 29 available Thursday - Required 25 and 25 available Friday - Required 22 and 23 available Sunday - Required 20 and 20 available