Assignment Instructions: Module 2 - The LP Model

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Question 01

Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week. a. Clearly define the decision variables b. What is the objective function? c. What are the constraints? d. Write down the full mathematical formulation for this LP problem.

```
data= matrix(c(3,1000,45,'$32',2,1200,40,'$24'), ncol=4, byrow=TRUE)

# specify the column names and row names of matrix
colnames(data) = c('Material','Sales','Labor','Profit')
rownames(data) <- c('Collegiate','Mini')

# assign to table
final=as.table(data)

# display
final
```

```
##            Material Sales Labor Profit
## Collegiate 3        1000  45    $32
## Mini       2        1200  40    $24
```

Management wishes to know what quantity of each type of backpack to produce per week.

Assume

The number of backpack of Collegiate

$$= B_c$$

The number of backpack of Mini

$$= B_m$$

Total labor hours

$$35 * 40 = 1400$$

(a) So the decision variables are

$$= B_c and B_m$$

(b) Objective function is to maximize the unit profit

$$Max \quad Z = 32B_c + 24B_m$$

(c) Constraints are

Material constraint:

$$3B_c + 2B_m \leq 5000$$

Labor constraint:

$$45B_c + 40B_m \leq 1400$$

Sales constraint:

$$B_c \leq 1000$$

$$B_m \leq 1200$$

(d) The full LP model of the given problem is

$$Max \quad Z = 32B_c + 24B_m$$

subject to:

Material constraint:

$$3B_c + 2B_m \leq 5000$$

Labor constraint:

$$45B_c + 40B_m \leq 1400$$

Sales constraint:

$$B_c \leq 1000$$

$$B_m \leq 1200$$

Non-negativity of the decision variables:

$$B_c \geq 0, B_m \geq 0, N_e \geq 0, N_h \geq 0$$

Question 02

The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes–large, medium, and small–that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

a. Define the decision variables
b. Formulate a linear programming model for this problem

```r
# specify the column names and row names of matrix
data <- data.frame(
  Plant = c("1", "1", "1", "2", "2", "2", "3", "3", "3"),
  Size = c("Large", "Medium", "Small", "Large", "Medium", "Small", "Large", "Medium", "Small"),
  Net_profit = c(420, 360, 300, 420, 360, 300, 420, 360, 300),
  Excess_capacity = c(750, NA, NA, 900, NA, NA, 450, NA, NA),
  Inprocess_storage = c(13000, NA, NA, 12000, NA, NA, 5000, NA, NA),
  Unit_sales = c(900,1200,750,900,1200,750,900,1200,750 ),
  Required_space = c(20, 15, 12, 20, 15, 12, 20, 15, 12)
)
 print(data)
```

```
##   Plant   Size Net_profit Excess_capacity Inprocess_storage Unit_sales
## 1     1  Large        420             750             13000        900
## 2     1 Medium        360              NA                NA       1200
## 3     1  Small        300              NA                NA        750
## 4     2  Large        420             900             12000        900
## 5     2 Medium        360              NA                NA       1200
## 6     2  Small        300              NA                NA        750
## 7     3  Large        420             450              5000        900
## 8     3 Medium        360              NA                NA       1200
## 9     3  Small        300              NA                NA        750
##   Required_space
## 1             20
## 2             15
## 3             12
## 4             20
## 5             15
## 6             12
## 7             20
## 8             15
## 9             12
```

Assume

The number of Large products in plant 1

$$= L1$$

The number of Medium products in plant 1

$$= M1$$

The number of Small products in plant 1

$$= S1$$

The number of Large products in plant 2

$$= L2$$

The number of Medium products in plant 2

$$= M2$$

The number of Small products in plant 2

$$= S2$$

The number of Large products in plant 3

$$= L3$$

The number of Medium products in plant 3

$$= M3$$

The number of Small products in plant 3

$$= S3$$

(a) So the decision variables are

$$= L1, M1, S1, L2, M2, S2, L3, M3, S3$$

(b) Objective function is to maximize the net profit

$$Max \quad Z = 420(L1 + L2 + L3) + 360(M1 + M2 + M3) + 300(S1 + S2 + S3)$$

$$Max \quad Z = 420L1 + 420L2 + 420L3 + 360M1 + 360M2 + 360M3 + 300S1 + 300S2 + 300S3$$

Constraints:

Excess capacity:

$$= L1 + L2 + L3 \leq 750$$
$$= M1 + M2 + M3 \leq 900$$
$$= S1 + S2 + S3 \leq 450$$

In process storage:

$$= 20L1 + 15M1 + 12S1 \leq 13000$$
$$= 20L2 + 15M2 + 12S2 \leq 12000$$
$$= 20L3 + 15M3 + 12S3 \leq 5000$$

Sales:

$$L1 + M1 + S1 \leq 900$$
$$L2 + M2 + S2 \leq 1200$$
$$L3 + M3 + S3 \leq 750$$

Due to they want to use the full excess capacity of all three plants in same percentage to avoid the lay-off,the linear programming model is:

$$900(L1 + M1 + S1) = 750(L2 + M2 + S2)$$
$$450(L2 + M2 + S2) = 900(L3 + M3 + S3)$$
$$750(L3 + M3 + S3) = 450(L1 + L2 + L3)$$

$$900(L1 + M1 + S1) - 750(L2 + M2 + S2) = 0$$
$$450(L2 + M2 + S2) - 900(L3 + M3 + S3) = 0$$
$$750(L3 + M3 + S3) - 450(L1 + L2 + L3) = 0$$

Non-negativity of the decision variables:

$$L1 \geq 0, M1 \geq 0, S1 \geq 0, L2 \geq 0, M2 \geq 0, S2 \geq 0, L3 \geq 0, M3 \geq 0, S3 \geq 0,$$