

QMM-Assignment 05

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#Question DATA

The Research and Development Division of the Emax Corporation has developed three new products. A decision now needs to be made on which mix of these products should be produced. Management wants primary consideration given to three factors: total profit, stability in the workforce, and achieving an increase in the company's earnings next year from the \$60 million achieved this year. In particular, using the units given in the following table, they want to Maximize $Z = P - 5C - 2D$, where P = total (discounted) profit over the life of the new products, C = change (in either direction) in the current level of employment, D = decrease (if any) in next year's earnings from the current year's level. The amount of any increase in earnings does not enter into Z , because management is concerned primarily with just achieving some increase to keep the stockholders happy. (It has mixed feelings about a large increase that then would be difficult to surpass in subsequent years.) The impact of each of the new products (per unit rate of production) on each of these factors is shown in the following table:

```
library(kableExtra)
data <- data.frame(
  Factor = c("Total Profit", "Employment Level", "Earnings Next Year"),
  X1 = c(15, 8, 6),
  X2 = c(12, 6, 5),
  X3 = c(20, 5, 4),
  Goal = c("Maximize", "= 70", ">= 60"),
  Units = c("Millions of Dollars", "Hundreds of Workers", "Millions of Dollars")
)

data %>%
  kable() %>%
  kable_styling(full_width = FALSE) %>%
  add_header_above(c(" " = 1, "Product Contributions" = 3, " " = 2))
```

Factor	Product Contributions			Goal	Units
	X1	X2	X3		
Total Profit	15	12	20	Maximize	Millions of Dollars
Employment Level	8	6	5	= 70	Hundreds of Workers
Earnings Next Year	6	5	4	>= 60	Millions of Dollars

01. Define Variables

Define x_1 , x_2 , x_3 , x_{1-} , x_{2-} , x_{3-} , x_{1+} , x_{2+} , x_{3+} as the production rates for Products 1, 2, and 3, respectively, and set.

• y_1 - Over and under deviations for employment level. • y_2 - Over and under deviations for next year's earnings.

The goal programming constraints are

$$y_1 + y_{1-} = 8x_1 + 6x_2 + 5x_3 - 70 \quad y_2 + y_{2-} = 6x_1 + 5x_2 + 4x_3 - 60$$

02. Objective function

$$Z = 15x_1 + 12x_2 + 20x_3 - 5(y_1 + y_{1-}) - 2y_{2-}$$

03. Formulate and Solve LP Model

```
# Load the library
library(lpSolveAPI)

# Initialize LP model with 2 constraints and 7 decision variables
lp_model <- make.lp(2, 7)

# Set objective function coefficients
set.objfn(lp_model, c(15, 12, 20, -5, -5, 0, -2))

# Define objective direction to maximize
lp.control(lp_model, sense = "max")
```

```
## $anti.degen
## [1] "fixedvars" "stalling"
##
## $basis.crash
## [1] "none"
##
## $bb.depthlimit
## [1] -50
##
## $bb.floorfirst
## [1] "automatic"
##
## $bb.rule
## [1] "pseudononint" "greedy"          "dynamic"          "rcostfixing"
##
## $break.at.first
## [1] FALSE
##
## $break.at.value
## [1] 1e+30
##
```

```

## $epsilon
##      epsb      epsd      epsel      epsint epsperturb      epspivot
##      1e-10      1e-09      1e-12      1e-07      1e-05      2e-07
##
## $improve
## [1] "dualfeas" "thetagap"
##
## $infinite
## [1] 1e+30
##
## $maxpivot
## [1] 250
##
## $mip.gap
## absolute relative
##      1e-11      1e-11
##
## $negrange
## [1] -1e+06
##
## $obj.in.basis
## [1] TRUE
##
## $pivoting
## [1] "devex"      "adaptive"
##
## $presolve
## [1] "none"
##
## $scalelimit
## [1] 5
##
## $scaling
## [1] "geometric"      "equilibrate" "integers"
##
## $sense
## [1] "maximize"
##
## $simplextype
## [1] "dual"      "primal"
##
## $timeout
## [1] 0
##
## $verbose
## [1] "neutral"

# Set up the constraints
# Employment constraint:  $8x_1 + 6x_2 + 5x_3 - y_{1+} + y_{1-} = 70$ 
set.row(lp_model, 1, c(8, 6, 5, -1, 1, 0, 0), indices = c(1, 2, 3, 4, 5, 6, 7))

# Earnings constraint:  $6x_1 + 5x_2 + 4x_3 - y_{2+} + y_{2-} = 60$ 
set.row(lp_model, 2, c(6, 5, 4, 0, 0, -1, 1), indices = c(1, 2, 3, 4, 5, 6, 7))

```

```

# Set right-hand side values
set.rhs(lp_model, c(70, 60))

# Set constraints type to equality
set.constr.type(lp_model, c("=", "="))

# Set non-negativity for decision variables
set.bounds(lp_model, lower = rep(0, 7))

# Set names for decision variables
colnames(lp_model) <- c("x1", "x2", "x3", "y1+", "y1-", "y2+", "y2-")

# Solve LP model
solve(lp_model)

```

```
## [1] 0
```

```

# Retrieve and display results
opt_value <- get.objective(lp_model)
opt_variables <- get.variables(lp_model)
list(Objective = opt_value, Variables = opt_variables)

```

```

## $Objective
## [1] 275
##
## $Variables
## [1] 0 0 15 5 0 0 0

```

04. Findings

$x_1 = 0$ $x_2 = 0$ $x_3 = 15$ $y_{1p} = 5$ $y_{1m} = 0$ $y_{2p} = 0$ $y_{2m} = 0$

The optimal production rates for Products x_1 , x_2 , and x_3 are 0, 0, and 15 units, respectively. This solution suggests that no units of Products x_1 and x_2 should be produced as both are 0. However, all production resources should be directed toward Product 3 ($x_3 = 15$).

For the positive and negative deviations in employment and earnings, the optimal values are 5 and 0, respectively. This outcome shows that the company will precisely meet its employment and earnings targets without any shortfall or surplus.

With these production levels and deviation values, the objective function Z reaches its maximum value of 275, which represents the optimal solution. Thus, the company can achieve a total profit of 275 million dollars while satisfying both its employment and earnings objectives.