Homework 14

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6.4:

8 $\binom{17}{8}$

14 If k even and $-100 \leq k \leq 100$, then $\binom{100}{\frac{k+100}{2}}$; else 0.

32 (a) *Proof.* $\square$

(b) $\binom{2n}{2} = \frac{2n\cdot(2n-1)}{2} = 2n^2 - n = n^2 - n + n^2 = 2\binom{n}{2} + n^2$

34 *Proof.* Suppose we are selecting a committee as described.

We can select a chairperson from among the n mathematics professors and then choose $n-1$ members from the other $2n-1$ people, yielding $n\binom{2n-1}{n-1}$ possible selections.

Alternatively, we must have some number of mathematics professors in the group, which must be at least 1 and at most n. For each number k from 1 to n, we select a chairperson, and choose k math profs and $n-k$ CS profs. So we have the total $\sum_{k=1}^{n} k\binom{n}{k}^2$.

Since both methods counted to the same number, they must be equal. $\square$

6.5:

8 $\binom{32}{12}$

16 (a) $\binom{22}{5}$

(b) $\binom{14}{5}$

(c) $\binom{34}{5} - \binom{28}{5}$

(d) $\binom{25}{5} - \binom{17}{5}$

18 $\binom{20}{2}\binom{18}{4}\binom{14}{3}\binom{11}{1}\binom{10}{2}\binom{8}{3}\binom{5}{2}\binom{3}{3}$

28 $6 \cdot \binom{8}{4}$

34 $6 \cdot \binom{5}{2} \cdot 3 \cdot 2$

40 (a) $\binom{40}{10}\binom{30}{10}\binom{20}{10}$

(b) $\binom{40}{10}\binom{30}{10}\binom{20}{10}/4!$

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