Homework 6

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20 Find two sets A and B such that $A \in B$ and $A \subset B$.
$A = \emptyset$, $B = \{\emptyset\}$

24 Can you conclude that $A = B$ if A and B are two sets with the same power set?
Yes.

28 Show that if $A \subset C$ and $B \subset D$ then $A \times B \subset C \times D$.

Proof. Let $x \in A$ and $y \in B$. (If either A or B is empty, then their cartesian product is the empty set, which is trivially a subset of $C \times D$.) Then (x, y) is in $A \times B$. Also, because $A \subset C$ and $B \subset D$, we know that $x \in C$ and $y \in D$, and so $(x, y) \in C \times D$. □

40 Show that $A \times B \neq B \times A$ when A and B are non empty and not equal.

Proof. Suppose A and B are non-empty inequal sets. Since A and B are not equal, one must contain an element the other does not. Without loss of generality, assume that A contains an element x that B does not. So for any element $y \in B$, the cartesian product $A \times B$ contains (x, y), whereas because x is not in B, the cartesian product $B \times A$ does not contain (x, y). □

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14 Find the sets A and B if $A - B = \{1, 5, 7, 8\}, B - A = \{2, 10\}$, and $A \cap B = \{3, 6, 9\}$.
$A = \{1, 3, 5, 6, 7, 8, 9\}$, $B = \{2, 3, 6, 9, 10\}$

26 Let A, B, and C be sets. Show that $(A - B) - C = (A - C) - (B - C)$.

Proof. □

32 Can you conclude that $A = B$ if A, B, and C are sets such that

a $A \cup C = B \cup C$?
No.

b $A \cap C = B \cap C$?
No.

c $A \cup C = B \cup C$ and $A \cap C = B \cap C$?
Yes.

44b Show that if A and B are sets, then $(A \oplus B) \oplus B = A$.

Proof. Suppose that A and B are sets, and x is an element.

$x \in A$	$x \in B$	$x \in A \oplus B$	$x \in (A \oplus B) \oplus B$	$x \in A = x \in (A \oplus B) \oplus B$.
1	1	0	1	1
1	0	1	1	1
0	1	1	0	1
0	0	0	0	1

In all cases, $x \in A = x \in (A \oplus B) \oplus B$, so $(A \oplus B) \oplus B = A$.

□

46 Determine whether the symmetric difference is associative; that is, if A, B, and C are sets, does it follow that $A \oplus (B \oplus C) = (A \oplus B) \oplus C$?
Yes, the symmetric difference is associative.