Homework 3

Chandler Swift

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Section 1.4

12 a True

b True

c False

d True

e False

f True

g False

24 Let $C(x)$ be the propositional function "x is in your class".

a $\forall x(P(x))$ and $\forall x(C(x) \rightarrow P(x))$, where $P(x)$ is the propositional function that a student has a phone.

b $\exists x(F(x))$ and $\exists x(C(x) \wedge (F(X))$, where $F(x)$ is the propositional function that a student has seen a foreign movie.

c $\exists x(\neg S(x))$ and $\exists x(C(x) \wedge \neg S(x))$, where $S(x)$ is the propositional function that a student can swim.

d $\forall x(Q(x))$ and $\forall x(C(x) \rightarrow Q(x))$, where $Q(x)$ is the propositional function that a student can solve a quadratic equation.

e $\exists x(\neg R(x))$ and $\exists x(C(x) \wedge \neg R(x))$, where $R(x)$ is the propositional function that a student wants to be rich.

32 a With the domain of dogs, and $F(x)$ representing the propositional function that a dog has fleas, the original is $\forall d(F(d))$. Negation is $\exists d(\neg F(d))$, or "Some dogs do not have fleas".

b With the domain of horses, and $A(x)$ representing that x can add, the original is $\exists h(A(h))$. The negation is $\forall h(\neg A(h))$, or, in English, "No horse can add".

c With the domain of koalas, and $C(x)$ being the propositional function of climbing ability, the original is $\forall k(C(k))$. The negation is $\exists k(\neg C(k))$, or "Some koalas cannot climb".

d With the domain of monkeys, and $F(x)$ being ability to speak French, $\forall m(\neg F(m))$. The negation is $\exists m(F(m))$, or "Some monkeys can speak French".

e With the domain of pigs, and $S(x)$ being ability to swim and $F(x)$ being the ability to catch fish, $\exists p(S(p) \wedge F(p))$. Negation: $\forall p \neg(S(p) \wedge F(p))$; that is, "No pig can both swim and catch fish".

36 b $\exists x(x < 0 \vee x \geq 5)$

c $\forall x(x < -1 \vee x > 1)$

54 a False

b False

c True

d False

Section 1.5

16 If $C(x)$ represents a student being in the class, and $F(x), Sop(x), J(x), Sen(x)$ represent being a freshman, sophomore, junior, and $CS(x), M(x)$ represent Computer Science and Mathematics majors respectively, with the domain of all people:

a $\exists s(C(s) \wedge J(s))$: True

b $\forall s(C(s) \rightarrow CS(s))$: False

c $\exists s(C(s) \wedge \neg M(s) \wedge \neg J(s))$: True

d $\forall s(C(s) \rightarrow (Sop(s) \vee CS(s)))$: False

e With the domain of majors, students, and year: $\exists m(\forall y(\exists s(y(s) \wedge m(s))))$: False

20 a $\forall x \forall y((y < 0 \wedge x < 0) \rightarrow x * y > 0)$

b $\forall x \forall y((x > 0 \wedge y > 0) \rightarrow \frac{x+y}{2} > 0)$

c $\exists x \exists y((x < 0 \wedge y < 0) \rightarrow \neg(x - y < 0))$

d $\forall x \forall y(\neg(|x + y| > |x| + |y|))$

28 a True

b False

c True

d False

e True

f False

g True

h False

i False

j True

30 c $\forall y(\neg Q(y) \vee \exists x(R(x, y))$

d $\forall y(\forall x \neg R(x, y) \wedge \exists x \neg S(x, y))$

40 a $x \neq 1$

b $x < -100$

c $x = y = 1$ or $x = y = 0$