Homework 13

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6.2:

4 (a) 5

(b) 13

12 *Proof.* For any four points, their x- and y-coordinates must be either (even, even), (even, odd), (odd, even), or (odd, odd). Adding another coordinate must then provide a duplicate of one of these combinations. So the new point and the point with the same even/odd combination has a midpoint at coordinates where both the x-coordinate and the y- coordinate have the same parity, and therefore its average is an integer (either $2j + 2k = 2(j + k)$, or $2j + 1 + 2k + 1 = 2(j + k + 1)$ for some j, k). □

16 (a) *Proof.* The first ten positive integers form five groups that sum to 11: 1 and 10, 2 and 9, 3 and 8, 4 and 7, and 6 and 5.

Since 7 numbers have been chosen from five groups, at least least two numbers must be in the same groups as other numbers. □

(b) No. (There must be one pair.)

28 With people A, B, C, D, E, if $(A, B), (A, C), (A, D), (D, E), (C, E)$ are friends and the rest are enemiees, this configuration results in a group where no three people are mutual friends nor enemies.

6.3:

12 (a) $\binom{10}{3}$

(b) $\binom{10}{3} + \binom{10}{2} + \binom{10}{1} + \binom{10}{0}$

(c) $\binom{10}{3} + \binom{10}{4} + \cdots + \binom{10}{10}$

(d) $\binom{10}{5}$

22 (a) 5040

(b) 720

(c) 120

(d) 120

(e) 24

(f) 0

32 (a) $\binom{7}{5} + \binom{7}{4}\binom{9}{1} + \binom{7}{3}\binom{9}{2} + \binom{7}{2}\binom{9}{3} + \binom{7}{1}\binom{9}{4}$

(b) $\binom{7}{4}\binom{9}{1} + \binom{7}{3}\binom{9}{2} + \binom{7}{2}\binom{9}{3} + \binom{7}{1}\binom{9}{4}$

34 (a) $26^6 - 25^6$

(b) $6 \cdot 5 \cdot 24 \cdot 23 \cdot 22 \cdot 21 = 7650720$

(c) $4 \cdot 24 \cdot 23 \cdot 22 \cdot 21$

(d) $(5 + 4 + 3 + 2 + 1)(24 \cdot 23 \cdot 22 \cdot 21)$

36 $\binom{15}{6} + (\binom{15}{5} \cdot \binom{10}{1}) + (\binom{15}{4} \cdot \binom{10}{2})$