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Control Charts for Short Runs: Nonconstant Process and Measurement Error

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When the usual assumption of constant variances does not hold, the recommended substitute for a deviations from nominal (DNOM) chart is to chart standardized deviations. This has the added difficulty of estimating different standard deviations for each subgroup. As an alternative to this, an approach to DNOM charts that allows more general process and measurement error models is presented. When applied to the constant variance case, the usual chart of differences from nominal results. When applied to the constant coefficient of variation case, a new DNOM chart based on ratios of subgroup means to nominals results. Both charts have the desirable feature of constant control limits.

Introduction

SHEWHART control charts were originally intended for application in high volume manufacturing. Schilling (1990, p. 183) has recently reminded us of Shewhart's admonition that statistical control over a process is difficult and "cannot be reached in the production of a product in which only a few pieces are manufactured". Nonetheless, because of its great success in mass production, attempts have been made to modify the control chart approach for application in the low-volume or short-run environment. This environment has become much more prevalent as technological changes such as flexible manufacturing and numerically controlled processes have caused increasing trends toward build-to-order production.

For monitoring short-run production the main recommendations to date have been: (1) to use a deviation control method (e.g., CUSUM or EWMA) with its increased power for early detection of small process shifts, (2) to use a chart based on individuals data to monitor control variables (e.g., monitoring a process variable such as chemical concentration, rather than a product variable such as plating thickness), or (3) to chart deviations from nominal (DNOM) when different part types are run through the same process. The last of these three methods is considered in this paper.

The details of the DNOM procedure can be found in Montgomery (1991, pp. 314–316). The general idea is that when many small batches of differing part types are run through a given process, the measured *differences* of these parts from their nominal (or target) dimensions can be plotted on the same control chart. Specifically, if X_{ij} ($j = 1, 2, \ldots, n$) represents the j^{th} measured value from a process with a target dimension of T_i, then the DNOM chart plots the average differences $\bar{X}_i - T_i$ for each set of data. Each subgroup consists of parts of the same type and each part type can occur any number of times in the sequence of subgroups. It is by combining deviations across all part types that the DNOM chart overcomes the data limitations that would normally preclude the use of a control chart procedure for short runs of any *single* part type.

In practice, each DNOM chart is not applied to different processes, but is instead restricted to different part types from the *same* process. For example, a DNOM chart might be maintained on a single metal plating process through which different short batches of printed circuit boards are run (each run might correspond to a different type of board). An obvious reason for restricting each DNOM chart to one process is that the plotted deviations must be measured in the same units, but the more important reason is that the variances of the $\bar{X}_i - T_i$ terms will stand a better chance of being approximately the same magnitude if only one process is involved.

In fact, the assumption of homogeneous variances (across part types) is critical to the usual DNOM procedure. Not only does it guarantee constant control limits for $\bar{X}_i - T_i$, but in most cases variance homogeneity is the only justification one has for accumu-

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lating data from different parts in order to construct the chart in the first place. When variances are not constant across all part types, it may be possible to use standardized DNOM charts (see Montgomery (1991, pp. 314–316)), which plot the statistic $(\bar{X}_i - T_i)/(\sigma_i/\sqrt{n})$, where σ_i is the standard deviation of the measured values in the i^{th} subgroup. Unfortunately, estimates of the σ_i's are usually based on control chart information for the individual parts. The problem with this is that it tends to beg the original short-run question, since it is very unlikely that the required control chart information on the *individual* parts will be available.

There are many processes for which nonconstant variances are more likely to be the rule, rather than the exception. That is, it is possible for both process variation and measurement error variation to depend upon the particular nominal dimension T_i. It is not uncommon, for example, to find a process whose variation is functionally dependent on the process average. Yamane (1967, p. 77) refers to the stability of the coefficient of variation for various biological characteristics. Colquhoun (1971, pp. 220–221) echoes this observation by stating it is quite common in practice to find that the relative scatter (as measured by the coefficient of variation) is more nearly constant than the absolute scatter (as measured by the standard deviation). In regression analysis, where nonhomogeneity of variances is called heteroskedasticity, Weisberg (1980, p. 122) states that it is intuitive to expect the variation in small values of the response variable to be smaller than the variation around the larger values of the response. For such processes, the assumption of homogeneous variances is obviously violated.

Secondly, there is the possibility that the measurement error itself may cause variance changes. In engineering metrology (see Sirohi and Radha Krishna (1980, p. 30)), measurement errors are specified either in absolute terms (as the maximum possible error over the full scale of the instrument) or in relative terms (as the percentage error of the true reading). It is common, for example, to see the rated accuracy of an instrument stated in terms of its maximum percent error. For such an instrument, it is apparent that the measurement error variation will change as the part type (and, hence, the nominal dimension) changes.

To address such problems two slightly different versions of the DNOM procedure are proposed. For processes with constant variance and measurement systems with constant error variance, the usual DNOM procedure that monitors how far $\bar{X}_i - T_i$ deviates from 0 is recommended. For processes with an approximately constant coefficient of variation coupled with measurement systems whose errors are reported as percentages of the instrument's reading, a DNOM chart that monitors how much $\bar{X}_i/T_i$ deviates from 1 is recommended. Just as $\bar{X}_i - T_i$ is interpreted as the distance from nominal, $\bar{X}_i/T_i$ has the convenient interpretation of percent of nominal. As will be shown below, both types of DNOM chart have the desirable feature of constant control limits.

Models for Measurement Error and Process Variation

The fundamental concern with the DNOM procedure is not so much the presence or absence of variance homogeneity, but rather the presence or absence of a suitable *model* for the variation. For example, the usual DNOM chart based on $\bar{X}_i - T_i$ tacitly assumes that both process and measurement error variation are constants that do not depend on the nominal dimension(s). However, if knowledge of the process and measurement system suggest that this is an inappropriate model, then this same knowledge will probably suggest an alternate model.

A more general approach (one that includes the usual DNOM chart as a special case) is to consider a process and measurement model of the form

$$X_m = X + \epsilon \tag{1}$$

where X_m is the measured value, X is the 'true' value of the characteristic being measured, and ϵ is the error of measurement. The X's are viewed as being generated by a process with a target (or nominal) dimension of T_i. Depending on the particular process and measurement error model, equation (1) can be used to generate estimates of the σ_i's required by the standardized DNOM chart. Specifically, it is shown in the Appendix that

$$\sigma_i^2 = Var(X_m|T_i) = Var(X|T_i) + E(Var(\epsilon|X)) \tag{2}$$

which holds under the mild assumptions that $E(X|T_i) = T_i$ and $E(\epsilon|X) = 0$ (i.e., for each T_i the process is centered at T_i, and for any particular process value X, the measurement errors have an expected value of zero).

Equation (2) can be applied to any set of assumptions, but here the discussion is limited to primarily two common models.

Model I

Model I describes the classical situation where it is assumed that the process variation does not change even if the nominal value does change, coupled with the assumption that measurement errors are independent of the magnitude being measured.

Process: $E(X \mid T_i) = T_i$ and $Var(X \mid T_i) = \sigma^2$ for any X and any process nominal value T_i; σ^2 is a constant that does not depend on the nominal dimension T_i.

Measurements: $E(\epsilon \mid X) = 0$ and $Var(\epsilon \mid X) = \sigma_\epsilon^2$ for any X; σ_ϵ^2 is a constant that does not depend on the value of X.

Model II

In Model II, both the measurement error and process variation are allowed to change. The standard deviation of the measurement error is assumed to be proportional to X, the quantity being measured, while the process standard deviation is assumed to be proportional to the process target value T_i.

Process: $E(X \mid T_i) = T_i$ and $Var(X \mid T_i) = K^2T_i^2$ for any X, any process nominal value T_i^2, and constant K (i.e., the standard deviation of the process is proportional to the nominal value). The constant K need not be known in advance.

Measurements: $E(\epsilon \mid X) = 0$ and $Var(\epsilon \mid X) = k^2X^2$ for any X and constant k (i.e., the standard deviation of the measurement error is proportional to the value of the characteristic measured). The constant k need not be known in advance. However, k can be estimated if desired (see the Appendix).

Additional Models

Additional models could be considered. Two such models are

Model III: $E(X \mid T_i) = T_i$, $Var(X \mid T_i) = \sigma^2$, $E(\epsilon \mid X) = 0$, and $Var(\epsilon \mid X) = k^2X^2$

Model IV: $E(X \mid T_i) = T_i$, $Var(X \mid T_i) = K^2T_i^2$, $E(\epsilon \mid X) = 0$, and $Var(\epsilon \mid X) = \sigma_\epsilon^2$.

Model III describes the case of a homogeneous variance process coupled with measurements reported as percentages of the true reading. Model IV describes a process with a constant coefficient of variation coupled with measurements having a constant variance. For completeness these models are briefly addressed in the Appendix.

It should be noted that the measured values X_m are assumed to follow a normal distribution in each of the models. However, in lieu of assuming the normality of X_m at the outset, one could alternatively assume for a given T_i that the process values X are normally distributed $N(T_i, \sigma_i)$ and that the errors are normal $N(0, \sigma_\epsilon)$, where σ_ϵ is allowed to depend on X. It then follows that the pair (X, ϵ) is bivariate normal (see Brownlee (1965, pp. 414–417)) and that their sum $X_m = X + \epsilon$ is normal (see Patel and Read (1982, p. 291)).

DNOM Charts

In this section DNOM charts are developed for Models I and II. The derivations and variance estimates are contained in the Appendix. It is assumed that subgroups of size n are available for each of m nominal values $T_1, T_2, \ldots, T_m$; where repeated T_i values are allowed. For each model the usual assumption is made that, for any fixed T_i, the process measurements follow approximately a normal distribution with mean T_i and variance σ_i^2. Thus, the quantity $(\bar{X}_i - T_i)/(\sigma_i/\sqrt{n})$ will follow approximately a standard normal distribution. Here, σ_i denotes the population standard deviation of measured values at nominal level T_i.

Since for Model I the variance is constant across the T_i values, the usual control chart procedure of pooling the within-subgroup data to obtain an estimate of σ_i is used. For this model the unbiased estimate

$$s^2 = \frac{1}{m}\sum_{i=1}^{m} s_i^2 \tag{3}$$

is used to estimate each σ_i^2 (since σ_i^2 is constant for all i), where s_i^2 denotes the sample variance of a group of n observations at nominal level T_i. Using s to estimate each σ_i, the standardized ratios $(\bar{X}_i - T_i)/(s/\sqrt{n})$ are expected to fall between −3 and +3. This condition can be rewritten as

$$-3s/\sqrt{n} \leq \bar{X}_i - T_i \leq 3s/\sqrt{n}.$$

That is, 3-sigma control limits for the differences $\bar{X}_i - T_i$ are given by $LCL = -3s/\sqrt{n}$ and $UCL = 3s/\sqrt{n}$. Alternatively, estimates of σ_i could be based on averaging the subgroup ranges (see Montgomery (1991, p. 314)).

For Model II each σ_i^2 is estimated by $\hat{\sigma}_i^2 = s^2T_i^2$ where s is found from

$$s^2 = \frac{1}{m}\sum_{i=1}^{m}\left(\frac{s_i}{T_i}\right)^2 \tag{4}$$

as shown in the Appendix. From the normality assumption, $(\bar{X}_i - T_i)/(\sigma_i/\sqrt{n})$ should then lie between −3 and +3. Substituting $\hat{\sigma}_i$ for σ_i, the resulting inequality $-3 \leq (\bar{X}_i - T_i)/(sT_i/\sqrt{n}) \leq 3$ can be rewritten in the more convenient form

$$1 - 3s/\sqrt{n} \leq \bar{X}_i/T_i \leq 1 - 3s/\sqrt{n}.$$

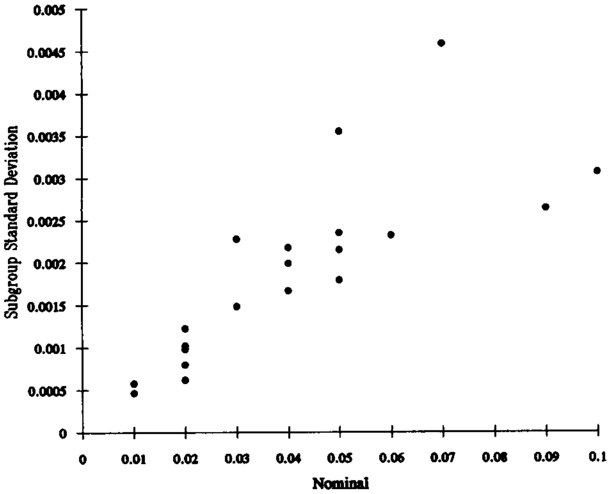

FIGURE 1. Plot of Subgroup Standard Deviations Versus Process Nominal Values.

Thus, for Model II the most convenient form of the DNOM chart would plot the subgroup statistics $\bar{X}_i / T_i$ on a chart with a center line of 1 and the constant control limits $LCL = 1 - 3s/\sqrt{n}$ and $UCL = 1 + 3s/\sqrt{n}$. Note that *no estimates of either K or k are required for constructing this chart.*

Example

Examples of DNOM charts for $\bar{X}_i - T_i$ under the assumptions of Model I can be found in many publications (e.g., see Montgomery (1991, pp. 314-315)), so only the DNOM chart for $\bar{X}_i / T_i$ will be illustrated. Table 1 shows simulated data from 20 subgroups of size 5 for a sequence of 20 process nominal values. The summary statistics for constructing the DNOM chart are given in Table 2 along with the particular nominal values used. The data was computer generated by assuming the process values were $N(T_i, \sigma_i)$. Error terms sampled from a $N(0, \sigma_\epsilon)$ distribution were then added to each process measurement to obtain the final measured values reported in Table 1. For this example $K = 0.05$ and $k = 0.001$ were used. The often encountered '10-to-1' rule (which recommends that measurement errors be about one-tenth of the stated accuracy in the dimension measured) was used to obtain k. Since process nominal values in the example were stated in hundredths, k was taken to be 0.001.

In practice, of course, one must decide which model is best to use for a given data set. Short of using a statistical test for variance homogeneity, perhaps the easiest way of doing this is simply to plot each subgroup s_i versus the corresponding nominal T_i. For the simulated data of this example, the plot of s_i versus T_i is shown in Figure 1. Since the graph indicates that subgroup variation increases with the value of T, Model II appears to be the appropriate choice for this data.

Next, s^2 was calculated using equation (4), yielding control limits of

$$LCL = 1 - 3s/\sqrt{n} = 1 - 3(0.050009)/\sqrt{5} = 0.932906$$

$$UCL = 1 + 3s/\sqrt{n} = 1 + 3(0.050009)/\sqrt{5} = 1.06709.$$

Figure 2 shows these control limits plotted along with the subgroup ratios $\bar{X}_i / T_i$ from Table 2. All points are within the control limits.

For comparison, if one had assumed (incorrectly) that the Model I assumptions applied to the data of Table 1, then the estimate s^2 from equation (3) would have been used, leading to control limits of $\pm 3s/\sqrt{n} = \pm 3(0.002150)/\sqrt{5} = \pm 0.002885$. In this case the charted statistic would be $\bar{X}_i - T_i$, and the resulting DNOM chart would show one point (subgroup 17) falling below the lower control limit. Thus, by using the usual DNOM procedure (Model I) on this data, the resulting control chart would have given a false indication of lack of control at subgroup 17.

Conclusion

DNOM charts are an attempt to adapt successful control chart methodology to the short-run production environment. Current DNOM practice plots the subgroup statistic $\bar{X}_i - T_i$ and assumes process variation to be a constant independent of the particular nominal dimension T_i. The resulting chart will have a center line of zero and constant control limits. When

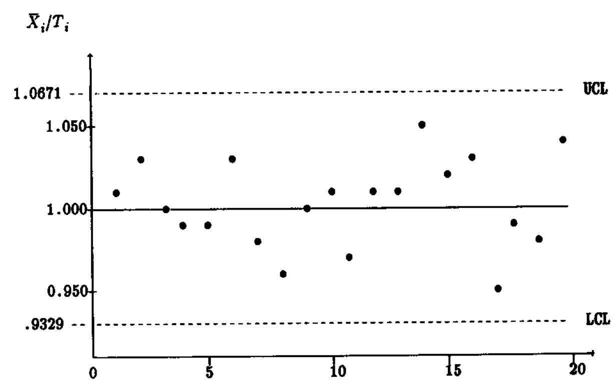

FIGURE 2. DNOM Chart of the Ratios $\bar{X}_i / T_i$ for the Data of Table 1.

process variation depends on T_i, it is recommended that standardized subgroup statistics be plotted instead.

In this paper the DNOM procedure is generalized to handle models of process and measurement error variation in addition to the constant variance model currently used. Practical experience and knowledge of the particular process should suggest the appropriate models of process and measurement error variance. Given such a model, the general formula (A1) can be used to find σ_i. The resulting formula will suggest the appropriate estimator for each σ_i. Finally, incorporating the estimates $\hat{\sigma}_i$, an appropriate DNOM chart can be developed from the requirement that the standardized subgroup statistics $(\bar{X}_i - T_i)/(\sigma_i/\sqrt{n})$ lie between -3 and $+3$.

This general procedure was applied to two common models, one that assumes constant process variances and one that assumes a constant process coefficient of variation. The former leads to the usual DNOM chart that plots $\bar{X}_i - T_i$ versus i. The latter results in a new DNOM chart that plots $\bar{X}_i/T_i$ versus i. Estimates of σ_i are simple, but necessarily different, for each chart and both charts enjoy the desirable property of having constant control limits.

Any effort to chart measurements of central tendency should also be accompanied by an appropriate variation control chart. Although we have not addressed this aspect of DNOM charts, it is apparent that a standard R or s chart for the subgroup deviations $X_{ij} - T_i$ would suffice when using Model I. Similarly, a variation control chart for Model II could be based on the variation measures s_i/T_i.

TABLE 1. 20 Subgroups from a Model II Process with K = 0.05 and k = 0.001

i	X_{i1}	X_{i2}	X_{i3}	X_{i4}	X_{i5}
1	0.03050	0.02798	0.02923	0.03397	0.02945
2	0.01024	0.01060	0.01033	0.00954	0.01073
3	0.04893	0.04748	0.05181	0.05123	0.05071
4	0.08753	0.09107	0.09154	0.08517	0.08946
5	0.09813	0.09508	0.10348	0.09754	0.09854
6	0.02901	0.03270	0.03199	0.03113	0.03003
7	0.03847	0.04215	0.03973	0.03776	0.03713
8	0.04660	0.04866	0.04530	0.04915	0.05135
9	0.01887	0.02090	0.01937	0.01920	0.02169
10	0.00962	0.00969	0.01073	0.01075	0.00976
11	0.06137	0.05569	0.05735	0.05732	0.06014
12	0.05554	0.04969	0.04997	0.05144	0.04569
13	0.02013	0.02053	0.01950	0.02012	0.02118
14	0.02078	0.02010	0.02017	0.02107	0.02252
15	0.04305	0.04046	0.03895	0.04205	0.03982
16	0.05437	0.05033	0.04879	0.05179	0.05270
17	0.06479	0.07049	0.06383	0.07218	0.06142
18	0.02027	0.02092	0.01940	0.01897	0.01934
19	0.04118	0.04164	0.03955	0.03707	0.03710
20	0.02055	0.02007	0.02021	0.02041	0.02255

TABLE 2. Summary Statistics for the Data of Table 1

i	T_i	$\bar{X}_i$	s_i	$\bar{X}_i/T_i$	s_i/T_i
1	0.03	0.030226	0.0022766	1.00753	0.0758861
2	0.01	0.010288	0.0004627	1.02880	0.0462677
3	0.05	0.050032	0.0017877	1.00064	0.0357537
4	0.09	0.088954	0.0026347	0.98838	0.0292748
5	0.10	0.098554	0.0030636	0.98554	0.0306359
6	0.03	0.030972	0.0014819	1.03240	0.0493966
7	0.04	0.039048	0.0019849	0.97620	0.0496224
8	0.05	0.048212	0.0023455	0.96424	0.0469104
9	0.02	0.020006	0.0012227	1.00030	0.0611336
10	0.01	0.010110	0.0005773	1.01100	0.0577278
11	0.06	0.058374	0.0023161	0.97290	0.0386024
12	0.05	0.050466	0.0035480	1.00932	0.0709595
13	0.02	0.020292	0.0006181	1.01460	0.0309059
14	0.02	0.020928	0.0009794	1.04640	0.0489686
15	0.04	0.040866	0.0016660	1.02165	0.0416490
16	0.05	0.051596	0.0021464	1.03192	0.0429278
17	0.07	0.066542	0.0045835	0.95060	0.0654783
18	0.02	0.019780	0.0007959	0.98900	0.0397948
19	0.04	0.039308	0.0021728	0.98270	0.0543212
20	0.02	0.020758	0.0010185	1.03790	0.0509245

As the example given shows, applying the usual constant variance DNOM chart to a process whose variation depends on T_i may lead to false indications of lack of statistical control. Thus, as a general practice, it is recommended that some effort first be expended to understand which type of process and measurement error model is operative. From such knowledge, the results of this paper may then be used to develop an appropriate DNOM chart.

Appendix

Because conditional expectation formulas are used to calculate $\sigma_i^2 = Var(X_m | T_i)$, in order to simplify the notation, the process nominal T_i will be notationally suppressed in some of the following derivations. That is, $E(X_m)$ and $Var(X_m)$ will be used to represent $E(X_m|T_i)$ and $Var(X_m|T_i)$, respectively. Similarly, $E(X|T_i) = T_i$ will be written as $E(X) = T_i$, and so forth. The minor effort of remembering that the expectations are conditional on T_i simplifies the development. No abbreviations are used for the measurement error assumptions. Using the conditional expectation formula and recalling that for both models $E(X) = T_i$ and $E(\epsilon | X) = 0$, one obtains

$$E(X_m) = E(E(X_m|X))$$
$$= E(E(X + \epsilon|X))$$
$$= E(E(X|X) + E(\epsilon|X))$$
$$= E(X + 0) = T_i.$$

Applying the well-known conditional variance formula (see Lindgren (1976, p. 130)), one obtains

$$Var(X_m) = Var(E(X_m|X)) + E(Var(X_m|X))$$
$$= Var(X) + E(Var(X + \epsilon|X))$$
$$= Var(X) + E(Var(\epsilon|X)).$$

Converting back the notation that shows dependence on T_i, one obtains

$$\sigma_i^2 = Var(X_m|T_i)$$
$$= Var(X|T_i) + E(Var(\epsilon|X)). \qquad (A1)$$

Finally, the particular process and measurement error assumptions for each model are substituted into equation (A1) to obtain formulas for σ_i.

Model I

Substituting the Model I assumptions into equation (A1) yields

$$\sigma_i^2 = \sigma^2 + \sigma_\epsilon^2.$$

Following the usual control chart procedure of pooling the within-subgroup data form the m samples, one could use the unbiased estimate $s^2 = \frac{1}{m}\sum_{i=1}^{m} s_i^2$ as an estimate of each σ_i^2 (since σ_i^2 is constant for all i), where s_i^2 denotes the sample variance of a group of n observations at nominal level T_i (duplicate T_i levels are allowed, and such data are treated as separate subgroups in order to keep the subgroup size constant).

Model II

Substituting the Model II assumptions into equation (A1) yields

$$\sigma_i^2 = K^2T_i^2 + E(k^2X^2)$$
$$= K^2T_i^2 + k^2(Var(X) + E(X)^2)$$
$$= K^2T_i^2 + k^2(K^2T_i^2 + T_i^2)$$
$$= (K^2 + k^2K^2 + k^2)T_i^2.$$

Since each s_i^2 is an estimate of σ_i^2, each ratio s_i^2/T_i^2 estimates the factor $K^2 + k^2K^2 + k^2$. Combining these estimates from all m subgroups, one obtains equation (4). Therefore, each σ_i is estimated by sT_i. Other estimates are possible, but those that combine the between-subgroup data with the within-subgroup data would not be appropriate for the purpose of control charting. As one example, the constant $K^2 + k^2K^2 + k^2$ could be estimated by using the mean square error from a regression of the transformed data X_{ij}/T_i on $1/T_i$ (see Neter, Wasserman, and Kutner (1983, pp. 171–172)). Such an estimate, however, would defeat the usefulness of the chart for detecting differences between subgroups.

For the purposes of the DNOM chart, *there is no need to separately estimate K and k,* although this could easily be accomplished if desired. One would just have to obtain k from the manufacturer's rating of the measuring instrument. For example, if an instrument were accurate to within 5% of the true value X being measured then, for normal errors, $0.05X$ would approximately equal $3\sigma_\epsilon$, resulting in $k = 0.05/3 = 0.017$. To obtain an estimate of K one would solve $s^2 = K^2 + k^2K^2 + k^2$ to find

$$K^2 = (s^2 - k^2)/(1 + k^2).$$

It is easy to show from using equation (A1) that for Model III

$$\sigma_i^2 = Var(X_m|T_i) = \sigma^2 + k^2(T_i^2 + \sigma_\epsilon^2)$$

and for Model IV

$$\sigma_i^2 = Var(X_m|T_i) = K^2T_i^2 + \sigma_\epsilon^2.$$

As in Model I (or Model II), unbiased estimates of σ_i can be generated for Model III (or Model IV) since σ_ϵ^2 (or k^2) can always be obtained from the instrument manufacturer's specifications. The development is straightforward and is not included here. It should also be noted that in many cases k should be small compared with T_i (e.g., when one follows the conservative '10-to-1' accuracy requirement for measurement systems). In that case, the DNOM procedure for Model I will serve as a reasonable approximation for Model III and the DNOM chart for ratios (Model II) should adequately handle the Model IV scenario.

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