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2.22 Predisposition for thrombosis. A genetic test is used to determine if people have a predisposition for *thrombosis*, which is the formation of a blood clot inside a blood vessel that obstructs the flow of blood through the circulatory system. It is believed that 3% of people actually have this predisposition. The genetic test is 99% accurate if a person actually has the predisposition, meaning that the probability of a positive test result when a person actually has the predisposition is 0.99. The test is 98% accurate if a person does not have the predisposition. What is the probability that a randomly selected person who tests positive for the predisposition by the test actually has the predisposition?

Has Thrombosis

Return of Test		True	False
	True	99%	2%
	False	1%	98%

$P(\text{Predis}) = 0.03$ $\quad P(\text{Pos} \mid \text{Predis}) = 0.99$

$P(\text{NoPredis}) = 1 - 0.03 = 0.97$ $\quad P(\text{Neg} \mid \text{NoPredis}) = 0.98$

$Pr(\text{Pos} \mid \text{NoPredis}) = 1 - 0.98 = 0.02$

$$P(\text{Predis} \mid \text{Pos}) = \frac{P(\text{Pos} \mid \text{Predis})\,P(\text{Predis})}{P(\text{Pos})}$$

$$P(\text{Predis} \mid \text{Pos}) = \frac{P(\text{Pos} \mid \text{Predis})\,P(\text{Predis})}{P(\text{Pos} \mid \text{Predis})\,P(\text{Predis}) + P(\text{Pos} \mid \text{NoPredis})\,P(\text{NoPredis})}$$

$$P(\text{Predis} \mid \text{Pos}) = \frac{(0.99)(0.03)}{(0.99)(0.03) + (0.02)(0.97)}$$

$Pr(\text{Predisposition} \mid \text{Positive test Result}) = 0.6048879837 \Rightarrow \boxed{60.49\%}$

2.26 Twins. About 30% of human twins are identical, and the rest are fraternal. Identical twins are necessarily the same sex – half are males and the other half are females. One-quarter of fraternal twins are both male, one-quarter both female, and one-half are mixes: one male, one female. You have just become a parent of twins and are told they are both girls. Given this information, what is the probability that they are identical?

IT = identical Twins

FT = Fraternal Twins

MM = both Male

FF = both Female

MF = mixed

$P(IT) = 0.30$ $\quad P(MM) = 0.25$

$P(FT) = 1 - P(IT) = 0.70$ $\quad P(FF) = 0.25$

$P(IT \cap FF) = (0.30)(0.50) = 0.15$ $\quad P(MF) = 0.50$

$$P(IT \mid FF) = \frac{P(IT \cap FF)}{P(FF)}$$

$$P(IT \mid FF) = \frac{0.15}{0.25} = 0.60 \Rightarrow \boxed{60\%}$$

2.28 Socks in a drawer. In your sock drawer you have 4 blue, 5 gray, and 3 black socks. Half asleep one morning you grab 2 socks at random and put them on. Find the probability you end up wearing

(a) 2 blue socks

(b) no gray socks

(c) at least 1 black sock

(d) a green sock

(e) matching socks

Total Socks = 12

$P(\text{Blue}) = 4/12 = \frac{1}{3} = 0.3\overline{3}$

$P(\text{Grey}) = 5/12 = 0.4167$

$P(\text{Black}) = 3/12 = 1/4 = 0.25$

a) $\underline{B}$ $\underline{B}$ ← $P(2^{nd}\text{ Blue}) = 3/11$ $P(2\text{ Blues}) = \frac{1}{3}\cdot\frac{3}{11} = \frac{1}{11} = 0.091 \Rightarrow \boxed{9.1\%}$

$P(\text{Blue}) = \frac{1}{3}$

b) $P(1^{st}) = 1 - P(\text{Grey}) = 1 - 5/12 = 7/12$

$P(2^{nd}) = 1 - 5/11 = 6/11$ $P(\emptyset\text{Grey}) = 7/12 \cdot 6/11 = \boxed{0.318}$

c) $P(\neg\text{Black}, \neg\text{Black}) = P(\neg\text{Black})\cdot(8/11) = (1 - 3/12)\cdot(8/11) = 9/12 \cdot 8/11 = 6/11$

$P(\geq 1\text{ Black}) = 1 - P(\neg\text{Black}, \neg\text{Black}) = 1 - 6/11 = 5/11 = \boxed{0.\overline{45}}$

d) $\boxed{\emptyset}$

e) $P(2\text{Blue}) = 1/11$

$P(2\text{Black}) = 1/22$

$P(2\text{Grey}) = P(\text{Grey})\cdot(4/11)$

$= 5/12 \cdot 4/11 = 5/33$

$P(\text{matching}) = P(2\text{Blue}) + P(2\text{Black}) + P(\text{Grey})$

$= 1/11 + 1/22 + 5/33$

$= \boxed{0.288}$

$\Rightarrow 28.8\%$

2.30 Books on a bookshelf. The table below shows the distribution of books on a bookcase based on whether they are nonfiction or fiction and hardcover or paperback.

Type		Format: Hardcover	Paperback	Total
	Fiction	13	59	72
	Nonfiction	15	8	23
	Total	28	67	95

(a) Find the probability of drawing a hardcover book first then a paperback fiction book second when drawing without replacement.

(b) Determine the probability of drawing a fiction book first and then a hardcover book second, when drawing without replacement.

(c) Calculate the probability of the scenario in part (b), except this time complete the calculations under the scenario where the first book is placed back on the bookcase before randomly drawing the second book.

(d) The final answers to parts (b) and (c) are very similar. Explain why this is the case.

a) $P(HC, PBF) = 28/95 \cdot 59/94 \approx \boxed{0.185} \Rightarrow 18.5\%$

b) $P(PBF, HC) = 59/95 \cdot 28/94 \approx 0.185$

$P(HCF, HC) = 13/95 \cdot 27/94 \approx 0.0363$

$P(F, HC) = P(PBF, HC) + P(HCF, HC) = 0.185 + 0.0363 \approx \boxed{0.22} \Rightarrow 22\%$

c) $P(PBF, HC) = 59/95 \cdot 28/95$

$P(HCF, HC) = 13/95 \cdot 28/95$

$P(F, HC) = P(PBF, HC) + P(HCF, HC) = (59/95 \cdot 28/95) + (13/95 \cdot 28/95) = 28/95(59/95 + 13/95)$

$= 28/95(72/95)$

$\approx \boxed{0.224} \Rightarrow 22.4\%$

d) The larger the sample size is, the less effect replacement will have on the output; as demonstrated in (b) and (c).

2.34 Ace of clubs wins. Consider the following card game with a well-shuffled deck of cards. If you draw a red card, you win nothing. If you get a spade, you win \$5. For any club, you win \$10 plus an extra \$20 for the ace of clubs.

(a) Create a probability model for the amount you win at this game. Also, find the expected winnings for a single game and the standard deviation of the winnings.

(b) What is the maximum amount you would be willing to pay to play this game? Explain your reasoning.

a) X = amount of money you can win

$x = \{0, 5, 10, 30\}$

x	$P(X=x)$
0	$1/2$
5	$1/4$
10	$3/13$
30	$1/52$
	$52/52$

$P(X=0) = P(\text{red}) = 26/52 = 1/2$

$P(X=5) = P(\text{Spade}) = 1/4$

$P(X=10) = P(\text{club} \cap \neg \text{Ace}) = (1/4)(48/52) = 12/52 = 3/13$

$P(X=30) = P(\text{Ace} \cap \text{club}) = (4/52)(1/4) = 1/52$

$\mu = E(x) = \sum_x P(x) x = (0 \cdot 1/2) + (5 \cdot 1/4) + (10 \cdot 3/13) + (30 \cdot 1/52) = 5/4 + 30/13 + 15/26 = 215/52 \approx \boxed{\$4.13}$

$\sigma^2 = Var(x) = \sum_x (x-\mu)^2 P(x) = (0-4.13)^2(1/2) + (5-4.13)^2(1/4) + (10-4.13)^2(3/13) + (30-4.13)^2(1/52) \approx 29.54$

$\sigma = SD(x) = \sqrt{\sigma^2} = \sqrt{29.54} \boxed{\approx \$5.44}$

b) Since the expected value is \$4.13, if you pay any more than that or the nearest 'x', \$5, you will be losing money. $\boxed{\$5}$

2.36 Is it worth it? Andy is always looking for ways to make money fast. Lately, he has been trying to make money by gambling. Here is the game he is considering playing: The game costs \$2 to play. He draws a card from a deck. If he gets a number card (2-10), he wins nothing. For any face card (jack, queen or king), he wins \$3. For any ace, he wins \$5, and he wins an *extra* \$20 if he draws the ace of clubs.

(a) Create a probability model and find Andy's expected profit per game.

(b) Would you recommend this game to Andy as a good way to make money? Explain.

a) X = Amount of money won

$P(X=0) = 9/13$

$P(X=3) = 3/13$

$P(X=5) = 1/13$

$P(X=25) = 1/52$

x	0	3	5	25
$P(X=x)$	$9/13$	$3/13$	$1/13$	$1/52$

$\mu = (0 \cdot 9/13) + 3(3/13) + 5(1/13) + 25(1/52) - 2$

$= 9/13 + 5/13 + 25/52 - 2 \boxed{\approx -\$0.44}$

b) No, I would not recommend this game to Andy because his expected value is negative here. Since, the negative is small we can round it to the nearest x, \$0, which would mean that Andy is expected to break even every game. Thus, this is not a great game to make money.

2.38 Baggage fees. An airline charges the following baggage fees: \$25 for the first bag and \$35 for the second. Suppose 54% of passengers have no checked luggage, 34% have one piece of checked luggage and 12% have two pieces. We suppose a negligible portion of people check more than two bags.

(a) Build a probability model, compute the average revenue per passenger, and compute the corresponding standard deviation.

(b) About how much revenue should the airline expect for a flight of 120 passengers? With what standard deviation? Note any assumptions you make and if you think they are justified.

a) X = Revenue for airline in dollars

x	0	25	60
$P(X=x)$	0.54	0.34	0.12

$P(X=0) = 0.54$

$P(X=25) = 0.34$

$P(X=60) = 0.12$

$\mu = E(X) = (0 \cdot 0.54) + (25 \cdot 0.34) + (60 \cdot 0.12) = \boxed{\$15.70}$

$\sigma^2 = (0-15.70)^2(0.54) + (25-15.70)^2(0.34) + (60-15.70)^2(0.12)$

$\sigma^2 = 398.01 \text{ dollars}^2$

$\sigma = SD(X) = \sqrt{\sigma^2} = \sqrt{398.01 \text{ dollars}^2}$

$\boxed{\sigma \approx \$19.95}$

b) $E(120X) = 120 \cdot E(X) = 120(15.7) = \$1884 \pm \$19.95$

The standard Deviation remains \$19.95 because we have not multiplied the payouts by a factor 120, merely the number of times that expected payout will be rendered.

2.10 European roulette. The game of European roulette involves spinning a wheel with 37 slots: 18 red, 18 black, and 1 green. A ball is spun onto the wheel and will eventually land in a slot, where each slot has an equal chance of capturing the ball. Gamblers can place bets on red or black. If the ball lands on their color, they double their money. If it lands on another color, they lose their money.

(a) Suppose you play roulette and bet $3 on a single round. What is the expected value and standard deviation of your total winnings?

(b) Suppose you bet $1 in three different rounds. What is the expected value and standard deviation of your total winnings?

(c) How do your answers to parts (a) and (b) compare? What does this say about the riskiness of the two games?

a) X= money won, 1 game

X	-3	3
$P(X=x)$	$19/37$	$18/37$

$P(X=-3)=P(\neg \text{Red})$

$= 1-P(\text{Red}) = 1-18/37 = 19/37$

$\mu = E(X) = (19/37)(-3)+(18/37)(3)$

$\boxed{\mu \approx \$0.08}$

$P(X=6)=P(\text{Red})=18/37$

$\sigma^2 = (-3+0.08)^2(19/37)+(3+0.08)^2(18/37) \approx 8.99$

$\sigma = \sqrt{\sigma^2} = \sqrt{8.99} = 2.998$

$\boxed{\sigma \approx \$3}$

b) $E(X_1+X_2+X_3) = E(X)+E(X)+E(X)$

X	-1	1
$P(X=x)$	$19/37$	$18/37$

$= (-0.03)-0.03-0.03$

$\boxed{\mu_3 \approx -0.08}$

$\mu = (-1)(19/37)+18/37 = -1/37 \approx -0.03$

$\sigma_3^2 = \sigma^2+\sigma^2+\sigma^2 = 0.999+0.999+0.999$

$\sigma^2 = (-0.97)^2(19/37)+(1.03)^2(18/37)$

$\sigma^2 = 0.999$

$\sigma_3^2 \approx 2.99781$

$\sigma_3 = \sqrt{\sigma_3^2} \approx \sqrt{2.99781}$

$\boxed{\sigma_3 \approx \$1.73}$

c) The second way of playing (b) is much safer since the standard deviation is lower.

R Questions

R1. In class, we described a game with a 1/2 chance of losing \$4, a 1/12 chance of gaining \$72, and a 5/12 chance of losing \$12. We are going to simulate playing this game 10,000 times. To start, create a vector that has -4 in it 6 times, +72 in it once, and -12 in it 5 times. You should learn the "rep" function to make this easier. Explain why sampling from this vector gives us the payouts in the ratios dictated by the probability model. [code, explanation]

```
# R1. Sampling from this vector gives us the payouts in
#     the ratios dictated by the probability model, because
#     the vector represents the expected number of occurences
#     of each 'x' out of 12 trials. Essentially giving the
#     probability of each occurance.
game <- c(rep(-4, 6), 72, rep(-12, 5))
```

R2. Use the sample function to simulate playing this game 10,000 times. Then, calculate the mean and sd of your 10,000 payouts. How do these results compare to our results from class? [code, answers, comparison to class results]

```
trials <- sample(game, 10000, replace = TRUE)
sum <- sum(trials)
mean <- sum / length(trials)

sqSums = 0
for (t in trials) {
  sqSums = sqSums + (t-mean)^2
}
variance <- sqSums/length(trials)
sd <- variance^(1/2)
```

```
[1] "The class value for E(x) =  -1  vs. My value for E(x) =  -1.1572"

[1] "The class value for sd =  22.34  vs. My value for sd =  22.1816520611077"
```

The values are slightly different because the sample function is psuedo-random, meaning the values in the sample will vary.

R3. In class, we discussed why 2X and X + X are not equal for random variables. You goal is to write a simulation that convinces you of this. First, design a setup that resembles 2X, where X is the game described in R1. As in R1 and R2, you'll make a vector and sample from it 10,000 times. Now, design a setup that resembles X + X. For each setup, you'll want to sample 10,000 times and store the payout results. Then calculate the mean and standard deviation in each set up. How do they compare? [code, means, standard deviations, comparison]

```
# R3.

# This is the mean for multiplying the values E(2X) = 2*E(X)
meanMul <- 2 * mean

# The Var(2X) = 2^2 * Var(X) = 4*Var(X)
varianceMul <- 4 * variance
sdMul <- varianceMul^(1/2)

# The mean for adding the games E(X1+X2) = E(X1) + E(X2)
meanAdd <- mean + mean

# The Var(X1+X2) = Var(X1) + Var(X2)
varianceAdd <- variance + variance
sdAdd <- varianceAdd^(1/2)
```

```
> meanAdd
[1] -2.3144
> meanMul
[1] -2.3144

> sdMul
[1] 44.3633
> sdAdd
[1] 31.36959
```