Math 183 HW5, Name: Chandler Burgess, **PID:** A98029477

3.20 With and without replacement. In the following situations assume that half of the specified population is male and the other half is female.

(a) Suppose you're sampling from a room with 10 people. What is the probability of sampling two females in a row when sampling with replacement? What is the probability when sampling without replacement?

(b) Now suppose you're sampling from a stadium with 10,000 people. What is the probability of sampling two females in a row when sampling with replacement? What is the probability when sampling without replacement?

(c) We often treat individuals who are sampled from a large population as independent. Using your findings from parts (a) and (b), explain whether or not this assumption is reasonable.

a) w/ replacement $\Rightarrow P(2F) = P(F) \cdot P(F) = 0.5^2 = 0.25$

w/o replacement $\Rightarrow P(2F) = P(F) * 4/9 = 5/10 \cdot 4/9 = 0.2\bar{2}$

b) w/ & w/o replacement $\Rightarrow P(2F) = P(F) \cdot P(F) = (0.5) \cdot (0.5) \approx 0.25$

$P(2F) = P(F) \cdot 4999/9999 \approx 0.25$

c) This assumption is reasonable because if the sample to replace is less than 10% of the population/sample in total, then the effect of not replacing becomes negligible.

3.22 Defective rate. A machine that produces a special type of transistor (a component of computers) has a 2% defective rate. The production is considered a random process where each transistor is independent of the others.

(a) What is the probability that the 10^{th} transistor produced is the first with a defect?

(b) What is the probability that the machine produces no defective transistors in a batch of 100?

(c) On average, how many transistors would you expect to be produced before the first with a defect? What is the standard deviation?

(d) Another machine that also produces transistors has a 5% defective rate where each transistor is produced independent of the others. On average how many transistors would you expect to be produced with this machine before the first with a defect? What is the standard deviation?

(e) Based on your answers to parts (c) and (d), how does increasing the probability of an event affect the mean and standard deviation of the wait time until success?

a) $P(10^{th}\text{ defective}) = q^9 \cdot p^1 = (0.98)^9 \cdot (0.02) \approx 0.017$

b) $P(X=0) = \binom{100}{0} q^{100} \cdot p^0 = (0.98)^{100} \approx 0.133$

c) $E(X) = \frac{1}{p} = \frac{1}{0.02} = 50$

$SD(X) = \sqrt{\frac{1-p}{p^2}} = \sqrt{\frac{0.98}{(0.02)^2}} \approx 49.5$

d) $E(X) = \frac{1}{p} = \frac{1}{0.05} = 20$

$SD(X) = \sqrt{\frac{0.95}{0.0025}} \approx 19.49$

e) Increasing the percentage of an event is inversly proportional to the number of trials until success. SD is inversely proportional, also.

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3.26 Chicken pox, Part I. The National Vaccine Information Center estimates that 90% of Americans have had chickenpox by the time they reach adulthood.[50]

(a) Is the use of the binomial distribution appropriate for calculating the probability that exactly 97 out of 100 randomly sampled American adults had chickenpox during childhood.

(b) Calculate the probability that exactly 97 out of 100 randomly sampled American adults had chickenpox during childhood.

(c) What is the probability that exactly 3 out of a new sample of 100 American adults have *not* had chickenpox in their childhood?

(d) What is the probability that at least 1 out of 10 randomly sampled American adults have had chickenpox?

(e) What is the probability that at most 3 out of 10 randomly sampled American adults have *not* had chickenpox?

a) Yes, because the trials are indepedent, meaning they all have equal probability of success in each trial, and the number of trials are fixed.

b) $P(X=97) = \binom{100}{97} q^3 \cdot p^{97} \approx 161,700\,(0.1)^3 \cdot (.9)^{97} \approx 0.0059$

c) $P(X=\neg 3) = P(X=97) \approx 0.0059$

d) $P(X=0) = q^{10} = (0.1)^{10} = 0.0000000001$

$P(X \geq 1) = 1 - P(X=0) = 0.9999999999$

e) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3) \approx 0.639$

$P(X=0) = 0.0000000001$

$P(X=1) = 10\,(0.1)^1\,(0.9)^9 = 0.38742$

$P(X=2) = 45\,(0.1)^2\,(0.9)^8 = 0.19371$

$P(X=3) = 120\,(0.1)^3\,(0.9)^7 = 0.0574$

3.32 Arachnophobia. A 2005 Gallup Poll found that 7% of teenagers (ages 13 to 17) suffer from arachnophobia and are extremely afraid of spiders. At a summer camp there are 10 teenagers sleeping in each tent. Assume that these 10 teenagers are independent of each other.[12]

(a) Calculate the probability that at least one of them suffers from arachnophobia.

(b) Calculate the probability that exactly 2 of them suffer from arachnophobia.

(c) Calculate the probability that at most 1 of them suffers from arachnophobia.

(d) If the camp counselor wants to make sure no more than 1 teenager in each tent is afraid of spiders, does it seem reasonable for him to randomly assign teenagers to tents?

a) $P(X \geq 1) = 1 - P(0) \approx 0.516$

$P(0) = q^{10} = (0.93)^{10} \approx 0.484$

b) $P(X=2) = \binom{10}{2} p^2 q^8 = 45 (0.07)^2 (0.93)^8 \approx 0.1234$

c) $P(X \leq 1) = P(X=0) + P(X=1) \approx 0.848$

$P(0) \approx 0.484$

$P(1) = 10 (0.07)(0.93)^9 \approx 0.3643$

d) Yes, because the probability of that working w/ a random group is ~85%.

$E(X) = 1.18$

3.42 Serving in volleyball. A not-so-skilled volleyball player has a 15% chance of making the serve, which involves hitting the ball so it passes over the net on a trajectory such that it will land in the opposing team's court. Suppose that her serves are independent of each other.

(a) What is the probability that on the 10^{th} try she will make her 3^{rd} successful serve?

(b) Suppose she has made two successful serves in nine attempts. What is the probability that her 10^{th} serve will be successful?

(c) Even though parts (a) and (b) discuss the same scenario, the probabilities you calculated should be different. Can you explain the reason for this discrepancy?

a) $P(3 \text{ in } 10) = \binom{10-1}{3-1} (q)^{9-2} p^{3-1} \cdot p$

$= \overset{36}{\binom{9}{2}} (0.85)^6 (0.15)^2 \cdot (0.15)$

$= 36 (0.86)^6 (0.15)^3$

$= 0.039$

b) $P(10^{th}) = P(S) = 0.15$

c) For b, the probabilities are independent so she is equally likely to make the serve on the 1^{st} and 10^{th} time. In part a it was not given that she had already 2/9 goals, thus we needed the odds of scoring $(2/9) \cap$ scoring on the 10^{th}.

3.44 Stenographer's typos, Part I. A very skilled court stenographer makes one typographical error (typo) per hour on average.

(a) What probability distribution is most appropriate for calculating the probability of a given number of typos this stenographer makes in an hour?

(b) What are the mean and the standard deviation of the number of typos this stenographer makes?

(c) Would it be considered unusual if this stenographer made 4 typos in a given hour?

(d) Calculate the probability that this stenographer makes at most 2 typos in a given hour.

a) The poisson probability distribution

b) $\mu = 1\ \text{typo}/\text{hr}$

$SD(X) = \sqrt{1}/\text{hr} = 1\ \text{typo}/\text{hr}$

c) Yes, because it is double the range of the standard deviation.

d) $P(X \leq 2) = P(0) + P(1) + P(2) = \frac{1}{4e}$

$P(0) = \frac{1}{e}$

$P(1) = \frac{1}{e}$

$P(2) = \frac{1}{2e}$