# Tutorial 13 - More Dynamic Modeling

# Population growth challenge #3 from lecture

Run a 250-year simulation where at year 50 the population receives 50 new individuals via immigration and at 250 years 90% of the population dies due to a disease outbreak.

# Modeling lake pollution

Imagine a lake that has a stream carrying water in and out of it. The stream delivers 25% of the lake volume to the lake each year . Many years ago a factory was constructed on the inflowing stream and it unintentionally leaked industrial waste into the stream. When the environmental protection agency discovered the pollutant in the lake and traced the source to the factory, they measured a concentration of 10 mg per cubic meter in the inflowing stream and assume that this had been the concentration in the stream for many years.

Challenge: Draw a conceptual model of the mass of pollutant in this lake. Try to translate your conceptual model into an equation.

# Modeling lake pollution

The conceptual model is actually not so different from our population model!

We should have input from the outflowing stream and loss of pollutant via the outflowing stream.

It might also be that we could consider bacterial degradation of the pollutant...

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$$C_{t+1} = C_t + Q_{in}C_{in} - Q_{out}C_{out}$$

or

$$C_{t+1} = C_t + Q_{in}C_{in} - Q_{out}C_{out} - dC_t$$

# Modeling lake pollution

$$C_{t+1} = C_t + \frac{Q_{in} C_{in} - Q_{out} C_{out}}{V}$$

A couple important assumptions (ignoring bacterial degradation for now) help to simplify the model:

- First, we are goign to assume that the lake volume and stream flow rates are constant through time. This means $Q_{out} = Q_{in}$.
- Second, we are going to assume the lake is completely and instantaneously mixed. This means $C_t = C_{out}$.

$$C_{t+1} = C_t + \frac{Q}{V}(C_{in} - C_t)$$

# Simulating our pollution model

$$C_{t+1} = C_t + \frac{Q}{V}(C_{in} - C_t)$$

Code up the equation above and run a simulation for what happened after the factory was constructed. Assume the leak began directly after construction and the lake had no pollution in it prior to the factory being constructed.

How many years does it take for the lake to reach an equilibrium concentration?

If the environmental protection agency stopped the leak, how many years would it take for the pollution concentration in the lake to reach 10% of the equilibrium concentration during factory operation?

# Simulating our pollution model

Try adding the bacterial degradation to your simulation. Assume that 10% of pollutant in a volume of water can be degraded by bacteria in one year.

How much would bacterial degradation reduce the equilibrium pollutant concentration during factory operation?

# Exercise 10

Exercise 10 is available on your TA's github repo.

**This will be due by 8 pm next Thursday (December 12th)**

Per usual, turn in your exercise via pull request on github.