Eigenvalues and Eigenvectors

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Outline

DEFINITION (CHARACTERISTIC POLYNOMIAL, CHARACTERISTIC EQUATION)

Let A be a square matrix of order n. The polynomial $\det(A - \lambda * I)$ is called the characteristic polynomial of A and is denoted by $p_A(\lambda)$ (in short, $p(\lambda)$, if the matrix A is clear from the context). The equation $p(\lambda) = 0$ is called the characteristic equation of A. If $\lambda \in \mathbb{F}$ is a solution of the characteristic equation $p(\lambda) = 0$, then λ is called a characteristic value of A.

THEOREM

Let $A \in \mathbb{M}_n(\mathbb{F})$. Suppose $\lambda = \lambda_0 \in \mathbb{F}$ is a root of the characteristic equation. Then there exists a non-zero $v \in \mathbb{F}^n$ such that $Av = \lambda_0 v$.

Definition of Eigenvalues and Eigenvectors

Definition

Let $A \in \mathbb{M}_n(\mathbb{F})$ and let the linear system $Ax = \lambda x$ has a non-zero solution $x \in \mathbb{F}^n$ for some $\lambda \in \mathbb{F}$. Then

- $\lambda \in \mathbb{F}$ is called an eigenvalue of A,
- $x \in \mathbb{F}^n$ is called an eigenvector corresponding to the eigenvalue λ of A, and
- the tuple (λ, x) is called an eigen-pair.

Cayley Hamilton Theorem

Theorem

Let A be a square matrix of order n. Then A satisfies its characteristic equation. That is, $A^n + a_{n-1}A^{n-1} + a_{n-2}A^{n-2} + ... + a_1 A + a_0 I = 0$ holds true as a matrix identity.

DEFINITION (MATRIX DIAGONALIZATION)

A matrix A is said to be diagonalizable if there exists a non-singular matrix P such that $P^{-1}AP$ is a diagonal matrix.

THEOREM (CONDITION FOR DIAGONALIZATION)

Let $A \in \mathbb{M}_n(\mathbb{R})$. Then A is diagonalizable if and only if A has n linearly independent eigenvectors