Uncertainty Quantification Part IV of Project

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1   SOBOL Indices

In our problem we have five input random parameters and two forces in which there is current and in other there is no current respectively. In our problem we have to evaluate which parameters has more effect on force 1 and force 2 respectively by using global sensitivity analysis. It actually the study in the output of model can be allocated to different sources of input parameters. In simple terms, global sensitive help us to know which input parameters has most impact on output. There are two kinds of sensitive analyse one is local and other is global. Global sensitive analyse deals with sobol indices which we calculate in our report and consider as measuring parameter to know about which input parameters has more effect on the forces. Sobol indices consider on entire domain. Mathematically this approach amounts to calculating the variance of the marginal expectations and expressing them in the fractional form.

2   Results and discussion

We had 4 major task to perform in this part of the project.

- The first task was to check which variables affects the force 1 and force 2 the most. To achieve this task, there was a need to calculate all the sobol indices. As we have 5 input random variables. Therefore, the total number of sobol indices is 31. This is a large number to be done, specially when we have 2 outputs, so basically we have 62 sobol indices. First of all the calculation of all the sobol indices was done according to the formulae provided in the lecture slides. Then we observe that ep is the most influential parameter for force 1 and e is the most influential parameter for force 2. (Using the fact that higher values of sobol indices corrrespond to higher sesitivity.)

- Next we have to calculate the total indices. Total indices of RV i is defined as the sum of all sobol indices containing i. Again the conclusion remains the same. Which shows the robustness of the analysis. The conclusion is that ep is the most influential parameter for force 1 and e is the most influential parameter for force 2. The results are presented in the table.

- Next we have to change the expermental error in the input parameters to obtain the value of sobol indices close to one another. We changed the uncertainty to be: 5 10 30 38 45 percent respectively for br, e, ep, haim and current and obtained a clustered sobol indices as possible for force 1. For force 2 we took the uncertainties to be 2, 12, 8, 10, 40 percent respectively in br, e, ep, haim and current and obtained a cluster sobol indices for force 2. There are many possibilities but the basic idea is to reduce the uncertainty of ep and e upto the point that sobol indices stop to change and after that we have to try different combinations and make the decisions on the go.

- In the last question of the project, it was aksed to reduce the standard deviation of the solution to 5 percent of the Crude Monte Carlo case by analysing the sobol indices. Our strategy was again the same, to just reduce the uncertainty in the parameters ep and e and then try a variety of combinations. out of the infinite possible cases we came to a case where the uncertainties were: 1 percent in br, 1 percent in e, 1 percent in ep, 35 percent in haim and 30 percent in current. This case satisfies the required conditions.

- All the results are shown in table next.

sobol indice	Sobol indices value
S1	0.15826517145371793
S2	0.3086276969206257
S3	0.88150658643181
S4	0.16301581929459705
S5	0.1629560435809936
S12	0.2966666270130993
S13	0.8598185843378146
S14	0.15826865757693265
S15	0.15826517145371793
S23	1.028678267291619
S24	0.30871521648906086
S25	0.3086276969206257
S34	0.8815776555308634
S35	0.88150658643181
S45	0.16301581929459705
S123	0.9999724055078734
S124	0.29669859781404434
S125	0.2966666270130993
S134	0.8598344053618364
S135	0.8598185843378146
S145	0.15826865757693265
S1234	1.0000000000000082
S1235	0.9999724055078734
S1245	0.29669859781404434
S1345	0.8598344053618364
S2345	1.0287565921542126
S12345	1.0000000000000082
S234	1.0287565921542126
S235	1.028678267291619
S245	0.30871521648906086
S345	0.8815776555308634

Table 1: force 1

sobol indice	Sobol indices value
S1	0.023050106497789512
S2	0.8229602309940888
S3	0.014550968028138836
S4	-0.010512568636192862
S5	-0.010806988987916842
S12	0.9453304888378417
S13	0.05264843235825355
S14	0.023362512221420346
S15	0.023050106497789512
S23	0.8646116741574911
S24	0.823965983518863
S25	0.8229602309940888
S34	0.014820193299405602
S35	0.014550968028138836
S45	-0.010512568636192862
S123	0.998734082057192
S124	0.9463375907768301
S125	0.9453304888378417
S134	0.05293354310888026
S135	0.05264843235825355
S145	0.023362512221420346
S1234	1.0
S1235	0.998734082057192
S1245	0.9463375907768301
S1345	0.05293354310888026
S2345	0.8658773767494344
S12345	1.0
S234	0.8658773767494344
S235	0.8646116741574911
S245	0.823965983518863
S345	0.014820193299405602

Table 2: force 2

sobol indice	total Sobol indices value
t1	9.259048898130654
t2	10.536230806381088
t3	15.080288993232077
t4	9.393733888443112
t5	9.39335832675911

Table 3: total force 1

sobol indice	total Sobol indices value
t1	8.084793511716414
t2	14.53563485418348
t3	7.7283525395175925
t4	7.433569262077281
t5	7.427863624981519

Table 4: total force 2

sobol indice	Sobol indices value (Force1)	Sobol indices value (Force2)
S1	-0.02993101855093307	0.8990117623063631
S2	-0.41679039111343213	0.9024274748708171
S3	0.49865105661894954	0.9079351543280313
S4	-0.46757436784572554	0.8665218194607095
S5	-0.45845366315186836	0.8611310666768924

Table 5: br,e,ep,haim,current-5,10,30,38,45

sobol indice	Sobol indices value (Force1)	Sobol indices value (Force2)
S1	-0.7381242013051807	0.6074423816823796
S2	-0.025352395770173383	0.9872749512173726
S3	0.29244553984734883	0.6096904187535296
S4	-0.7423935818745057	0.6011121158572884
S5	-0.745489583861811	0.6015768373303467

Table 6: br,e,ep,haim,current-2,12,8,10,40

R	ratio of force 1	ratio of force 2
10	0.09971372755652874	0.07025423729866993
100	0.04974493516900481	0.051919760705885
1000	0.051080128580251405	0.040033049237389935
10,000	0.050103258255467915	0.042083061978078505
100,000	0.04973189899035581	0.04138434115530612

Table 7: Ratio of force 1 and force 2