Uncertainty Quantification Part III of Project

Abhishek Chandra

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1 Improving Crude Monte Carlo

Crude MC method is exceptionally well with the results but it has a major drawack, which is its convergence rate. With the number of increasing random variables in the model and growth in the interets og high-performance computing, no method can sustain with such a low rate of convergence even with such robust results. Also, Monte Carlo is more than 50 years oild and lot of progress has already been made to devise alternative techniques to Monte Carlo, which could outperform MC in convergence rate. Among these methods, in this report, three methods have been discussed section wise. Compresive comparison of these methods has been carried out in the last section.

2 Antithetic Variates

The first method which is discussed to improve the speed of MC method is the Antithetic variates. Antithetic variates focusses on generating the samples of two random variables X and Y, where Y = 1-X. Both random variables are considered on the interval [0,1]. In case the interval is [a,b], Y is calculated as Y = a+b-X. The algorithm of this method is presented in the figure below. The results are presented in the table.

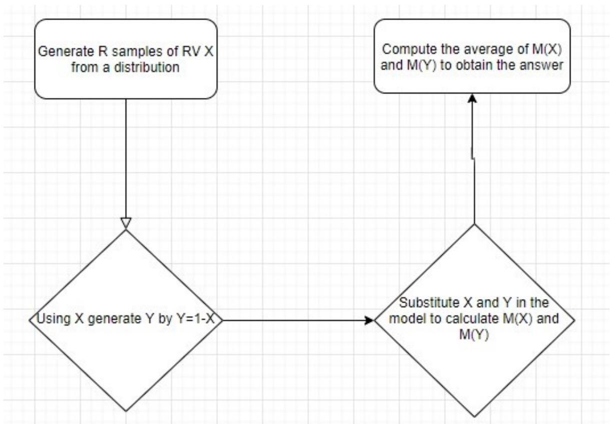

Fig 1: Antithetic Variates Algorithm

R	S1	S2	S3	conv
100	147.3833606857812	147.43131268864943	147.3967041591269	0.8296135761491448
1000	147.4422407130798	147.44704986050286	147.4535174245216	0.2505858093158745
10000	147.437151419219	147.42965096056736	147.44761219670457	0.07954409830263023
100000	147.44001069415438	147.43597236453982	147.438823207554	0.025238388406292143

Table 1: e ; force 1

R	S1	S2	S3	conv
100	1.1696665504225274	1.108927592002519	1.194106592294967	0.11749291155634782
1000	1.0721129410473544	1.057035958639897	1.0561897664452768	0.034256736805207286
10000	1.0828320775044449	1.0895630971771286	1.0862837987148697	0.010818516644486784
100000	1.0905962277703927	1.0826772067533605	1.0822808718711174	0.0034192761833354447

Table 2: e ; force 2

R	S1	S2	S3	conv
100	150.25253608488487	149.78544433987463	150.21177646499495	2.220098931533893
1000	150.0686531151387	150.1178290582321	149.989531931062	0.6813941231367602
10000	150.09358247700777	150.08354017151325	150.06202545264924	0.2155718638183268
100000	150.07896497497083	150.07037412643686	150.06954514286667	0.06798118331603682

Table 3: ep ; force 1

3 Stratified Sampling

Stratified sampling is another method which is used for to increase the computation speed. The method is based on the assumption that we do not need the samples to be equally districuted. Our choice of sample distribution should depend upon the PDF of the RV X. If we make a precise judgement on how to generate random samples efficiently, it could help in improving the computational cost. For example, in case of stratified sampling, the domain of dependence os districuted in such a way that different numer of samples are called from each districuted interval. The user then has to decide in which domains he needs more samples and in which domains he need less samples. Then Model is applied on these samples concatenating the outputs gives rise to the complete random output. In a nutshell, the algorithm can be studied from the figure below. The tables below show the results for this case.

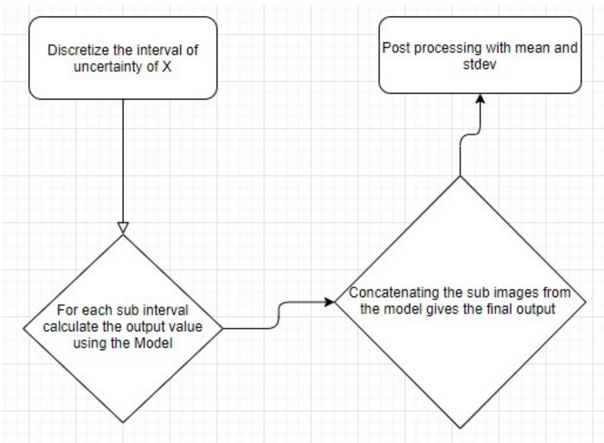

Fig 2: Stratified Sampling Algorithm

4 Importance Sampling

Importance sampling is an approach which is based on the assumption of reducing the variance, which directly increases the cconvergence rate. To minimize the variance, an optimisation problem is solved, and the output model is changed a little. The output model is multiplied by the pdf of X and divided by the variance minimising function g. In practice, it is extremely difficult to calculate g precisely, but fortunately we were provided a similar g for 2 input parameters, which upon modifications gave very good results for our problem. Below, the figure describes the algorithm for the process and tables show the convergence of the method.

R	S1	S2	S3	conv
100	0.00016120358662441314	0.00016070245122570164	0.00016115985626826137	2.381909282530199e-06
1000	0.00016100630147359554	0.00016105906157075502	0.0001609214135976004	7.310570551194362e-07
10000	0.00016103304779449147	0.00016102227356261573	0.00016099919075858156	2.312836676726132e-07
100000	0.00016101736490608745	0.00016100814791959321	0.00016100725851613784	7.2935944104968e-08

Table 4: ep ; force 2

R	S1	S2	S3	conv
100	150.02414066704077	150.26601784371155	149.7306853505839	2.596593266955689
1000	149.83599977724487	149.8684760049459	149.88861782309675	0.7559168609706008
10000	149.8067363057543	149.86644251053605	149.86844897018784	0.24380129158800973
100000	149.82104094758148	149.83238036339668	149.83126436754756	0.07695816820686315

Table 5: all random ; force 1

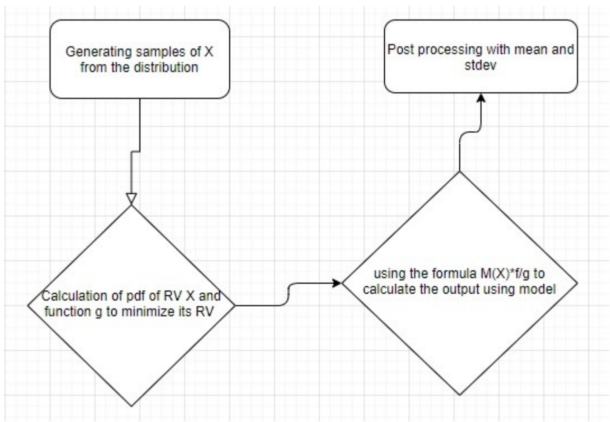

Fig 3: Importance Sampling Algorithm

5 Overall Analysis

Overall, the methods exhibit a very close convergene but the stratified and Antithetic almost give the same rate of convergence. In some cases there is a tradeoff between the mean and variance values. For example in Table 20, Stratified performs better if we consider the convergence but mean is also important, in which antithtic is more closer to what one would expect. Hence, these two methods provide a great rise in convergence rate as compared to Crude Monte Carlo algorithm.

R	S1	S2	S3	conv
100	1.8537119480761233	1.6429449596441135	1.432495221276942	0.219925523881926
1000	1.7252430039597768	1.808131063205819	1.71718366588981	0.070427484211383
10000	1.7583105615891954	1.7276554827209032	1.7275435004271629	0.022286845683252562
100000	1.7316591275340878	1.7298552477469784	1.7383774446211864	0.00703602354628807

Table 6: all random ; force 2

R	S1	S2	S3	conv
100	147.16458172790354	147.3800720685669	147.21149478078902	0.8099591273336184
1000	147.48602419540202	147.54048197473657	147.4439154491292	0.24943306651737657
10000	147.44077507875934	147.43366183118326	147.42679786858213	0.07930414644546487
100000	147.43719651349423	147.43538687917945	147.42959956505882	0.025167233386197457
1000000	147.43626228248226	147.43660888098384	147.43793060053372	0.007963322764195703

Table 7: e ; force 1

R	S1	S2	S3	conv
100	1.1749625255919918	1.1060244883577475	1.1997505507094786	0.11402323352786026
1000	1.127517238042638	1.1251794233942791	1.1511526564465433	0.0332463450939305
10000	1.1413868512637817	1.1537952423422506	1.1552471743980166	0.0106863324768872
100000	1.1494603531311078	1.1539703272359843	1.151615188607741	0.003409215174427287
1000000	1.150840092375203	1.1513138207477431	1.150414319358087	0.0010783374359141771

Table 8: e ; force 2

R	S1	S2	S3	conv
100	150.29957749527975	149.84426002107617	150.70770506954622	2.27954838343086
1000	150.05736276686878	150.2445196840891	150.41461857470574	0.6831364890001358
10000	150.03454774349115	150.0068498768856	150.0562345702291	0.21494588226253358
100000	150.0442614329806	150.07980474632674	150.07728717365248	0.06785220764564463

Table 9: ep ; force 1

R	S1	S2	S3	conv
100	0.00016125405661495786	0.00016076555364650933	0.00016169193028076344	2.4456916659651948e-06
1000	0.0001609941882362235	0.00016119498595387697	0.00016137748238264098	7.329264120947448e-07
10000	0.00016096971035589467	0.00016093999374973945	0.0001609929778117643	2.3061206189088062e-07
100000	0.00016098013202081867	0.0001610182658835887	0.00016101556481939867	7.279756813344523e-08

Table 10: ep ; force 2

R	S1	S2	S3	conv
100	148.77712500007604	149.1739290109338	149.8250199690889	0.7981275415241923
1000	149.2556172211472	149.5819668162486	149.27216633773136	0.24781412791309865
10000	149.224756900373	149.22900122857973	149.2981367668484	0.0779025827165367
100000	149.29344869778524	149.2965882265527	149.2430596503005	0.02488702133165043

Table 11: all as random force 1

R	S1	S2	S3	conv
100	0.655315484458012	0.564701126734198	0.5979740160551965	0.07055615305585015
1000	0.6495425928163934	0.6240409128305637	0.6509881508609359	0.023622631942853148
10000	0.6294291084974643	0.6177342118809899	0.6219973503336643	0.007122233353644568
100000	0.6236107177426425	0.625949959241611	0.6243905449492281	0.0022290894240190695

Table 12: all ; force 2

R	S1	S2	S3	conv
100	146.07952	145.82222	146.36597	0.82118
1000	146.91307	146.24403	147.33725	0.25223
10000	146.62619	146.79737	146.7712	0.07978
100000	146.71001	146.75125	146.72767	0.02524

Table 13: e ; random force 1

R	S1	S2	S3	conv
100	1.2874	1.32545	1.14907	0.11782
1000	1.17353	1.18257	1.11809	0.03552
10000	1.18622	1.16191	1.16994	0.01108
100000	1.18297	1.18021	1.182008	0.0035

Table 14: e ; random force 2

R	S1	S2	S3	conv
100	157.97035	153.69862	154.67204	2.07235
1000	155.19063	154.95136	155.82625	0.69055
10000	155.41324	155.4666	155.58076	0.2178
100000	155.45142	155.40212	155.33932	0.06901

Table 15: ep ; random force 1

R	S1	S2	S3	conv
100	0.000169	0.000164	0.000165	2.20e-06
1000	0.000166	0.000166	0.000167	7.40e-07
10000	0.000166	0.000166	0.000166	2.34e-07
100000	0.000166	0.000166	0.000166	7.40e-08

Table 16: ep ; random force 2

R	S1	S2	S3	conv
100	154.10506	153.75737	153.84013	0.51245
1000	153.66738	153.76286	153.89638	0.16214
10000	153.59309	153.5554	153.57207	0.05119
100000	153.59489	153.59883	153.56071	0.01622

Table 17: all ; random force 1

R	S1	S2	S3	conv
100	0.61066	0.605318	0.51952	0.05924
1000	0.57714	0.579403	0.57627	0.01831
10000	0.57295	0.57598	0.57571	0.00567
100000	0.57657	0.57917	0.5775	0.00179

Table 18: all ; random force 2

methods	mean	conv
M C	147.47139	0.07969
SS	147.43719	0.02516
AS	147.43715	0.07954
IS	146.62619	0.07978

Table 19: e ; force 1

methods	mean	conv
MC	1.15112	0.01069
SS	1.15161	0.00340
AS	1.08283	0.01081
IS	1.18622	0.01108

Table 20: e ; force 2

methods	mean	conv
M C	150.50391	0.21620
SS	150.05623	0.21494
AS	150.06202545264924	0.2155718638183268
IS	155.58076	0.2178

Table 21: ep ; force 1

methods	mean	conv
MC	0.00016	2.3e-07
SS	0.000160	2.3061206189088062e-07
AS	0.00016	2.312836676726132e-07
IS	0.000166	2.34e-07

Table 22: ep ; force 2

methods	mean	conv
MC	149.52385	0.24487
SS	149.29813	0.07790
AS	149.82104	0.07695
IS	153.59309	0.05119

Table 23: all ; force 1

methods	mean	conv
MC	1.77449	0.02177
SS	0.62199	0.00712
AS	1.72754	0.02228
IS	0.57571	0.00567

Table 24: all ; force 2