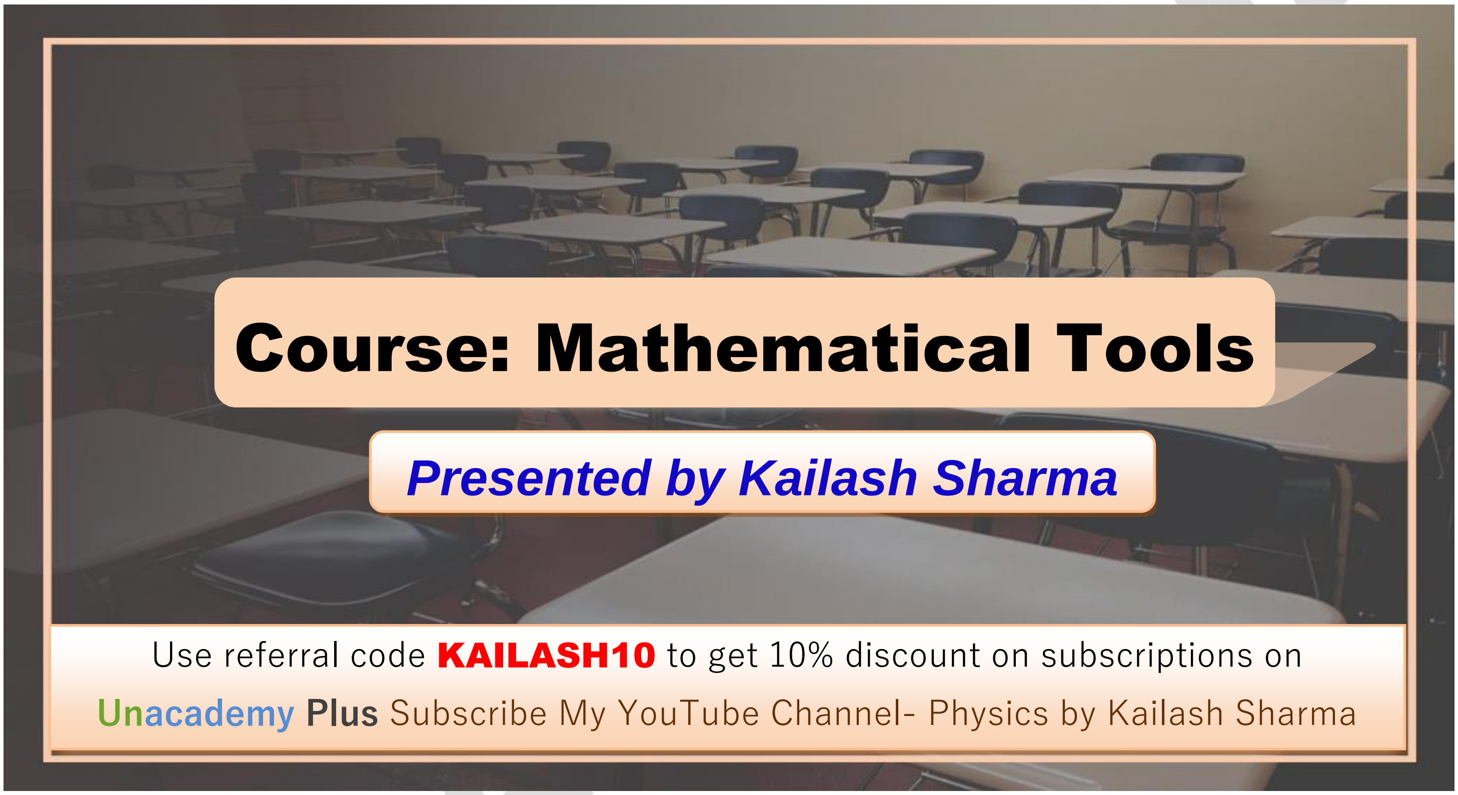
Course: Mathematical Tools
Presented by Kailash Sharma
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MATHEMATICAL TOOLS

PART-I

TRIGONOMETRY & FUNCTIONS

1. If $f(x) = 3x + 4x^2 - 2$, then value of $f(-1)$ is
(A) 1 (B) –1 (C) 2 (D) 5

2. If $g(x) = e^{2x} + e^x - 1$ and $h(x) = 3x^2 - 1$, the value of $g(h(0))$ is:
(A) $\frac{1}{e^2}+e-1$ (B) $\frac{1}{e^2}+\frac{1}{e}-1$ (C) $e^2 + e - 1$ (D) $\frac{1}{e^2}+\frac{1}{e}$

3. If $f(x)=\dfrac{\frac{1}{x}+1}{\frac{1}{x}-1}$;
The value of $f(x) + f(-x)$ is
(A) $2(1 + x^2)$ (B) $2\frac{(1-x^2)}{1+x^2}$ (C) $2\frac{(1+x^2)}{1-x^2}$ (D) $\frac{1+x^2}{1-x^2}$

4. If $f(x) = \frac{x-1}{x+1}$, Find the value of
(i) $f(1)$ (ii) $f(0)$ (iii) $f(f(1))$ (iv) $f(2)$

5. If $f(x) = x^3$; $g(y) = y - 1$; $h(z) = z + 1$
The value of $f(g(h(x)))$ is:
(A) $x^3 - 1$ (B) $x^3 + 1$ (C) $x + 1$ (D) x^3

6. If $f(x) = \frac{x^3-1}{x^2+1}$, then the value of $f(f(1))$ is
(A) 2 (B) –2 (C) 1 (D) –1

7. If $f(x) = \frac{x}{(x^2+a^2)^{3/2}}$, where a is a constant. The value of $f\left(\frac{a}{\sqrt{2}}\right)$ is
(A) $\frac{2^{3/2}}{3a^2}$ (B) $\frac{2a^2}{3\sqrt{3}}$ (C) $\frac{2}{3\sqrt{3}a^2}$ (D) $\frac{3\sqrt{3}}{2}a^2$

8. If $f(x) = x^6$; $h(y) = -y^2 + 1$; $g(z) = z - 1$
(A) $f(x) - h(x^3) = 1$ (B) $f(x) + h(x^3) = 1$
(C) $g(y) + h(\sqrt{y}) = 0$ (D) $g(y^6) - f(y) = -1$

9. If f(x) = $x^2 - 1$ and g(x) = $\frac{1}{x}+1$; the value of $f\left(\frac{1}{g(x)}\right)$ is

10. If f(x) = 4x + 3. Find f(f(2)).
(A) 24 (B) 27 (C) 37 (D) 47

11. f(x) = log x^3 and g (x) = log x.
Which of the following statement is / are true ?
(A) f(x) = g(x) (B) 3f(x) = g(x) (C) f(x) = 3g(x) (D) f(x) = $(g(x))^3$

12. tan15° is equivalent to :
(A) $(2-\sqrt{3})$ (B) $(5+\sqrt{3})$ (C) $\left(\frac{5-\sqrt{3}}{2}\right)$ (D) $\left(\frac{5+\sqrt{3}}{2}\right)$

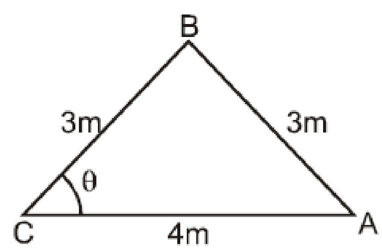
13. $\sin^2\theta$ is equivalent to :
(A) $\left(\frac{1+\cos\theta}{2}\right)$ (B) $\left(\frac{1+\cos 2\theta}{2}\right)$ (C) $\left(\frac{1-\cos 2\theta}{2}\right)$ (D) $\left(\frac{\cos 2\theta-1}{2}\right)$

14. sinA . sin(A + B) is equal to
(A) $\cos^2A$. cosB + sinA $\sin^2B$
(B) $\sin^2A$. cosB + $\frac{1}{2}\cos^2A$. sinB
(C) $\sin^2A$. cosB + $\frac{1}{2}\sin^2A$. sinB
(D) $\sin^2A$. sinB + cosA $\cos^2B$

15. –sinθ is equivalent to:
(A) $\cos\left(\frac{\pi}{2}+\theta\right)$ (B) $\cos\left(\frac{\pi}{2}-\theta\right)$ (C) sin (θ – π) (D) sin (π + θ)

16. If $x_1 = 8\sin\theta$ and $x_2 = 6\cos\theta$ then
(A) $(x_1 + x_2)_{max} = 10$
(B) $x_1 + x_2 = 10\sin(\theta + 37°)$
(C) $x_1x_2 = 24\sin2\theta$
(D) $\frac{x_1}{x_2} = \frac{4}{3}\tan\theta$

17. θ is angle between side CA and CB of triangle, shown in the figure then θ is given by :



(A) $\cos\theta = \frac{2}{3}$ (B) $\sin\theta = \frac{\sqrt{5}}{3}$ (C) $\tan\theta = \frac{\sqrt{5}}{2}$ (D) $\tan\theta = \frac{2}{3}$

18. If f(x) = 5x – 5, g(x) = $\sin^3 x + 2\cos^3 x$; The value of f(g(f(1))) is
(A) 5 (B) 0 (C) 10 (D) –5

19. If f(x) = x + 1; g(z) = z^2; h(y) = 3y, The value of f(h(g(a))) is:
(A) $(3a + 1)^2$ (B) $3a^2 + 1$ (C) $3(a^2 + 1)$ (D) $3a^2$

20. Equation of straight line is 2x + 3y = 5. Slope of the straight line is:
(A) $\frac{3}{2}$ (B) $\frac{2}{3}$ (C) $-\frac{2}{3}$ (D) $-\frac{3}{2}$

21. If f(x) = sin x + cos x. Then $\frac{f(x)+f(-x)}{f(x)-f(-x)} =$
(A) $\frac{\sin x+\cos x}{\sin x-\cos x}$ (B) cot x (C) tan x (D) $\frac{\sin x-\cos x}{\sin x+\cos x}$

22. f(x) = cos x + sin x. Find $f\left(\frac{\pi}{2}\right)$.
(A) 0 (B) 1 (C) 2 (D) 3

23. If f(x) = x^2 and g(x) = sin(2x); the value of g(f($\sqrt{y}$)) =
(A) sin y (B) sin (2y) (C) $\sin(\sqrt{y})$ (D) $\sin^2(2y)$

24. $\sin^2\theta =$
(A) $\frac{1+\cos 2\theta}{2}$ (B) $\frac{1-\cos 2\theta}{2}$ (C) $1 - \cos^2\theta$ (D) $\sin(2\theta)$

25. $\cos 2\theta =$
(A) $2\cos^2\theta - 1$ (B) $1 - 2\sin^2\theta$ (C) $\cos^2\theta - \sin^2\theta$ (D) $\cos^2\theta + \sin^2\theta$

26. $\cos^2\theta =$
(A) $\frac{1+\cos 2\theta}{2}$ (B) $\frac{1-\cos 2\theta}{2}$ (C) $1 - \sin^2\theta$ (D) $\cos(2\theta)$

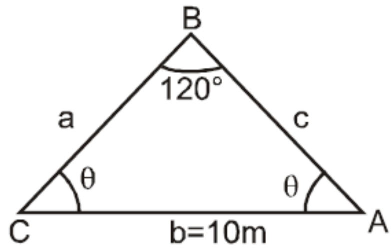
27. If f(x) = $\sin^3 x - \cos(2x)$, then the value of $f\left(\frac{\pi}{2}\right)$ is:
(A) 0 (B) 2 (C) 1 (D) –2

28. If f(x) = $\sin^2 x - \cos^2 x$. Then find $f\left(\frac{\pi}{12}\right)$.
(A) $\frac{\sqrt{3}}{2}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

29. If $f(x) = \dfrac{x-1}{x+1}$ then find f {f(x).

30. $y = f(x) = \dfrac{2x-3}{3x-2}$. Find f(y).

31. For a triangle shown in the figure, side CA is 10 m, angle $\angle A$ and angle $\angle C$ are equal then :



(A) side a = side c = 10m

(B) side a $\neq$ side c

(C) side a = side c = $\dfrac{10\sqrt{3}}{3}$ m

(D) side a = side c = $\dfrac{10}{\sqrt{2}}$ m

32. If $y_1 = A\sin\theta_1$ and $y_2 = A\sin\theta_2$ then

(A) $y_1 + y_2 = 2A\sin\left(\dfrac{\theta_1+\theta_2}{2}\right)\cos\left(\dfrac{\theta_1-\theta_2}{2}\right)$

(B) $y_1 + y_2 = 2A\sin\theta_1\sin\theta_2$

(C) $y_1 - y_2 = 2A\sin\left(\dfrac{\theta_1-\theta_2}{2}\right)\cos\left(\dfrac{\theta_1+\theta_2}{2}\right)$

(D) $y_1.y_2 = -2A^2\cos\left(\dfrac{\pi}{2}-\theta_1\right).\cos\left(\dfrac{\pi}{2}-\theta_2\right)$

33. Which of following are true

(A) sin37° + cos37° = sin53° + cos53°

(B) sin37° – cos37° = cos53° – sin53°

(C) tan37° + 1 = tan 53° – 1

(D) tan37° × tan53° = 1

34. If $\mathbf{R^2 = A^2 + B^2 + 2AB\cos\theta}$, if |**A**| = |**B**| then value of magnitude of **R** is equivalent to :

(A) $2A\cos\theta$

(B) $A\cos\dfrac{\theta}{2}$

(C) $2A\cos\dfrac{\theta}{2}$

(D) $2B\cos\dfrac{\theta}{2}$

PART-II

DIFFERENTIATION

1. A particle is moving along x-axis such that its position 'x' varies with time (t). Find the velocity (v) and acceleration (a) of particle if its position w.r.t time is given by:

(i) $x = t^2$ (ii) $x = \frac{1}{t}$ (iii) $x = \frac{1}{\sqrt{t}}$ (iv) $x = t^{3/2}$

(v) $x = t^{5/2}$ (vi) $x = \sqrt{2}t^2$ (vii) $x = 5000$ (viii) $x = t^2 + t + 5$

(ix) $x = 4t^3 + 3$ (x) $x = 3t + \frac{2}{t}$

2. Find $\frac{dy}{dt}$

(i) $y = \sin(t + 2)$ (ii) $y = \sin(\omega t + \phi)$ where ω and ϕ are constant

(iii) $y = \cos 2\theta$, where $\frac{d\theta}{dt} = \omega$ (iv) $y = \sin(2\theta + 3)$ where $\frac{d\theta}{dt} = \omega$

(v) $y = 2x^2 + 3x + 4$ where $\frac{dx}{xt} = V_x$ (vi) $y = (2t + 4)^3$

(vii) $y = \sin^2 t$ (viii) $y = \cos^2 t$

3. Find $\frac{dy}{dx}$.

$x^3 + y^3 = 18\,xy$

4. Find the derivative of given functions w.r.t. corresponding independent variable.
$y = \tan x + \cot x$

5. $q = \sqrt{2r - r^r}$, find $\frac{dq}{dr}$

6. Find the derivative of given functions w.r.t. corresponding independent variable.
$y = x^2 + x + 8$

7. Find the first derivative and second derivative of give functions w. r.t. the independent variable x.
$y = \ell n x^2 + \sin x$

8. Find the first derivative & second derivative of given functions w.r.t. corresponding independent variable.
$y = \sin x + \cos x$

9. Find derivative of given functions w.r.t. the respective independent variable.

$$\frac{\ell nx + e^x}{\tan x}$$

10. Find the first derivative & second derivative of given functions w.r.t. corresponding independent variable.

$$y = \ell nx + e^x$$

11. Find derivative of given functions w.r.t. the respective independent variable.

$$y = \frac{\cot x}{1 + \cot x}$$

12. Find derivative of given functions w.r.t. the independent variable x.

$$y = e^x \ell nx$$

13. Find $\frac{dy}{dx}$ as a function of x.

d $y = \sin^3 x + \sin 3x$

14. Find derivative of given functions w.r.t. the independent variable x.

$y = \sin x \cos x$

15. Find $\frac{dy}{dx}$.

$$x^2y + xy^2 = 6$$

16. Find derivative of given functions w.r.t. the corresponding independent variable.

$$y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$$

17. Find $\frac{dy}{dx}$ as a function of x

$y = (4 - 3x)^9$

18. Find derivative of given functions w.r.t. the independent variable.

$$y = \frac{2x + 5}{3x - 2}$$

19. Find $\frac{dy}{dx}$ as a function of x

$\sin^2(x^2 + 1)$

20. Find derivative of given functions w.r.t. the independent variable.

$$y = \frac{\ell nx}{x}$$

21. Find $\frac{dy}{dx}$ as a function of x

$y = 2\sin(\omega x + \phi)$ where ω and ϕ constants

22. Find derivative of given functions w.r.t. the corresponding independent variable.

$r = (1 + \sec\theta)\sin\theta$

23. Find derivative of given functions w.r.t. the independent variable

$(\sec x + \tan x)(\sec x - \tan x)$

24. Find the first derivative and second derivative of give functions w. r.t. the independent variable x.

$y = \sqrt[7]{x} + \tan x$

25. Find $\frac{dy}{dx}$ as a function of x

$y = \sin 5x$

26. Suppose u and v are functions of x that are differentiable at x = 0 and that

$u(0) = 5$, $u'(0) = -3$, $v(0) = -1$ $v'(0) = 2$

Find the values of the following derivatives at x = 0.

(a) $\frac{d}{dx}(uv)$ (b) $\frac{d}{dx}\left(\frac{u}{v}\right)$ (c) $\frac{d}{dx}\left(\frac{v}{u}\right)$ (d) $\frac{d}{dx}(7v - 2u)$

27. Find $\frac{dy}{dx}$.

$(x + y)^2 = 4$

28. Given that $a = v\frac{dv}{dx}$ then find 'a' as a function of 'x' if

(i) $v = kx + c$, where 'k' and 'c' are constant (ii) $v = k\sqrt{x}$, where 'k' is constant

(iii) $v = A\sin kx$, where 'A' and 'k' are constant (iv) $v = \sqrt{1+x^2}$

(v) $v = \frac{1}{\sqrt{1+x^2}}$

29. Find $\frac{dy}{dx}$

(i) $y = x\sin x$ (ii) $y = e^x\cos x$ (iii) $y = \frac{x}{1+x}$ (iv) $y = \sqrt{x}\,\sin x$

30. Given that $\alpha = \omega \dfrac{d\omega}{d\theta}$ find 'α' if

(i) $\omega = 2\theta^2 + \theta + 1$ (ii) $\omega = 4 \sin 2\theta$ (iii) $\omega = 2 + \cos\theta$

31. Given y = f(u) and u = g(x) , find $\dfrac{dy}{dx}$

$y = 2u^3$, $u = 8x - 1$

32. Given y = f(u) and u = g(x) , find $\dfrac{dy}{dx}$

$y = \sin u$, $u = 3x + 1$

33. Given y = f(u) and u = g(x) , find $\dfrac{dy}{dx}$

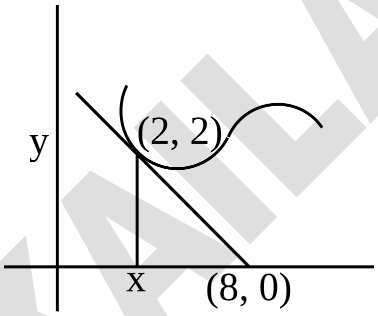
$y = 6u - 9$, $u = (1/2)\, x^4$

34. Given y = f(u) and u = g(x) , find $\dfrac{dy}{dx}$

$y = \cos u$, $u = -\dfrac{x}{3}$

35. Momentum of a body moving in a straight line is $p = (t^2 + 2t + 1)$ kg m/s. Find the force acting on a body at t = 2sec.

36. What is $\dfrac{dy}{dx}$ at (2, 2) in shown figure?



(A) $\dfrac{2}{3}$ (B) $\dfrac{2}{5}$ (C) $\dfrac{1}{3}$ (D) $-\dfrac{1}{3}$

37. If $S = \dfrac{t^3}{3} - 2t^2 + 3t + 4$, then

(A) at t = 1, S is minimum (B) at t = 1, S is maximum
(C) at t = 3, S is maximum (D) at t = 3, S is minimum

38. The charge flowing in a conductor varies with time as $Q = at - \frac{1}{2}bt^2 + \frac{1}{6}ct^3$, where a, b, c are positive constant. Then, the current $i = \frac{dQ}{dt}$

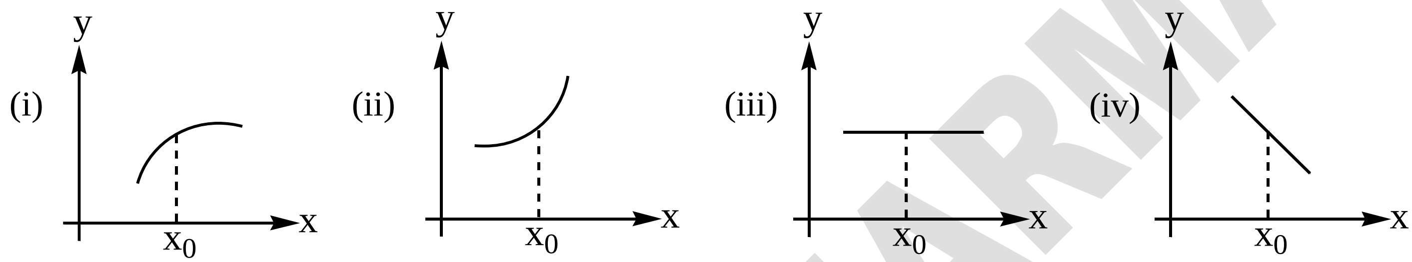
(A) Has an initial value a

(B) Reaches a minimum value after time b/c

(C) Reaches a maximum value after time b/c

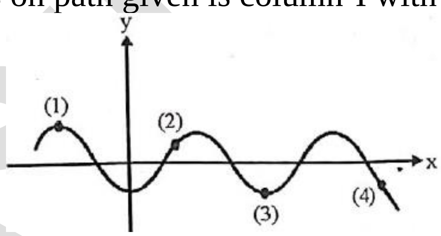
(D) Has a minimum value $\left(a - \frac{b^2}{2c}\right)$

39. Which of the following statements are true based on graphs of y-versus x as shown below:



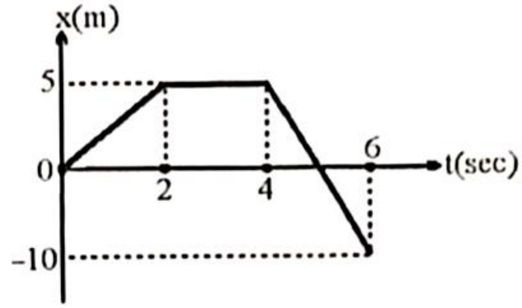
(A) Slope at x_0 is positive and non-zero in graph (i) and (ii)
(B) Slope is constant in (iii)
(C) Slope at x_0 is negative in (iv) at x_0
(D) Slope at x_0 is negative in (ii)

40. Consider the motion of a particle in a x-y plane as shown in the diagram. Match the property of the curvilinear path at different points on path given is column-I with the properties given in column-II



Column-I	**Column-II**
(A) Point (1)	(P) x is positive
(B) Point (2)	(Q) y is positive
(C) Point (3)	(R) Slope $\left(\frac{dy}{dx}\right)$ is positive
(D) Point (4)	(S) Slope $\left(\frac{dy}{dx}\right)$ is zero
	(T) Slope $\left(\frac{dy}{dx}\right)$ is non-zero

41. A particle is moving according to the position time (x-t) graph as shown. Find velocity of particle at t = 1 sec., 3 sec., 5 sec.



42. Suppose that the radius r and area $A = \pi r^2$ of a circle are differentiable functions of t. Write an equation that relates $\frac{dA}{dt}$ to $\frac{dr}{dt}$.

43. Suppose that the radius r and area $S = 4\pi r^2$ of a circle are differentiable functions of t. Write an equation that relates $\frac{ds}{dt}$ to $\frac{dr}{dt}$.

44. Particle's position as a function of time is given by $x = -t^2 + 4t + 4$ find the maximum value of position coordinate of particle.

45. Find the values of function $2x^3 - 15x^2 + 36x + 11$ at the points of maximum and minimum

46. The radius r and height h of a circular cylinder are related to the cylinder's volume V by the formula $V = \pi r^2 h$.
(a) If height is increasing at a rate of 5 m/s while radius is constant, Find rate of increase of volume of cylinder.
(b) If radius is increasing at a rate of 5 m/s while height is constant, Find rate of increase of volume of cylinder.
(c) If height is increasing at a rate of 5 m/s and radius is increasing at a rate of 5 m/s, Find rate of increase of volume of cylinder.

47. Find two positive numbers x & y such that x + y = 60 and xy is maximum –

48. A sheet of area 40 m^2 in used to make an open tank with a square base, then find the dimensions of the base such that volume of this tank is maximum.

PART-III

INTEGRATION

1. Evaluate the following indefinite integrals.

(i) $\int dx$ (ii) $\int x dx$ (iii) $\int \left(\frac{1}{x^2}\right) dx$ (iv) $\int x^{5/2} dx$

(v) $\int \sqrt[3]{x^2} dx$ (vi) $\int x^2 dx$ (vii) $\int (3\sin x + 2) dx$ (viii) $\int \frac{3}{5} x^{5/3} dx$

(ix) $\int (x^2 - 2x + 1) dx$ (x) $\int \left(\frac{x^{-3}}{2} + x^2\right) dx$ (xi) $\int \frac{dx}{4x}$ (xii) $\int \left(2 - \frac{5}{x^2}\right) dx$

2. Find integrals of given functions
$x^2 - 2x + 1$

3. Find integrals of given functions
$\sqrt{x} + \frac{1}{\sqrt{x}}$

4. Find integrals of given functions
$\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$

5. Find integrals of given function $\sec^2 x$.

6. Find integrals of given functions $\operatorname{cosec}^2 x$

7. Find integrals of given functions sec x tan x

8. Find integrals of given functions $\frac{1}{3x}$

9. Integrate by using the substitution suggested in bracket.
$\int x \sin(2x^2) dx$, (use, $u = 2x^2$)

10. Integrate by using the substitution suggested in bracket.
$\int \sec 2t \tan 2t \; dt$, (use, $u = 2t$)

11. Integrate by using a suitable substitution
$\int \frac{3}{(2-x)^2} dx$

12. Integrate by using a suitable substitution

$\int \sin(8z-5)\,dz$

13. Find integrals of given functions.

$\int x^{-3}(x+1)dx$

14. Find integrals of given functions.

$\int (1-\cot^2 x)\,dx$

15. Find integrals of given functions.

$\int \cos\theta(\tan\theta+\sec\theta)\,d\theta$

16. Integrate by using the substitution suggested in bracket.

$\int 12(y^4+4y^2+1)^2(y^3+2y)\,dy,$ (use, $u = y^4 + 4y^2 + 1$)

17. Integrate by using the substitution suggested in bracket.

$\int \frac{dx}{\sqrt{5x+8}}$

(a) Using $u = 5x + 8$ (b) Using $u = \sqrt{5x+8}$

18. Integrate by using the substitution.

$\int \sqrt{3-2s}\,ds$

19. Integrate by using the substitution.

$\int \sec^2(3x+2)dx$

20. Integrate by using the substitution.

$\int \operatorname{cosec}\left(\frac{\upsilon-\pi}{2}\right)\cot\left(\frac{\upsilon-\pi}{2}\right)d\upsilon$

21. Integrate by using the substitution.

$\int \frac{6\cos t}{(2+\sin t)^3}\,dt$

22. Evaluate the following definite integrals.

(i) $\int_1^2 x^2 dx$ (ii) $\int_0^4 x^{3/2} dx$ (iii) $\int_2^4 \frac{1}{x} dx$ (iv) $\int_0^2 \left(2x^2+3x+1\right)dx$

(v) $\int_0^{\pi/2} \left(\cos 2x+\sin 2x\right)dx$ (vi) $\int_0^1 \frac{1}{4-2x}dx$ (vii) $\int_1^2 (2+3x)^3\,dx$

23. Definite integration. $\int_{-4}^{-1} \frac{\pi}{2} d\theta$

24. Definite integration $\int_{\sqrt{2}}^{5\sqrt{2}} r\, dr$

25. Definite integration $\int_{0}^{1} e^{x}\, dx$

26. Definite integration $\int_{\pi}^{2\pi} \theta\, d\theta$

27. Definite integration $\int_{0}^{\sqrt[3]{7}} x^{2} dx$

28. Definite integration $\int_{0}^{\sqrt{\pi}} x \sin x^{2} dx$

29. Definite integration $\int_{0}^{1} \frac{dx}{3x+2}$

30. Use a definite integral to find the area of the region between the given curve and the x–axis on the interval [0,b]
$y = 2x$

31. Use a definite integral to find the area of the region between the given curve and the x–axis on the interval [0,b]
$y = \frac{x}{2} + 1$

32. Use a definite integral to find the area of the region between the given curve and the x–axis on the interval $[0, \pi]$
$y = \sin x$

33. Use a definite integral to find the area of the region between the given curve and the x–axis on the interval $[0, \pi]$
$y = \sin^2 x$

34. $I = \int_0^{2\pi} \sin(\theta + \phi).d\theta$ where ϕ is a constant. Then value of I:

(A) may be positive
(B) may be negative
(C) may be zero
(D) Always zero for any value of ϕ

35. If $x_1 = 3\sin\omega t$ and $x_2 = 4\cos\omega t$ then

(A) $\frac{x_1}{x_2}$ is independent of t

(B) Average value of $< x_1^2 + x_2^2 >$ from $t = 0$ to $t = \frac{2\pi}{\omega}$ is 12.5

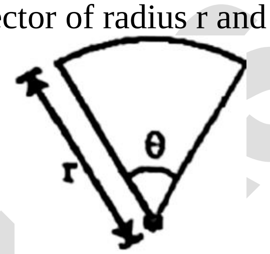
(C) $\left(\frac{x_1}{3}\right)^2 + \left(\frac{x_2}{4}\right)^2 = 1$

(D) Average value of $< x_1 \cdot x_2 >$ from t = 0 to $t = \frac{2\pi}{\omega}$ is zero

36. $I = \int_0^{\pi} \sin(\theta + \phi), d\theta$, where ϕ is non zero constant then the value of I :

(A) may be positive
(B) may be negative
(C) may be zero
(D) always zero if $\phi = \frac{\pi}{4}$

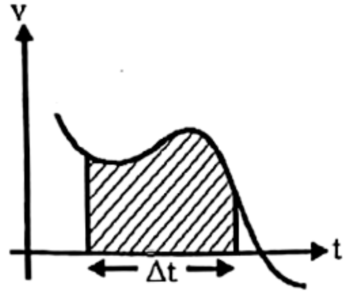
37. Find the area of sector of radius r and angle θ by integration.



38. Find the volume of the right circular solid cone of radius R and height h by integration.

39. Find the surface area of a sphere by integration.

40. Figure shows a graph of velocity versus time for a particle in one dimensional motion. Which of the following statements is correct ?



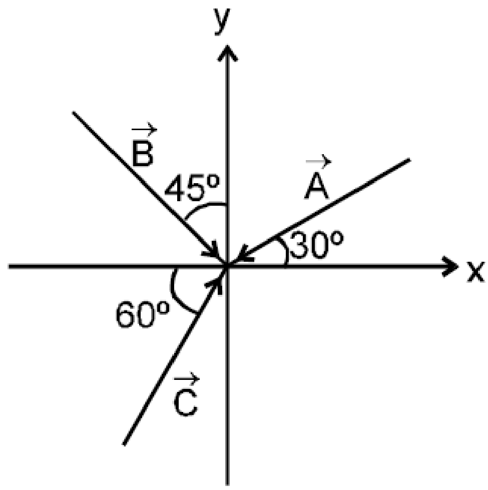
(A) The shaded area represents distance traveled by particle in time interval Δt
(B) The shaded area represents the acceleration of during time interval Δt
(C) The acceleration is constant during time internal Δt
(D) During time interval Δt particle first moves away from initial position and then returns back

41. Use a definite integral to find the area of the region between the given curve and the x–axis on the interval [0,b]
$y = 3x^2$

PART-IV

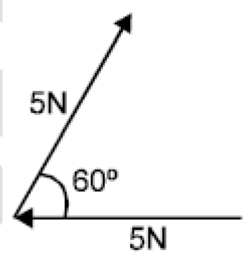
VECTOR

1. Vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are shown in figure. Find angle between



(i) $\vec{A}$ and $\vec{B}$ (ii) $\vec{A}$ and $\vec{C}$ (iii) $\vec{B}$ and $\vec{C}$

2. The forces, each numerically equal to 5N, are acting as shown in the figure. Find the angle between forces?



3. Rain is falling vertically downwards with a speed 5m/s. if unit vector along upward is defined as $\hat{j}$, represent velocity of rain in vector form.

4. A man walks 40 m north, then 30 m east and then 40 m south. Find the displacement form the starting point?

5. A vector of magnitude 30 and direction eastwards is added with another vector of magnitude 40 and direction northwards. Find the magnitude and direction of resultant with the east.

6. Two vectors $\vec{a}$ and $\vec{b}$ inclined at an angle θ w.r.t. each other have a resultant $\vec{c}$ which makes an angle β with $\vec{a}$. If the directions of $\vec{a}$ and $\vec{b}$ are interchanged, then the resultant will have the same

(A) magnitude
(B) direction
(C) magnitude as well as direction
(D) neither magnitude nor direction

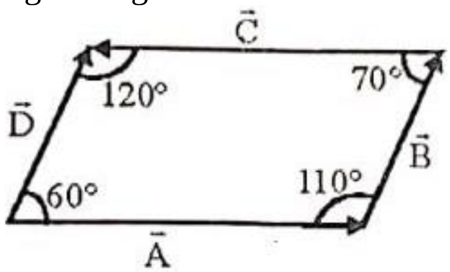
7. Two vectors $\vec{A}$ and $\vec{B}$ lie in a plane. Another vector $\vec{C}$ lies outside this plane. The resultant $\vec{A}+\vec{B}+\vec{C}$ of these three vectors

(A) can be zero
(B) cannot be zero
(C) lies in the plane of $\vec{A}$ & $\vec{B}$
(D) lies in the plane of $\vec{A}$ & $\vec{A}+\vec{B}$

8. The vector sum of the forces of 10 N and 6N can be

(A) 2 N (B) 8 N (C) 18 N (D) 20 N

9. In the given figure



(A) Angle between $\vec{A}$ and $\vec{B}$ is 110°
(B) Angle between $\vec{C}$ and $\vec{D}$ is 60°
(C) Angle between $\vec{B}$ and $\vec{C}$ is 110°
(D) Angle between $\vec{B}$ and $\vec{C}$ is 70°

10. Which of the following forces cannot be a resultant of 5N force and 7N force?

(A) 2N (B) 10N (C) 14N (D) 5N

11. A man moves in an open field such that after moving 10m in a straight line, he makes a sharp turn of 60° to his left. Find the net displacement of the man after 7 such turns.

(A) 10 m (B) 20 m (C) 70 m (D) 30 m

12. If the angle α between two forces of equal magnitude is reduced to $(\alpha - \pi/3)$, then the magnitude of their resultant becomes $\sqrt{3}$ times of the earlier one. The angle α is

(A) $\pi/2$ (B) $2\pi/3$ (C) $\pi/4$ (D) $4\pi/5$

13. A particle is moving westward with a velocity $\vec{v}_1$ = 5 m/s. Its velocity changed to $\vec{v}_2$ = 5 m/s northward. The change in velocity vector $\left(\Delta\vec{V} = \vec{v}_2 - \vec{v}_1\right)$ is:

(A) $5\sqrt{2}$ m/s towards north east
(B) 5 m/s towards north west
(C) Zero
(D) $5\sqrt{2}$ m/s towards north west

14. Consider east as positive x-axis, north as positive y-axis. A girl walks 10 m east first time then 10 m in a direction 30° west of north for the second time and then third time in unknown direction and magnitude so as to return to her initial position. What is her third displacement in unit vector notation.

(A) $-5\hat{i} - 5\sqrt{3}\hat{j}$
(B) $5\hat{i} - 5\sqrt{3}\hat{j}$
(C) $-5\hat{i} + 5\sqrt{3}\hat{j}$
(D) She cannot return

15. The sum of three forces $|\vec{F}_1| = 100N, |\vec{F}_2| = 80N$ & $|\vec{F}_3| = 60N$ acting on a particle is zero. The angle between $\vec{F}_1$ & $\vec{F}_2$ is nearly

(A) 53° (B) 143° (C) 37° (D) 127°

16. The direction of three forces 1N, 2N and 3N acting at a point are parallel to the sides of an equilateral triangle taken in order. The magnitude of their resultant is.

(A) $\sqrt{3}N$ (B) $\frac{\sqrt{3}}{2}$ N (C) $\frac{3}{2}$ N (D) Zero

17. $\vec{A} = \hat{i} + \hat{j} - \hat{k}; \vec{B} = 2\hat{i} + 3\hat{j} + 5\hat{k}$ angle between $\vec{A}$ and $\vec{B}$ is

(A) 120° (B) 90° (C) 60° (D) 30°

18. Given the vectors

$$\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$$
$$\vec{B} = 3\hat{i} - 2\hat{j} - 2\hat{k}$$
$$\vec{C} = p\hat{i} + p\hat{j} + 2p\hat{k}$$

Find the angle between $(\vec{A} - \vec{B})$ & $\vec{C}$

(A) $\theta = \cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (B) $\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (C) $\theta = \cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$ (D) None of these

19. A set of vectors taken in a given order given a closed polygon. Then the resultant of these vectors is a

(A) scalar quantity (B) pseudo vector (C) unit vector (D) null vector.

20. The vector sum of two force P and Q is minimum when the angle θ between their positive directions , is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) π

21. The vector sum of two vectors $\vec{A}$ and $\vec{B}$ is maximum, then the angle θ between two vectors is

(A) 0° (B) 30° (C) 45° (D) 60°

22. Given: $\vec{C} = \vec{A} + \vec{B}$. Also, the magnitude of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 12, 5 and 13 units respectively. The angle between $\vec{A}$ and $\vec{B}$ is

(A) 0° (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

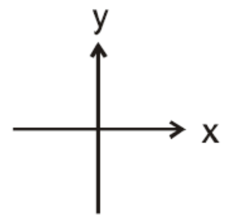
23. If $\vec{P} + \vec{Q} = \vec{P} - \vec{Q}$ and θ is the angle between $\vec{P}$ and $\vec{Q}$, then

(A) $\theta = 0°$ (B) $\theta = 90°$ (C) P = 0 (D) Q = 0

24. Find the magnitude of $3\hat{i}+2\hat{j}+\hat{k}$?

25. If $\vec{A}=3\hat{i}+4\hat{j}$ then find $\hat{A}$

26. What are the x and the y components of a 25 m displacement at an angle of 210° with the x-axis (anti clockwise)?



27. One of the rectangular components of a velocity of 60km h^{-1} is 30km h^{-1}. Find other rectangular component?

28. If $0.5\hat{i}+0.8\hat{j}+C\hat{k}$ is a unit vector. Find the value of C

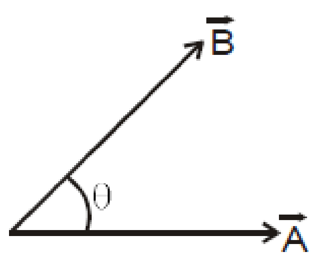
29. The rectangular components of a vector are (2, 2). The corresponding rectangular components of another vector are $(1, \sqrt{3})$. find the angle between the two vectors

30. The x and y components of a force are 2N and –3N. the force is**:**

(A) $2\hat{i}-3\hat{i}$ (B) $2\hat{i}+3\hat{j}$ (C) $-2\hat{i}-3\hat{j}$ (D) $3\hat{i}+2\hat{j}$

31. If $\vec{A}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{B}=2\hat{j}+\hat{j}$ find (a) $\vec{A}.\vec{B}$ (b) $\vec{A}\times\vec{B}$

32. If $|\vec{A}|=4$, $|\vec{B}|=3$ and $\theta = 60°$ in the figure, Find (a) $\vec{A}.\vec{B}$ (b) $|\vec{A}\times\vec{B}|$



33. Three non zero vectors $\vec{A}$, $\vec{B}$ & $\vec{C}$ satisfy the relation $\vec{A}.\vec{B}=0$ & $\vec{A}.\vec{C}=0$. Then $\vec{A}$ can be parallel to:

(A) $\vec{B}$ (B) $\vec{C}$ (C) $\vec{B}.\vec{C}$ (D) $\vec{B}\times\vec{C}$

34. The magnitude of scalar product of two vectors is 8 and that of vector product is $8\sqrt{3}$. The angle between then is :

(A) 30° (B) 60° (C) 120° (D) 150°

35. Which of the following is perpendicular to $\hat{i}-\hat{j}-\hat{k}$?

(A) $\hat{i}+\hat{j}+\hat{k}$ (B) $-\hat{i}+\hat{j}+\hat{k}$ (C) $\hat{i}+\hat{j}-\hat{k}$ (D) None of these

36. The vector having a magnitude of 10 and perpendicular to the vector $3\hat{i}-4\hat{j}$ is

(A) $4\hat{i}+3\hat{j}$ (B) $5\sqrt{2}\hat{i}-5\sqrt{2}\hat{j}$ (C) $8\hat{i}+6\hat{j}$ (D) $8\hat{i}-6\hat{j}$

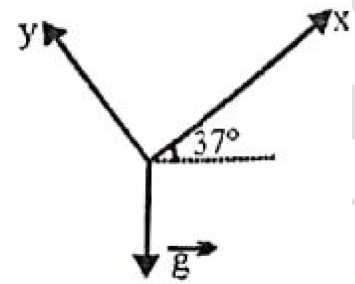
37. $\vec{A}+\vec{B}=2\hat{i}$ and $\vec{A}-\vec{B}=4\hat{j}$ then angle between $\vec{A}$ and $\vec{B}$ is

(A) 127° (B) 143° (C) 53° (D) 37°

38. The projection of the vector $3\hat{i}+4\hat{k}$ on y axis is:

(A) Zero (B) 5 (C) 4 (D) 3

39. What are component of $\vec{g}$ along x and y axis respectively $|\vec{g}| = 10\ m/s^2$

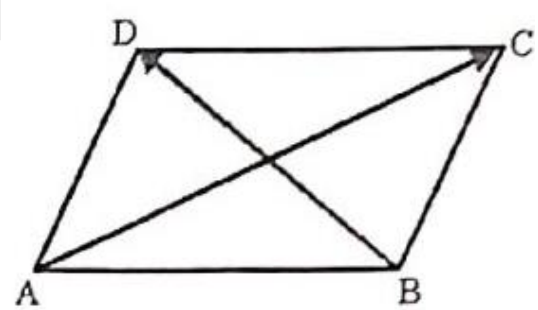


(A) (–6, –8) (B) (6, 8) (C) (8, 6) (D) (–3, –4)

40. There are two vectors of same magnitude 5 and addition of them have magnitude $5\sqrt{3}$ then what is magnitude of their difference

(A) 5 (B) 0 (C) 10 (D) 8

41. If in the shown figure $\vec{AC}=\hat{i}+2\hat{j}+4\hat{k}$ and $\vec{BD}=\hat{i}-3\hat{j}+\hat{k}$ then $\vec{BC}$ is



(A) $\frac{3}{2}\hat{i}-\frac{1}{2}\hat{j}+5\hat{k}$ (B) $\frac{3}{2}\hat{i}-2\hat{j}+3\hat{k}$ (C) $2\hat{i}-\hat{j}+5\hat{k}$ (D) $\hat{i}-\frac{1}{2}\hat{j}+\frac{5}{2}\hat{k}$

42. A vector that is perpendicular to both the vector $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}-\hat{j}+\hat{k}$ is

(A) $-\hat{i}+\hat{k}$ (B) $-\hat{i}-2\hat{j}+\hat{k}$ (C) $\hat{i}-2\hat{j}+\hat{k}$ (D) $\hat{i}+\hat{k}$

43. $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})$ is equal to

(A) 0
(B) $\vec{a}\times\vec{b}$
(C) $2(\vec{a}\times\vec{b})$
(D) $|a|^2 + |b|^2$

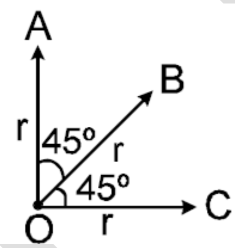
44. A vector $\vec{A}$ points vertically downward and $\vec{B}$ points towards north. The vector product $\vec{A} \times \vec{B}$ is:

(A) Along west
(B) Along east
(C) Zero
(D) Vertically upward

45. Two vectors acting in the opposite directions have a resultant of 10 units. If they act at right angles to each other, then the resultant is 50 units. Calculate the magnitude of two vectors.

46. The angle θ between directions of forces $\vec{A}$ and $\vec{B}$ is 90° where A = 8 dyne and B = 6 dyne. If the resultant $\vec{R}$ males an angle α with $\vec{A}$ then find the value of 'α' ?

47. Find the resultant of the three vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ each of magnitude r as shown in figure ?



48. If $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = \hat{i} + \hat{j} + 2\hat{k}$ then find out unit vector along $\vec{A}+\vec{B}$.

49. The x and y components of vector $\vec{A}$ are 4m and 6m respectively. The x, y components of vector $\vec{A}+\vec{B}$ are 10 m and 9m respectively. Find the length of $\vec{B}$ and angle that $\vec{B}$ makes with the x axis.

50. A vector is not changed if

(A) it is displaced parallel to itself
(B) it is rotated through an arbitrary angle
(C) it is cross-multiplied by a unit vector
(D) it is multiplied by an arbitrary scalar

51. If the angle between two forces increases, the magnitude of their resultant

(A) decreases
(B) increases
(C) remains unchanged
(D) first decreases and then increases

52. A car is moving on a straight road due north with a uniform speed of 50 km h^{-1} when it turns left through 90°. If the speed remains unchanged after turning, the change in the velocity of the car in the turning process is

(A) zero
(B) $50\sqrt{2}$ km h^{-1} S-W direction
(C) $50\sqrt{2}$ km h^{-1} N-W direction
(D) 50 km h^{-1} due west

53. Which of the following sets of displacements might be capable of bringing a car to its returning point?
(A) 5, 10, 30 and 50 km (B) 5, 9, 9 and 16 km
(C) 40, 40, 90 and 200 km (D) 10, 20, 40 and 90 km

54. When two vector $\vec{a}$ and $\vec{b}$ are added, the magnitude of the resultant vector is always
(A) greater than (a + b) (B) less than or equal to (a + b)
(C) less than (a + b) (D) equal to (a + b)

55. If $|\vec{A}+\vec{B}| = |\vec{A}| = |\vec{B}|$, then the angle between $\vec{A}$ and $\vec{B}$ is
(A) 0º (B) 60º (C) 90º (D) 120º

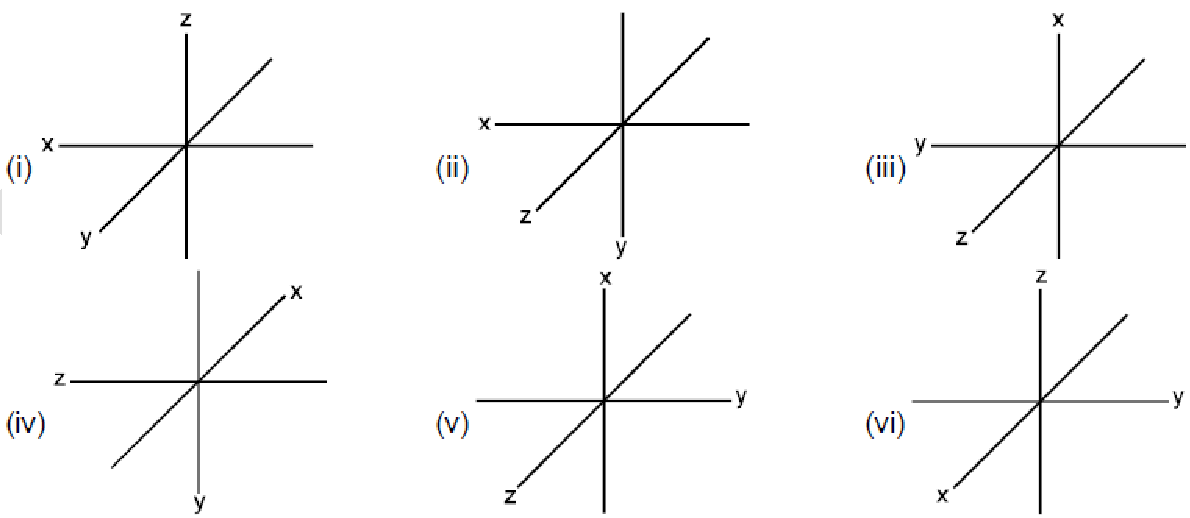
56. Given: $\vec{a}+\vec{b}+\vec{c}=0$. Out of the three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ two are equal in magnitude. The magnitude of the third vector is $\sqrt{2}$ times that of either of the two having equal magnitude. The angles between the vectors are :
(A) 90º, 135º, 135º (B) 30º, 60º, 90º (C) 45º, 45º, 90º (D) 45º, 60º, 90º

57. Vector $\vec{A}$ is of length 2 cm and is 60° above the x-axis in the first quadrant. Vector $\vec{B}$ is of length 2cm and 60° below the x-axis in the fourth quadrant. The sum $\vec{A}+\vec{B}$ is a vector of magnitude-
(A) 2 along + y-axis (B) 2 along + x-axis (C) 1 along – x axis (D) 2 along – x axis

58. Six forces, 9.81 N each, acting at a point are coplanar. If the angles between neighboring forces are equal, then the resultant is
(A) 0 N (B) 9.81 N (C) 2 × 9.81 N (D) 3 × 9.81 N

59. A vector $\vec{A}$ points vertically downward and $\vec{B}$ points towards east, then the vector product $\vec{A}\times\vec{B}$ is
(A) along west (B) along east (C) zero (D) along south

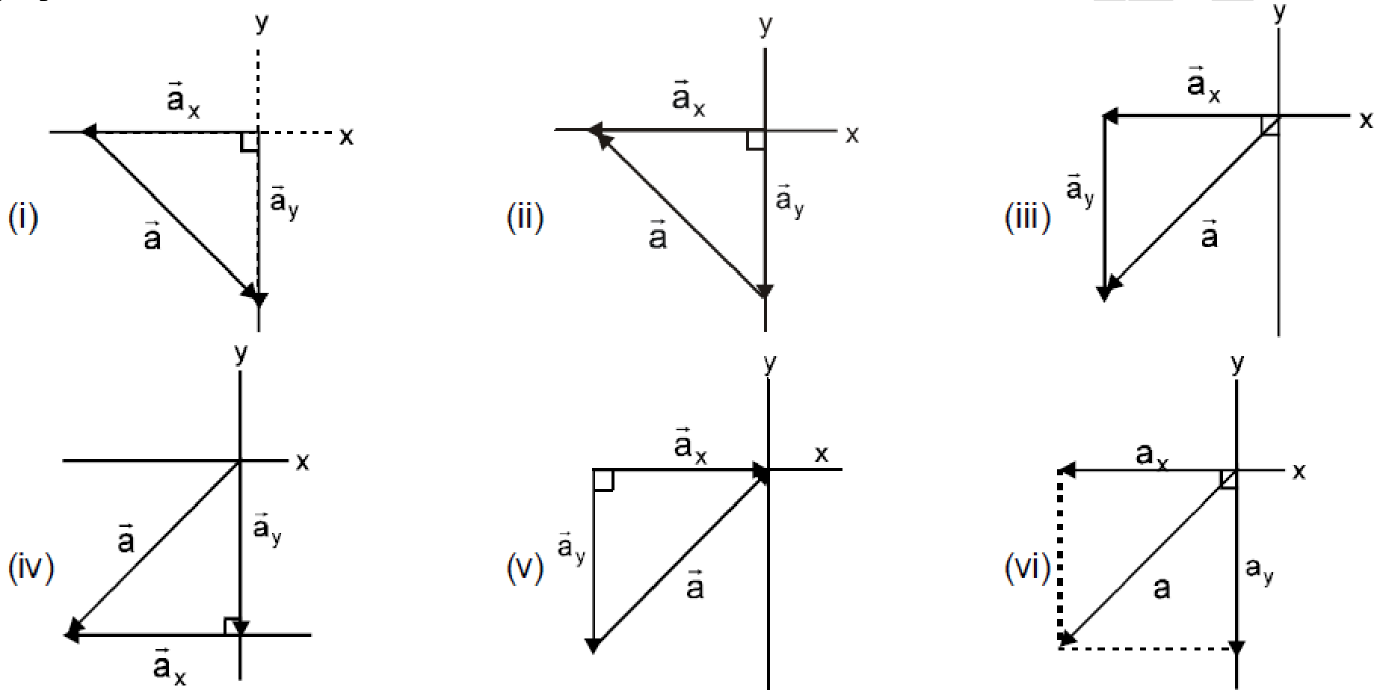
60. Which of the arrangement of axes in Fig. can be labelled "right-handed coordinate system"? As usual, each axis label indicates the positive side of the axis.



(A) (i), (ii) (B) (iii), (iv) (C) (vi) (D) (v)

61. Which of the following is a true statement
(A) A vector cannot be divided by another vector
(B) Angular displacement can either be a scalar or a vector.
(C) Since addition of vectors is commutative therefore vector subtraction is also commutative.
(D) The resultant of two equal forces of magnitude F acting at a point is F if the angle between the two forces is 120º.

62. In the Figure which of the ways indicated for combining the x and y components of vector a are proper to determine that vector?



(A) (iii) (B) (iv) (C) (vi) (D) (i), (ii) and (v)

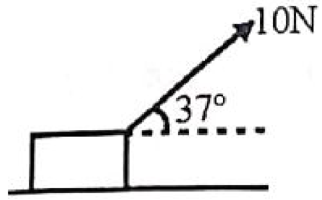
63. Let $\vec{a}$ and $\vec{b}$ be two non-null vectors such that $|\vec{a}+\vec{b}|=|\vec{a}-2\vec{b}|$. Then the value of $\frac{|\vec{a}|}{|\vec{b}|}$ may be:

(A) $\frac{1}{4}$ (B) $\frac{1}{8}$ (C) 1 (D) 2

64. Three forces are acting on a body. In which of following option(s) can the body be in equilibrium?
(A) 2N, 4N, 5N (B) 1N, 2N, 3N (C) 3N, 3N, 7N (D) 5N, 2N, 1N

65. There are four forces $\vec{F}_1, \vec{F}_2, \vec{F}_3, \vec{F}_4$ acting on particle such that particle is in equilibrium. Suddenly $\vec{F}_4$ vanishes. The resultant of remaining forces acting in particle is
(A) $\vec{F}_1 + \vec{F}_2$ (B) $F_1 + F_2 + F_3$ (C) $\vec{F}_1 + \vec{F}_2 + \vec{F}_3$ (D) $-\vec{F}_4$

66. A boy is pulling a block by a force of 10 N at an angle of 37° to the horizontal.



(A) The component of this force in the horizontal direction is 6N
(B) The component of this force in the horizontal direction is 8N
(C) The component of this force in the vertical direction is 6N
(D) The component of this force in the vertical direction is 8N

67. Mark the INCORRECT option.
(A) If $\vec{d} - \vec{e} = \vec{f}$ and f = d + e then $\vec{d}$ and $\vec{e}$ are opposite in direction
(B) If $\vec{d} + \vec{e} = \vec{f}$ and f = $\sqrt{2}$ d ; d = e then $\vec{d}$ and $\vec{e}$ are perpendicular
(C) If $\vec{d} - \vec{e} = \vec{f}$ and f = d + e then $\vec{d}$ and $\vec{e}$ are in the same direction
(D) If $\vec{d} + \vec{e} = \vec{f}$ and f = $\sqrt{2}$ d ; d = e then $\vec{d}$ and $\vec{e}$ are in opposite direction

ANSWER KEY

MATHEMATICAL TOOL

Part-I
TRIGONOMETRY & FUNCTIONS

1. B **2.** B **3.** C **4.** (i) 0 (ii) –1 (iii) –1 (iv) $\frac{1}{3}$ **5.** D **6.** D **7.** C

8. BCD **9.** $\frac{-2x-1}{(x+1)^2}$ **10.** D **11.** C **12.** A **13.** C **14.** C **15.** ACD

16. ABCD **17.** ABC **18.** A **19.** B **20.** C **21.** B **22.** B **23.** B **24.** C

25. ABC **26.** AC **27.** B **28.** B **29.** $-\frac{1}{x}$ **30.** x **31.** C **32.** AC **33.** ABD **34.** CD

Part-II
DIFFERENTIATION

1. (i) 2t, 2 (ii) $-\frac{1}{t^2}, \frac{2}{t^3}$ (iii) $-\frac{t^{-3/2}}{2}, \frac{3t^{-5/2}}{4}$ (iv) $\frac{3t^{1/2}}{2}, \frac{3t^{-1/2}}{4}$

(v) $\frac{5t^{3/2}}{2}, \frac{15t^{1/2}}{4}$ (vi) $2\sqrt{2}t, 2\sqrt{2}$ (vii) 0, 0 (viii) 2t + 1, 2

(ix) $12t^2$, 24t (x) $3-\frac{2}{t^2}, \frac{4}{t^3}$

2. (i) cos (t + 2) (ii) $\omega \cos(\omega t + \phi)$ (iii) $-2\omega \sin 2\theta$ (iv) $2\omega \cos(2\theta + 3)$

(v) V_x (4x + 3) (vi) $6(2t + 4)^2$ (vii) 2 sin t cos t (viii) –2 cos t sin t

3. $\frac{dy}{dx} = \frac{18y - 3x^2}{3y^2 - 18x}$ **4.** $\sec^2 x - \text{cosec}^2 x$ **5.** $\frac{1-r}{\sqrt{2r - r^2}}$ **6.** $\frac{dy}{dx} = 2x + 1$

7. $\frac{dy}{dx} = \frac{2}{x} + \cos x, \frac{d^2y}{dx^2} = \frac{-2}{x^2} - \sin x$ **8.** $\frac{dy}{dx} = \cos x - \sin x, \frac{d^2y}{dx^2} = -\sin x - \cos x$

9. $\frac{\tan x\left(e^x + \frac{1}{x}\right) - \sec^2 x(e^x + \ell nx)}{\tan^2 x}$ **10.** $\frac{dy}{dx} = \frac{1}{x} + e^x, \frac{d^2y}{dx^2} = -\frac{1}{x^2} + e^x$

11. $\frac{-\csc^2 x}{(1+\cot x)^2}$ **12.** $e^x \ell nx + \frac{e^x}{x}$ **13.** $3\sin^2 x \cos x + 3\cos 3x$

14. $\cos^2 x - \sin^2 x$ **15.** $\frac{-2xy - y^2}{x^2 - 2xy}$ **16.** $\frac{dy}{dx} = 1x + 2x + \frac{2}{x^3} - \frac{1}{x^2}$

17. $\frac{dy}{dx} = -27(4-3x)^8$ **18.** $y' = \frac{-19}{(3x-2)^2}$ **19.** $4x \sin(x^2+1)\cos(x^2+1)$

20. $\frac{1}{x^2} - \frac{\ell nx}{x}$ **21.** $2\omega \cos(\omega x + \phi)$ **22.** $\frac{dr}{d\theta} = \cos\theta + \sec^2\theta$

23. $\frac{dy}{dx} = 0$ **24.** $\frac{dy}{dx} = \frac{x^{-\frac{6}{7}}}{7} + \sec^2 x, \frac{d^2y}{dx^2} = \frac{-6}{49} x^{\frac{-13}{7}} + 2\tan x \sec^2 x$

25. $5 \cos 5x$ **26.** (a) 13 (b) – 7 (c) $\frac{7}{25}$ (d) 20

27. $\frac{dy}{dx} = -1$ **28.** (i) $k(kx + c)$ (ii) $\frac{k^2}{2}$ (iii) $A^2 k \sin kx \cos kx$ (iv) x (v) $-x(1+x^2)^{-2}$

29. (i) $(x \cos x + \sin x)$ (ii) $e^x \cos x - e^x \sin x)$ (iii) $\frac{1}{(1+x)^2}$ (iv) $\left(\frac{\sin x}{2\sqrt{x}} + \sqrt{x}\cos x\right)$

30. (i) $(4\theta + 1)(2\theta^2 + \theta + 1)$ (ii) $(15 \sin 4\theta)$ (iii) $(-\sin\theta)(2 + \cos\theta)$

31. $\frac{dy}{dx} = 48(8x-1)^2$ **32.** $3 \cos(3x + 1)$ **33.** $12x^3$ **34.** $\frac{dy}{dx} = -\frac{1}{3}\sin\frac{x}{3}$

35. 6N **36.** D **37.** BD **38.** ABD

39. ABC **40.** (A) Q,S (B) P,Q,R,T (C) P,S (D) P,T

41. 2.5 m/s, zero, –7.5 m/s **42.** $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$. **43.** $\frac{ds}{dt} = 8\pi r\frac{dr}{dt}$. **44.** 8

45. $y_{max} = 39$, $y_{min} = 38$

46. (a) $\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} = 5\pi r^2$ (b) $\frac{dV}{dt} = 2\pi hr\frac{dr}{dt} = 10\pi rh$ (c) $\frac{dV}{dt} = \pi r^2\frac{dh}{dt} = 2\pi hr\frac{dr}{dt} = 5\pi r^2 + 10\pi rh$

47. $x = 30$ & $y = 30$ **48.** $\sqrt{\frac{40}{3}}m$

PART-III
INTEGRATION

1. (i) $x + c$ (ii) $\frac{x^2}{2} + c$ (iii) $-\frac{1}{x} + c$ (iv) $\frac{2}{7}x^{7/2} + c$

(v) $\frac{3}{5}x^{5/3} + c$ (vi) $\frac{x^3}{3} + c$ (vii) $(2x - 3\cos x + c)$ (viii) $\frac{9}{40}x^{8/3} + c$

(xi) $\frac{x^3}{5} - x^2 + x + c$ (x) $-\frac{1}{4}x^{-2} + \frac{x^3}{3} + c$ (xi) $\frac{1}{4}\ln x$ (xii) $\left(2x + \frac{5}{x} + c\right)$

2. $\frac{x^3}{3} - x^2 + x + c$ **3.** $\frac{2\sqrt{x^3}}{3} + 2\sqrt{x} + c$ **4.** $\frac{3x^{4/3}}{4} + \frac{3x^{2/3}}{2} + c$ **5.** $\tan x + c$

6. $-\cot x + c$ **7.** $\sec x + c$ **8.** $\frac{1}{3}\ell nx + c$ **9.** $-\frac{1}{4}\cos(2x^2) + C$

10. $\frac{1}{2}\sec 2t + C$ **11.** $\frac{2}{2-x} + C$ **12.** $-\frac{\cos(8z-5)}{8} + C$ **13.** $-\frac{1}{x} - \frac{1}{2x^2} + C$

14. $2x + \cot x + C$ **15.** $-\cos\theta + \theta + C$ **16.** $(y^4 + 4y^2 + 1)^3 + C$

17. $\left[\frac{2}{5}\sqrt{5x+8}\right] + C$ **18.** $-\frac{1}{3}(3-2s)^{3/2} + C$ **19.** $\frac{1}{3}\tan(3x+2) + C$

20. $-2\operatorname{cosec}\left(\frac{\upsilon - \pi}{2}\right) + C$ **21.** $\frac{-3}{(2+\sin t)^2} + C$

22. (i) $\frac{7}{3}$ (ii) $\frac{64}{5}$ (iii) $\ln 2 = 0.693$ (iv) $\frac{40}{3}$

(v) 1 (vi) $\frac{\ln 2}{2}$ (vii) 289.25

23. $\frac{3\pi}{2}$ **24.** 24 **25.** $e - 1$ **26.** $\frac{3\pi^2}{2}$

27. $\frac{7}{3}$ **28.** 1 **29.** $\frac{1}{3}\ell n\frac{5}{2} = \ell n\left(\frac{5}{2}\right)^{\frac{1}{3}}$

30. Using n subintervals of length $\Delta x = \frac{b}{n}$ and right-endpoint values:

$$\text{Area} = \int_0^b 2x\,dx = b^2 \text{ units}$$

31. $\frac{b^2}{4}+b=\frac{b(4+b)}{4}$ units **32.** 2 units **33.** $\pi/2$ units **34.** D

35. BCD **36.** ABC **37.** $r^2\theta/2$ **38.** πR^2h

39. $4\pi r^2$ **40.** A

41. Using n subintervals of lengths $\Delta x = \frac{\mathbf{b}}{\mathbf{n}}$ and right-end point values:

$$\text{Area} = \int_0^b 3x^2dx = b^3$$

PART-IV
VECTOR

1. (i) 105°, (ii) 150° (iii) 105°. **2.** 120° **3.** $\vec{V}_R = -5\hat{j}$

4. 30 m East **5.** 50, 53° with East **6.** A

7. B **8**. B **9.** C **10.** C

11. A **12.** B **13.** A **14.** A

15. B **16.** A **17.** B **18.** C

19. D **20.** D **21.** A **22.**C

23. D **24.** $\sqrt{14}$ **25.** $\frac{3\hat{i}+4\hat{j}}{5}$

26. – 25 cos 30° and –25sin 30° **27.** $30\sqrt{3}\,\text{km h}^{-1}$ **28.** $\pm\frac{\sqrt{11}}{10}$

29. 15° **30.** A **31.** (a) 3, (b) $-\hat{i}+2\hat{j}-\hat{k}$ **32.** (a) 6, (b) $6\sqrt{3}$

33. D **34.** BC **35.** D **36.** C

37. A **38.** A **39.** A **40.** A

41. D **42.** A **43.** C **44.** B

45. P = 40 ; Q = 30 **46.** 37° **47.** r(1 + $\sqrt{2}$) **48.** $\frac{4\hat{i}+5\hat{j}+2\hat{k}}{\sqrt{45}}$

49. $3\sqrt{5}, \tan^{-1}\frac{1}{2}$ **50.** A **51.** A **52.** B

53. B **54.** B **55.** D **56.** A

57. B **58.** A **59.** D **60.** ABC

61. ABD **62.** ABC **63.** CD **64.** AB

65. CD **66.** BC **67.** CD