3(b). Given a potential biased coin with

α = Probability of getting heads.

prior distribution of α $\rightarrow$ $P(\alpha) = 30\,\alpha^2 (1-\alpha)^2$

$\rightarrow$ Taking Bernoulli's distribution;

Posterior of α = $P(\alpha \mid x) = \dfrac{P(\alpha) \cdot P(x \mid \alpha)}{P(x)}$

$$= \frac{30\,\alpha^2 (1-\alpha)^2 \cdot \alpha^{N_1} (1-\alpha)^{(N-N_1)}}{\int_0^1 30\,\alpha^2 (1-\alpha)^2\, \alpha^{N_1} (1-\alpha)^{N-N_1} \cdot d\alpha}$$

where N_1 = number of success & N = Total

$$= \frac{30\,\alpha^{2+N_1} (1-\alpha)^{(N-N_1+2)}}{\int_0^1 30\,\alpha^{2+N_1} (1-\alpha)^{(N-N_1+2)} \cdot d\alpha}$$

~~let~~ [illegible]

$$= \text{let } \quad c(m,k) = \int_{\alpha=0}^{1} \alpha^m (1-\alpha)^k \, d\alpha$$

2) Posterior of $\alpha = P(\alpha|x) = \dfrac{30 \cdot \alpha^{N_1+2} (1-\alpha)^{N-N_1+2}}{30 \cdot \int_0^1 \alpha^{N_1+2} (1-\alpha)^{(N-N_1+2)}}$

let $N_1+2 = m$; $N-N_1+2 = k$

$$\Rightarrow \boxed{P(\alpha|x) \propto \alpha^m (1-\alpha)^k}$$

EAP:

EAP of $\alpha = E\left[P(\alpha|x)\right] = \int_0^1 \alpha \, P(\alpha|x) \cdot d\alpha$

$$= \frac{\int_0^1 \alpha^{m+1} (1-\alpha)^k \cdot d\alpha}{\int_0^1 \alpha^m \cdot (1-\alpha)^k \cdot d\alpha}$$

$$= \frac{c(m+1,k)}{c(m,k)} = \frac{c(m+1,k)}{c(m,k)}$$

From lecture slides: $c(m,k) = \dfrac{m!\,k!}{(m+k+1)!}$

$$\Rightarrow \frac{(m+1)\cdot \cancel{m!\,k!}}{(m+k+2)\,\cancel{(m+k+1)!}} \times \frac{\cancel{(m+k+1)!}}{\cancel{m!\,k!}}$$

$$= \frac{m+1}{m+k+2}$$

$$= \frac{N_1+2+1}{N_1+2+N-N_1+2+2} = \frac{N_1+3}{N+6}.$$

$$\boxed{EAP = \frac{N_1+3}{N+6}}$$