Assignment 6

Machine Learning COMS 4771

Spring 2017, Itsik Pe'er

Assigned: April 10th Due: Wednesday, April 24th, 1:10pm

Submission: Courseworks.

1) **Conditional independence:**
Chessboard City has 64 rectangular blocks arranged in an 8x8 grid, bounded from the south by Machine St., and from the west by Learning Ave. The rest of the streets are numbered from south to north by numbers `1,...,8`, and avenues from west to east by letters `a,...,h`. Each block is named `a1,...,h8` after its northeast corner. You are in charge of public health in the city and are concerned regarding an airborne epidemic that spreads easily from block to an adjacent block or diagonally adjacent block by the south and west winds. Specifically, each city block may either be contaminated or not, and it has a (potentially empty) set of size 3 blocks that can infect it: an adjacent block to its south (unless it faces Main St.), an adjacent block to its west (unless it faces Learning Ave.) and a diagonally-adjacent block to its southwest (if it faces neither Main St. nor Learning Ave.). Each block *x* has a 5% chance to be spontaneously contaminated, i.e. even if *x* has no contaminated blocks that can infect it. However, each such potentially infecting neighbor block that is contaminated additively increases the chance of *x* being infected by 30 percentage points.

   a) Draw the Bayes Net for the joint distribution of contaminated blocks across Chessboard City, and write the conditional probability tables [20pt]

   b) City Hall is located in blocks `c5,d5` and City Hall Neighborhood contains these two blocks along with the 6 blocks that are adjacent to them. You have verified that neither City Hall Neighborhood nor any of the 10 blocks adjacent to it are contaminated.
   What blocks depend on `a1`? Explain. [15 pt]
   How would your answer change if you know `g7`? Explain. [15pt]

2) **Chordal graphs:**
   a) Prove that if you remove a node (and its incident edges) from a chordal graph the result is a chordal graph [10pt]
   b) Prove that the number of maximal cliques in a chordal graph is linear in the number of vertices, but that's not necessarily the case for general graphs[15pt]
   c) Prove a property we relied on in class, that when constructing an overlap graph of maximal cliques, with edge weights being the size of respective overlaps, the maximal spanning tree has the running-intersection property. [15pt]

3) **Comparing likelihood of structures:**
The ALARM Bayesian network http://bit.ly/2nZSUMZ describes vitals sign and similar diagnostic measurements, along with the accepted belief regarding their relationship. Consider datasets of 50,000 (http://bit.ly/2oivEfA) and 500,000 (http://bit.ly/2oYyIfk) samples observing some of these [link] observing some of these variables. Do these datasets each support this accepted network, or can you improve its topology and/or parameterization? Feel free to use libraries such as BayesPy (bayespy.org) to answer this question. [50pt]

4) **Exact inference across a tree:**
Consider a knockout tournament of six rounds. Teams are ranked 0 (best) through 63 (worst). The 1st round pits pairs of teams whose ranks sum up to 63. The rest of the matchups are set in advance such that if favorites (better-ranking) win, the r-th round pits pairs of teams whose ranks sum up to $2^{7-r}-1$.
   a) Assuming a general function $W(i,j,r)$ that tells you the probability of team i winning a match over team j in round r, write the tournament as a Bayesian network. [15pt]
   b) Assume each team i has a quality α_i (monotonically decreasing with i) such that
   $W(i,j,r) = \text{logistic}((\alpha_i - \alpha_j)/r)$
   Write specific pseudocode to answer queries:
   What is the probability of team i winning a match in round r assuming team j won a match in round s? [25pt]