Assignment 2

Machine Learning COMS 4771

Spring 2017, Itsik Pe'er

Assigned: Feb 6^{th} Due: Class time, Feb 13^{th}

Submission: Courseworks

1. Legendre polynomials (en.wikipedia.org/wiki/Legendre_polynomials) are a basis for the space of real functions, with unique properties for approximating functions in [-1,1]. Repeat the simulation question 2 from Assignment 1 run #3, this time learning an approximation for the function in [-1,1] using L2 regularized regression with a regularization weight λ, and using as a basis the seven Legendre polynomials even of degree ≤6. This regression task involves fitting them with a vector θ of 7 respective coefficients. Perform this regression fitting 5 times with the same data using different values $\lambda_1 = 0, \lambda_2, \dots, \lambda_5$ respectively. You should choose $\lambda_2, \dots, \lambda_5$ to show a range of behaviors till a high enough lambda that essentially ignores the data. On a single plot, show the original polynomial, the training points and the 5 inferred approximations.

   Your code should save the plot file `ApproximationPlot.pdf` as well as a **pickle file** called **problem1.pkl** containing a Python dictionary with the following entries:

   `xtrain` - training input (as numpy array or pandas dataframe)

   `xtest` - testing input

   `ytrain` - training output

   `ytest` - testing output

   `ThetaStar_[Lambda]` - one entry for each of your 5 choices of `Lambda` each with the fit coefficients for the Legendre polynomials, as a numpy array.

   `Risktrain`,

   `Risktest` - training and testing empirical risk values for the 5 polynomials

   The function to do all of this should be called should be called `RegularizedFitPoly()` in a file `RegularizedFitPoly.py` within a submitted folder called `Assignment02_Problem01`

   [40 points]

2. Men are from Mars, Women are from Venus, but when you land on Saturn you discover a new gender whose height distribution you wish to parameterize. You assume it is a Gaussian, with standard deviation of σ, but you don't know the mean μ. You have a prior assumption about μ. You assume it is distributed normally around some value μ_{prior} with standard deviation σ_{prior} . Compute the posterior distribution of μ, the maximum-a-posteriori value and the expected-a-posteriori value given sample $x_1, \dots, x_N$,

   [20 points]

3. You need to compute the posterior distribution of the distribution parameter and the expected-a-posteriori value, given data $x_1, \dots, x_N$ for the following cases:

   a. Exponential distribution of rate λ with assumed prior $p(\lambda) = \frac{\lambda^3 e^{-\lambda}}{6}$. You may use the notation $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt$

   b. A potentially biased coin, with probability of Heads α , for which you assume the prior $p(\alpha) = 30\alpha^2(1-\alpha)^2$.

   [40 points]

Good luck!