

## Problem Set - 1

Date : \_\_\_\_\_

### Question 1

Using ID number and tutorial section, we get

$$C = 0.1(3+6) \text{ nF}$$

$$C_1 = 0.9 \text{ nF}$$

$$R_2 = 94 + (4 \times 3)$$

$$= 106$$

M2 is a PMOS, active region is achieved when the value of  $|V_{ds}| > |V_{gs}| - |V_{th}|$  with all these values being negative. The value of  $V_s$  is set to 5 through connection to  $V_{DD}$ . Thus, the value of  $V_g = V_{bias}$  has to be less than 5 while also satisfying the aforementioned equation.

For proper function,  $V_{ds}$  could be any value between  $-0.2$  and  $-4.8 \text{ V}$ .

① At  $-0.2 \text{ V}$ ,

$$0.2 > |V_{bias} - 5| - 0.92$$

$$1.12 > 5 - V_{bias}$$

$$V_{bias} > 3.88 \text{ V}$$

② At  $-4.8 \text{ V}$ ,

$$4.8 > |V_{bias} - 5| - 0.92$$

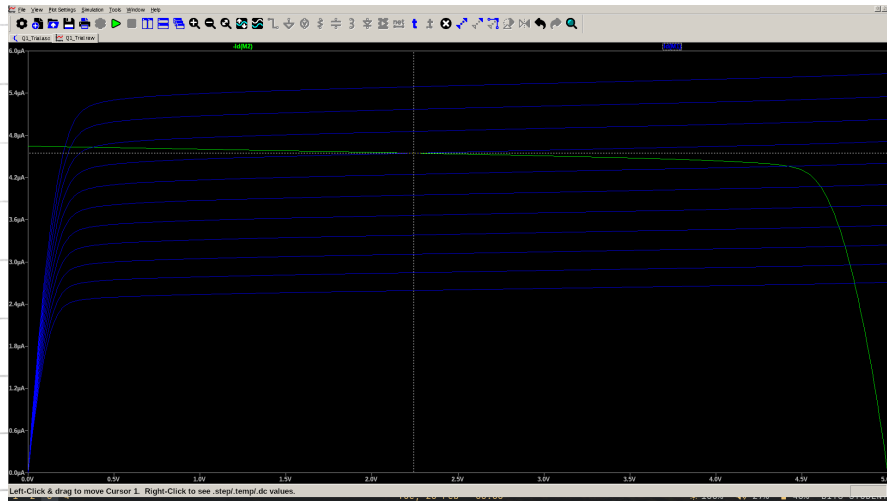
$$5.72 > 5 - V_{bias}$$

$$V_{bias} > -0.72 \text{ V}$$

Thus, the setting of the value of  $V_{bias}$  to  $3.5 \text{ V}$  will ensure that M2 never goes outside the saturation region as long as  $V_{ds}$  for the PMOS does not go higher than  $-0.58 \text{ V}$  which is achievable as we will show in the following sections.

The Q-point is determined by plotting the  $V_{ds}$  graphs of both the loads and finding their intersection.

The below figure does that for  $V_{bias}$  set to 3.5V. Here, we can see the intersection point set is found at 1.07V. Thus, the DC operating point of the amplifier is 1.07V. (which is plotted on different lines)

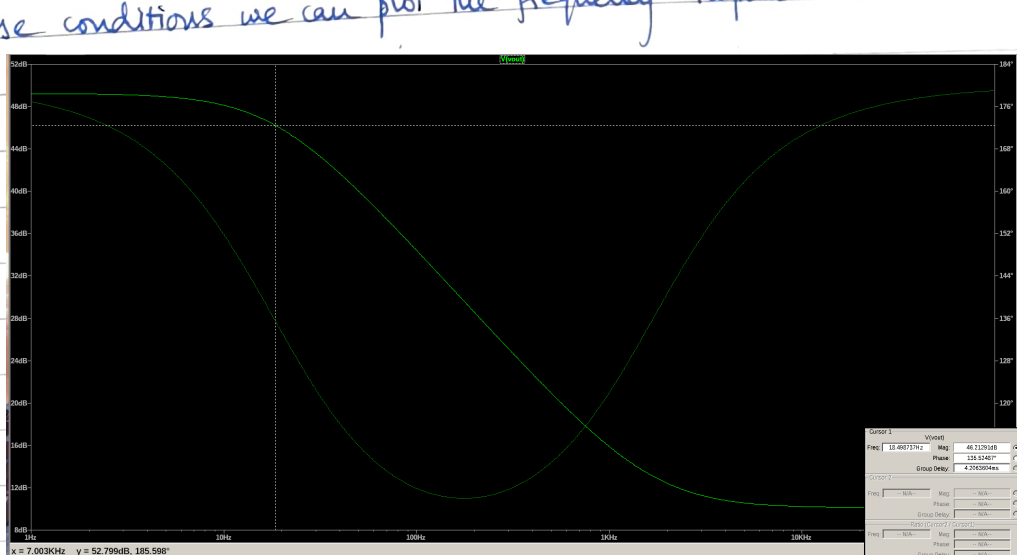


Here, we can see that the  $V_{ds1}$  is always less than 4.42V which is the condition which we used to choose 3.5V.

The DC point for the MOSFET M1 is thus 1.07V and the DC point for M2 is 3.5V.

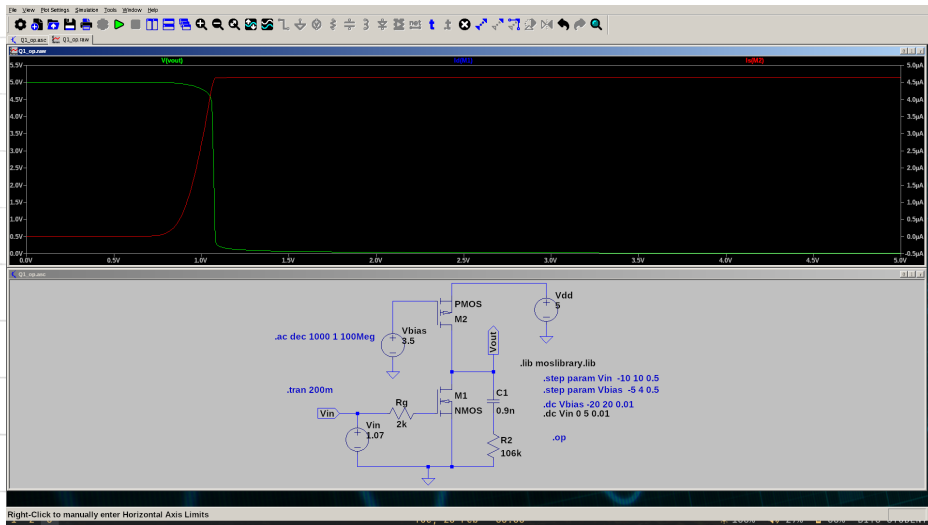
This maximises the gain but it clips the wave because of the large amplitude of 1V. These are just some of the drawbacks to deal with when designing.

With these conditions we can plot the frequency response.



In the diagram we can see that this acts as a low pass filter with 3dB value of 18.5Hz.

We will now plot the input, output characteristics to find the small signal parameters.



At  $V_{in} = 1.07V$ ,  $V_{out} = 2.08V$

$$I_{ds1} = 4.55 \mu A, I_{ds2} = 4.55 \mu A.$$

However, the values of  $x_0$  should be found individually.

For PMOS,

$$-9.7598253 \text{ V}$$

618.5198  $\mu A$ .

- 20V

630.35148  $\mu\text{A}$ .

For NMOS,

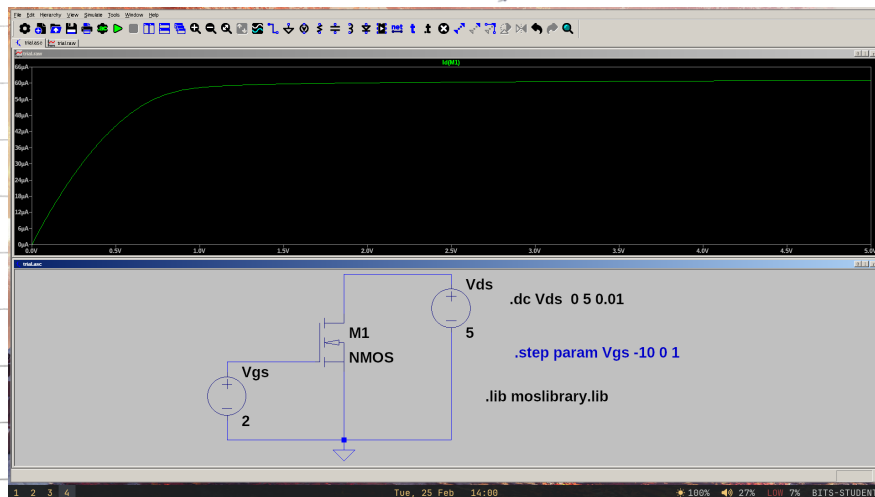
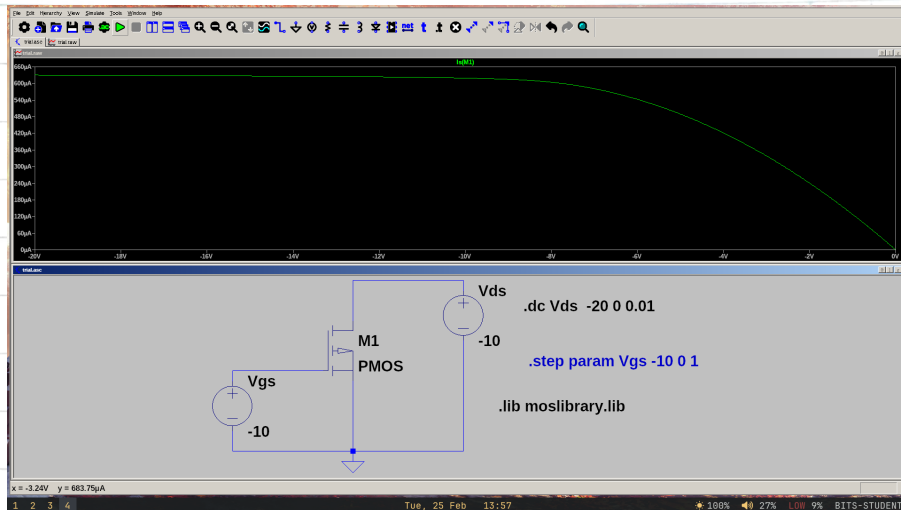
1.6122893V

59. 813022  $\mu\text{A}$ .

5V

61.104969  $\mu A$ .

And generally  $\gamma = 0.287\sqrt{v}$



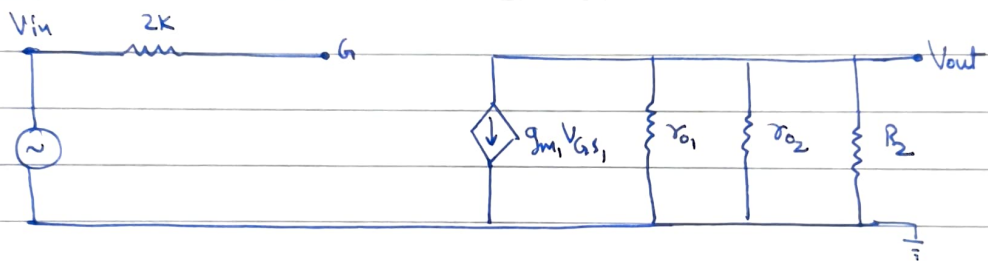
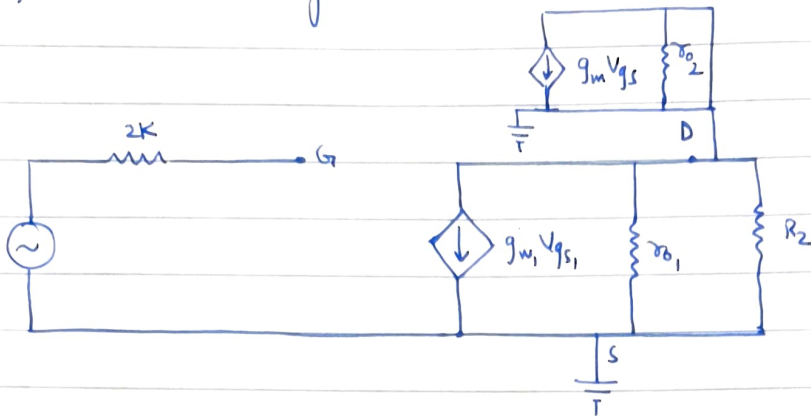
$$g_{m1} = \frac{2I_D}{V_{ov}} = \frac{2 \times 4.55}{107 - 0.67} = 22.75 \mu S$$

$$r_{o1} = \frac{5 - 1.612}{61.105 - 59.813} = 2.622 M\Omega$$

$$g_{m2} = \frac{2I_D}{V_{ov}} = \frac{2 \times 4.55}{5 - 3.5 - 0.921} = 15.71 \mu S$$

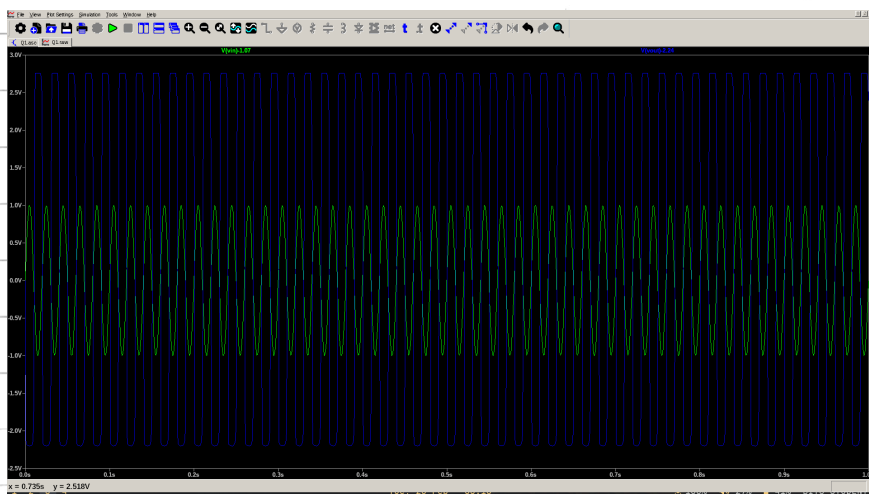
$$R_{o2} = \left| \frac{-9.760 - (-20)}{618.512 - 630.351} \right| = 0.865 M\Omega$$

Now for the small signal model,



$$\text{Gain} = -g_m (\tau_{o1} \parallel \tau_{o2} \parallel R_2) = -22.75 \mu\text{V} \times 0.0911 \text{ M}\Omega = -2.07$$

This can be seen in the input and output waves of an example wave





Problem Set - 1Question 2

→ Hand calculations for  $Z, y, h$  parameters for low and high frequencies:-

For Low Frequencies (1K Hz):-

Z-Parameters (Open circuit Impedance parameters):-

$$\text{We know :- } V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\Rightarrow Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} \quad Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0}$$

$$Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0} \quad Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0}$$

At low frequencies, capacitors act as open circuits

$$\Rightarrow C_1 = C_2 \rightarrow \infty$$

$$\Rightarrow R_{\text{total}} = R_1 + R_2 + R_3 + R_4$$

Using resistances, we find Z-parameters of the circuit.

$$Z_{11} = R_1 + R_2 = 36 \text{ k}\Omega + 44 \text{ k}\Omega = \underline{80 \text{ k}\Omega}$$

$$Z_{12} = R_2 = \underline{44 \text{ k}\Omega}$$

$$Z_{21} = R_3 = \underline{22 \text{ k}\Omega}$$

$$Z_{22} = R_3 + R_4 = 22 \text{ k} + 50 \text{ k} = \underline{72 \text{ k}\Omega}$$

$$\Rightarrow \text{Z Parameters} \begin{bmatrix} 80 \text{ k} & 44 \text{ k} \\ 22 \text{ k} & 72 \text{ k} \end{bmatrix} \Omega$$

Y Parameters (Short circuit Admittance Parameters):

We know :-  $I_1 = y_{11} V_1 + y_{12} V_2$

$$I_2 = y_{21} V_1 + y_{22} V_2$$

Also,  $[Y] = [Z]^{-1}$

$$\Rightarrow y_{11} = \frac{z_{22}}{\Delta}, \quad y_{12} = -\frac{z_{12}}{\Delta}$$

$$y_{21} = -\frac{z_{21}}{\Delta}, \quad y_{22} = \frac{z_{11}}{\Delta}$$

where  $\Delta = z_{11} z_{22} - z_{12} z_{21}$

From previous calculations, we have

$$z_{11} = 80k\Omega; \quad z_{12} = 44k\Omega; \quad z_{21} = 22k\Omega; \quad z_{22} = 72k\Omega$$

$$\begin{aligned} \Rightarrow \Delta &= (80k)(72k) - (44k)(22k) \\ &= 5760M\Omega^2 - 968M\Omega^2 \\ &= \cancel{4792M\Omega^2} 4792M\Omega^2 \end{aligned}$$

$$y_{11} = \frac{72k}{4792M} \approx 15.03\mu S$$

$$y_{12} = \frac{-44k}{4792M} \approx -9.18\mu S$$

$$y_{21} = \frac{-22k}{4792M} \approx -4.59\mu S$$

$$y_{22} = \frac{80k}{4792M} \approx 16.7\mu S$$

$$\Rightarrow [Y] = \begin{bmatrix} 15.03\mu & -9.18\mu \\ -4.59\mu & 16.7\mu \end{bmatrix} S$$

## h-parameters (Hybrid Parameters):-

We know :-  $V_1 = h_{11} I_1 + h_{12} V_2$   
 $I_2 = h_{21} I_1 + h_{22} V_2$

We can express h parameters in terms of Z parameters by:-

$$h_{11} = Z_{11} \quad , \quad h_{12} = \frac{Z_{12}}{Z_{22}}$$

$$h_{21} = \frac{Z_{21}}{Z_{11}} \quad , \quad h_{22} = \frac{1}{Z_{22}}$$

From previous calculations, we have.

$$Z_{11} = 80k\Omega; \quad Z_{12} = 44k\Omega; \quad Z_{21} = 22k\Omega; \quad Z_{22} = 72k\Omega$$

$$h_{11} = Z_{11} = 80k\Omega$$

$$h_{12} = \frac{44k}{72k} = 0.611$$

$$h_{21} = \frac{22k}{80k} = 0.275$$

$$h_{22} = \frac{1}{72k} = 13.89\mu S$$

$$[H] = \begin{bmatrix} 80k\Omega & 0.611 \\ 0.275 & 13.89\mu S \end{bmatrix}$$



For high frequency (10 MHz).

At high frequencies, capacitors act as short circuits  
i.e.  $C_1 = C_2 = 0$ .

Z Parameters :- (Open circuit impedance parameters)

$$Z_{11} = R_1 = 36 \text{ k}\Omega \text{ [as } C_2 \text{ is shorted, } R_2 \text{ is redundant]}$$

$$Z_{12} = R_2 = 0$$

$$Z_{21} = 0 \text{ [no coupling]}$$

$$Z_{22} = R_2 \parallel (R_3 + R_4)$$

$$= 36 \text{ k} \parallel 72 \text{ k}$$

$$= \frac{36 \text{ k} \times 72 \text{ k}}{36 \text{ k} + 72 \text{ k}} = 24 \text{ k}\Omega$$

$$\Rightarrow [Z] = \begin{bmatrix} 36 \text{ k} & 0 \\ 0 & 24 \text{ k} \end{bmatrix} \Omega$$

Y Parameters (Short circuit impedance parameters).

$$\text{We know } [Y] = [Z]^{-1}$$

$$\text{For a diagonal Matrix } [Z] = \begin{bmatrix} Z_{11} & 0 \\ 0 & Z_{22} \end{bmatrix}$$

$$\text{inverse is } [Y] = \begin{bmatrix} 1/Z_{11} & 0 \\ 0 & 1/Z_{22} \end{bmatrix}$$

$$\Rightarrow [Y] = \begin{bmatrix} 1/36 \text{ k} & 0 \\ 0 & 1/24 \text{ k} \end{bmatrix}$$

$$\Rightarrow [Y] = \begin{bmatrix} 27.78 \mu & 0 \\ 0 & 4.167 \mu \end{bmatrix} \text{ S}$$

H-parameters :- (Hybrid parameters).

We know,

$$h_{11} = Z_{11}$$

$$h_{12} = \frac{Z_{12}}{Z_{22}}$$

$$h_{21} = \frac{Z_{21}}{Z_{11}}$$

$$h_{22} = \frac{1}{Z_{22}}$$

$$\Rightarrow h_{11} = 36k\Omega$$

$$h_{12} = \frac{0}{24k} = 0$$

$$h_{21} = \frac{0}{36k} = 0$$

$$h_{22} = \frac{1}{24k} = 41.67\mu S$$

$$\Rightarrow [h] = \begin{bmatrix} 36k\Omega & 0 \\ 0 & 41.67\mu S \end{bmatrix}$$

Value of  $I_1$  in high frequency:-

We know:-

$$\rightarrow I_1 = \frac{V_1}{R_1}$$

$$\rightarrow V_1 = 15V$$

$$\rightarrow R_1 = 36k\Omega = 36 \times 10^3 \Omega$$

$$\Rightarrow I_1 = \frac{15}{36 \times 10^3}$$

$$I_1 = 416.67 \mu A$$

→ For maximum power transfer, the value of load connected to the output should be equal to the value of internal resistance.

$$\Rightarrow R_s = R_L$$

$$\Rightarrow \underline{\underline{R_L = 36k\Omega}}$$

→ The value of  $V_2$  remains constant with the change in capacitance  $C_1$ .