Sl.No. 1005

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VI Semester B.Sc. Examination, April/May - 2018
(Semester Scheme)
MATHEMATICS (Paper - VII)
(2015-16 Batch and onwards)
Algebra IV and Calculus III

Time : 3 Hours **Max. Marks : 80**

Instructions : **Answer all the sections.**

SECTION - A

I. Answer any eight questions. Each question carries two marks.

a) Prove that the vectors (1,2,3), (1,1,1) and (0,1,0) are linearly independent.

b) Express (1,7,–4) as a linear combination of (1,–3,2) and (2,–1,1) in $V_3(R)$.

c) Determine whether $S = \{(x, y, z) / x, y, z \in R, x = y = z\}$ is a subspace of $V_3(R)$

d) Discuss the linearity of $T : R \to R^2$ defined by $T(x) = (x + 1, x + 5)$.

e) Find the matrix of the linear transformation $T(x,y,z) = (-x+y+z, x-y+z)$ where $T : V_3(R) \to V_2(R)$.

f) Find $T^2(x, y)$ of $T : V_2(R) \to V_2(R)$ defined by $T(x,y) = (x,x-y)$

g) Prove that $\Gamma(n+1) = n\Gamma(n)$

h) Prove that $\beta(m,n) = \beta(m+1,n) + \beta(m,n+1)$

i) Find $\int_0^1 x^2(1-x)^3\,dx$ using Beta function.

j) If $\phi = \log(x^2 + y^2 + z^2)$ then find $\Delta\phi$ at (1,1,1).

k) Show that the vector $(6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational

l) If $\vec{F} = x^2 y\hat{i} + 2xyz\hat{j} + y^2 z\hat{k}$ find $div\vec{F}$

SECTION - B

II. Answer any eight questions. Each question carries four marks.

a) Show that the set $Q\left(\sqrt{2}\right)=\left\{a+b\sqrt{2}\,/\,a,b\in Q\right\}$ is a vector space over Q under the operations usual addition and scalar multiplication

b) Show that the set {(1,1,0), (1,0,1), (0,1,1)} forms a basis of the vector space $V_3(R)$.

c) Show that any two bases of a finite dimensional vector space V have the same finite number of vectors.

d) Construct the addition table for $V_2(Z_2)$ and list all its subspaces.

e) If 'n' vectors spans a vector space V containing 'r' linearly independent vectors in V, then prove that $n \geq r$

f) Show that $T: R^3 \rightarrow R^2$ defined by $T(x,y,z) = (x+z,\ x+y+z)$ is a linear transformation.

g) Find a linear transformation $T: V_2(R) \rightarrow V_2(R)$ such that T(2,1) = (3,4), T(–3,4) = (0,5).

h) Find the range, rank, kernel, null space of the linear transformation $T: V_2(R) \rightarrow V_3(R)$ defined by $T(x,y) = (x-y, y, x+y)$

i) Prove that every vector space V over F of dimension n is isomorphic to $V_n(R)$.

j) Show that the linear map $T: V_3(R) \rightarrow V_3(R)$ defined by

$T(e_1) = e_1+e_2$, $T(e_2) = e_2+e_3$, $T(e_3) = e_1+e_2+e_3$ is non singular and find its inverse.

SECTION - C

III. Answer any eight questions. Each question carries four marks.

a) Prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$

b) Evaluate $\int_0^1 x^5(1-x^3)^{10}dx$

c) Show that $\int_0^{\pi/2} \frac{1}{\sqrt{\sin\theta}}d\theta \times \int_0^{\pi/2} \sqrt{\sin\theta}\ d\theta = \pi$

d) Show that $\int_0^{\infty} \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx = 2\beta(m,n)$

e) Prove that $\frac{\Gamma\left(n+\frac{1}{2}\right)}{\Gamma(n+1)} = \frac{1.3.5...(2n-1)}{2.4.6.....2n}\sqrt{\pi}$

f) Find the directional derivative of the function $\phi = x^2y + y^2z - xyz$ at the point (2,–4,6) in the direction of the vector $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$.

g) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ show that div $\left[f(r)\vec{r}\right] = rf^1(r) + 3f(r)$

h) If $\vec{f} = xyz\,\hat{j}$ and $\vec{g} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ find

i) div $\left(\vec{f}\times\vec{g}\right)$

ii) $\Delta div\left(\vec{f}\times\vec{g}\right)$

i) If ϕ is a scalar function and $\vec{A}$ is a vector function then prove that div $\left(\phi\vec{A}\right)$ = grad $\phi.\vec{A} + \phi(div\,\vec{A})$

j) Verify Green's theorem for the function P = $xy + y^2$, Q = x^2 over the closed curve C of the region bounded by $y = x^2$ and $y = x$.

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