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Sl.No. 0479

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VI Semester B.Sc. Examination, September - 2021
(Semester Scheme) (CBCS)
MATHEMATICS
Algebra - IV and Complex Analysis - I (DSE)

Time : 3 Hours **Max. Marks : 80**

Instructions : 1) *Answer all the five questions.*

2) *First question carries 20 marks and remaining questions carry 15 marks.*

1. Answer any TEN questions. Each question carries two marks.

a) Prove that the vectors (1, 2, 3), (1, 1, 1) and (0, 1, 0) are linearly independent.

b) In a vector space V over F, prove that
$c(\alpha-\beta)=c\alpha-c\beta \,\forall c\in F, \alpha, \beta \in V$

c) Define basis and dimension of a vector space.

d) If λ is an eigen value of an invertible linear transformation T, then prove that λ^{-1} is an eigen value of T^{-1} where $\lambda \neq 0$.

e) Find $T^2(x, y, z)$ of the transformation $T: R^3 \to R^3$, given that $T(x, y, z) = (-z, y, -x)$ relative to the standard transformation.

f) Find the eigen values of the linear transformation $T: V_2(R) \to V_2(R)$ defined by $T(x, y) = (x, x+y)$.

g) Find the equation to the straight line through the points $1-5i$ and $-i$.

h) Evaluate $\lim\limits_{Z\to e^{i\pi/3}} \dfrac{Z^3}{Z^6+Z^3+1}$.

i) Show that the function $f(z)=e^{\bar{z}}$ is not analytic.

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j) Find the cross ratio of the points $1, -1, i, -i$

k) Find the Jacobian of the transformation $f(z) = 2z^2$.

l) Find the fixed points of the transformation $f(z) = \frac{4z-3}{z}$.

2. Answer any THREE questions. Each question carries five marks.

a) Show that $V = \{a + b\sqrt{2} \mid a, b \in Q\}$ is a vector space over the field of rational numbers under the operations addition and multiplication.

b) Find the basis and dimension of the subspace spanned by the vectors (2, 4, 2), (1, –1, 0) and (0, 3, 1).

c) In $V_3(Z_3)$ how many vectors are spanned by (1, 2, 1) and (2, 1, 1).

d) Show that dim (V/W) = dim V-dim W where W is a subspace of a finite dimensional vector space V over the field F.

e) Show that the sum of any two subspaces of a vector space is also a subspace.

3. Answer any THREE questions. Each question carries five marks.

a) Find the linear transformation

$T : V_2(R) \to V_3(R)$ such that

$T(-1, 1) = (-1, 0, 2)$ and

$T(2, 1) = (1, 2, 1)$

b) Find the matrix of the linear transformation $T : V_3(R) \to V_2(R)$ with respect to the standard basis defined by

$T(x, y, z) = (z-2y, x + 2y - z)$

c) Find the range space, null space rank and nullity of the linear transformation $T(x, y, z) = (x, 2y, 3z)$.

d) Find the inverse of the matrix $\begin{bmatrix} 1 & 3 & 0 \\ 2 & 4 & 1 \\ 1 & 3 & 0 \end{bmatrix}$ using linear transformation.

e) If V is a finite dimensional vector space over field F, then prove that $T : V \rightarrow V$ is invertible if and only if T is non-singular.

4. Answer any THREE questions. Each question carries five marks.

a) Find the equation of the circle passing through the points $2, -1 + 3i, 1 - i$.

b) Find the derivative of $f(z) = \dfrac{2z - 1}{z + 2i}$ at $z = -i$ by the definition.

c) Derive Cauchy - Riemann equations in polar form.

d) If $f(z) = u + iv$ is analytic and $u - v = e^x(\cos y - \sin y)$, find $f(z)$ in terms of z.

e) Prove that $u = e^x(x\cos y - y\sin y)$ is harmonic and find its harmonic conjugate.

5. Answer any THREE questions. Each question carries five marks.

a) Show that a bilinear transformation transforms circles into circles or straight lines.

b) Find a bilinear transformation which maps $(1, i, -1)$ into $(i, 0, -i)$.

c) Prove that the transformation $W = \dfrac{i(z-i)}{z+i}$ maps the upper half of the z - plane into the interior of the unit circle in the W-plane.

d) Discuss the transformation $W = \sin Z$.

e) Show that the transformation $W = iz$ is a rotation of the z - plane through the angle $\dfrac{\pi}{2}$. Find the image of the infinite strip $0 < x < 1$.

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