20654

VI Semester B.Sc. Examination, May/June 2014
(Semester Scheme)
Mathematics (Paper – X)
Complex Analysis

Time : 3 Hours   Max. Marks : 80

Instructions : *i) Section* ***A*** *is* ***compulsory****.*
ii) ***Five full*** *questions to be answered from Sections* ***B*** *and* ***C****, choosing* ***atleast two*** *from each Section.*
iii) ***All*** *questions in Section* ***B*** *and* ***C*** *carry* ***equal*** *marks.*

SECTION – A

1. Answer **any ten** questions. **Each** question carries **two** marks :

a) Express $\frac{3-2i}{1-i}$ in the form a + ib.

b) If $z = 3-2i$, find $\left|\frac{1-\bar{z}}{1-z}\right|$

c) If $\sin(A+iB) = x+iy$, then prove that $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$.

d) Find the equation of the line through the points 8 – i and 2 + 2i.

e) If the sum and difference of two complex numbers z1 and z_2 are real and purely imaginary, show that $z_2 = \bar{z}_1$.

f) Find $\lim_{z\to 2+i} \left(z^2+5z+4\right)$.

g) Find the points of discontinuity of $f(z) = \frac{z}{z^2-2z+2}$.

h) Prove that $u = \frac{x}{x^2+y^2}$ is harmonic.

i) Prove that f(z) = sinz is analytic, where z = x + iy.

j) If $u = e^x \cos y$ and $v = e^x \sin y$, show that the curves $u = c_1$ and $v = c_2$ intersect orthogonally.

k) Find $\int_c \bar{z}\, dz$, where c is the line segment joining –i to i.

l) Find $\int_C \frac{e^{3z}}{z^2 - 4} dz$, where $C : |z| = 1$.

m) Define inversion and reflection of a transformation.

n) Find the invariant points of the transformation $f(z) = \frac{3z - 5}{z + 1}$.

o) Determine the poles of $f(z) = \frac{z^2}{(z-1)^2 (z+2)}$.

SECTION – B

2. a) Find the complex equation to a line in the form $\bar{\mu}z + \mu\bar{z} = c$, c being real.

b) Show that the equation $\text{Re}\left(\frac{z-4}{z-2i}\right) = 0$ represents a circle. Find its centre and radius.

c) Find whether the points (2, 1), (3, 5), (–2, 0) and (1, –1) are concyclic or not.

3. a) Find the derivative of $f(z) = \cos z$ at $z = i$ by the definition.

b) If f(z) is analytic, show that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$.

c) Find the analytic function whose real part is $x^2 - y^2 - y$.

4. a) Prove that the real and imaginary parts of an analytic function are harmonic.

b) Verify that $u = x^3 - 3xy^2$ is harmonic. Find v such that $f(z) = u + iv$ is analytic.

c) Find the orthogonal trajectories of the family of curves $u = \sin x \cosh y = c$, c being a parameter.

5. a) Evaluate $\int_{(0,3)}^{(2,4)} (x^2 + 2y)\,dx + (3x - y)dy$ along the parabola $x = 2t$, $y = t^2 + 3$.

b) Evaluate $\int_C \frac{z^2 - 1}{z^2 + 1} dz$; where $C : | z - i | = 1$.

c) If f(z) is analytic in a region R between two closed contours C_1 and C_2, prove that

$$\int_{C_1} f(z)dz = \int_{C_2} f(z)dz .$$

SECTION – C

6. a) Prove that every polynomial equation of degree ≥ 1 has atleast one root.

b) Evaluate $\int_C \frac{z \cos z}{\left(z - \frac{\pi}{2}\right)^2} dz$; where C is the circle $|z - 1| = 1$.

c) Evaluate (without using integral formula) $\oint_C \frac{1}{(z - a)^2} dz$ where a is any point with in the simple closed contour C.

7. a) Define inverse points with respect to a circle $|z| = r$. Show that the equation of a circle with z_1 and z_2 as inverse points is $\left|\frac{z - z_1}{z - z_2}\right| = k$, for $k \neq 1$.

b) Prove that the transformation $W = \frac{1}{z}$ transforms circles into circles or to straight lines.

c) Find the image of the triangle, formed by the points $2 + 3i$, $-3 + i$ and $2 - 2i$ in the z-plane under the transformation $w = 2z + 1 + i$ in the w-plane.

8. a) If $f(z) = u + iv$ is analytic in a region R, prove that its Jacobian is $| f'(z) |^2$.

b) Prove that the cross ratio of four points remain invariant under a bilinear transformation.

c) Find the bilinear transformation which maps 0, 1, ∞ onto –5, –1, 3.

9. a) Evaluate $\int_C \frac{z}{(z+1)(z-3)^2} dz$; where C : $|z| = 3$ by using Cauchy's residue theorem.

b) Show that $\int_0^{2\pi} \frac{d\theta}{a + b\cos\theta} = \frac{2\pi}{\sqrt{a^2 - b^2}}$, for $a > b > 0$.

c) If f(z) has a pole of order m at z = a, then show that the residue at z = a is

$$\lim_{z \to a} \frac{1}{\boxed{m-1}} \frac{d^{m-1}}{dz^{m-1}} \left[(z-a)^m f(z)\right].$$

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