20654 **M-357**

Sl.No. 0429

Total No. of Pages : 4

VI Semester B.Sc. Examination, May/June - 2016
(Semester Scheme)
MATHEMATICS (Paper - X)
Complex Analysis

Time : 3 Hours **Max. Marks : 80**

Instructions : *1) Section A is compulsory.*

2) Five full questions to be answered from sections B and C choosing atleast two from each section.

3) All questions in sections B & C carry equal marks.

SECTION - A

Q1) Answer any Ten questions. Each question carries two marks.

a) Find the modulus of $\frac{(1+i)^2}{3-i}$.

b) Find the real and imaginary parts of log (1 + i).

c) Sketch the graph of $\arg z = \frac{\pi}{4}$.

d) If $z_1 = 1+2i$ and $z_2 = 2-i$, find $z_1.z_2$ and $z_1 \times z_2$.

e) Find the area of the triangle whose vertices are 1 + 2i, 4 – 3i, –4 – i.

f) Express the equation $2x + y = 5$ in complex conjugate co - ordinates.

g) Find the equation of the straight line passing through the points $z_1 = -1+3i$ and $z_2 = 2-3i$.

h) Find the derivative of $f(z) = \cos z_1$ at $z = i$.

i) Show that the function $f(z) = z^2 + 3z - 2$ is continuous at $z_0 = 2+i$.

j) Prove that the function $f(z) = e^{\bar{z}}$ is not analytic.

P.T.O.

k) Evaluate $\int_0^{1+i}(x^2 - iy)dz$ along the curve $y = x$.

l) Find the Jacobian of the transformation $f(z) = x^2 + iy^2$.

m) Find the fixed points of the transformation $f(z) = \dfrac{4z-3}{z}$

n) Show that $f(z) = \overline{z}$ is not conformal.

o) Determine the poles & its order of $f(z) = \dfrac{3z+1}{z^2(z^2+1)}$.

SECTION - B

Q2) a) Show that $\log\left(\dfrac{a+ib}{a-ib}\right) = 2i\tan^{-1}\left(\dfrac{b}{a}\right)$.

b) Show that $\operatorname{amp}\left[\dfrac{z-2+i}{z+2i}\right] = \dfrac{\pi}{4}$ represents a circle. Find its centre and radius.

c) Find the equation of a circle described on the join of z_1 & z_2 as diameter.

Q3) a) Define limit of a function $f(z)$ and find $\displaystyle\lim_{z \to e^{i\frac{\pi}{4}}}\left[\dfrac{z^2}{z^4+z^2+1}\right]$.

b) Show that $f(z) = \begin{cases} \dfrac{xy^2(x+iy)}{x^4+y^4}, & \text{for } z \neq 0 \\ 0, & \text{for } z = 0 \end{cases}$ is not differentiable at $z = 0$.

c) Derive Cauchy - Riemann equations in polar form.

Q4) a) Show that $f(z) = z^n$ is analytic where, n is a +ve integer.

b) Prove that an analytic function with constant modulus is constant.

c) Find the derivative of $f(z) = \log z$ using C - R equations.

Q5) a) If $f(z) = u + iv$ is analytic in the domain D then prove that u and v are harmonic functions.

b) Prove that the function $u = x^3 - 3xy^2$ is harmonic and find its harmonic conjugate.

c) Find the orthogonal trajectory of the curve $e^x \sin y = c$.

SECTION - C

Q6) a) Evaluate $\int_c (x - y + ix)^2 \, dz$ along the straight line Joining $z = 0$ and $z = 1 + i$.

b) Evaluate $\int_c (\overline{z})^2 dz$ around the circle $|z - 1| = 1$.

c) State and prove Cauchy's integral theorem.

Q7) a) Evaluate $\int_c \frac{z\,dz}{(z^2+1)(z^2-9)}$ where, C is the circle $|z| = 2$

b) Evaluate $\int_c \frac{\cos z}{(z-\pi)^3} dz$ where, C is the circle $|z| = 4$.

c) State and prove Liouville's theorem.

Q8) a) Discuss the transformation, $w = e^z$.

b) Show that $w = \frac{z-i}{z+i}$ transforms, upper half of the Z - plane to interior of the unit circle.

c) Show that any bilinear transformation having two fixed points a and b can be written in the form

$$\frac{w-a}{w-b} = k\left(\frac{z-a}{z-b}\right)$$

Q9) a) Find the bilinear transformation which maps the points $z = 2, i, -2$ onto the points $w = 1, i, -1$.

b) If $f(z)$ has a pole of order m at $z = a$, then show that the residue at $z = a$ is ,

$$\frac{1}{(m-1)!} \underset{z \to a}{\mathrm{Lt}} \left\{ \frac{d^{m-1}}{dz^{m-1}} \left[(z-a)^m f(z) \right] \right\}$$

c) Using Cauchy's Residue theorem evaluate $\int_C \frac{z^2}{(z+1)(z-1)^2} dz$ where C is the circle $|z| = 2$.

♦♦♦♦