EXPERIMENT - 05

Matrix of linear transformation w.r.t Standard basis

1 Find the matrix of linear transformation $T : V_2(R) \to V_2(R)$ defined by $T(x,y) = (2x+3y, 4x-5y)$ with respect to standard bases

Maxima Code :

```
kill(all)$
T(x):=[2*x[1]+3*x[2], 4*x[1]-5*x[2]];
u[1]:[1,0]$
u[2]:[0,1]$
v[1]:[1,0]$
v[2]:[0,1]$
for i:1 thru 2 do
(
eq[i]:v[1][i]*x+v[2][i]*y
)$
for k:1 thru 2 do
(
soln:solve([eq[1]=T(u[k])[1], eq[2]=T(u[k])[2]],
 [x,y]),
a[k]:ev(x,soln),
b[k]:ev(y,soln)
)$
print("The matrix of the linear transformation
 is")$
print("The matrix of the linear transformation is")$
A:matrix([a[1],a[2]],[b[1],b[2]]);
```

1. $T(x,y) = (2x+3y, 4x-5y)$

W.K.T $e_1 = (1,0)$ $e_2 = (0,1)$

$T(1,0) = T(e_1) = (2,4)$

$T(0,1) = T(e_2) = (3,-5)$

$\therefore$ The matrix of linear transformation is given by $m(T) = A \begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix}$

Output :-

The matrix of the linear transformation is

$$\begin{bmatrix} 2 & 3 \\ 4 & -5 \end{bmatrix}$$

2. $T(x, y, z) = (x+z,\ x+2y,\ x+y+z)$

w.k.t $e_1 = (1,0,0)$, $e_2 = (0,1,0)$ $e_3 = (0,0,1)$

$T(1,0,0) = T(e_1) = (1,1,1)$

$T(0,1,0) = T(e_2) = (0,2,1)$

$T(0,0,1) = T(e_3) = (1,0,1)$

$\therefore$ Matrix of linear transformation is given by

$$m(T) = A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Output :-

The matrix of the linear transformation is

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Scanned with OKEN Scanner

2 Find the matrix of linear transformation $T: V_3(R) \to V_3(R)$ defined by $T(x, y, z) = (x+z, x+2y, x+y+z)$

Maxima code:

```
kill(all)$
T(x):=[x[1]+x[3],x[1]+2*x[2], x[1]+x[2]+x[3]];
u[1]: [1,0,0]$
u[2]: [0,1,0]$
u[3]: [0,0,1]$
v[1]: [1,0,0]$
v[2]: [0,1,0]$
v[3]: [0,0,1]$
for i:1 thru 3 do
eq[i]:v[1][i]*x+v[2][i]*y +v[3][i]*z
)$
for k:1 thru 3 do
(
soln:solve([eq[1]=T(u[k])[1], eq[2]=T(u[k])[2],
eq[3]=T(u[k])[3]], [x,y,z]),
a[k]: ev(x,soln),
b[k]: ev(y, soln),
c[k]: ev(z, soln)
)$
Print ("The matrix of the linear transformation is")$
M: matrix([a[1], a[2], a[3]], [b[1], b[2], b[3]],
[c[1], c[2], c[3]]);
```

3. $T(x, y) = (y, -x)$

WKT $e_1 = (1, 0)$ $e_2 = (0, 1)$

$T(1, 0) = T(e_1) = (0, -1)$

$T(0, 1) = T(e_2) = (1, 0)$

$\therefore$ Matrix of L.T is given by

$$M(T) = A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

output :-

$T(x) := (x_2, -x_1)$

The matrix of the linear transformation is

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

Date

Expt. No.

3 Find the matrix of the linear transformation $T: R^2 \to R^2$ defined by $T(x, y) = (y, -x)$ relative to standard basis

maxima code:

```
kill(all)$
T(x):=[x[2], -x[1]];
u[1]:[1,0]$
u[2]:[0,1]$
v[1]:[1,0]$
v[2]:[0,1]$
for i:1 thru 2 do
(
eq[i]:v[1][i]*x + v[2][i]*y
)$
for k:1 thru 2 do
(
soln:solve([eq[1]=T(u[k])[1], eq[2]=T(u[k])[2]],
[x,y]);
a[k]:ev(x,soln),
b[k]:ev(y,soln)
)$
print("The matrix of the linear transformation is")$
A:matrix([a[1],a[2]],[b[1],b[2]]);
```

Teacher's Signature :

4. $T(x, y) = (x, x+y)$

W k T $\quad e_1 = (1, 0) \quad e_2 = (0, 1)$

$$T(1,0) = T(e_1) = (1, 1)$$

$$T(0,1) = T(e_2) = (0, 1)$$

$\therefore$ Matrix of LT is given by

$$m(T) = A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Output :-

$T(x) : = [x_1, x_1 + x_2]$

The matrix of the linear transformation is

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Date

Expt. No. Page No. 31

4 Find the matrix of the linear transformation $T: R^2 \to R^2$ defined by $T(x, y) = (x, x+y)$

Maxima code:

```
kill(all)$
T(x):=[x[1], x[1]+x[2]];
u[1]:[1,0]$
u[2]:[0,1]$
v[1]:[1,0]$
v[2]:[0,1]$
for i:1 thru 2 do
(
eq[i]: v[1][i]*x + v[2][i]*y
)$
for k:1 thru 2 do
(
soln:solve([eq[1] = T(u[k])[1], eq[2] = T(u[k])[2]],
  [x,y]),
a[k]: ev(x, soln),
b[k]: ev(y, soln)
)$
print("The matrix of the linear transformat
  -ion is ")$
A: matrix([a[1], a[2]], [b[1], b[2]]);
```

11/07/2022

Teacher's Signature :

1) $2x+5y,\ 4x+3y$

$$\begin{bmatrix} 2 & 5 \\ 4 & 3 \end{bmatrix}$$

$|A-\lambda I| = 0$

$$\left| \begin{bmatrix} 2 & 5 \\ 4 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 2 & 5 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$(2-\lambda)(3-\lambda) - 20 = 0$

$6 - 2\lambda - 3\lambda + \lambda^2 - 20 = 0$

$\lambda^2 - 5\lambda + 6 - 20 = 0$

$\lambda^2 - 5\lambda - 14 = 0$

$\lambda^2 - 7\lambda + 2\lambda - 14 = 0$

$\lambda(\lambda-7) + 2(\lambda-7) = 0$

$(\lambda-7)(\lambda+2) \Rightarrow \langle \lambda = 7 \rangle \langle \lambda = -2 \rangle$

If $\lambda = 7$

$$\begin{bmatrix} 2-\lambda & 5 \\ 4 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2-7 & 5 \\ 4 & 3-7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -5 & 5 \\ 4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-5x_1 + 5x_2 = 0 \rightarrow (1)$

$4x_1 - 4x_2 = 0 \rightarrow (2)$

$4x_1 = 4x_2$

$x_1 = x_2$

Let $x_1 = k$, (or)

$x_2 = k$.

Expt. No. 6 Date ________ Page No. 32

EXPERIMENT - 06

EIGEN VALUES AND EIGEN VECTORS

1 Find the Eigen values and Eigen vectors of the transformation $T(x, y) = (2x+5y, 4x+3y)$

```
kill(all)$
T(x):=[2*x[5*x[2], 4*x[1]+3*x[2]];
u[1]:[1,0]$
u[2]:[0,1]$
v[1]:[1,0]$
v[2]:[0,1]$
for i:1 thru 2 do
(
 eq[i]: v[1][i]*x + v[2][i]*y
)$
for k:1 thru 2 do
(
Soln: solve([eq[1] = T(u[k])[1], eq[2] = T(u[k])
        [2]], [x,y]),
a[k]:ev(x, soln),
b[k]: ev(y,soln)
)$
print("The matrix of the linear transformation'\x")$
M: matrix([a[1], a[2]], [b[1], b[2]]);
charpoly(M,x);
eigenvalues(M);
eigenvectors(M);
```

Teacher's Signature : ________

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} k \\ k \end{bmatrix}$$

Eigen vectors

If $\lambda = -2$

$$\begin{bmatrix} 2+2 & 5 \\ 4 & 3+2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$4x_1 + 5x_2 = 0$ $\qquad$ $4x_1 + 5x_2 = 0$

$4x_1 = -5x_2$

$x_1 = -5/4 \, x_2$ $\quad$ or $\quad$ $x_2 = \frac{-4}{5} x_1$

Let $x_1 = k_1$, $x_2 = \frac{-4}{5} k$

Eigen vectors $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} k \\ -4/5k \end{bmatrix}$

Output :-

$T(x) := [3x_1 + 3x_2, 4x_1 + 3x_2]$

The matrix of the linear transformation is $\begin{pmatrix} 2 & 5 \\ 4 & 3 \end{pmatrix}$

$(2-x)(3-x) - z\ 0$

[[7, -2], [1, 1]]

[[[7, -2], [1, 1]], [[[1, 1]], [[1, -4/5]]]].

2) $3x + 3y$, $x + 5y$

$$\begin{bmatrix} 3 & 3 \\ 1 & 5 \end{bmatrix}$$

$|A - \lambda I| = 0$

$$\left| \begin{bmatrix} 3 & 3 \\ 1 & 5 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 3-\lambda & 3 \\ 1 & 5-\lambda \end{bmatrix} \right| = 0$$

2 Find the Eigen values and eigen vectors of the linear transformation $T: R^2 \to R^2$ defined by $T(x, y) = (3x + 3y, x + 5y)$ relative to standard basis.

```
kill(all)$
T(x) := [3*x[1] + 3*x[2], x[1] + 5*x[2]];
u[1]: [1,0]$
u[2]: [0,1]$
v[1]: [1,0]$
v[2]: [0,1]$
for i:1 thru 2 do
```

Teacher's Signature :

$(3-\lambda)(5-\lambda) - 3 = 0$

$15 - 3\lambda - 5\lambda + \lambda^2 - 3 = 0$

$\lambda^2 - 8\lambda + 12 = 0$

$\lambda^2 - 6\lambda - 2\lambda + 12 = 0$

$\lambda(\lambda-6) - 2(\lambda-6) = 0$

$(\lambda-6)(\lambda-2) = 0$

$\lambda = 6, \lambda = 2$

Put $\lambda = 6$

$$\begin{vmatrix} 3-\lambda & 3 \\ 1 & 5-\lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 3-6 & 3 \\ 1 & 5-6 \end{vmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-3x_1 + 3x_2 = 0$

$x_1 - x_2 = 0$

Consider $x_1 - x_2 = 0$

$x_1 = x_2$

Let $x_1 = k$

$x_2 = k$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} k \\ k \end{bmatrix}$$

put $\lambda = 2$

$$\begin{vmatrix} 3-2 & 3 \\ 1 & 5-2 \end{vmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_1 + 3x_2 = 0$

$x_1 + 3x_2 = 0$

Expt. No. ____________ Date ____________ Page No. 34

```
(
  eq[i] : v[1][i]*x + v[2][i]*y
)$
for k:1 thru 20 do
(
soln : solve([eq[1] = T(u[k])[1], eq[2] = T(u[k])[2]),
[x,y]),
a[k] : ev(x, soln),
b[k] : ev(y, soln),
)$
print("The matrix of the linear transformation is")$
M:matrix([a[1], a[2]], [[b[1], b[2]]]);
charpoly(M, x);
eigenvalues(M);
eigenvectors(M);
```

Teacher's Signature : ____________

$\langle x_1 = -3x_2 \rangle$ or $\lambda\ \lambda_2$ 3

let $x_1 = k$

$\langle x_2 = \frac{-1}{3} k \rangle$

Output :-

$T(x) := [3x_1 + 3x_2,\ x_1 + 5x_2]$

The matrix of the linear transformation is $\begin{bmatrix} 3 & 3 \\ 1 & 5 \end{bmatrix}$

$(3-\lambda)(5-\lambda) - 3$

[[6, 2], [1, 1]]

[[[6, 2], [1, 1]], [[[1, 1]], [[1, -1/3]]]]

3) $x+y,\ y$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = 0$$

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\begin{vmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(1-\lambda) = 0$$

$$(1-\lambda)^2 = 0$$

$$1 + \lambda^2 - 2\lambda = 0$$

$$\lambda^2 - \lambda - \lambda + 1 = 0$$

$$\lambda(\lambda-1) - 1(\lambda-1) = 0$$

$$(\lambda-1)(\lambda-1) = 0$$

$$\langle \lambda = 1 \rangle$$

put $\lambda = 1$

$$\begin{bmatrix} 1-1 & 1 \\ 0 & 1-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_2 = 0$ $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

3 Find the eigen values and eigen vectors of the linear transformation $T(x,y) = (x+y, y)$

```
kill(all)$
T(x): [x[1] + x[2] , x[2] ];
u[1]: [1, 0] $
u[2]: [0, 1] $
v[1] : [1, 0] $
v[2]: [0, 1] $
for i : 1 thru 2 do
(
eq[i] : v[1][i] * x + v[2][i] * y
)$
for k : 1 thru 2 do
(
soln : solve ([eq[1] = T(u[k])[1], eq[2] = T(u[k])[2]],
[x, y]),
a[k] : ev(x, soln),
b[k] : ev(y, soln),
)$
print ("The matrix of the linear transformation is")$
```

Output

T(x): [$x_1 + x_2$, x_2]

The matrix of the linear transformation is $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$(1-x)^2$

[[1], [2]]

[[[1], [2]], [[[1, 0]]]]


```
M:matrix ([a[1], a[2]], [b[1], b[2]]);
charpoly (M, x);
eigenvalues (M);
eigenvectors (M);
```

19/08/2022

Date

Expt. No. 7.1

7.1 Cauchy's Riemann Equation in Cartesian Form

1 cos z

```
kill(all)$
w: cos(x+%i*y)$
u: realpart(w)
v: imagpart(w);
if diff(u,x) = diff(v,y) and diff(u,y) = -diff(v,x)
print("By CR equation given function Analytic")
else
print("Given function is not Analytic")$
```

2 z Im(z)

```
kill(all)$
w: (x+i*y)*imagpart(x+%i*y)$
u: realpart(w);
v: imagpart(w);
if diff(u,x) = diff(v,y) and diff(u,y) = -diff(v,x)
print("By CR equation given function Analytic")
else
print("Given function is not analytic")$
```

Teacher's Signature :

1) cos z

$f(z) = w = \cos z$

$= \cos(x+iy)$

$= \cos x \cdot \cos(iy) - \sin x \sin(iy)$

$= \cos x \cosh y - \sin x (i * \sinh y)$

$u = \cos x \cosh y \qquad v = -\sin x \sinh y$

$\frac{\partial u}{\partial x} = -\sin x \cosh y \qquad \frac{\partial v}{\partial x} = -\cos x \sinh y$

$\frac{\partial u}{\partial y} = \cos x \sinh y \qquad \frac{\partial v}{\partial y} = -\sin x \cosh y$

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ & $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

Function is Analytic

output :-

cos(x) cosh(y)

-sin(x) sin h(y)

By CR equation given function is analytic

2) $f(z) = w = z\,Im(z)$

$= (x+iy)(y)$

$f(z) = xy + iy^2$

$u = xy \qquad v = y^2$

$\frac{\partial u}{\partial x} = y \qquad \frac{\partial v}{\partial x} = 0$

$\frac{\partial u}{\partial y} = x \qquad \frac{\partial v}{\partial y} = 2y$

$\therefore \frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$ & $\frac{\partial u}{\partial y} \neq -\frac{\partial v}{\partial x}$

$\therefore$ Function is not analytic

output :-

$x\ y$

y^2

Given function is not analytic

3) e^z

$f(z) = w = e^{x+iy}$

$= e^x . e^{iy}$

$= e^x (\cos y + i \sin y)$

$= e^x \cos y + i e^x \sin y$

$u = e^x \cos y \qquad v = e^x \sin y$

$\frac{\partial u}{\partial x} = e^x \cos y \qquad \frac{\partial v}{\partial x} = e^x \sin y$

$\frac{\partial u}{\partial y} = -e^x \sin y \qquad \frac{\partial v}{\partial y} = e^x \cos y$

$\therefore \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ & $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

$\therefore$ function is analytic

Output :-

%e^x cos y

%e^x sin y

By CR equation given functions is analytic

4) 1/z

$f(z) = w = \frac{1}{z} = \frac{1}{x+iy}$

$= \frac{1}{x+iy} \times \frac{(x-iy)}{(x-iy)}$

$= \frac{x-iy}{x^2-(-1)y^2} = \frac{x-iy}{x^2+y^2}$

Expt. No. .... Date ______

3 e^z

```
kill(all)$
w: %e^(x+%i*y)$
u: realpart(w);
v: imagpart(w);
if diff(u,x) = diff(v,y) and diff(u,y) = -diff(v,y) then
print("By CR equation given function is analytic")
else
print("Given function is not analytic")$
```

4 1/z

```
kill(all)$
w: 1/(x+%i*y)$
u: realpart(w);
v: imagpart(w);
if diff(u,x) = diff(v,y) and diff(u,y) = -diff(v,y) then
print("By CR equation given function is analytic")
else
print("Given function is not analytic")$
```

Teacher's Signature : ______

$$u = \frac{x}{x^2+y^2} \qquad v = \frac{-y}{x^2+y^2}$$

$$\frac{\partial u}{\partial x} = \frac{(x^2+y^2)\,1 - x\,(2x)}{(x^2+y^2)^2} = \frac{x^2+y^2-2x^2}{(x^2+y^2)^2}$$

$$\frac{\partial v}{\partial x} = \frac{-y}{(x^2+y^2)} \times (2x)$$

$$\frac{\partial v}{\partial x} = \frac{-2xy}{x^2+y^2}$$

$$\frac{\partial u}{\partial y} = \left(\frac{x}{x^2+y^2}\right) \times 2y$$

$$\frac{\partial v}{\partial y} = \frac{(x^2+y^2)(-1) - (-y)(2y)}{(x^2+y^2)^2} = \frac{-(x^2+y^2)+2y^2}{(x^2+y^2)^2}$$

$$\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y} \quad \& \quad \frac{\partial u}{\partial y} \neq -\frac{\partial v}{\partial x}$$

Function is not analytic

Output :-

$x / y^2 + x^2$

$-y / x^2+y^2$

Given function is not analytic

~~Output~~

~~x/y^2+x^2~~

~~$-y/x^2+y^2$~~

~~Given function is not analytic~~

19/08/2022

1) $u = r^2\cos 2\theta - r\sin\theta$

$v = r^2\sin 2\theta + r\cos\theta$

$\frac{\partial u}{\partial r} = 2r\cos 2\theta - \sin\theta$

$\frac{\partial v}{\partial \theta} = 2r^2\cos 2\theta - r\sin\theta$

$\frac{\partial u}{\partial \theta} = 2r\sin\theta - r\cos\theta$

$\frac{\partial v}{\partial r} = 2r\sin 2\theta + \cos\theta$

$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$

$2r\cos\theta - \sin\theta = \frac{1}{r}[2r^2\cos 2\theta - r\sin\theta]$

$= 2r\cos\theta - \sin\theta = \frac{\partial u}{\partial r}$

$\frac{\partial u}{\partial \theta} = -r\frac{\partial v}{\partial r}$

$-2r^2\sin\theta - r\cos\theta = -r\{2r\sin 2\theta + \cos\theta\}$

$= -2r^2\sin 2\theta - r\cos\theta = \frac{\partial u}{\partial \theta}$

$\therefore \frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ & $\frac{\partial u}{\partial \theta} = -r\frac{\partial v}{\partial r}$

Hence it is analytic

output :-

The function is analytic

2) $\log z$

$z = x + iy$

$x = r\cos\theta$

$y = r\sin\theta$

$z = r\cos\theta + ir\sin\theta$

$= r(\cos\theta + i\sin\theta)$

$\{z = re^{i\theta}\}$

Date

Expt. No. 7.2 Page No. 40

7.2 Cauchy Riemann Equation in polar Form

1 Check the analyticity of the function $(r^2\cos 2\theta - r\sin\theta) + i(r^2\sin 2\theta + r\cos\theta)$

```
kill(all)$
w:r^2*cos(2*t)-r*sin(t)+%i*(r^2*sin(2*t)+r*cos(t))$
u:realpart(w)$
v:imagpart(w)$
ur:ratsimp(diff(u,r));
ut:ratsimp(diff(u,t));
vr:ratsimp(diff(v,r));
vt:ratsimp(diff(v,t));
if ur=ratsimp(1/r*vt) and ut=ratsimp(-r*vr) then
disp("The function is analytic")
else
disp("The function is not analytic")$
```

2 Check the analyticity of the function $\log z$

```
kill(all)$
w: log(r*%e^(%i*t))$
u:realpart(w)$
v:imagpart(w)$
ur:ratsimp(diff(u,r));
ut:ratsimp(diff(u,t));
```

Teacher's Signature :

Let $w = \log z$

$= \log(re^{i\theta})$

$= \log r + \log e^{i\theta}$

$w = \log r + i\theta$

$u = \log r$ $\qquad$ $v = \theta$

$\frac{\partial u}{\partial r} = \frac{1}{r}$ $\qquad$ $\frac{\partial v}{\partial \theta} = 1$

$\frac{\partial u}{\partial \theta} = 0$ $\qquad$ $\frac{\partial v}{\partial r} = 0$

$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \Rightarrow \frac{1}{r} 1 = \frac{1}{r} = \frac{\partial u}{\partial r}$

$\frac{\partial u}{\partial \theta} = -r\frac{\partial v}{\partial r} \Rightarrow -r.0 = 0 = \frac{\partial u}{\partial \theta}$

$\therefore \frac{\partial u}{\partial \theta} = \frac{1}{r}\frac{\partial v}{\partial r} + \frac{\partial u}{\partial r} = -r\frac{\partial v}{\partial \theta}$

∴ Hence Analytic

Output:-

The function is analytic

3) $u = \frac{1}{r}\cos\theta,$ $\qquad$ $v = -\frac{1}{r}\sin\theta$

$\frac{\partial u}{\partial r} = -\frac{1}{r^2}\cos\theta$ $\qquad$ $\frac{\partial v}{\partial \theta} = -\frac{1}{r}\cos\theta$

$\frac{\partial u}{\partial \theta} = -\frac{1}{r}\sin\theta$ $\qquad$ $\frac{\partial v}{\partial r} = \frac{1}{r^2}\sin\theta$

$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \Rightarrow \frac{1}{r}\left(-\frac{1}{r}\cos\theta\right) = -\frac{1}{r^2}\cos\theta = \frac{\partial u}{\partial r}$

$\frac{\partial u}{\partial t} = -r\frac{\partial v}{\partial r} \Rightarrow -r\left(\frac{1}{r^2}\sin\theta\right) = -\frac{1}{r}\sin\theta = \frac{\partial u}{\partial \theta}$

∴ Hence it is Analytic

Output:-

The function is Analytic

Date ____

Expt. No. ____ $\qquad$ Page No. 41

```
vr:ratsimp(diff(v,r));
vt:ratsimp(diff(v,t));
if ur=ratsimp(1/r*vt) and ut=ratsimp(-r*vr)
then
disp("The function is analytic")
else
disp("The function is not analytic")$
```

3 Check the analyticity of the function $\frac{1}{r}\cos\theta - \frac{i}{r}\sin\theta$

```
kill(all)$
w: 1/r*cos(t)-(%i/r*(sin(t))$
u:realpart(w)$
v:imagpart(w)$
ur:ratsimp(diff(u,r));
ut:ratsimp(diff(u,t));
vr:ratsimp(diff(v,r));
vt:ratsimp(diff(v,t));
if ur=ratsimp(1/r*vt) and ut=ratsimp(-r*vr) then
disp("The function is analytic")
else
disp("The function is not analytic")$
```

19/08/2025

Teacher's Signature : ____

Expt. No. 08 Date ______ Page No. 42

8. Construction of Analytic Function using Milne Thomson method

1 Construct an analytic function whose real part is $u = \frac{1}{2}\log(x^2+y^2)$

```
kill(all)$
u:(log(x^2+y^2))/2;
ux: diff(u,x);
uy: diff(u,y);
f1 : subst([x=z, y=0], ux)$
f2: subst([x=z, y=0], uy)$
ratsimp(integrate(f1, z)-%i*integrate(f2,z));
```

2 Construct an analytic function whose imaginary part is $v = x\sin x \sinh y - y\cos x\cosh y$

```
kill(all)$
v:x* sin(x)* sinh(y)-y*cos(x)*cosh(y);
vx: diff(v,x);
vy: diff(v,y);
f1 : subst([x=z, y=0], vx]$
f2: subst([x=z, y=0], vy]$
ratsimp(integrate(f2,z)+%i*integrate(f1,z));
```

Teacher's Signature : ______

1) $u = \frac{1}{2}\log(x^2+y^2)$

diff u w.r.t x partially.

$$\frac{\partial u}{\partial x} = \frac{1}{2}\frac{2x}{x^2+y^2} \Rightarrow \frac{x}{x^2+y^2}$$

$$\frac{\partial u}{\partial y} = \frac{1}{2}\frac{2y}{x^2+y^2} \Rightarrow \frac{y}{x^2+y^2}$$

WKT $f(z) = u+iv$ $\qquad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

$$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$$

$$= \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y}$$

$$f'(z) = \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$$

put $x=z$ & $y=0$ in the above equation

$$f'(z) = \frac{z}{z^2} - i\left(\frac{0}{z^2}\right) = \frac{1}{z}$$

on integration we get

$f(z) = \log(z)+c$

Output :- $\log z + c$

$\log 1 + c$

2) $v = x\sin x\sinh y - y\cos x\cosh y$

$$\frac{\partial v}{\partial x} = \sinh y(x\cos x + \sin x) - y\cosh y(-\sin x)$$
$$= x\sinh y\cos x + \sinh y\sin x + y\cosh y\sin x$$

$$\frac{\partial v}{\partial y} = x\sin x\cosh y - \cos x(y\sinh y + \cosh y)$$
$$= x\sin x\cosh y - y\cos x\sinh y - \cos x\cosh y$$

WKT $f(z) = u+iv$, $f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$ $\qquad \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

$$f'(z) = \frac{\partial v}{\partial y} + i\frac{\partial v}{\partial x}$$

put $x=z$ & $y=0$

$f'(z) = z\sin z - \cos z + i(0)$ $\quad = \int z\sin z - \int\cos z$

$= \sin z + z(-\cos z) - \int(-\cos z)dz = \sin z + z\cos z - \sin z$

$f(z) = -z \cos z$

Output :-

$-z \cos z$

3) $u = 2x - x^3 + 3xy^2$

$\frac{\partial u}{\partial x} = 2 - 3x^2 + 3y^2$

$\frac{\partial u}{\partial y} = 6xy$

$f(z) = u + iv, \quad f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$

$= \frac{\partial u}{\partial x} - i\frac{\partial u}{\partial y} = 2 - 3x^2 + 3y^2 - i(6xy)$

Put $x = z, \; y = 0$

$f'(z) = 2 - 3z^2 - i(0) = 2 - 3z^2$

on integrating $f'(z)$ we get

$f(z) = 2z - z^3$

Output :-

2z - z^3

4) $v = y + e^x \cos y$

$\frac{\partial v}{\partial x} = e^x \cos y$

$\frac{\partial v}{\partial y} = 1 - e^x \sin y$

$f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x} = \frac{\partial v}{\partial y} + i\frac{\partial v}{\partial x}$

Put $x = z, \; y = 0$

$f'(z) = 1 - e^z \sin(0) + i \, e^z \cos(0) = 1 + ie^z$

Integrate $f'(z)$,

$f(z) = z + ie^z$

Output :- z + %i e^z

3 Construction an analytic function whose real part is $u = 2x - x^3 + 3xy^2$

```
kill(all)$
u: 2*x - x^3 + 3*x*y^2;
ux: diff(u,x);
uy: diff(u,y);
f1 : subst([x=z, y=0], ux)$
f2 : subst([x=z, y=0], uy)$
ratsimp(integrate(f1, z) - %i*integrate(f2, z));
```

4 Construct an analytic function whose imaginary part is $v = y + e^x \cos y$

```
kill(all)$
v: y + %e^(x)*cos(y);
vx: diff(v, x);
vy: diff(v, y);
f1 : subst([x=z, y=0], vx)$
f2 : subst([x=z, y=0], vy)$
ratsimp(integrate(f1, z) + %i * integrate(f2, z));
```

17/08/2022

Teacher's Signature : ______

1) Co-efficient matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} R_3 \to R_3 - R_1$$

$$\sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} R_3 \to R_3 - R_2$$

$\therefore$ Rank = 2

Nullity = 1

dim(domain) = 3

$\therefore$ Rank + nullity = dim(domain)

2+1=3

3=3

$\therefore$ Rank - nullity theorem is verified

Range Space = $\{T(\alpha) \mid \alpha \in V\}$

= Span $\{(1,1,0), (0,1,1), (0,0,0)\}$

= $\{x(1,1,0) + y(0,1,1) + z(0,0,0)\}$

Range Space = $(x, x+y, y)$

Expt. No. 09 Page No. 44

Date

9. The Rank nullity Theorem

1 If linear transformation $T: R^3 \to R^3$ is defined by $T(e_1) = (1,0,0)$ $T(e_2) = (0,1,1)$ and $T(e_3) = (1,2,1)$ then find range space, rank and nullity of T and hence verify rank - nullity theorem.

```
kill(all)$
A: matrix([1,1,0],[0,1,1],[1,2,1])$
dim :3$
echelon(A);
r : rank(A)$
print("Dimension of range space is ", r)$
nullspace(A);
n: nullity(A)$
print("Dimension of null space is ", n)$
if dim = r+n then
print(" Rank-nullity theorem is verified")$
```

Output:-

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

dimension of range Space is 2

$$\text{Span}\left(\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}\right)$$

dimension of null space is 1

rank - nullity theorem is verified.

Teacher's Signature :

3) co-efficient matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix} R_2 \to R_2 - R_1$

$\sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix} R_2 \to -R_2$

$\sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix} R_3 \to R_3 - R_2$

$\sim \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ ~~$R_3 \to \frac{R_3}{2}$~~ ~~$R_3 \to \frac{R_3}{2}$~~ $R_3 \to \frac{R_3}{2}$

Rank = 3

nullity = 0

dim(domain) = 3

∴ rank + nullity = dim (domain)

3 + 0 = 3

3 = 3

∴ Rank - Nullity theorem is verified

2 If linear transformation $T: R^3 \to R^3$ is defined by $T(e_1) = (1,1,0)$ $T(e_2) = (1,0,1)$ and $T(e_3) = (0,1,1)$ then find range space, rank and nullity of T and hence verify rank - nullity theorem.

```
kill(all)$
A: matrix([1,1,0],[1,0,1],[0,1,1])$
dim:3$
echelon(A);
r:rank(A)$
print("Dimension of range space is",r)$
nullspace(A);
n: nullity(A)$
print("Dimension of null space is", n)$
if dim = r+n then
print("Rank-nullity theorem is verified")$
```

output:-

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

dimension of range space is 3

span()

dimension of null space is 0

rank-nullity theorem is verified.

Teacher's Signature : ______

3) $m(T) = A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} R_2 \to R_2/2$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} R_3 \to R_3/3$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Rank = 3
nullity = 0
dim (domain) = 3

∴ Rank and nullity theorem is verified.

WKT

$R(T) = \{T(\alpha) / \alpha \in V\}$

$R(T)$ is spanned by $(1,0,0), (0,1,0), (0,0,1)$

$R(T) = x(1,0,0) + y(0,1,0) + z(0,0,1)$

$= (x,0,0) + (0,y,0) + (0,0,z)$

$R(T) = (x,y,z)$

w.k.T

$N(T) = \{\alpha \in V \mid T(\alpha) = 0\}$

$N(T) = \{(x,y,z) = 0\}$

$N(T) = \{(x,y,z) = (0,0,0)\}$

$= \{x=0, y=0, z=0\}$

$N(T) = \{0,0,0\}$

3 Find the range, rank, kernel, nullity and verify the rank nullity theorem for the transformation $T(x,y,z) = (x, 2y, 3z)$.

```
kill(all)$
T(x):=[x[1]+2*x[2]+3*x[3]];
u[1]:[1,0,0]$
u[2]:[0,1,0]$
u[3]:[0,0,1]$
v[1]:[1,0,0]$
v[2]:[0,1,0]$
v[3]:[0,0,1]$
for i:1 thru 3 do
(
eq[i]:v[1][i]*x+v[2][i]*y+v[3][i]*z
)$
for k:1 thru 3 do
(
soln:solve([eq[1]=T(u[k])[1],eq[2]=T(u[k])[2],
eq[3]=T(u[k])[3]],[x,y,z]),
a[k]:ev(x,soln),
b[k]:ev(y,soln),
c[k]:ev(z,soln)
)$
print("The matrix of the linear transformation is")
M:matrix([a[1],a[2],a[3]],[b[1],b[2],b[3]],
[c[1],c[2],c[3]]);
dim:3$
echelon(M);
r:rank(M)$
```

Output :-

The matrix of the linear transformation is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Expt. No.


```
print ("Dimension of range space is ", r)$
nullspace (M);
n: nullity (M)$
print ("Dimension of null space is", n)$
if dim = r+n then
print ("Rank-nullity theorem is verified")$
```

17/08/2023

Teacher's Signature :

1 $u = e^x \sin y$ $\quad v = -e^x \cos y$

$\frac{\partial u}{\partial x} = e^x \sin y$ $\quad \frac{\partial v}{\partial x} = -e^x \cos y$

$\frac{\partial^2 u}{\partial x^2} = e^x \sin y$ $\quad \frac{\partial^2 v}{\partial x^2} = -e^x \cos y$

$\frac{\partial u}{\partial y} = e^x \cos y$ $\quad \frac{\partial v}{\partial y} = e^x \sin y$

$\frac{\partial^2 u}{\partial y^2} = -e^x \sin y$ $\quad \frac{\partial^2 v}{\partial y^2} = e^x \cos y$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = e^x \sin y - e^x \sin y = 0$$

$$\frac{\partial^2 v}{\partial y} + \frac{\partial^2 v}{\partial y^2} = -e^x \cos y + e^x \cos y = 0$$

$\therefore$ Given Function is Harmonic

2 $u = (x-1)^3 - 3xy^2 + 3y^2$

$v = 3(x-1)^2 y - y^3$

$\Rightarrow$ diff u w.r.t x partially we get

$\frac{\partial u}{\partial x} = 3(x-1)^2 \cdot 1 - 3y^2 + 0$

$\frac{\partial u}{\partial x} = 3(x-1)^2 - 3y^2$

Again diff w.r.t x partially we get

$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial^2 u}{\partial x^2} = 6(x-1) - 0$

$\frac{\partial^2 u}{\partial x^2} = \langle 6x-6 \rangle$

Date

Expt. No. .... Page No. 49

10. Harmonic Functions

1 Show that $u = e^x \sin y$ and $v = -e^x \cos y$ are harmonic

```
kill(all)$
u: %e^x * sin(y);
v: -%e^x * cos(y);
uxx: diff(u,x,2);
uyy: diff(u,y,2);
vxx: diff(v,x,2);
vyy: diff(v,y,2);
if ratsimp(uxx+uyy)=0 and ratsimp(vxx+vyy)=0 then
print("u and v are harmonic")
else
print("u and v are not harmonic")$
```

Output :-

%e^x sin(y)

-%e^x cos(y)

u and v are harmonic

2 Show that $u = (x-1)^3 - 3xy^2 + 3y^2$ and $v = 3(x-1)^2 y - y^3$

```
kill(all)$
u: (x-1)^3 - 3*x*y^2 + 3*y^2;
v: 3*(x-1)^2*y - y^3;
uxx: diff(u,x,2);
uyy: diff(u,y,2);
vxx: diff(v,x,2);
vyy: diff(v,y,2);
if ratsimp(uxx+uyy)=0 and ratsimp(vxx+vyy)=0 the
print("u and v are harmonic")
```

Teacher's Signature :

$$\frac{\partial u}{\partial y} = 0 - 6xy + 6y$$

Again diff w.r.t y partially we get

$$\frac{\partial^2 u}{\partial y^2} = -6x + 6$$

Consider

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \cancel{6x} - \cancel{6} - \cancel{6x} + \cancel{6}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{condtd.}$$

$$\frac{\partial v}{\partial x} = 6y - 0$$

$$\frac{\partial^2 v}{\partial x^2} = (6xy - 0)$$

$$\frac{\partial^2 v}{\partial x^2} = 6y$$

$$\frac{\partial v}{\partial y} = 3(x-1)^2 - y^3$$

$$\frac{\partial^2 v}{\partial y^2} = 0 - 6y$$

Consider

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \cancel{6y} - \cancel{6y}$$

$$= 0$$

$\therefore$ Given Function is Harmonic

else

print ("u and v are not harmonic")$

Output :-

$-3xy^2 + 3y^2 + (x-1)^3$

$3(x-1)^2 y - y^3$

u and v are harmonic

Teacher's Signature : ______

3 $u = \frac{1}{2}\log(x^2+y^2)$ $\qquad$ $v = 2xy$

$\frac{\partial u}{\partial x} = \frac{x}{x^2+y^2}$ $\qquad$ $\frac{\partial v}{\partial x} = 2y$ $\qquad$ $\frac{\partial^2 v}{\partial x^2} = 0$

$\frac{\partial^2 u}{\partial x^2} = \frac{y^2-x^2}{(x^2+y^2)^2}$ $\qquad$ $\frac{\partial v}{\partial y} = 2x$ $\qquad$ $\frac{\partial^2 v}{\partial y^2} = 0$

$\frac{\partial u}{\partial y} = \frac{y}{x^2+y^2}$ $\qquad$ $\therefore \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ $\qquad$ $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$

$\frac{\partial^2 u}{\partial y^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$ $\qquad$ $\therefore \Delta^2 u = 0$ $\qquad$ $\therefore \Delta^2 v = 0$

~~∴ u~~ $\qquad$ ∴ v is Harmonic

∴ u is Harmonic

∴ u and v are Harmonic

output :-

1/2 log (x²+y²)

2xy

u and v are Harmonic

4) $2z^2 - iz^3$

$2(x+iy)^2 - i(x+iy)^3$

$2(x^2+i2xy-y^2) - i(x^3-iy^3+3x^2iy-3xy^2)$

$2x^2+i4xy-2y^2-x^3i-y^3+3x^2y+3ixy^2$

$(2x^2-2y^2-y^3+3x^2y)+i(4xy-x^3+3xy^2)$

$u = 2x^2-2y^2-y^3+3x^2y$ $\qquad$ $V = 4xy-x^3+3xy^2$

$\frac{\partial u}{\partial x} = 4x+6xy$ $\qquad$ $\frac{\partial v}{\partial x} = 4y-3x^2+3y^2$

$\frac{\partial^2 u}{\partial x^2} = 4+6y$ $\qquad$ $\frac{\partial^2 v}{\partial x^2} = -6x$

$\frac{\partial u}{\partial y} = -4y-3y^2+3x^2$ $\qquad$ $\frac{\partial v}{\partial y} = 4x+6xy$

$\frac{\partial^2 u}{\partial y^2} = -4-6y$ $\qquad$ $\frac{\partial^2 v}{\partial y^2} = 6x$

$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ $\qquad$ $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$

Date

Expt. No. $\qquad$ Page No. 50

3)
```
kill(all)$
u: 1/2 * log (x^2 +y^2);
v: 2*x*y;
uxx:
uyy:
vxx:
vyy:
if ratsimp( uxx +uyy) =0  and ratsimp (vxx+vyy)=0
then
print (" u and v are harmonic")
else
print (" u and v are not harmonic")$
```

4
```
kill(all)$
z: x+%i*y$
w: 2*z^2- %i*z^3;
u :real part (w)$
v: imagpart (w)$
uxx: diff (u,x, 2)$
uyy: diff (u,y,2)$
vxx: diff (v,x, 2)$
vyy: diff (v,y,2)$
if ratsimp (uxx +uyy)=0 and ratsimp (uxx +vyy)
=0 then
print (" u and v are harmonic")
else
print (" u and v are not harmonic ")$
```

Teacher's Signature :

$-6x + 6x = 0$

$\therefore \Delta^2 v = 0$

$\therefore$ u is Harmonic

$\therefore$ v is Harmonic

$\therefore$ u and v are Harmonic

Output :-

2 (%iy + x)^2 - %i (%iy + x)^3

u and v are harmonic

19/08/2022

1) 1, -1, i, -i

$\Rightarrow z_1 = 1, \quad z_2 = -1, \quad z_3 = i, \quad z_4 = -i$

$$\text{Cross ratio} = \frac{(z_1 - z_2)(z_3 - z_4)}{(z_1 - z_4)(z_3 - z_2)}$$

$$= \frac{(1+1)(i+i)}{(1+i)(i+1)} = \frac{2(2i)}{(1+i)^2}$$

$$= \frac{4i}{1+i^2+2i} = \frac{4i}{\cancel{1}-\cancel{1}+2i} = 2$$

Output :- 2

2) 2, 2i, -1, -i

$z_1 = 2, \quad z_2 = 2i, \quad z_3 = -1, \quad z_4 = -i$

$$\text{Cross ratio} = \frac{(z_1 - z_2)(z_3 - z_4)}{(z_1 - z_4)(z_3 - z_2)}$$

$$= \frac{(2-2i)(-1+i)}{(2+i)(-1-2i)} = \frac{-2+2i+2i-2i^2}{-2-4i-i-2i^2}$$

$$= \frac{-\cancel{2}+4i+\cancel{2}}{-2-5i+2} = \frac{-4}{5}$$

Output :- -4/5

3 $z = x+iy$

$$= \frac{(4i-2)}{(-1+i)} \times \frac{-1-i}{-1-i} = \frac{-4i+4+2+2i}{1-i^2}$$

$$= \frac{(4i-2)(-1-i)}{(-1)^2-(i)^2} = \frac{-2i+6}{1+1}$$

$$= \frac{\cancel{2}(-i+3)}{\cancel{2}} = 3-i$$

Output :- 3-i

11. Cross Ratio

1 Find the cross ratio of four points 1, -1, i, -i

```
kill(all)$
z1 : 1$
z2 : -1$
z3 : %i$
z4 : -%i$
CR : rectform(fullratsimp(((z1-z2)*(z3-z4))/
     ((z2-z3)*(z4-z1))));
```

2 Find the cross ratio of four points 2, 2i, -1, -i

```
kill(all)$
z1 : 2$
z2 : 2*%i$
z3 : -1$
z4 : -%i$
CR : rectform(fullratsimp(((z1-z2)*(z3-z4))/
     ((z2-z3)*(z4-z1))));
```

3
```
kill(all)$
z1 : 1$
z2 : -1$
z3 : %i$
CR : rectform(fullratsimp(((z1-z2)*(z3-z4))/
     ((z2-z3)*(z4-z1))));
```

19/08/2022

Teacher's Signature :

1) $\frac{z-1}{z+1}$

Let

$w = z$ be the fixed point then we have

$$z = \frac{z-1}{z+1}$$

$$z(z+1) = z-1$$
$$z^2 + z = z-1$$
$$z^2 + z - z + 1 = 0$$
$$z^2 + 1 = 0$$
$$z^2 = -1$$
$$z = \sqrt{-1}$$
$$z = \pm i$$
$$z = -i, i$$

2 $w = \frac{3z-4}{z}$

Let $w = z$ be the fixed point then we have

$$z = \frac{3z-4}{z}$$

3

$$z(z) = 3z - 4$$
$$z^2 = 3z + 4 = 0$$
$$= \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{3 \pm \sqrt{9 - 4 \times 4}}{2 \times 1}$$
$$= \frac{3 \pm \sqrt{9-16}}{2}$$

Expt. No. ______ Date ______ Page No. 53

12. Bilinear Transformation

1 Find fixed points of the transformation

$w = \frac{z-1}{z+1}$

```
kill(all)$
w: (z-1)/(z+1);
FP: rectform(fullratsimp(w=z));
solve(FP, z);
```

output :-

$$\frac{z-1}{z+1}$$

$$\frac{z-1}{z+1} = z$$

$[z = -\%i, z = \%i]$

02 Find the fixed points of the transformation

$w = \frac{3z - 4}{z}$

```
kill(all)$
w: (3*z-4)/4;
FP: rectform(fullratsimp(w=z));
solve(FP, z);
```

Teacher's Signature : ______

$$= \frac{3 \pm \sqrt{-7}}{2}$$

$$= \frac{3 \pm i\sqrt{7}}{2}$$

$$z = \frac{3 - i\sqrt{7}}{2}, \quad \frac{3 + i\sqrt{7}}{2}$$

3 $w = \frac{2z-5}{z+4}$

Let $w = z$ be the fixed points then we have

$$z = \frac{2z-5}{z+4}$$

$$z(z+4) = 2z-5$$

$$z^2 + 4z = 2z - 5$$

$$z^2 + 4z = -2z + 5 = 0$$

$$z^2 + 2z + 5 = 0$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{4 - 4 \times 5}}{2 \times 1}$$

$$= \frac{-2 \pm \sqrt{4-20}}{2}$$

$$= \frac{-2 \pm \sqrt{-16}}{2}$$

$$z = \frac{-2 \pm \sqrt{-16}}{2}$$

$$z = \frac{-2 \pm 4i}{2}$$

$$z = \frac{\cancel{2}(-1+2i)}{\cancel{2}}$$

Output :-

$$\frac{3z-4}{z}$$

$$\frac{3z-4}{z} = z$$

$$z = \frac{-\sqrt{7}\,\%i+3}{2}, \quad z = \frac{\sqrt{7}\,\%i+3}{2}$$

3 Find fixed points of the transformation

$$w = \frac{2z-5}{z+4}$$

```
kill(all)$
w: (2*z-5) / (z+4);
FP: rectform (fullratsimp (w = z));
solve (FP, z);
```

Output :-

$$\frac{2z-5}{z+4}$$

$$\frac{2z-5}{z+4} = z$$

$[z = -2\%i, \ i = -1, \ z = 2, \ \%i = -1]$

$z = -1 + 2i$

$z = 2i - 1$

4) $w = 3z$

Let $w = 3z$ be the fixed point then we have

$z = 3z$

$3z - z = 0$

$2z = 0$

$z = 0$

4 Find fixed points of the transformation $w = 3z$

```
kill(all)$
w: 3*z;
FP: rectform(fullratsimp(w=z));
solve (FP, z);
```

Output :-

$3z$

$3z = z$

$z = 0$

19/08/2022

Teacher's Signature : ______