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VI SEMESTER B.Sc. EXAMINATION, SEPTEMBER/OCTOBER 2022

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SCHEME: SEMESTER – CBCS

MATHEMATICS

ALGEBRA-IV AND COMPLEX ANALYSIS-I (DSE)

Time: 03 Hours **Max Marks: 80**

Instructions: 1. Answer all the five questions.
2. First question carries 20 marks and remaining questions carry 15 marks.

1. Answer any TEN questions. Each question carries two marks. **10x2=20**

a) Express *(1,7,-4)* as a linear combination of the vectors *(1,-3,2)* and *(2,-1,1)* in V_3 *(R)*.

b) Show that vectors (a_1, a_2) and (b_1, b_2) in V_2 *(F)* are linearly dependent if and only if $a_1b_2 - a_2b_1 = 0$.

c) Determine whether $S = \{(x, y, z) | x, y, z \in R \ \ x = y = z\}$ is a subspace of V_3 *(R)*.

d) Define basis and dimension of a vector space.

e) Find $T^2(x, y, z)$ of the transformation $T: V_3(\mathrm{R}) \to V_3(\mathrm{R})$ defined by $T(x, y, z) = (z, -x, y)$.

f) Find the Eigen values of the linear transformation $T: R^2 \to R^2$ defined by $T(x, y) = (x + y, x - y)$.

g) Find the equation to the straight line through the points $1 + 2i$ and $-i$.

h) Evaluate: $\lim_{Z \to 2} \left\{\frac{Z^3 - 2Z^2 + 2Z - 4}{Z^2 - 3z + 2}\right\}$.

i) Prove that $f(z) = z^2$ is analytic.

j) Find the cross ratio of the four points *1, -1, i, -i*.

k) Find the Jacobian of the transformation $w = x^2 + iy^2$.

l) Find the fixed points of the transformation $f(z) = \frac{2z-3}{z-2}$.

2. Answer any THREE questions. Each question carries five marks. **3x5=15**

a) Show that the set of complex numbers is a vector space over the field of real numbers R.

b) Show that the set {(1,1,0), (1,0,1), (0,1,1)} forms a basis of the vector space V_3(R).

c) In $V_3(Z_3)$ how many vectors are spanned by (2,1,1) and (1,2,2).

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d) If ξ_1, ξ_2 *and* ξ_3 are linearly independent in $V_3(R)$ then show that $\xi_1+\xi_2$, $\xi_2+\xi_3$ and $\xi_3+\xi_1$ are also linearly independent in $V_3(R)$.

e) Prove that any two basis of a finite dimensional vector space have same number of elements.

3. Answer any THREE questions. Each question carries five marks. 3x5=15

a) Show that $T: R^3 \to R^2$ defined by $T(x,y,z) = (x+z,\ x+y+z)$ is a linear transformation.

b) Find a linear transformation $T: V_3(R) \to V_3(R)$ such that $T(2,1) = (3,4)$ and $T(-3,4) = (0,5)$.

c) Find the range, rank, kernel, nullity and verify the rank-nullity theorem for the linear transformation $T: V_3(R) \to V_3(R)$ defined by $T(x,y,z) = (x+y,\ x-y,\ 2x+z)$.

d) Find the inverse of the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 1 & -1 \\ 1 & 1 & 4 \end{bmatrix}$ using linear transformation.

e) If A is any square matrix, prove that A and A^T have same Eigen values.

4. Answer any THREE questions. Each question carries five marks. 3x5=15

a) Derive the equation of a circle passing through three non-collinear points z_1, z_2 and z_3.

b) Using the definition of derivative find $f'(z)$ at $z = i$, where $f(z) = \frac{z+1}{z-1}$.

c) Derive the Cauchy-Riemann equations in polar form.

d) Show that real and imaginary parts of an analytic function are harmonic.

e) Find the analytic function whose imaginary part is $\tan^{-1}\left(\frac{y}{x}\right)$.

5. Answer any THREE questions. Each question carries five marks. 3x5=15

a) Prove that the cross ratio of four points remain invariant under a bilinear transformation.

b) Find the bilinear transformation which maps $(0,1,\infty)$ onto $(-5,-1,3)$.

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c) Find the image of the triangle formed by the points $2 + 3i, -3 + i$ and $2 - 2i$ in the z-plane under the transformation $w = 2z + 1 + i$ in the w-plane.

d) Discuss the transformation $w = \frac{1}{2}\left(z + \frac{1}{z}\right)$.

e) Show that the transformation $w = iz$ is a rotation of the z-plane through the angle $\frac{\pi}{2}$. Find the image of the infinite strip $0 < x < 1$.

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