VI Semester B.Sc. Examination, November/December 2013
(Semester Scheme)
Mathematics (Paper – X)
Complex Analysis

Time : 3 Hours Max. Marks : 80

Instructions : *1) Section* ***A*** *is* ***compulsory.***
2) ***Five full*** *questions to be answered from Sections* ***B*** *and* ***C*** *choosing* ***atleast two*** *from* ***each*** *Section.*
3) ***All*** *questions in Sections* ***B*** *and* ***C*** *carry* ***equal*** *marks.*

SECTION – A

1. Answer **any ten** questions. **Each** question carries **two** marks.

a) Find the real and imaginary parts of $e^{-\frac{i\pi}{2}}$.

b) If $z_1 = 3 - 4i$ and $z_2 = 4 + i$ find $z_1 \odot z_2$ and $z_1 \times z_2$.

c) Find the equation of the straight line passing through the points $1 + i$ and $2 - i$.

d) Find the Locus represented by $z\bar{z} - 2z - 2\bar{z} + 8 = 0$.

e) If $z = x + iy$, find the modulus and amplitude of e^{iz}.

f) Show that $\lim\limits_{z \to 0} \dfrac{xy}{x^2 + y^2}$ does not exist.

g) Prove that $\sin iz = i \sinh z$.

h) Show that $f(z) = z I_m z$ is not analytic.

i) Show that the function $u = \log |z|$ is harmonic.

j) Evaluate $\int_0^{1+i} (x^2 - iy)\, dz$ along the path $y = x^2$.

k) Evaluate $\int_c (\bar{z})^2 dz$ where $c : |z| = 1$.

l) Find the fixed points of the transformation $w = \frac{1}{z - 2i}$.

m) Find the Jacobian of $f(z) = x^3 - iy^2$.

n) Find the cross ratio of the points 2, 2i, –1, – i.

o) Determine the poles of $f(z) = \frac{e^z}{z^2(z^2+9)}$.

SECTION – B

2. a) If z_1 and z_2 are non-zero complex numbers such that $\frac{z_1}{z_2}$ is purely imaginary, prove that $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$.

b) Show that $\operatorname{amp}\left(\frac{z-2+i}{z+2i}\right) = \frac{\pi}{4}$ represents a circle. Find its centre and radius.

c) Verify whether the points (– 3, – 4), (3, – 4), (2, 5) and (–5, 0) are concyclic or not.

3. a) Evaluate $\lim\limits_{z \to e^{i\pi/3}} \left[\frac{(z - e^{i\pi/3})z}{z^3+1}\right]$.

b) If $f(z) = \frac{z-1}{z+1}$ find $f'(z)$ at $z = 2 - i$ using definition of differentiability.

c) Derive the polar form of Cauchy-Riemann equations.

4. a) Show that $f(z) = e^z$ is analytic.

b) Find the analytic function f(z) whose real part is $u = \sin x \cosh y$.

c) Show that the function $u = 2x + x^3 + 3xy^2$ is harmonic and find the function v such that $f(z) = u + iv$ is analytic.

5. a) Evaluate $\int_c (x+2y)dx + (4-2x)dy$ around an ellipse $x = 4\cos\theta,\ y = 3\sin\theta$

   $0 \leq \theta \leq 2\pi$. Where C is taken in anticlockwise direction.

   b) Evaluate $\int_c \frac{1}{z(z-1)} dz$ where $c : |z| = 3$.

   c) State and prove Cauchy's integral formula for the derivative of a function f(z).

SECTION – C

6. a) If a function f(z) is analytic at all points within and on a closed contour C in a simply connected region R of the complex plane, then prove that $\int_c f(z)dz = 0$

   b) Evaluate $\int_c \frac{e^{2z}}{(z+1)^4} dz$ where $c : |z| = 3$.

   c) Evaluate $\int_c \frac{\sin 3z}{z + \frac{\pi}{2}} dz$ where $c : |z| = 5$.

7. a) Show that a bilinear transformation transforms circles into circles or straight lines.

   b) Find the bilinear transformation which maps $-1, 1, \infty$ on to $-i, \infty, 1$.

   c) Find the image of the circle $|z| = 3$ under the transformation $w = \frac{3z-1}{z+1}$.

8. a) Show that the bilinear transformation $w = \frac{z-i}{z+i}$ maps the upper half of the z – plane into the interior of the unit circle in the w-plane.

   b) Show that the transformation $w = iz$ is a rotation of the z-plane through an angle $\frac{\pi}{2}$. Find the image of the infinite strip $0 < x < 1$.

   c) Discuss the transformation $w = \sin z$.

9. a) Evaluate $\int_c \frac{z^2}{(z-2)(z+3)} dz$; Where c : | z | = 4 using Cauchy's residue theorem.

b) If f(z) has a pole of order 1 at z = a, then prove that Res $f(a) = \lim_{z \to a} [(z-a)f(z)]$.

c) Show that $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta} d\theta = \frac{\pi}{12}$.