



VI Semester B.Sc. Examination, April/May - 2018
(Semester : Scheme)

MATHEMATICS (Paper - VIII)
Complex Analysis and Numerical Analysis
(2015-16 Batch and Onwards)

Time : 3 Hours

Max. Marks : 80

Instruction: Answer all the sections.

SECTION - A

- I. Answer any eight questions . Each question carries two marks.
 - a) Find the equation of the line joining the points $2+3i$ and $1+2i$.
 - b) If $f(z) = \sin z$. find $f'(z)$ at $z = i$ using the definition of derivative.
 - c) Prove that $f(z) = z^2$ is analytic.
 - d) Evaluate $\int_0^{1+i} (x^2 - iy) dz$ along the curve $y = x$
 - e) Evaluate $\int_C \frac{1}{z-2} dz$ where C is the circle $|z|=4$
 - f) State fundamental theorem of Algebra.
 - g) Find an interval in which a real root of $x^3 - x - 4 = 0$ lies.
 - h) Use Newton - Raphson method to find $\sqrt{17}$ correct to three decimal Places.
 - i) Solve $\frac{dy}{dx} = y - x$, by Picard's method upto two approximations given $y(0) = 1$.
 - j) Prove that $(1 + \Delta)(1 - \nabla) = 1$
 - k) Construct the forward difference table for $f(x) = x^3 + 1$ for $x = 0$ (1)5
 - l) State simpson's $\frac{1}{3}$ rd rule for 'n' intervals.

SECTION - B

II. Answer any eight questions. Each question carries four marks.

- Define continuity of $f(z)$ at $z = z_0$ and show that $f(z) = \frac{z^2}{z^4 + z^2 + 1}$ is continuous at $z = e^{i\pi/2}$
- Find whether the points $(2,1), (3, 5), (-2,0)$ and $(1,-1)$ are concyclic or not.
- State and prove Cauchy - Riemann equations in polar form.
- Find the analytic function $f(z)$ whose real part is $e^x \cos y$ and find its imaginary part.
- Prove that the function $u = x^3 - 3xy^2$ is harmonic and find its harmonic conjugate.
- State and prove Cauchy's integral formula.
- Evaluate $\int (\bar{z})^2 dz$ around the circle $|z-1| = 1$.
- Show that $\int_C \frac{z^2 - 4}{z(z^2 + 9)} dz = \frac{-8\pi i}{9}$ where 'C' is the circle $|z| = 1$
- Evaluate $\int_C \frac{z \cos z}{(z - \frac{\pi}{2})^2} dz$ where C is the circle $|z - i| = 3$.
- State and prove Cauchy's inequality.

SECTION - C

III. Answer any eight questions. Each question carries four marks.

- Find a real root of the equation $x^3 - 2x - 5 = 0$ by Bisection method correct to three decimal places.
- Find a real root of the equation $\cos x - 3x + 1 = 0$ correct to three decimal places by the method of false position.

- c) Use modified Euler's method to solve $\frac{dy}{dx} = x - y^2$, given that $y(0) = 1$ for $x = 0.2$ with $h = 0.1$

- d) Apply Runge - Kutta fourth order method to solve

$$\frac{dy}{dx} = 2x - y \text{ with } y(0) = 1, \text{ for } x = 0(0.5)1$$

- e) Estimate the population for the year 1995 from the given table.

Year	1960	1970	1980	1990	2000
Population in crores	46	66	81	93	101

- f) Use Newton - Gregory formula to find a polynomial in x for the data.

x	0	1	2	3
$f(x)$	2	3	12	35

- g) Using Lagrange's interpolation formula find $f(5)$ given that $f(1) = 2, f(2) = 4, f(3) = 8$ and $f(7) = 128$

- h) Derive general quadrature formula.

- i) Evaluate $\int_0^1 \frac{x}{1+x^4} dx$ with $n=4$ using trapezoidal rule, hence find an approximate value of π .

- j) Evaluate $\int_0^3 (x^4 + x) dx$ with $n=6$ by using weddle's rule.

