Photon geodesics on Semi-classically corrected Schwarzschild metric

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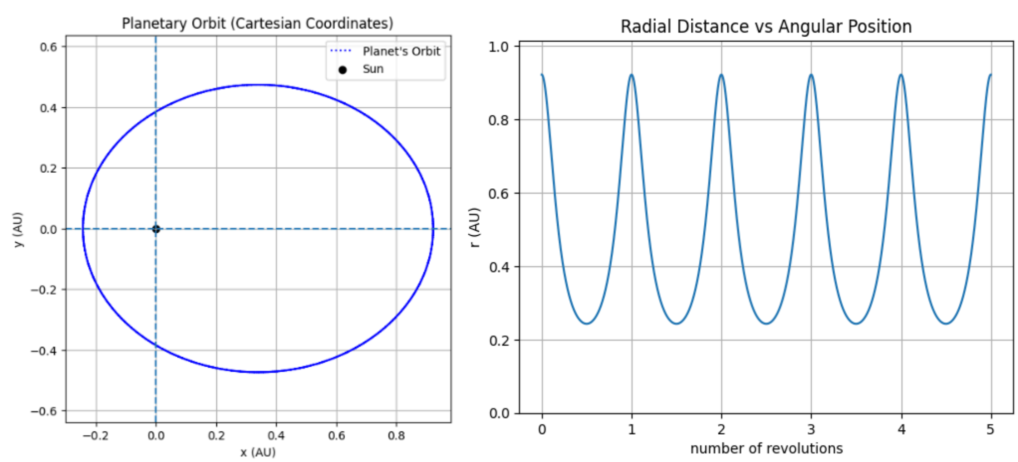
This document contains a brief summary of the python plots I obtained during the Summer of 2024 (May-July) while working under Prof. Arundhati Dasgupta, University of Lethbridge, Alberta.

Overview

For completeness I have incorporated the simulated plots of the following :

1. Geodesics for massive test particle in Newtonian gravitation
2. Geodesics for massive test particle in Schwarzschild background
3. Geodesics for light in Schwarzschild background
4. Geodesics for light in Semi-classically corrected Schwarzschild metric

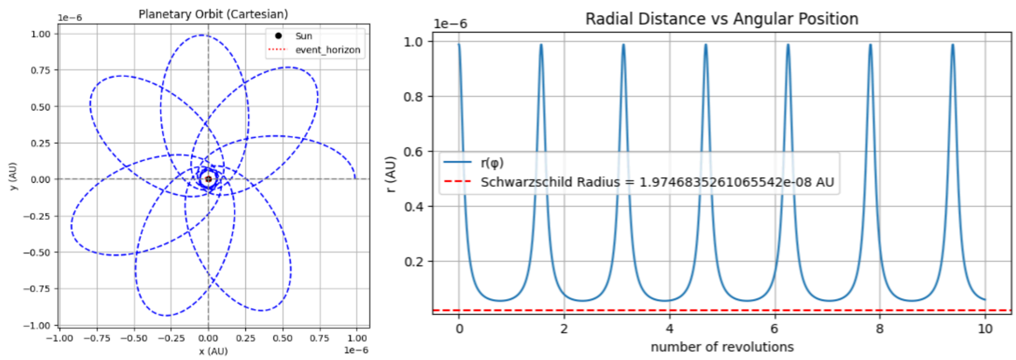
Newtonian Gravitation



Remarks. The test particle starts at the aphelion, and is simulated for 5 periods of revolution.

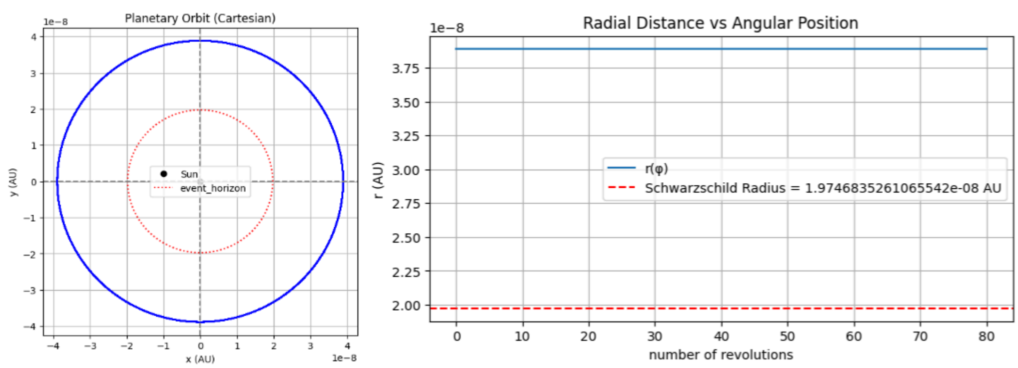
Schwarzschild Metric : Test particle has non-zero mass

1. Precessional Motion for an orbiting test particle



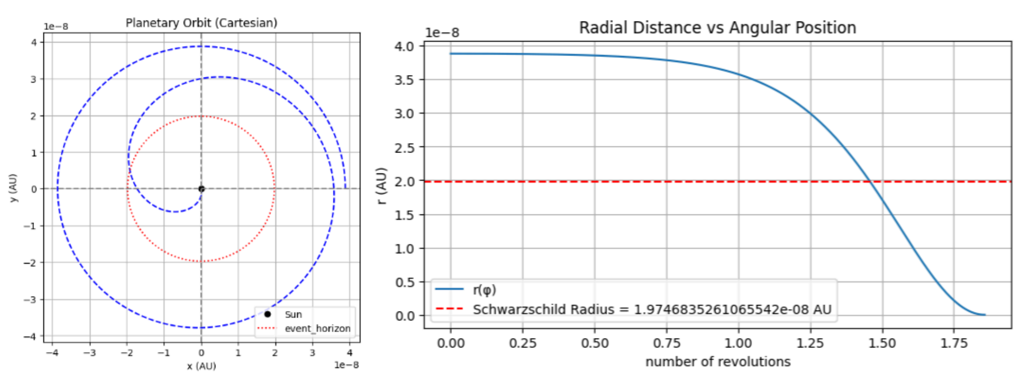
Remarks. Orbit simulated for a massive test particle shows precession of the orbit axes as predicted by the theory. As before, the particle started at the aphelion.

2. Circular Orbits



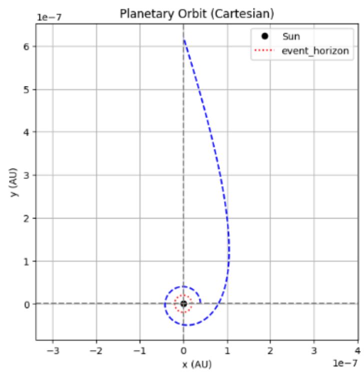
Remarks. For the Schwarzschild background two circular orbits exist (one stable and the other unstable). The unstable circular orbit has been simulated 80 revolutions.

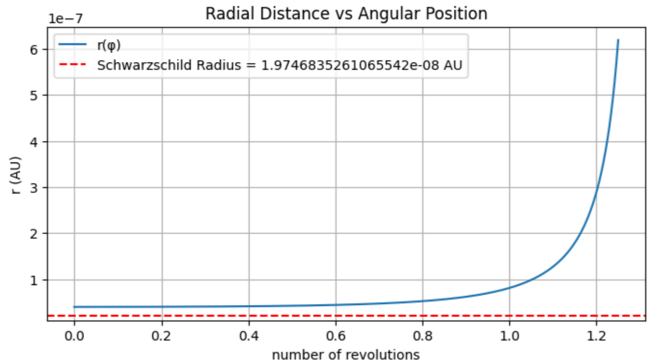
3. Test Particle crosses the event horizon and falls inside



Remarks. During the second revolution, the test particle crosses the event horizon and thus, falls inside the black hole

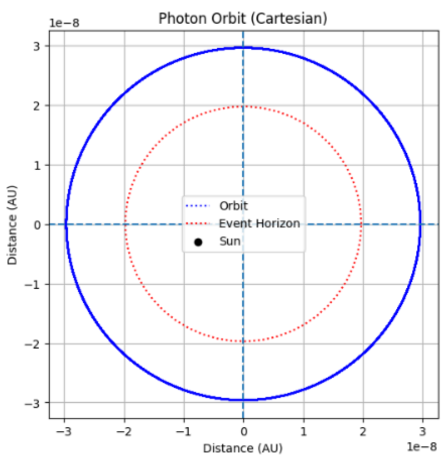
4. <u>Test particle starts tangentially near the black hole and escapes it</u>

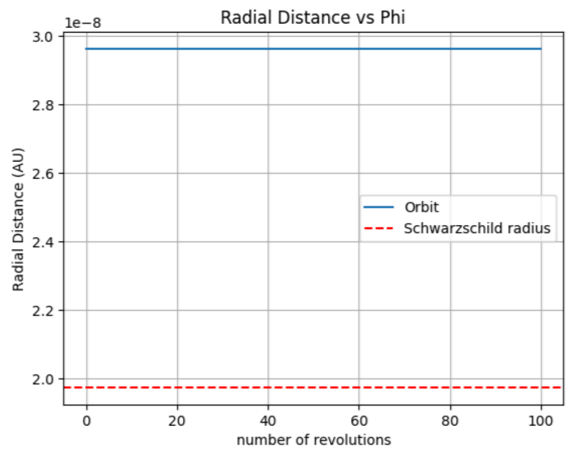




Schwarzschild Metric : Photon Geodesics

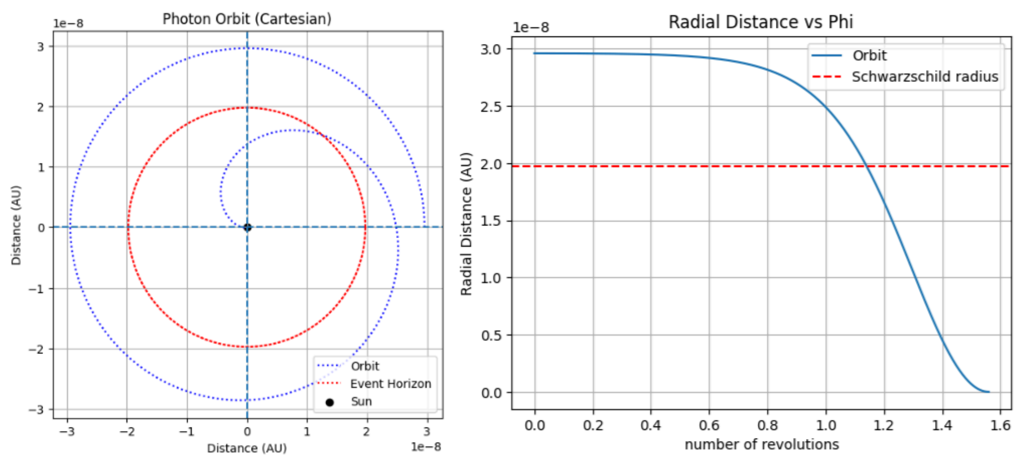
1. <u>Circular Orbits</u>





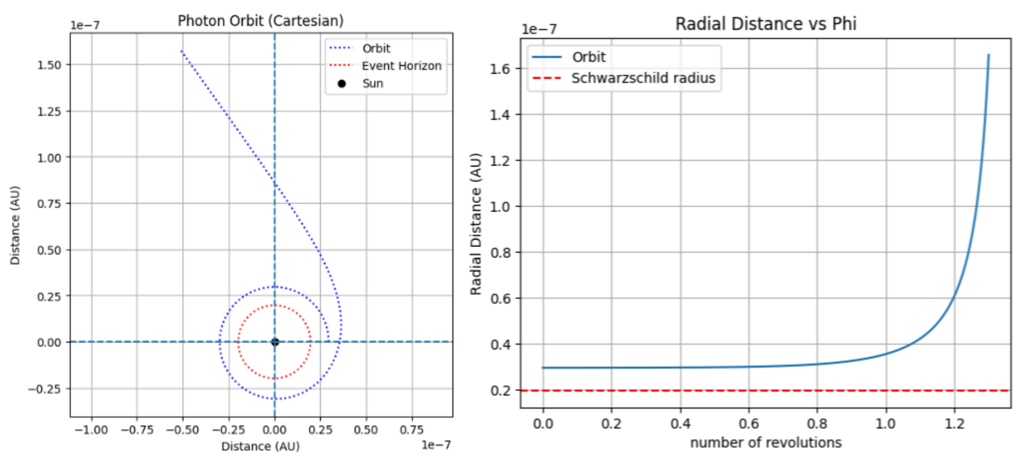
Remarks. Unlike the geodesics for massive test particles, the circular photon geodesic can only be unstable. This happens when the periastron distance is 1.5 times the Schwarzschild radius. The simulation shows the stability of the circular orbit for 100 revolutions.

2. <u>Photon falls inside the event horizon</u>



Remarks. As seen in the right subplot, the test photon starts at a radial distance slightly less than 1.5 r_s = 3 scales. This corresponds to an unstable inward swirling trajectory as per theory. The simulation agrees with the known prediction.

3. Test photon starts tangentially near the black hole and escapes it



Remarks. The trajectory starts with a radial distance slightly more than 1.5 r_s = 3 scales, thus leading to the test photon escaping the blackhole.

Semi Classically corrected Schwarzschild background

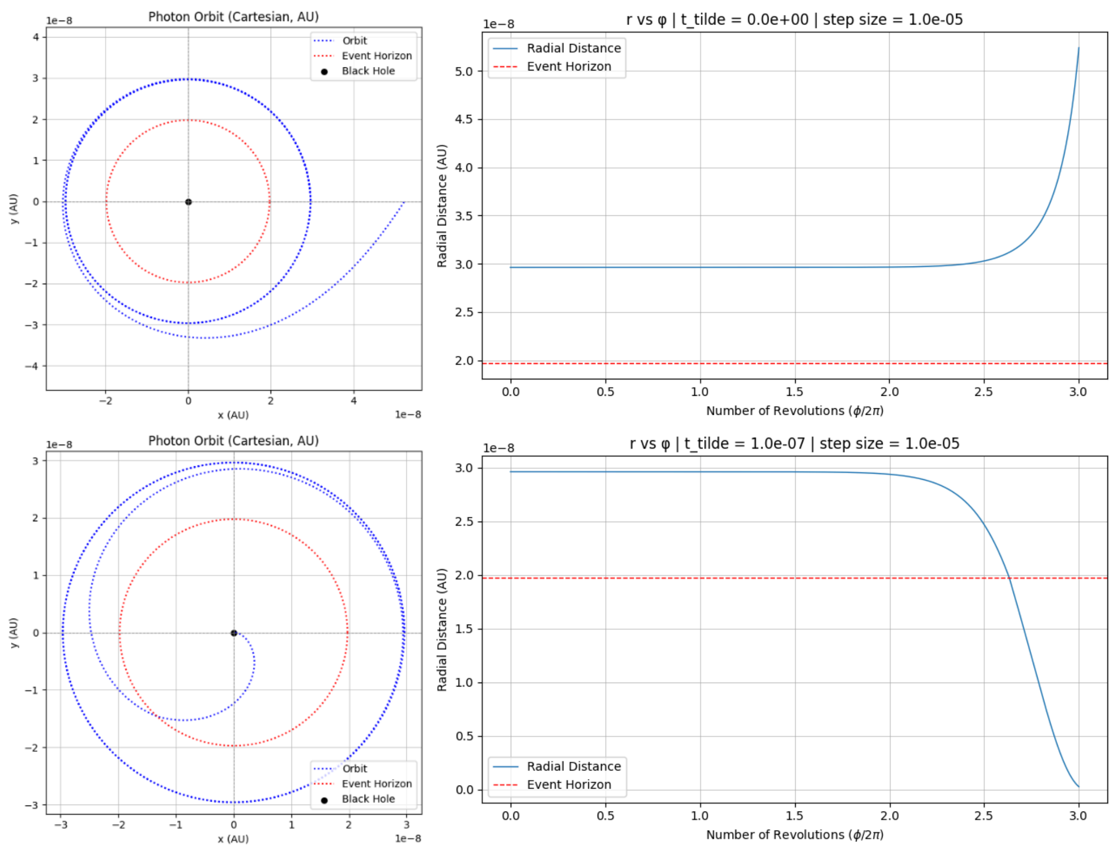
Finally, simulations were run to obtain the nature of photon geodesics when the Schwarzschild metric is modified using semiclassical corrections originating from a previous paper of LQG. Analytic and numerical calculations concerning some of the key concepts of this corrected metric were shown in Swayamsiddha's paper.

In Swayamsiddha's paper, the expression for the critical value of periastron distance was obtained as below :

$$P_{critical} = 1.5r_s\,[1 + \alpha\,\bar{t}]$$

where $\alpha \approx 2.030/3 \approx 0.677$

1. <u>Comparison between Corrected and Uncorrected metric</u>



Remarks. The figure above shows simulation results for two values of the parameter $\bar{t}$, i.e. the parameter that controls the quantum effects.

a. The subplots in the first row correspond to $\bar{t}$ = 0, i.e. the case of an uncorrected background. The test photon starts tangentially at a radial separation of

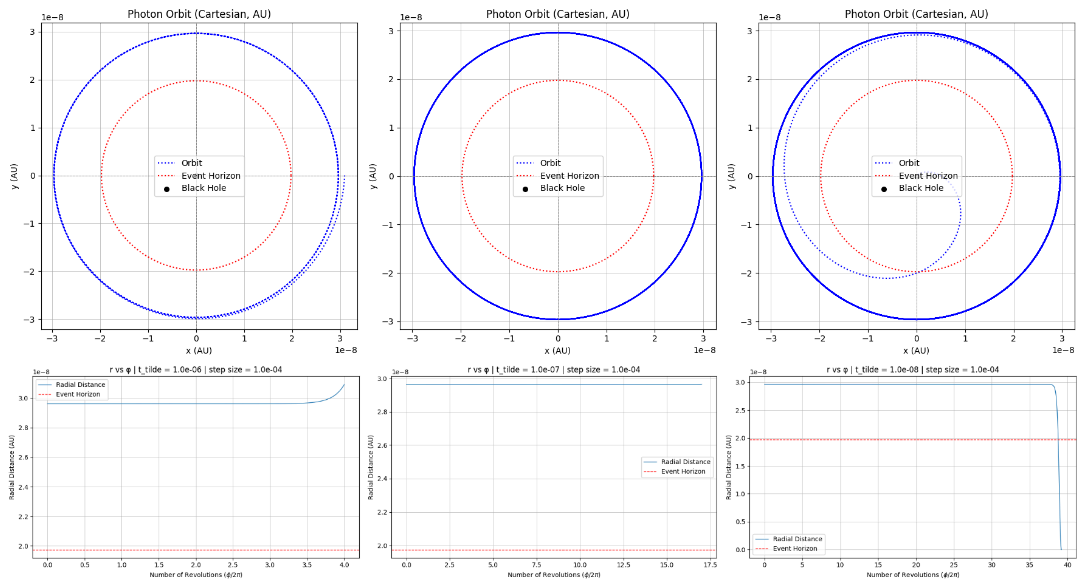
   r_0 = r_s (1.5+10^{-8})

   i.e. very slightly outside the event horizon. This leads to the photon escaping the blackhole after circling it a certain number of times.

b. The subplots in the second row correspond to $\bar{t}$ = 10^{-7}, i.e. the case of a corrected background. Note that all parameters other than $\bar{t}$ are held the same. The test photon starts tangentially with the same radial separation as above. But for this case, the photon falls inside the event horizon. This suggests that the *critical value of periastron distance must have increased*.

2. <u>Circular Orbit for corrected metric</u>

As inferred from the previous section, the semiclassical corrections push the surface of circular orbits outward as compared to the ones without any correction. We try to simulate the circular (or atleast nearly circular) orbits for the corrected metric and find the new value of the critical parameter (periastron distance, P) by manually tweaking it until a circular orbit is obtained.



Remarks. The three columns show the simulated trajectories for $\bar{t}$ values : 10^{-6},10^{-7} and 10^{-8} respectively from left to right.

a. $\bar{t} = 10^{-6}$.

The test particle traverses a fairly circular orbit for 3 revolutions. The value of 'α' lies between 0.630804 and 0.630808.

b. $\bar{t} = 10^{-7}$.

The trajectory of the test photon is circular for 17 revolutions. On the 18th revolution, the orbit becomes unstable and the test photon escapes. The value of 'α' is estimated to be 0.6308085.

c. $\bar{t} = 10^{-8}$.

The trajectory of the test photon stays sufficiently circular for 38 revolutions. On the 39th revolution, the orbit becomes unstable and the test photon falls into the event horizon. The value of 'α' is estimated to be 0.630808.

Since the value of $\bar{t}$ lies between 10^{-66} and 10^{-9} , we will give the most preference to the last of the three columns. And even if not that, the three values/ranges of the values of ‘α’ are consistent with each other up to 5th decimal.

Conclusion

Swayamsiddha’s paper reported the value of ‘α’ to be around 0.677, while our estimate of the same using simulated plots and tweaking the parameter gives ≈ 0.63080. Although the two values aren’t exactly the same, they are close. Further, various approximations were made to obtain that value in Swayamsiddha’s paper, which might be the cause behind the slight mismatching of the two values.
