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Stress Analysis of an Euler-Bernoulli Beam

ES221-Mechanics of Solids

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Problem Statement

We are creating a program to analyze stress in simply supported beams subjected to point loads and uniformly distributed loads (UDLs). Users will input the beam length, load magnitudes, and their positions, and the program will generate diagrams showing shear force (SFD), bending moment (BMD), and the beam's deflection.

Objectives

- Create an interactive interface where users can input beam span length, point load details, and UDL parameters, and receive detailed structural analysis results.
- Generate clear and accurate plots showing the Shear Force Diagram (SFD) and Bending Moment Diagram (BMD) based on the loading configuration.
- Develop algorithms to calculate support reactions, shear forces, bending moments, and deflections along the beam using numerical and symbolic methods.
- Provide a visual representation of the beam's deflection under the specified loads and material properties.

Methodology

Beam Model
The beam is modeled as a *simply supported beam* with supports at the ends.
Loads considered:

- Point Loads: single forces at specific locations.
- UDLs (Uniformly Distributed Loads): distributed forces between two positions.

Theoretical Basis: Equilibrium equations:

$$\sum F_y = 0, \quad \sum M_A = 0$$

are used to determine reactions at supports.

Shear Force Calculation:
Shear at a section is computed as:

$$V(x) = R_A - \sum P_i - \int w(x)\, dx$$

Bending Moment Calculation:
Moment at a section is calculated as:

$$M(x) = R_A \cdot x - \sum P_i(x - a) - \int w(x)(x - x_c)\, dx$$

Deflection:
From bending theory,

$$\frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

The deflection curve is obtained by successive numerical integration using the trapezoidal rule.

MATLAB Implementation

- *Input Section*: Beam length, number of point loads and UDLs, their magnitudes, and positions.
- *Reaction Calculation*: Symbolic solution of equilibrium equations.
- *Discretization*: Beam divided into small intervals (dx = 0.01 m).
- *Calculation Loop*: Shear force and bending moment at each section are computed.
- *Deflection Module*: Uses numerical integration of $M/(EI)$ twice to obtain slope and deflection, enforcing boundary conditions (zero deflection at both ends).
- *Plotting Section*: Plots Shear Force Diagram (SFD), Bending Moment Diagram (BMD), and Deflection Curve.

Results and Discussion

The program was tested for a simply supported beam of length 10 meters, subjected to two point loads (1000 N at 1 m and 2000 N at 4 m from the left support) and one uniformly distributed load (6000 N/m from 3 m to 8 m). The material and geometric properties entered were Young's Modulus $E = 2 \times 10^{11}$ Pa and Moment of Inertia $I = 0.0001$ m^4.

The MATLAB Command Window output for this loading case is shown below. It highlights the step-by-step inputs and reports the maximum deflection of 29.76 mm at $x = 5.06$ m.

```
Command Window
Enter span length of beam (m): 10
Number of point loads? 2
Load magnitude (N): 1000
Distance from left support (m): 1
Load magnitude (N): 2000
Distance from left support (m): 4
Number of UDLs? 1
UDL intensity (N/m): 6000
Start position (m): 3
End position (m): 8
Enter Young's Modulus E (Pa): 200000000000
Enter Moment of Inertia I (m^4): 0.0001
Maximum deflection: 29.76 mm at x = 5.06 m
fx >>
```

Figure 1: Sample Command Window showing input parameters and computed maximum deflection

The program generates clear visualizations for the shear force, bending moment, and deflection diagrams along the beam. Figure 2 presents these diagrams for the specified loading case, marking points of interest including point load locations, UDL start/end positions, zero shear location, and position of maximum moment.

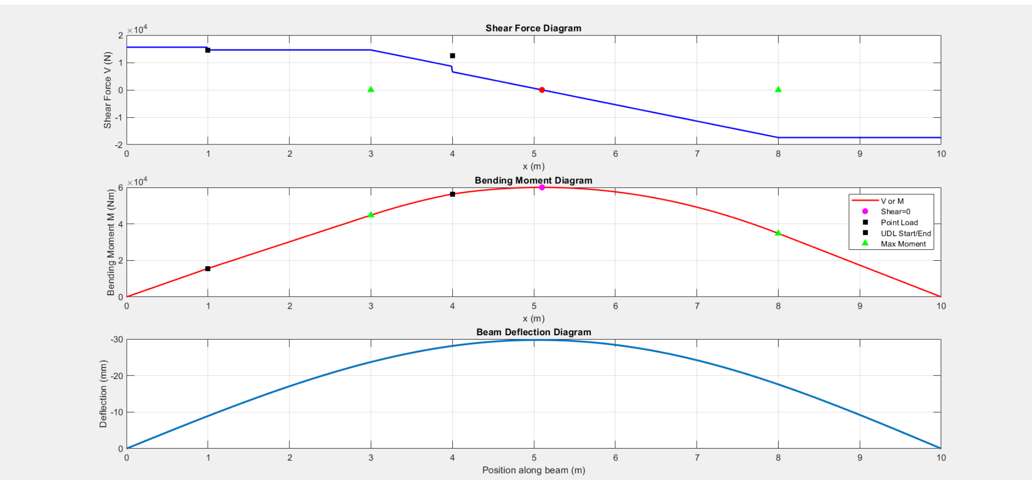


Figure 2: Shear Force Diagram (top), Bending Moment Diagram (middle), and Beam Deflection Diagram (bottom) for the specified loading case

These results demonstrate that the software efficiently computes support reactions, internal force distributions, and beam deflections for complex loading scenarios, providing both numerical values and intuitive graphical insights. The inclusion of visual outputs makes interpretation straightforward and enhances the utility of the tool for design and analysis purposes.

Web Application for Beam Analysis

To enhance accessibility and interactive visualization, the MATLAB-based analysis tool was extended into a web application:
`https://beam-analysis-calculator.vercel.app/`

The web interface allows users to input beam and loading configurations easily, and instantly visualize the structural response.

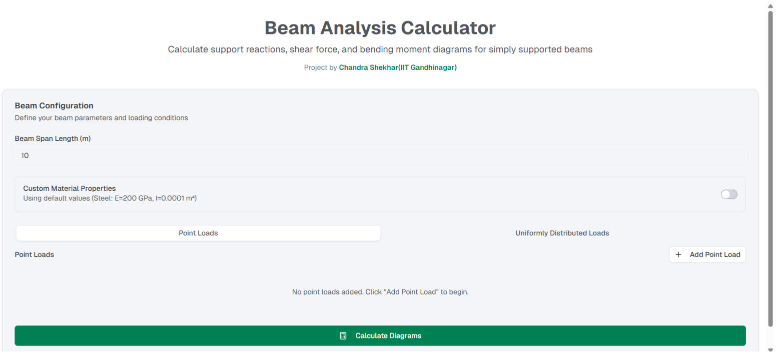
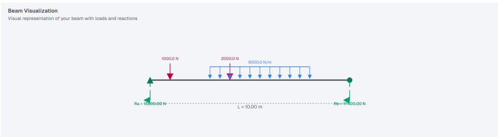


Figure 3: Home interface of the Beam Analysis Calculator web application showing initial configuration options for the user.

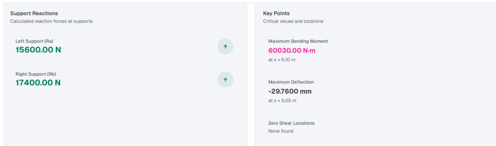
Figure 3 presents the main landing page of the web application, where users enter beam span length, specify custom material properties, and add point loads or distributed loads through a user-friendly interface.

For the same example analyzed earlier (a 10 m beam with two point loads and one UDL), Figures 4 summarize the graphical and numerical outputs delivered by the website:

The web application dynamically computes and displays support reactions, the position and magnitude of the maximum bending moment, and maximum deflection for any combination of loads. High-quality plots for SFD, BMD, and deflection are generated instantly, making it a powerful tool for both learning and design validation. The results shown were cross-validated with MATLAB outputs, confirming consistency and reliability across platforms.



(a) Beam visualization with applied loads and computed support reactions.



(b) Key numerical results: support reactions, maximum bending moment, maximum deflection, and their locations.

(c) Graphical output: Shear Force Diagram (SFD), Bending Moment Diagram (BMD), and Deflection Curve, as computed by the web app.

Figure 4: Composite view of the web application: (a) beam visualization, (b) key numerical results, and (c) graphical diagrams.

Limitations and Future Improvements

This program is currently limited to simply supported beams under vertical point loads and uniform distributed loads. Future improvements could include:

- Extending analysis to other beam types such as cantilever and fixed beams.
- Including non-uniformly distributed loads such as triangular or varying intensity loads.
- Refining numerical integration methods for improved accuracy in deflection calculation.
- Adding a graphical user interface (GUI) to enhance user experience.

Learning Outcomes

- Gained a thorough understanding of static equilibrium relations to calculate beam support reactions.
- Applied numerical methods to determine shear force and bending moment distributions.
- Integrated Euler-Bernoulli beam theory for calculation of deflection using numerical integration.
- Improved proficiency in MATLAB programming for structural analysis and visualization.
- Developed skills in interpreting structural response diagrams like SFD, BMD, and deflection curves.

Overall, the project has strengthened understanding of structural mechanics principles and computational techniques essential for practical beam analysis.

References

1. R. C. Hibbeler, *Structural Analysis*, 10th Edition, Pearson Education, 2016. A comprehensive textbook covering equilibrium, shear force and bending moment diagrams, and deflection of beams.

2. S. P. Timoshenko, *Strength of Materials*, Part II, 3rd Edition, D. Van Nostrand Company, 1956. Classic text on beam theory including Euler-Bernoulli bending equations used to determine deflections.

3. J. M. Gere and S. P. Timoshenko, *Mechanics of Materials*, 4th Edition, PWS Publishing Company, 1997. Contains detailed explanations of internal forces, moments, and beam deflection calculations.