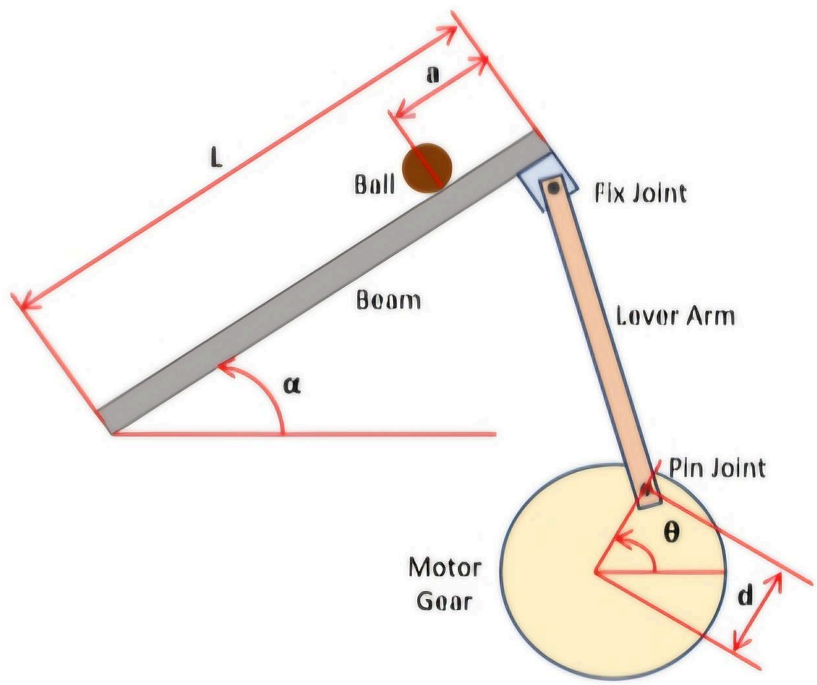
Ball and Beam (Part 2)



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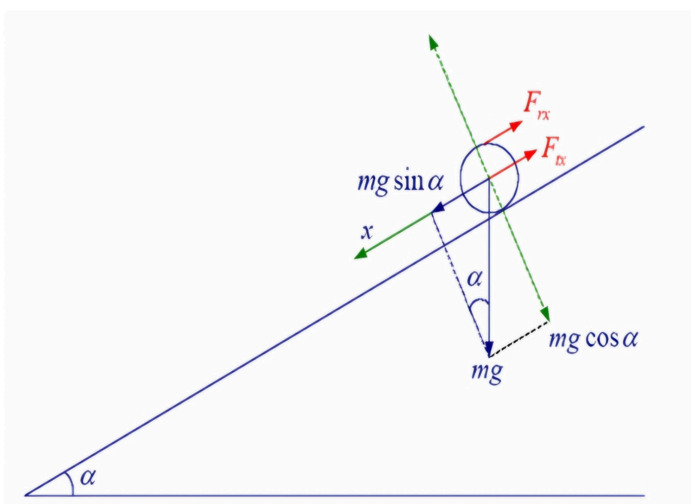
1. INTRODUCTION

The Ball and Beam system is a classic control problem involving stabilizing a ball rolling on a beam. A servo motor adjusts the beam's angle, and the objective is to control the ball's position by manipulating the servo gear angle. This system is naturally unstable, and the ball will roll off the beam without control.
In this project, we aim to model the system dynamics, linearize the equations, and design a controller to stabilize the ball at a desired position. The project includes MATLAB simulations, controller design (PID), and building a physical system to implement and test the control strategy.

2. OVERVIEW

- **Defining the System**

Mass of the ball: m = 0.011 kg
Radius of the ball: R = 0.015 m
Lever arm offset: d = 0.09 m
Gravitational acceleration: g = 9.8 m/s2
Length of the beam: L = 3.0 m
Ball position coordinate: x
Beam angle coordinate: α
Servo gear angle: θ



According to Newton's Second Law along the inclined beam:

$$F_{tx} + F_{rx} = mg \sin \alpha$$

where

$$F_{tx} = m\ddot{x}, \quad F_{rx} = (J/R^2)\ddot{x}$$

$$m\ddot{x} + (J/R^2)\ddot{x} = mg \sin \alpha$$

The original system is inherently nonlinear due to the presence of the term sin(α) in the governing equation of the ball's motion:

$$m\ddot{x} + (J/R^2)\ddot{x} = mg \sin(\alpha)$$

Nonlinear systems are generally more complex to analyze and control, especially when designing controllers using classical or modern control theory, which often rely on linear system models.

To simplify the analysis and design of controllers, we linearize the system around an equilibrium point. In this case, the system operates near a small angle α, so we can use the small angle approximation:

sin(α) ≈ α (for small angles, where α is in radians).

By making this approximation, we transform the original nonlinear equation into a linear form:

$$(m + (J/R^2))\ddot{x} = mg\alpha$$

From geometry, the value of $\alpha = (d/L)\theta$. Thus, we have:

$$(m + (J/R^2))\ddot{x} = mg(d/L)\theta$$

The equation of motion is:

$$\ddot{x} = (mgd/(L(m + (J/R^2))\ddot{x}))\theta$$

Taking the Laplace Transform:

$$s^2X(s) = (mgd/(L(m + (J/R^2)))\theta(s)$$

The transfer function is:

$$P(s) = X(s)/\theta(s) = (mgd/(L(m + J/R^2)))(1/s^2)$$

Task 6: Root Locus and Bode Plot of Ball-Beam System

6.1 Using MATLAB, plot the following for the system's open-loop transfer function:

- **Plot root locus and comment on the system's stability.**
  The plot for the root locus of the designed ball and beam system is as follows:

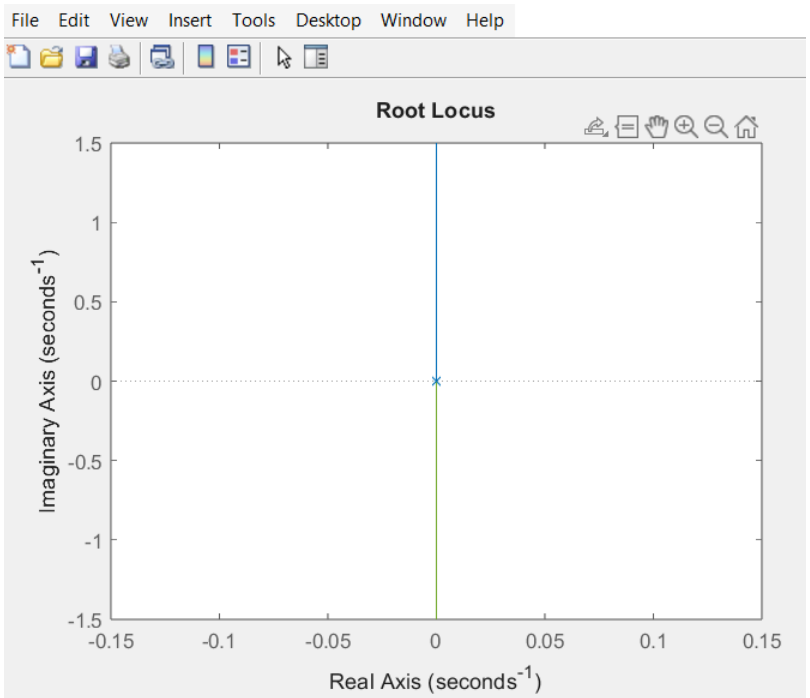


Fig. Root locus plot for the system

From the above plot, we can infer that the root is on the imaginary axis at s=0 with no other branches. Thus, the system appears to be marginally stable. Since there are no poles in the left half of the plane, the system is not strictly unstable, but it is also not asymptotically stable (no poles in the left half plane. This marginal stability suggests that without further feedback or control, any disturbances may cause sustained oscillations.

- **Plot the bode plot and report the system's phase margin and gain margin.**
  The bode plot for the designed system is as follows:

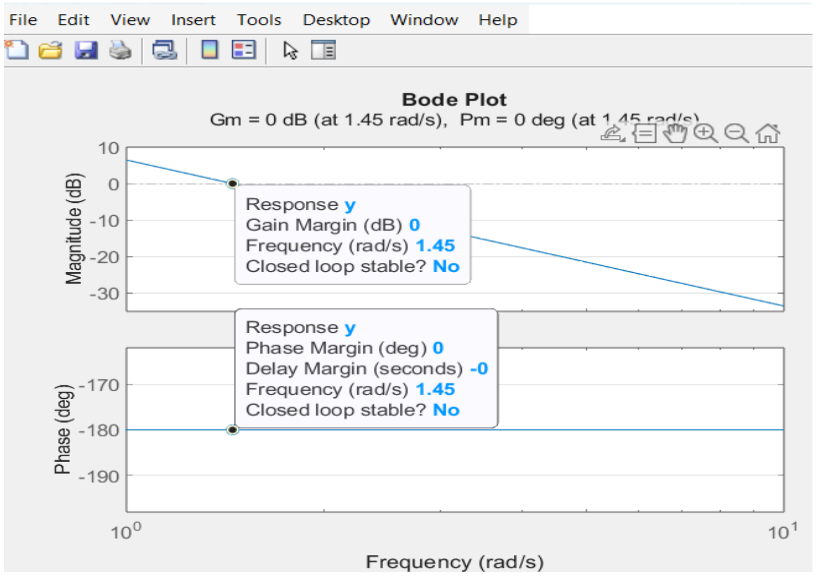


Fig. Bode plot for the system

From the bode plot, we can deduce that the gain margin comes out to 0 dB.
The Phase margin is 0°.

6.2 Consider the design criteria: less than 5% overshoot and settling time less than 3 seconds within a 2% tolerance band. Complete the following:

- **Root Locus: Plot the design criteria using MATLAB (Hint: use grid command) and recommend the appropriate compensator to achieve the desired performance.**

  The plot of the root locus of the system with the design criteria of overshoot less than 5% and settling time less than 3 seconds with a tolerance band of less than 2% is as follows:

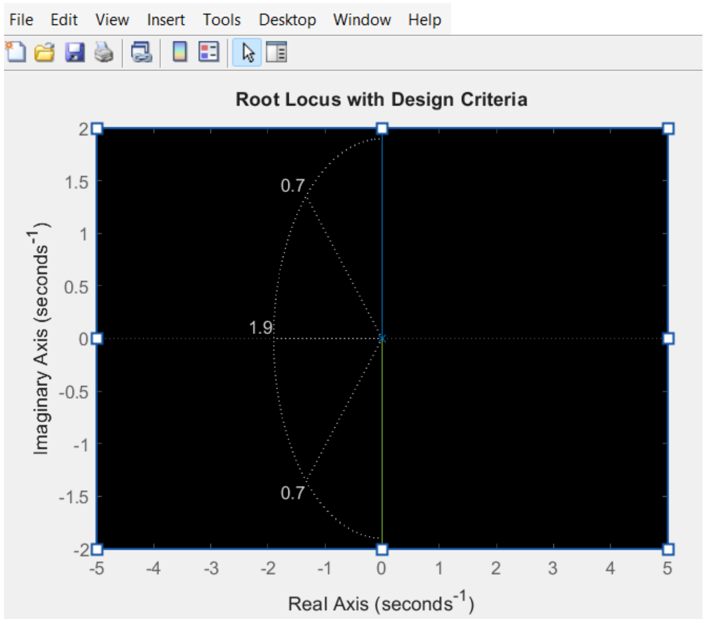


Fig. Root locus plot with the given design criteria

To meet the design criteria of less than 5% overshoot and a settling time of less than 3 seconds (within a 2% tolerance band), we have set the damping ratio to 0.7 and a natural frequency of at least 1.9 rad/s, which aligns with these requirements. This is done because the initial root locus plot shows that the system was marginally stable with a pole at the origin, which does not satisfy the design criteria. We shift the poles into the left half-plane with sufficient damping and natural frequency to achieve the desired transient response. Therefore, we use a **Lead Compensator**. A lead compensator can improve the damping ratio and the transient response by moving the poles to the left in the s-plane.

- **Bode Plot: analyze the plot to recommend the compensator to meet the design criteria.**

  The bode plot of the system with the required design criteria is as follows:

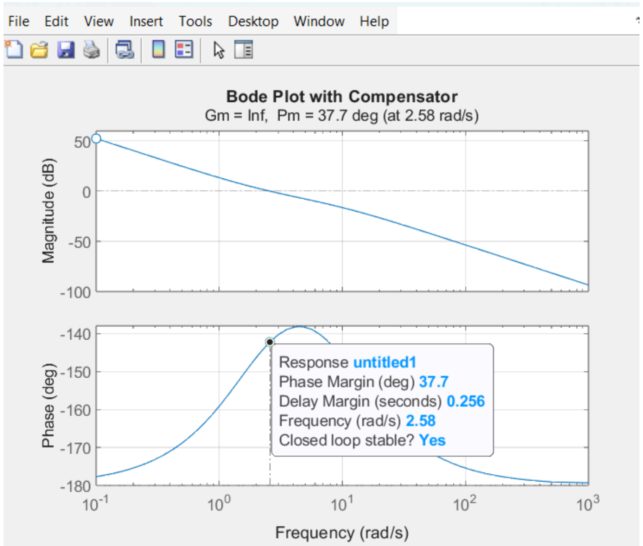


Fig. Bode plot with the given design criteria

The analysis of the system is:

- ➔ Phase Margin: The current phase margin is 37.7°, corresponding to a damping ratio of approximately 0.7. Based on standard second-order system approximations, this might result in an overshoot close to or slightly above 10%.
- ➔ Settling Time: From the given delay margin of 0.256 seconds, the bandwidth and pole location seem insufficient to achieve the desired settling time of less than 3 seconds.
- ➔ Thus, the system is stable but needs further tuning for improved dynamic performance.

To design a compensator to satisfy the design specification of an overshoot of less than 5% and a settling time of less than 3 seconds with a tolerance band of less than 2%, we require an increase of the phase margin and an improvement of settling time. The lead compensator is thus to boost phase margin by adding one zero and one pole but ensuring z<p. We need to adjust the compensator gain and pole-zero placement to improve settling time and increase the system's bandwidth. The target bandwidth should support a dominant time constant of around 1 second (to meet settling time < 3s).

The recommended Lead Compensator design will be:

$C(s) = K\frac{s+z}{s+p},\ where\ z < p\ and\ \frac{z}{p} = 0.1\ to\ 0.5$

Through this compensator, the increased phase margin (target: 50°–60°) will result in a damping ratio of 0.6–0.7, reducing overshoot to below 5%. Higher bandwidth will decrease settling time to meet the requirement of under 3 seconds. The compensator maintains closed-loop stability.

Task 7: Controller Design using Root Locus and Bode Plot Approaches

7.1 Based on the previous recommendations, design a first-order lead/lag compensator to meet the design criteria:

The first-order lead/lag compensator with the design criteria of less than 5% overshoot and a settling time of less than 3 seconds (within a 2% tolerance band), is done using MATLAB and the compensator transfer function is mentioned in its code.

- Plot the root locus of the system with the compensator.

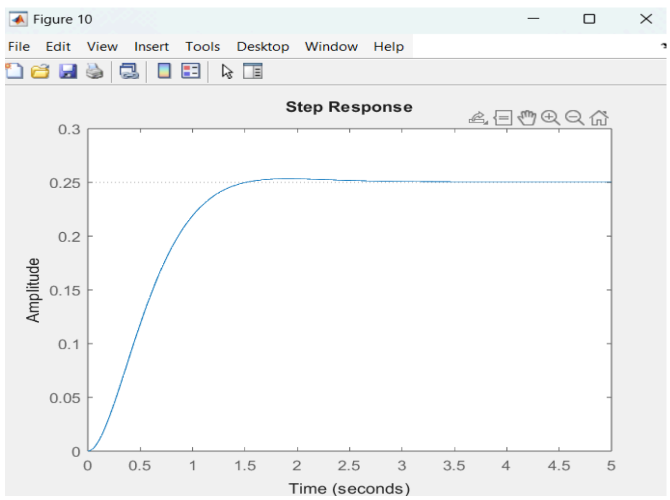


Fig. Step response with compensator

The root locus plot with the compensator is as follows:

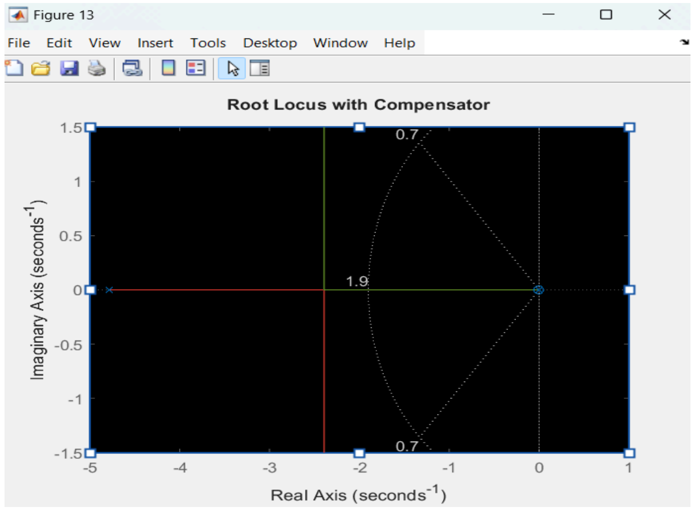


Fig. Root locus plot with the compensator

```
Maximum overshoot time : 1.32 %
Settling time : 1.3450 seconds
>>
```

Fig. The maximum overshoot and settling time of the system

- Plot the bode plot of the system with the compensator.

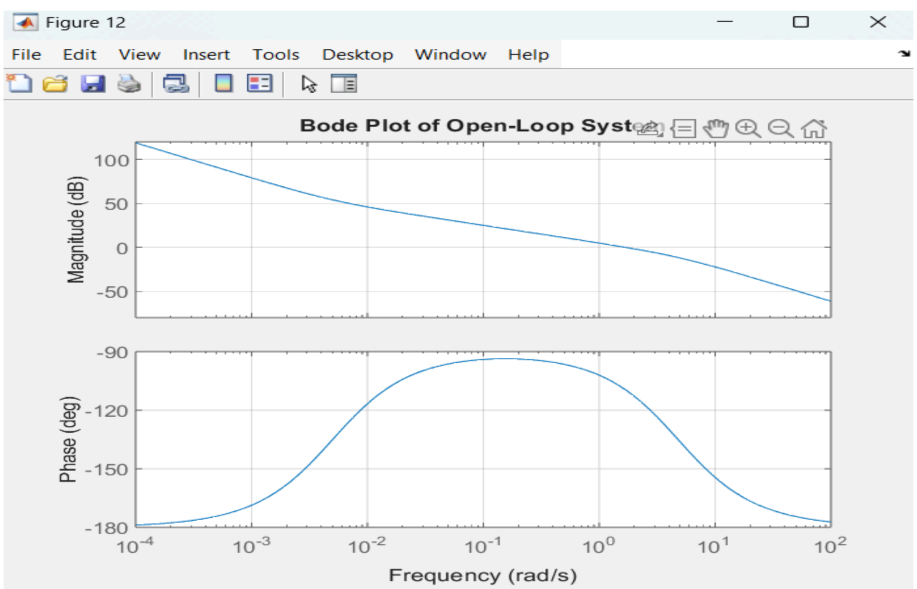


Fig. Bode plot with the compensator

7.2 Comment on the pole-zero placement strategy used in both approaches and explain how the compensator pole-zero placement affects the root locus and bode plot.

In the Lead compensator are used to change the transient condition of the system and the system faster response. Since we want to change our transient condition such that overshoot of the open loop transfer function should be less than 5% and settling time should be less than 3 seconds.

$$\therefore\ G(s) = \frac{mgd}{L\left(m + \frac{J}{R^2}\right)s^2}$$

$\therefore\ m = \cancel{0.002}\ 0.011\ kg$

$g = 9.81\ m/s^2$

$d = 0.09\ m$

$L = 0.3\ m$

$R = 0.015\ m$

$J = \frac{2}{5} mR^2$

After putting values →

$$G(s) = \frac{2.21}{s^2}$$

lead compensator:

$$G_c(s) = k_c \alpha \frac{(Ts+1)}{(\alpha Ts+1)}$$

In the root locus approach, we'll find the angle deficiency and found the zeros and poles after applying the magnitude condition the compensator found as -

$G_c(s) = k_c(s+Z)/(s+p) = (4.0206(s+0.005))/(s+4.79)$.

The root locus plotted in the above question illustrates the movement of the closed-loop system poles as the gain is varied. There are two initial poles on the real axis, indicating an underdamped system. The presence of a zero at the origin suggests that the system has a type 1 response, implying zero steady-state error for step inputs. The root loci move towards the left half-plane as the gain increases, potentially improving system stability. However, the loci also move towards the imaginary axis, which could lead to instability if the gain is too high.

7.3 Plot the system's closed-loop response (with the compensator) for a step input. Verify that the system satisfies the design criteria using both the root locus and bode plot approaches.

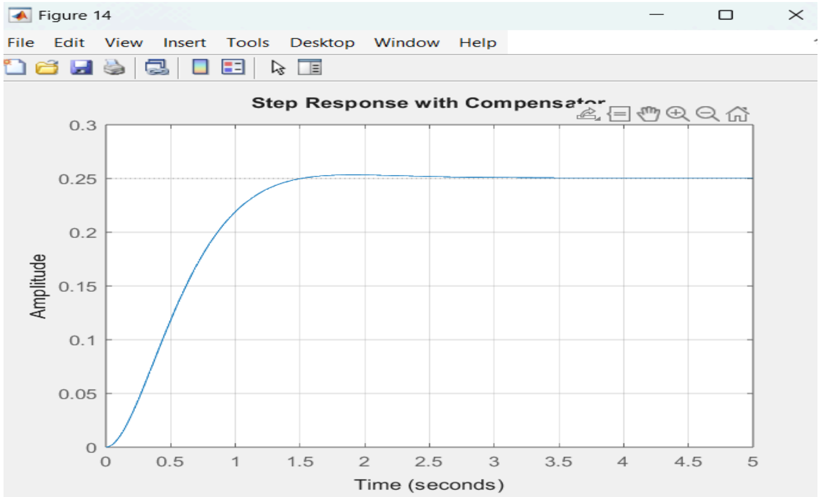
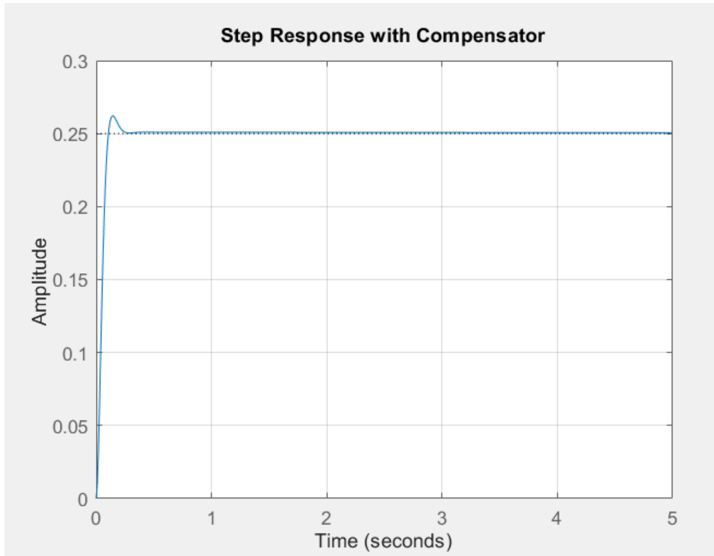


Fig. Step response with the compensator

```
Maximum overshoot: 1.32 %
Settling time: 1.3450 seconds
Design meets the criteria: less than 5% overshoot and settling time less than 3 seconds.
>>
```

Fig: Step Response for a step input with Compensator designed using Root Locus

This design meets the criteria. The overshoot is well below the 5% limit at 1.32%, and the system settles in just over 1.34 seconds, comfortably within the 3-second requirement. The response shows a smooth rise and steady-state approach, indicating a stable and well-damped system. The root locus design has effectively tuned the compensator to achieve these favourable performance characteristics.

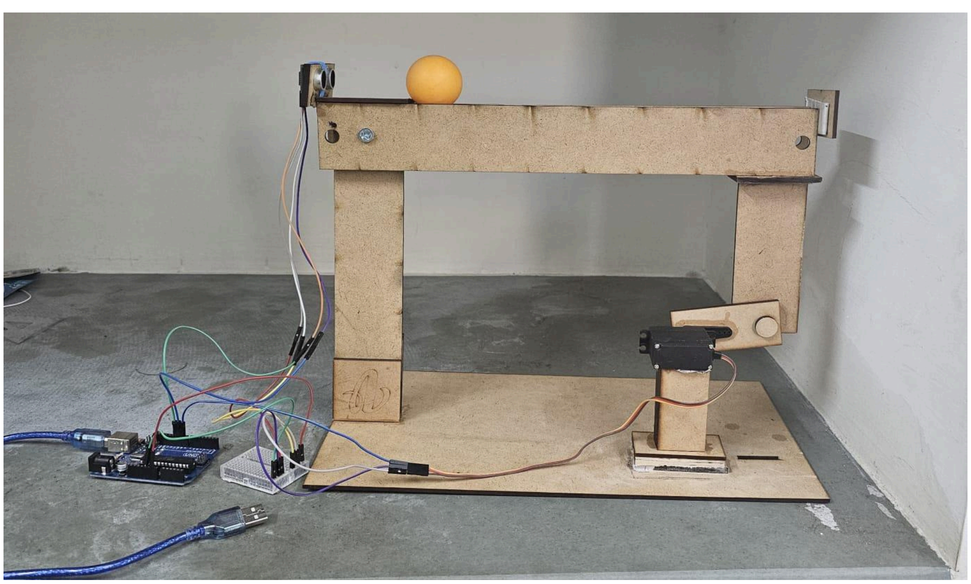



```
Maximum overshoot: 4.89 %
Settling time: 0.2000 seconds
```

Fig: Step Response for a step input with Compensator designed using Bode Plot

This design also meets the criteria. The overshoot is 4.89%, slightly below the 5% threshold, and the settling time is exceptionally fast at just 0.200 seconds, well within the 3-second requirement. The quick response exhibits minimal oscillations, suggesting that the Bode plot's compensator provides a well-balanced, fast, and stable system response.

Task 8: Demonstration of the Physical Project Setup



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