BER Performance of DS-CDMA System Over a Frequency Selective Multipath Rayleigh Fading Channel

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***Abstract*— In this paper, we present a study of the bit error rate (BER) performance of DS-CDMA system over a frequency selective multipath Rayleigh fading channel with perfect power control. The standard Gaussian approximation (SGA) is used to evaluate the BER performance for the DS-CDMA. The performance of DS-CDMA over the frequency selective fading channel is examined with the varying numbers of multipath components, varying numbers of interfering cells, and various process gain. From the simulation results we have seen that the BER performance is affected by these parameters.**

I. INTRODUCTION

Code division multiple access (CDMA) is a radio communication technique to allow multiple users to share the same spectrum simultaneously. It is the most investigated application of spread spectrum techniques [1]. In DS-CDMA (direct-sequence code division multiple access), the narrowband message is multiplied by a large bandwidth signal, which is called the spreading signal. The spreading signal is generated by convolving a pseudo-noise (PN) code with a chip waveform whose duration is much smaller than the symbol duration [2], [1]. By assigning different code sequences to each user, it is possible to allow many users to share the same channel and frequency simultaneously [1]. However an approximate orthogonality constraint on the code sequence is employed to guarantee acceptable performance [2]. Since PN codes are used and synchronization of user signals is not possible, it is not possible to achieve perfect orthogonality between the spreading sequences of different users, therefore, the signal of another user may appears as noise in some other user's signal. This phenomenon is called the multiple access interference (MAI)[1]. MAI causes degradation in bit error rate (BER) and system performance. There has been a significant amount of research conducted on this subject since it is widely used in wireless communication systems [11], [4], [5].

Multiple access interference (MAI) is a factor which limits the capacity and performance of DS-CDMA systems. MAI refers to the interference between direct sequence users. This interference is the result of the random time offsets between signals, which make it impossible to design code waveforms to be completely orthogonal. While the MAI caused by any one user is generally small, as the number of interferers or their power increases, MAI becomes substantial. Therefore, any analysis of performance of a CDMA system has to take into account the amount of MAI and its effects on the parameters that measure the performance (most notably the signal-to-interference-and-noise ration (SINR) at the receiver and the related bit error probability on the information bit stream). Much work has been reported on the calculation of the user average bit error rate (BER) for DS-CDMA systems. The most widely used and popular approach is the Gaussian approximation (GA) [6] and its variants.

In this paper, we study the bit error rate (BER) performance of an asynchronous DS-CDMA system over a frequency selective multipath Rayleigh fading channel with perfect power control. The standard Gaussian approximation (SGA) is used to evaluate the BER performance for the DS-CDMA. This approximation is the most widely cited and most widely used [7], [8], [9], [10] because of its simplicity. The performance of DS-CDMA over the frequency selective Rayleigh fading channel is examined with varying numbers of multipath components, varying numbers of interfering cells, and various process gain.

II. SYSTEM MODEL

In this section we provide a mathematical description of an asynchronous DS-CDMA system. We consider the reverse link (mobiles to base station) of a M_c cells asynchronous DS-CDMA system that supports K active users. This system is shown in Fig. 1.

A. Transmitted Signal

We assume that there are K active users transmitting signals in DS-CDMA system. Each of them transmits a signal which

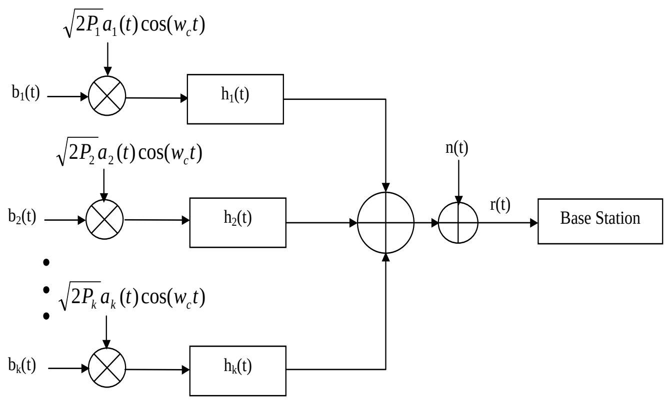

Fig. 1. reverse link DS-CDMA System Model.

is described by [11]

$$s_k(t-\acute{\tau_k}) = \sqrt{2P_k}b_k(t-\acute{\tau_k})a_k(t-\acute{\tau_k})\cos(\omega_c t+\theta_k) \quad (1)$$

where $b_k(t)$ is a binary data sequence, $a_k(t)$ is a pseudo-random sequence, P_k is the power of the transmitted signal, ω_c is the carrier angular frequency, $\acute{\tau_k}$ is the time delay that accounts for the lack of synchronism between the transmitters, and θ_k is the phase angle of the kth carrier. The kth user's data signal is a sequence of unit amplitude rectangular pulses of duration T_b, taking values from {-1,+1} with equal probability. This sequence can be expressed as

$$b_k(t) = \sum_{j=-\infty}^{\infty} b_j^k p_{T_b}(t-jT_b) \quad (2)$$

where p_{T_b}= 1, for $0 \leq t < T_b$, and p_{T_b}= 0, otherwise. The spreading signal $a_k(t)$ can be expressed as

$$a_k(t) = \sum_{i=-\infty}^{\infty} a_i^k \psi(t-iT_c) \quad (3)$$

where $\psi(t)$ is a chip waveform that is time-limited to [0,T_c] and normalized to have energy T_c, where $\int_0^{T_c}\psi^2(t)dt = T_c$ is the chip period, and $a_i^{(k)}$ is the i$^{\text{th}}$ chip value of the kth user; this chip value can be either -1 or +1. There are N chips per bit and thus $N = \frac{T_b}{T_c}$ is the process gain for user k. We assume that the desired user is k=0 and all other users contribute to MAI.

B. Channel Model

We assume that the channel, $h_k(t)$, between the k$^{\text{th}}$ and the base station is a multipath Rayleigh fading channel. We also assume that the channel is a frequency selective where the chip rate $\frac{1}{T_c}$ is higher than the channel coherence bandwidth. The delay difference between any two different paths are larger than the chip duration T_c. The complex low-pass equivalent impulse response of this channel is given by

$$h_k(t) = \sum_{l_k=1}^{L_k} \alpha_{k,l_k} e^{j\phi_{k,l_k}}\delta(t-\tau_{k,l_k}) \quad (4)$$

where ϕ_{k,l_k} is the phase of the multipath component, τ_{k,l_k} is the path delay, and L_k is the number of multipath components. α_{k,l_k} is the magnitude of the l$^{\text{th}}$ multipath components with Rayleigh distribution as follows

$$f_\alpha(\alpha) = \frac{\alpha}{\sigma_\alpha^2} \quad (5)$$

where the parameter σ in (5) is equal to half the average path power.

The received signal at the input of the correlation receiver is given by

$$\begin{aligned} r(t) \; &= \sum_{k=0}^{K-1}\sum_{l_k=1}^{L_k}\sqrt{2P_k}\alpha_{k,l_k}b_k(t-\tau_{k,l_k}) \\ &\times a_k(t-\tau_{k,l_k})\cos(\omega_c t+\phi_{k,l_k})+n(t) \quad (6) \end{aligned}$$

where $n(t)$ is additive white Gaussian noise (AWGN) with a two-sided power density of $\frac{N_0}{2}$. Note that the value of θ_k is absorbed into the channel phase ϕ_{k,l_k}, while the values of $\acute{\tau_k}$ is included in τ_{k,l_k}.Without loss of generality, we assume that the reference user is denoted by k = 0 (user of interest). The decision statistic at the output of the correlator is given by [12], [13]

$$\begin{aligned} Z_0 \; &= \int_0^{T_b} r(t)a_0(t-\tau_{0,0})\cos(\omega_c t)dt \\ &= b_0\,\alpha_{0,0}\sqrt{\frac{P_0}{2}}\,T_b + \sum_{k=0}^{K-1}\sum_{[l_k=1,l_0\neq 1]}^{L_k}\Upsilon_{k,l_k}+\zeta \quad (7) \end{aligned}$$

where b_0 is the transmitted bit from user 0, $\alpha_{0,0}$ is the amplitude of the desired multipath component, P_0 is the transmitted power of the desired user, and

$$\zeta = \int_0^{T_b} n(t)a_0(t-\tau_{0,0})\cos(\omega_c t)dt \quad (8)$$

is a zero-mean Gaussian random variable with variance $\sigma_\zeta^2 = \frac{N_0T_b}{4}$.

We can re-write the decision statistic in (7) as

$$Z_0 = D_0 + \Upsilon + \zeta \quad (9)$$

where D_0 is the desired signal component (first term in (7)), Υ is the MAI (second term in (7)), and ζ is the AWGN as quantified in (8).

III. SYSTEM PERFORMANCE

The use of the Gaussian Approximation to determine the bit error rate (BER) for a CDMA communication system is based on the argument that the bit decision statistic Z_0 in (9) may be modelled as a Gaussian random variable [14]. The resulting estimation is called the standard Gaussian approximation (SGA). This approximation is widely used in the literatures [7], [8], [9], [10].

In SGA, a central limit theorem (CLT) is used to approximate the sum of the MAI signals as an additive white-Gaussian process additional to the background Gaussian noise process. The receiver design, thus, consists of a conventional single user matched filter (correlation receiver) to detect the desired user signal. The average variance of the MAI over all possible operation conditions is used to compute the signal-to-noise ratio (SNR) at the correlator output. The SGA is more accurate as the number of interfering users increases.

In this paper, we model the propagation channel as a frequency-selective channel generating L multipaths per user, each of them independently faded with Rayleigh statistics. In the case of M_c interfering cells equipped by a conventional correlation-type receiver at the base station and perfectly implementing the power control, the BER using the SGA approach can be given by [2]

$$BER_L|_{\alpha_{0,0}} = Q\left(\sqrt{\frac{\alpha_{0,0}^2}{\frac{N_o}{2E_b} + \frac{2\sigma^2}{3N}[(1+\frac{M_c}{5})LK - 1]}}\right) \quad (10)$$

where M_c is the number of interfering cells, σ^2 is the variance of the Rayleigh random variable modelling the fading process, and $Q(n) = \frac{1}{\sqrt{2\pi}}\int_n^\infty e^{\frac{-u^2}{2}} du$. Averaging over the distribution of $\alpha_{0,0}$ in (10) with respect to the Rayleigh distribution and using the integral identity, the average BER in frequency selective Rayleigh fading using the SGA as [15]

$$BER_L = \frac{1}{2} - \frac{1}{2\sqrt{1 + \frac{N_o}{2E_b\sigma^2} + \frac{2}{3N}[(1+\frac{M_c}{5})LK - 1]}}. \quad (11)$$

IV. SIMULATION RESULTS

In this section, we present and discuss the numerical results of the BER performance of an asynchronous DS-CDMA system over a frequency-selective multipath Rayleigh fading channel. The numerical results are based on the standard Gaussian approximation (SGA) for the multipath and multiple access interference (MAI) (eq. (11)) with perfect power control. Each cell is equipped with a conventional correlation receiver. The received power of the desired signal is normalized to 1.

Fig. 2 shows the BER performance over a frequency selective multipath Rayleigh fading channel with perfect power control, as a function of the number of interfering cells M_c. In this simulation, the number of multipaths is set to L_k = 5, the signal-to-noise ratio (SNR) $\frac{E_b}{N_o} = 20$, the process gain $N = 84$, and $\sigma = 1$. As illustrated in Fig. 2, we observed that the BER increases when the number of interfering cells is less. It is evident that the performance of the DS-CDMA system depends on the number of interfering cells.

Fig. 3 shows the BER performance over a frequency selective multipath Rayleigh fading channel with perfect power control, as a function of the number of the multipath components (L_k = 3, 5, and 10). The number of interfering cells is set to M_c = 4, the SNR = 20, the process gain $N = 84$, and $\sigma = 1$. From the figures, it is clear that varying the multipath components, L_k, has a significant influence on the BER performance using the SGA approximation.

Fig. 4 shows the BER performance over a frequency selective multipath Rayleigh fading channel with perfect power control, as a function of the process gain ($N = 32$, 84, and 128). The number of interfering cells is set to M_c = 4, the SNR = 20, the number of multipath components is set to $L_k = 4$, and $\sigma = 1$. From the figures, it is clear that varying the process gain, N, has a significant influence on the BER performance using the SGA approximation.

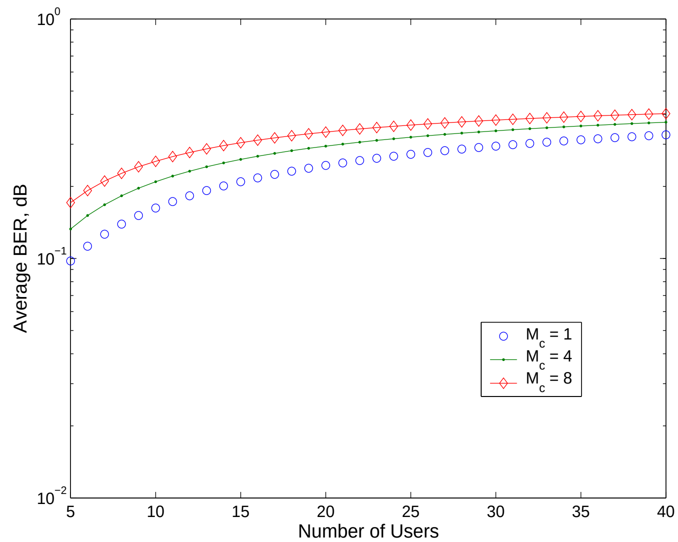

Fig. 2. BER performance over a frequency selective multipath Rayleigh fading channel with perfect power control, as a function of the number of interfering cells with L_k = 5, the process gain N = 84, and SNR = 20.

Fig. 4 shows the average BER performance obtained by simulation over a frequency selective multipath Rayleigh fading and AWGN channel. From the simulation, we find that averaging over more experiments and using a larger symbol size will produce results closer to the theoretical results. The symbol size used in this simulation is 10,000.

V. CONCLUSION

In this paper, the reverse link (mobile to base station) performance of an asynchronous DS-CDMA cellular system over a frequency selective multipath Rayleigh fading channel is evaluated. The standard Gaussian approximation (SGA) in case of perfect power control has been used to evaluate the BER performance for the DS-CDMA system. From the simulation results we conclude that the BER performance is affected by the number of multipath components, the value of the gain process and the number of interfering cells.

REFERENCES

[1] Ojanper, Tero, *Wideband CDMA for third generation mobile communications* , Boston : Artech House, 1998.

[2] T. S. Rappaport, *Wireless Communication- Principles and Practice*, Prentice-Hall, 1996.

[3] J. S. Lehnert, "An efficient technique for evaluating DS-SS multiple access communications," *IEEE Transactions on Communications*, vol. 37, no. 8, pp. 851-858, Aug. 1989.

[4] Nazari, Nersi, Ziemer, and E. Rodger, "Computationally efficient bounds for the performance of direct-sequence spread-spectrum multiple access communications system in jamming environment," *IEEE Transactions on Communications*, vol. 36, no. 5, pp. 577-587, May 1998.

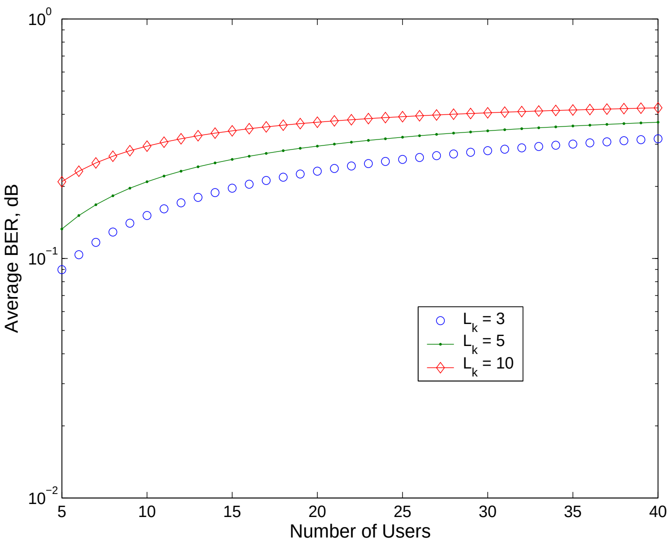

Fig. 3. BER performance over a frequency selective multipath Rayleigh fading channel with perfect power control, as a function of the number of the multipath components (L_k = 3, 5, and 10) with the process gain N = 84, and SNR = 20

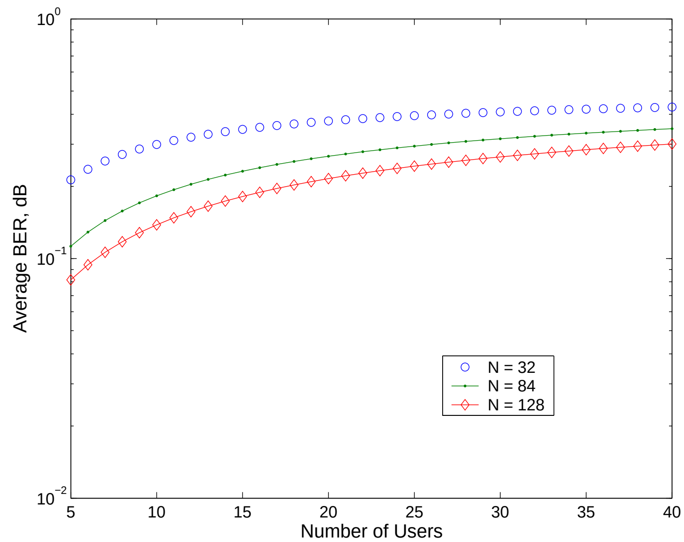

Fig. 4. BER performance over a frequency selective multipath Rayleigh fading channel with perfect power control, a as a function of the process gain (N = 32, 84, and 128) with the multipath components L_k = 4, and SNR = 20

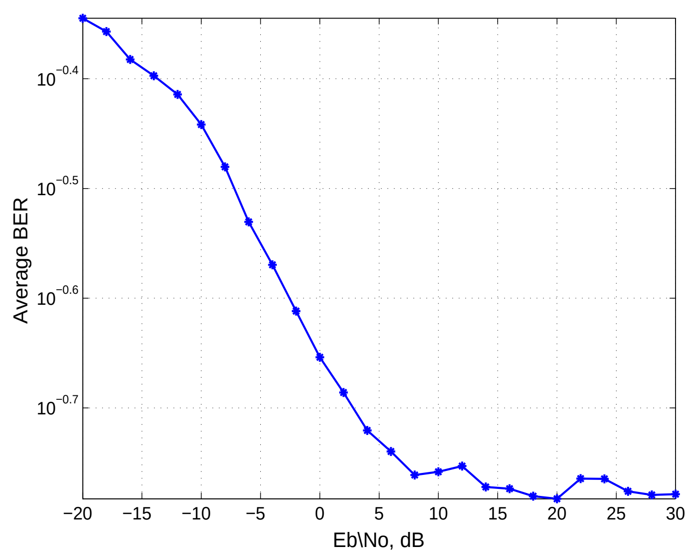

Fig. 5. Simulated BER performance over a frequency selective multipath Rayleigh fading and AWGN channel

[5] T. M. Lok, and J. S. Lehnert, "Error probabilities for generalized quadriphase DS-SSMA communication systems with random signature sequences," *IEEE Transactions on Communications*, vol. 44, no. 7, pp. 876-885, July 1996.

[6] J. M. Holtzman, "A simple, accurate method to calculate spread-spectrum multiple-access error probabilities," *IEEE Transactions on Communications*, vol. 40, no. 3, pp. 461-464, March 1992.

[7] M. B. Pursley, "Performance evaluation for phase-coded spread-spectrum multiple-access communication-Part I: Syatem analysis," *IEEE Transactions on Communications*, vol. COM-25, pp. 795-799, Aug. 1977.

[8] D. E. Borth, and M. B. Pursley, "Analysis of direct-sequence spread spectrum multiple-access communication over Rician fading channels," *IEEE Transactions on Communications*, vol. COM-27, pp. 1566-1577, Oct. 1979.

[9] O. K. Tonguz, and M. M. Wang, "Cellular CDMA networks impaired by Rayleigh fading: System performance with power control," *IEEE Transactions on Veh. Technol.*, vol. 43, pp. 515-527, Aug. 1994.

[10] K. L. Cheah, S. W. Oh, and K. H. Li, "Efficient performance analysis of asynchronous cellular CDMA over Rayleigh-fading channels," *IEEE Commun. Letters*, vol. 1, pp. 71-73, May. 1997.

[11] J. S. Lehnert, and M. B. Pursley, "Error probabilities for binary direct-sequence spread-spectrum communications with random signature sequences," *IEEE Transactions on Communications*, vol.COM-35, no. 1, pp. 87-98, Jan. 1987.

[12] M. O. Sunay, and P. J. McLane, "Calculation error probabilities for DS-CDMA systems: When not to use the Gaussian approximation," *IEEE Global Telecommunications Conference, Globecom'96*, vol. 3, pp. 1744-1749, Nov. 1996.

[13] J. Mar, and H. Y. Chen, "Performance analysis of cellular CDMA networks over frequency-selective fading channel," *IEEE Transactions on Vehicular Technology*, vol. 47, no. 4, pp. 1234 -1244, Nov. 1998.

[14] R. K. Morrow Jr., and J. S. Lehnert, "Bit-to-bit error dependence in slotted DS/SSMA packet systems with random signature sequences," *IEEE Transactions on Communications*, vol. 37, no. 10, pp. 1052 -1061, Oct. 1989.

[15] J. Cheng, and N. C. Beaulieu, "Accurate DS-CDMA bit-error probability calculation in Rayleigh fading," *IEEE on Wireless Communications*, vol. 1, no. 1, pp. 3-15, Jan. 2002.