A Cluster-based Approach to Maximize Number of Users in Wireless Multi-Cell Networks

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Abstract—**In this paper, we consider the problem of maximizing the total number of users in a multi-cell wireless network subject to power and independent set constraints on the base stations (BSs). More precisely, we impose the condition that no two adjacent BSs can operate simultaneously due to interference requirements. We propose equivalent mixed integer linear and quadratic programming formulations for this problem and compute upper and lower bounds as well as optimal solutions for instances with up to 2000 users and 30 BSs so far. The equivalent quadratic model is obtained by penalizing the independent set constraints leading to a quadratic problem with non-convex objective function that is hard to solve. To overcome this difficulty, we derive an equivalent quadratic concave objective function which allows to solve the problem to optimality using CPLEX. Finally, we propose an efficient greedy heuristic. In our numerical experiments, we consider realistic disk graph based network instances with radial transmission ranges of 40 to 10 ms for each base station. Network deployments are generated randomly within an area of 50*50 ms^2. Our preliminary numerical results indicate that for different instances, either the linear or quadratic model allows to find the optimal solution more efficiently. Whilst the proposed greedy heuristic allows to obtain tight near optimal solutions for most part of the instances with gaps which are lower than 5% from the optimal solution in less than one second.**

Keywords—**Multi-Cell wireless networks, mixed integer programming, greedy heuristic, cluster-based networks, disk graph instances.**

I. Introduction

Multi-cell wireless networks has become a hot topic research field within last decades. It is expected that these type of networks will be continuously and mandatorily required as part of future technology developments in order to connect the world under the Internet of Things (IoT) paradigm [9], [6]. A multi-cell wireless network is basically composed of a mobile or fixed set of base stations (BSs) acting as a backbone network and a set of electronic devices such as mobile phones, laptop computers, tablets, and desktop computers for example, which are to be connected to the BSs. Two important issues arise when designing multi-cell wireless networks. The first one is related with the problem of saving power consumption in each base station while simultaneously maximizing the total number of users to be attended on. Whilst the second one is related with the Inter-cell Interference (ICI) problem that emerges when placing two base stations close together [3], [4], [5], [13], [8], [12]. In particular, this is a high complex problem that severely affects power consumption and user capacities in the network. In order to overcome the ICI problem, recently some strategies in the literature have led to the development of cluster-based approaches. For instance in [8], some of the users are chosen to act as relays to other users in the network in order to increase the capacity of a particular cell by using Long Term Evolution (LTE) technology. As a consequence, a two-hop topology network configuration is obtained where some users are directly connected to the BSs while others are connected through other users to the BSs. A survey of different approaches for interference management using orthogonal frequency division multiple access (OFDMA) technology for femtocell based networks is presented in [11]. There, the authors recognize the importance of handling efficiently the ICI problem between neighbouring femto-cells, and between femto-cells and macro-cells as well. Finally, the authors present a qualitative comparison of different approaches and discuss future challenges to design efficient interference management schemes using OFDMA.

In this paper, we consider the problem of maximizing the total number of users in a multi-cell wireless network subject to power and independent set constraints on the BSs. More precisely, we impose the condition that no two adjacent BSs can operate simultaneously due to interference requirements. For this purpose, we formulate the multi-cell allocation problem by means of equivalent mixed integer linear and quadratic programming problems (Resp. MILP and MIQP). Thus, we compute upper and lower bounds as well as optimal solutions for instances with up to 2000 users and 30 BSs so far. These are realistic dimensions for wireless sensor networks [1]. In particular, the independent set constraints avoid activating simultaneously two BSs which are neighbors. Notice that in general, any network can be represented by means of a graph composed of a set of nodes or vertices, and a set of links (edges) connecting pairs of nodes in the network. Consequently, our proposed models emerge as a combination of two classical combinatorial optimization problems, namely the p-Median and Maximum Independent Set (MIS) problems which both have proved to be NP-Hard in the literature [2], [7]. Similarly as for the MIS problem, the equivalent quadratic model we propose is obtained by penalizing the independent set constraints leading to a non-convex objective function that is hard to solve in general. To overcome this difficulty, we derive an equivalent quadratic concave objective function and solve the problem to optimality using CPLEX [10]. Finally, we propose a greedy heuristic that allows to compute tight bounds and near optimal solutions in remarkably short CPU time. In our numerical experiments, we consider realistic disk graph

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based network instances with radial transmission ranges of 40 to 10 ms for each base station. Network graph deployments are generated randomly within an area of 50*50 ms^2. As far as we know, mathematical programming models while including power consumption and independent set constraints in order to maximize the total number of users in a multi-cell wireless network have not been investigated so far in the literature.

The paper is organized as follows. In Section 2, we give a brief system description of the problem and introduce the new mathematical programming formulations for the multi-cell allocation problem. Subsequently, in Section 3, we present and explain the greedy heuristic. Then, in Section 4 we present preliminary numerical results while comparing all the proposed models together with the greedy approach. Finally, in Section 5 we give the main conclusions of the paper and discuss some future research directions.

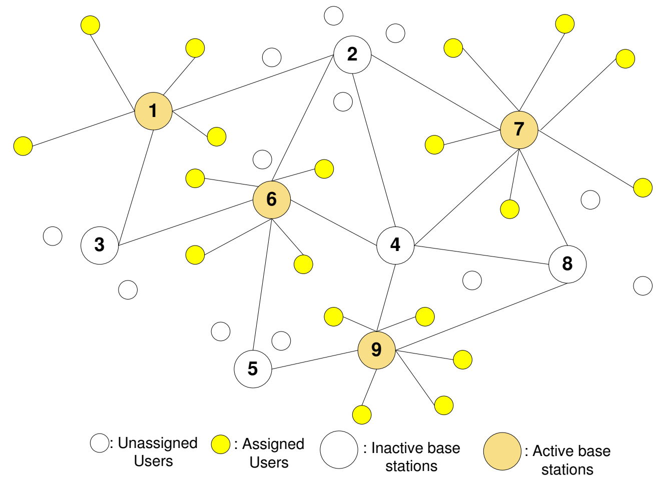

Fig. 1: An input graph representing a feasible solution.

II. Problem description and mathematical formulations

We represent a multi-cell wireless network by means of a connected input graph $G = (B \cup E)$ where B and E denote the set of BSs and edges connecting the BSs, respectively. We also consider a set K of users which are to be connected to the BSs in B. Thus, the main goal is to assign the maximum number of users to the BSs such that each user is connected to at most one BS. A user can only be connected to a particular BS if it is activate, otherwise none of the users can be connected to it. Whereas a BS can only be activated if their neighboring BSs are inactive. Finally, each BS, if it is active has to respect a maximum power constraint which limits the maximum number of users that can be assigned to it. As mentioned earlier, we consider realistic disk graph based network instances with radial transmission ranges of 40 to 10 ms for each BS. This means that if we activate a particular BS, then we cannot activate any other BS which is closer to it within the predefined radial transmission range, i.e., we cannot connect any pair of BSs (v_1, v_2) where $v_1, v_2 \in \mathcal{V}$, $v_1 \neq v_2$ such that v_1 is in the radial transmission range of v_2 and vice versa. A feasible network solution for the problem is depicted in Figure 1. In Figure 1, large circle nodes represent the set of BSs whilst smaller ones represent users. Notice that large white circles denote inactive BSs while coloured ones represent active BSs. Similarly, small white circles represent users which cannot be attended by the network operator whilst coloured ones represent assigned users. Also notice that the subset of large coloured nodes form an independent set which means that any of these pairs of nodes are not reachable between them directly and thus inter-cell interference is completely eliminated. Finally, observe that each user is connected to a unique colored BS and requires a certain power consumption cost from the BS to establish communication. Consequently, we aim to find the optimal solution while maximizing the total number of users that can be attended by the whole network. Another observation is that a feasible solution like the one shown in Figure 1 does not need to form an independent set of maximum cardinality since each BS is constrained by its own amount of power which is different for each BS. We propose the following MILP formulation for this problem

$$P_1 : \max_{\{x,y\}} \quad \sum_{k \in K} \sum_{b \in B} x_{kb} \tag{1}$$

$$\text{s.t.} \quad x_{kb} \leq y_b, \forall k \in K, b \in B \tag{2}$$

$$\sum_{b \in B} x_{kb} \leq 1, \forall k \in K \tag{3}$$

$$\sum_{k \in K} P_{kb} x_{kb} \leq Pot_b, \forall b \in B \tag{4}$$

$$y_b + y_c \leq 1, \forall (b,c) \in E \tag{5}$$

$$x \in \{0,1\}^{|K \times B|}, y \in \{0,1\}^{|B|} \tag{6}$$

where x and y are binary decision variables. The x variables are defined as $x_{kb} = 1$ if and only if user $k \in K$ is assigned to BS $b \in B$, otherwise $x_{kb} = 0$. Similarly, $y_b = 1$ if and only if BS $b \in B$ is in the network solution and $y_b = 0$ otherwise. The objective function (1) maximizes the total number of users in the network. Constraints (2) ensure that any user k can only be connected to BS b if it is active. Constraints (3) ensure that each user is assigned to at most one BS. Next, constraints (4) impose the condition that the sum of power required by the users to connect to BS $b \in B$ must be at most Pot_b, $\forall b \in B$. Here, each input power entry in matrix $P = (P_{kb})$ represents the amount of power required to connect user $k \in K$ with BS $b \in B$. Finally, constraints (5) impose the condition that two adjacent BSs which are reachable between them cannot be simultaneously active in the solution while constraints (6) are domain constraints for the decision variables.

In order to derive an equivalent quadratic model for P_1, we remove constraints (5) from it and add them to its objective function in the form of new penalized quadratic terms leading to $\sum_{k \in K} \sum_{b \in B} x_{kb} - \mathcal{M} \sum_{(b,c) \in E} y_b y_c$ where $\mathcal{M}$ is a bigM positive real value [7]. However, it is easy to see that each quadratic term $y_b y_c$ is not convex. In order to overcome this difficulty, we prove the following proposition.

Proposition 1: Each term $y_b y_c$ can be equivalently written as the following convex expression

$$y_b y_c = \frac{1}{2} \left[(y_b + y_c)^2 - (y_b + y_c) \right]$$

Proof 1: Notice that y_b and $y_c \in \{0,1\}$, then there exists only four possible combinations for which the equality holds. $\square$

Consequently, Proposition 1 leads us to write the following equivalent quadratic model

$$\begin{aligned} Q_1 : \max_{\{x,y\}} \quad & \sum_{k\in K}\sum_{b\in B} x_{kb} - \\ & \frac{\mathcal{M}}{2}\sum_{(b,c)\in E}\Big[(y_b+y_c)^2-(y_b+y_c)\Big] \quad (7) \\ \text{s.t.} \quad & x_{kb} \le y_b, \forall k\in K, b\in B \\ & \sum_{b\in B} x_{kb} \le 1, \forall k \in K \\ & \sum_{k\in K} P_{kb}x_{kb} \le Pot_b, \forall b\in B \\ & x\in\{0,1\}^{|K\times B|}, y\in\{0,1\}^{|B|} \end{aligned}$$

We note that the relaxation of Q_1 is convex and thus it allows to obtain optimal solutions with Off-The-Shelf solvers like CPLEX [10]. Hereafter, we denote by LP_1 and $QRel_1$ the linear and convex programming relaxations of P_1 and Q_1, respectively.

III. Greedy heuristic approach

In this section, we propose a simple and efficient greedy heuristic approach that allows to compute tight bounds and near optimal solutions for most of the input graph instances. The proposed algorithm is basically motivated by the fact that near optimal solutions can be obtained by simply maximizing the cardinality of the independent set obtained with the active BSs. By doing so, somehow we intuitively increase the total amount of power that is available from the BSs forming the independent set. The algorithm is simple and it is depicted in Algorithm III.1 as follows. The algorithm receives as an input the matrix (P_{kb}), the vector (Pot_b) and the graph $G(B,E)$ where B is the set of BSs and E is the set of edges connecting BSs in B. Two steps are performed. The first one is concerned with initializing an empty set S where the active BSs will be stored. Then, it starts by choosing the BS with minimum degree in order to remove the least possible number of neighboring BSs in each iteration of the while loop. This process continues until the set G is empty. Subsequently, in step two, we solve a reduced MILP problem which is obtained by considering only the BSs accumulated in $S \subset B$. More precisely, we solve the following optimization problem

Algorithm III.1: Greedy heuristic

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Data: Input power data P = (P_kb), Pot = (Pot_b) and graph
      G(B, E).
Result: A feasible solution for P_1, Q_1.
Step 1;
S ← ∅;
while G is not empty do
    Let v be a node of minimum degree in G;
    S ← S ∪ {v};
    Remove v and its neighbors from G;
Step 2;
Solve P_R(S) and let Z_opt = Val(P_R(S)) be the optimal
  objective function value of P_R(S);
Return Z_opt;
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$$\begin{aligned} P_R(S) : \max_{\{x\}} \quad & \sum_{k\in K}\sum_{b\in S} x_{kb} \\ \text{s.t.} \quad & \sum_{b\in S} x_{kb} \le 1, \forall k\in K \\ & \sum_{k\in K} P_{kb}x_{kb} \le Pot_b, \forall b\in S \\ & x\in\{0,1\}^{|K\times S|} \end{aligned}$$

Notice that the greedy Algorithm III.1 allows to obtain a feasible solution and that solving the linear programming relaxation of $P_R(S)$ allows to obtain an upper or lower bound for the problem. In our preliminary numerical results presented in the next section, we compute both feasible solutions as well as upper and lower bounds for the problem.

Theorem 1: The greedy Algorithm III.1 allows one to obtain an independent set S with at least $|S| \ge \frac{|B|}{\Delta+1}$ where Δ is the maximum degree of any node in graph $G(B,E)$.

Proof 2: Notice that a node u is in $B\backslash S$ as it is removed as a neighbor of some particular node $v\in S$ when v is added to S. Thus, we can assign node u to node v. Since a node $v \in S$ can be assigned at most Δ times, then we have that $|B\backslash S| \le \Delta|S|$. It is also obvious that $|S| + |B\backslash S| = |B|$ and consequently $|S| \ge \frac{|B|}{\Delta+1}$. $\square$

In the next section, we compare all the proposed models P_1, Q_1, LP_1, and $QRel_1$ together with the greedy heuristic approach in terms of CPU times, number of branch and bound nodes, and upper and lower bounds obtained.

IV. Preliminary numerical results

In this section, we present preliminary numerical results in order to compare all the proposed models together with the greedy Algorithm III.1 for different values of $|B| = \{10,20,30\}$ and with radial transmission ranges of $\{40,30,20,10\}$ for $|K| = \{1000,1500,2000\}$ users. In particular, in Table I we present numerical results for P_1, Q_1, LP_1, and $QRel_1$ whereas in Table II, we present numerical results for the greedy heuristic compared to the best solution obtained with either P_1 or Q_1. Connected disk graph instances are randomly generated with coordinates inside an area of $50*50$ ms^2. Finally, each entry in the input power matrix is randomly drawn from the interval $(0;1)$ mWs. Whilst the maximum power value of each BS is generated as $Pot_b = \frac{\max_{\{j\in K\}}\{P(j,b)|K|\}}{\delta|B|}$ where δ is an arbitrary integer value drawn from the interval $[1,8]$. We implement a Matlab program using CPLEX 12.6 [10] to solve the mixed integer linear and quadratic programming problems together with their linear programming and convex relaxations, respectively. The numerical experiments have been carried out on an Intel(R) 64 bits core(TM) with 3.40 Ghz and 8 gigabytes of RAM.

In Tables I-II, the set of instances we solve is the same. In Table I, the legend is as follows. In column 1, we present the instance number. Then, in columns 2-4, we present the radial transmission range in ms, the number of base stations and number of users, respectively. In columns 5-9 and 10-14, we present the optimal solution or best solution found with CPLEX, number of branch and bound nodes, CPU time in seconds, the optimal value of the relaxation and its CPU time in seconds for P_1 and Q_1, respectively. We limit the maximum CPU time for CPLEX to one hour. Thus, when the CPU time reported equals 3600s, then it means that it is the best solution found with CPLEX and we cannot certificate its optimality. On the opposite, if the CPU time is less than 3600, then we found the optimal solution. Finally, in columns 15-16 we report gaps that we compute by $\frac{LP_1 - P_1}{P_1} * 100$ and $\frac{QRel_1 - Q_1}{Q_1} * 100$, respectively. In particular, we subtract the penalty term from the objective function of $QRel_1$. In Table II, the legend is as follows. In columns 1-4, for the sake of clarity we present exactly the same information as in Table I. Subsequently in columns 5-6 we present the optimal or best solution obtained with P_1 or Q_1 and its minimum CPU time in seconds to obtain it. Next, in columns 7-9 we present the lower bound obtained with the greedy heuristic, the number of branch and bound nodes required by CPLEX to solve $P_R(S)$ and CPU time in seconds for the greedy heuristic. Notice that this is in fact a lower bound for the original problem. Similarly, in columns 10-11, we present the bound obtained with the greedy heuristic while solving the linear relaxation of $P_R(S)$ and the CPU time in seconds as well. Notice that in this last case, we cannot ensure that the value obtained when solving the linear relaxation of $P_R(S)$ will always be a lower bound. As it can be observed for some of the instances, we obtain upper bounds while for most of them we obtain lower bounds. In columns 12-13 we report gaps that we compute by $\frac{\max\{P_1,Q_1\} - LB(IP)}{\max\{P_1,Q_1\}} * 100$ and $\frac{|\max\{P_1,Q_1\} - B(IP)|}{\max\{P_1,Q_1\}} * 100$, respectively. Finally, in columns 14-15 we report the cardinality number of active BSs obtained with the greedy heuristic $|S|$ and the lower bound obtained due to Theorem 1. From Table I, we mainly observe that the instances are harder to solve with both P_1 and Q_1 when the number of users and BSs increase. This is somehow reflected in terms of branch and bound nodes and CPU times. In particular, we notice that when the input graphs are sparse, then solving the problem to optimality is even harder. Next, we see that the linear and convex relaxations are not tight at all when compared to the optimal solutions obtained which is reflected by the gaps. Further, we note that all the optimal solutions obtained with the relaxations are equal to the total input number of users. This clearly evidences the need to improve both relaxations by using some cutting plane approach as part of future research. We also see that for three instances the gaps equal zero. This can be explained by the fact that all users can be attended on by the set of chosen BSs. Consequently, in this case the problem is easy to solve. For these cases, we note that the integer solutions obtained are equal to the optimal solutions obtained with the relaxations. However, this is not the case for all remaining instances. Regarding the CPU times required by CPLEX to solve the relaxations, we observe that the linear ones grow more rapidly than quadratic ones. More precisely, we see that the linear ones go up to 25 seconds whereas the quadratic ones only require 7 seconds at most for the worst case instances. Finally, we observe that for some of the instances, the quadratic model allows to obtain the optimal solution in significantly less CPU time than the linear one. While the opposite goes for remaining instances. From Table II, we mainly observe that the greedy heuristic allows one to obtain near optimal solutions with gaps which are lower than 5% for most of the instances. This is an interesting result since the CPU times required to achieve these solutions are very low, e.g., less than one second for most part of the instances. In particular, few instances show that solving $P_R(S)$ is not easy all the time. This is evidenced by the number of branch and bound nodes and by the CPU times. On the opposite, we see that the linear programming relaxation of $P_R(S)$ can be solved in less than one second for all the instances. Surprisingly, these bounds are very tight when compared to the optimal objective function values obtained with $P_R(S)$. This is also an interesting result as it suggests that it would be convenient to consider the linear relaxation for new variants of the proposed greedy approach. Finally, the gap columns confirm these observations and the last two columns verify the condition of Theorem 1.

V. Conclusion

In this paper, we considered the problem of maximizing the total number of users in a multi-cell wireless network subject to power and independent set constraints on the base stations. In particular, we imposed the condition that no two adjacent base stations can operate simultaneously due to interference requirements. We proposed mixed integer linear and quadratic programming formulations for this problem and computed upper and lower bounds as well as optimal solutions for instances with up to 2000 users and 30 base stations. The proposed quadratic model is obtained by penalizing the independent set constraints leading to a quadratic problem with non-convex objective function that is hard to solve. To overcome this difficulty, we derived an equivalent quadratic concave objective function which allowed to solve the problem to optimality using CPLEX. Finally, an efficient greedy heuristic was proposed. In our numerical experiments, we considered realistic disk graph based network instances with radial transmissions ranges of 40 to 10 ms for each base station. Our preliminary numerical results indicated that for particular instances, either the linear or quadratic models allow to find the optimal solution more efficiently. Whilst the proposed heuristic allowed to obtain optimal and tight near optimal solutions in significantly short CPU time when compared to the optimal solutions obtained with the proposed models.

As future research, we plan to formulate new variants of the proposed models as well as stochastic versions of them. Finally, new meta-heuristic based procedures will also be proposed to solve large scale instances of the problem with significantly higher number of users and base stations.

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TABLE I: Numerical results for P_1 and Q_1.

#	R(ms)	$\|B\|$	$\|K\|$	P_1	#B&b	CPU (s)	LP_1	CPU(s)	Q_1	#B&b	CPU(s)	$QRel_1$	CPU(s)	Gap$_1$(%)	Gap$_2$(%)
1	40	10	1000	599	11	21.93	1000.00	0.59	599	363	99.19	1000.00	0.42	66.94	66.94
2	40	10	1500	1236	602	81.11	1500.00	1.47	1236	10762	140.35	1500.00	0.58	21.36	21.36
3	40	10	2000	1665	41	126.56	2000.00	1.42	1665	195	221.10	2000.00	0.64	20.12	20.12
4	30	10	1000	844	620	33.21	1000.00	0.64	844	514	56.46	1000.00	0.42	18.48	18.48
5	30	10	1500	1077	0	29.97	1500.00	1.61	1077	942	397.80	1458.33	0.75	39.28	35.41
6	30	10	2000	1684	72	83.07	2000.00	1.56	1684	873	202.10	2000.00	0.66	18.76	18.76
7	20	10	1000	1000	0	2.36	1000.00	0.56	1000	0	2.04	1000.00	0.50	0	0
8	20	10	1500	1249	11	73.77	1500.00	1.12	1249	1609	418.30	1500.00	0.58	20.10	20.10
9	20	10	2000	1682	27161	787.93	2000.00	1.53	1682	1028	327.48	2000.00	0.69	18.91	18.91
10	10	10	1000	1000	633	57.28	1000.00	0.62	1000	2090	98.23	1000.00	0.42	0	0
11	10	10	1500	1419	90	67.80	1500.00	1.01	1419	25870	527.00	1500.00	0.55	5.71	5.71
12	10	10	2000	1877	15224	1298.29	2000.00	1.75	1877	1043	305.51	2000.00	0.78	6.55	6.55
1	40	20	1000	550	3	77.89	1000.00	1.54	550	457	261.93	1000.00	0.72	81.82	81.82
2	40	20	1500	598	40	201.16	1500.00	2.71	598	262	283.87	1500.00	1.12	150.84	150.84
3	40	20	2000	789	13	387.21	2000.00	5.73	789	221	267.91	2000.00	1.44	153.49	153.49
4	30	20	1000	717	0	94.83	1000.00	1.26	717	1605	464.35	1000.00	0.73	39.47	39.47
5	30	20	1500	1058	20	141.45	1500.00	2.75	1058	1535	1195.82	1500.00	1.11	41.78	41.78
6	30	20	2000	1417	37040	1166.71	2000.00	6.79	1417	44302	2658.53	2000.00	7.32	41.14	41.14
7	20	20	1000	962	3850	900.42	1000.00	1.75	962	7020	202.47	1000.00	1.00	3.95	3.95
8	20	20	1500	1306	16323	1624.63	1500.00	5.65	1306	4647	585.36	1500.00	1.22	14.85	14.85
9	20	20	2000	1909	412	473.17	2000.00	4.57	1909	576	1156.49	2000.00	1.64	4.77	4.77
10	10	20	1000	906	1655	84.80	1000.00	2.73	906	10815	409.56	1000.00	0.78	10.38	10.38
11	10	20	1500	1287	11508	286.52	1500.00	5.41	1287	23570	489.44	1500.00	1.31	16.55	16.55
12	10	20	2000	1927	1200317	3600	2000.00	11.39	1928	274936	852.76	2000.00	1.78	3.79	3.73
1	40	30	1000	309	0	110.18	1000.00	7.60	309	1118	458.67	1000.00	1.05	223.62	223.62
2	40	30	1500	468	239	258.29	1500.00	17.16	468	2454	1100.07	1500.00	1.65	220.51	220.51
3	40	30	2000	790	3	425.44	2000.00	29.39	790	6384	2272.08	2000.00	2.56	153.16	153.16
4	30	30	1000	403	211	119.56	1000.00	8.32	403	2479	763.18	1000.00	1.05	148.14	148.14
5	30	30	1500	599	290	310.58	1500.00	15.94	599	45583	2199.55	1500.00	1.65	150.42	150.42
6	30	30	2000	797	0	642.24	2000.00	31.04	797	1518	3155.18	2000.00	2.60	150.94	150.94
7	20	30	1000	591	7473	283.56	1000.00	8.19	591	10890	296.00	1000.00	1.06	69.20	69.20
8	20	30	1500	871	5612	421.81	1500.00	15.54	871	3067	2193.43	1500.00	1.83	72.22	72.22
9	20	30	2000	1155	2376	1000.63	2000.00	33.37	1155	43504	2602.16	2000.00	2.89	73.16	73.16
10	10	30	1000	961	1112508	3600	1000.00	6.04	961	271174	3600	1000.00	6.44	4.06	4.06
11	10	30	1500	1256	252947	3600	1500.00	15.30	1256	173331	3600	1500.00	5.38	19.43	19.43
12	10	30	2000	2000	1465	1303.82	2000.00	24.09	2000	1532	1352.74	2000.00	2.73	0	0

TABLE II: Numerical results for the Greedy Heuristic Approach.

#	R(ms)	$\|B\|$	$\|K\|$	$\max\{P_1, Q_1\}$	Min. CPU(s)	$LB(IP)$	#B&b	CPU(s)	$B(LP)$	CPU(s)	Gap$_{LB}$(%)	Gap$_B$(%)	$\|S\|$	$\frac{\|B\|}{\Delta+1}$
1	40	10	1000	599	21.93	591	0	0.19	591.37	0.14	1.33	1.27	2	1.00
2	40	10	1500	1236	81.11	1233	0	0.25	1233.39	0.17	0.24	0.21	3	1.00
3	40	10	2000	1665	126.56	1626	0	0.28	1626.80	0.20	2.34	2.29	3	1.11
4	30	10	1000	844	33.21	828	0	0.20	828.97	0.16	1.89	1.78	3	1.43
5	30	10	1500	1077	29.97	1061	0	0.23	1061.87	0.22	1.48	1.40	4	1.25
6	30	10	2000	1684	83.07	1662	0	0.34	1662.40	0.20	1.30	1.28	3	1.11
7	20	10	1000	1000	2.04	1000	0	0.23	1000.00	0.17	0	0	5	1.67
8	20	10	1500	1249	73.77	1238	0	0.31	1238.75	0.17	0.88	0.82	3	1.67
9	20	10	2000	1682	327.48	1681	0	0.30	1681.43	0.20	0.05	0.03	3	1.67
10	10	10	1000	1000	57.28	987	0	0.25	987.34	0.17	1.30	1.26	4	1.25
11	10	10	1500	1419	67.80	1380	0	0.34	1380.27	0.22	2.74	2.72	4	2.00
12	10	10	2000	1877	305.51	1874	0	0.27	1874.96	0.23	0.15	0.10	4	1.67
1	40	20	1000	550	77.89	550	0	0.19	550.89	0.16	0	0.16	3	1.00
2	40	20	1500	598	201.16	598	0	0.20	598.31	0.17	0	0.05	2	1.00
3	40	20	2000	789	267.91	761	0	0.20	761.86	0.20	3.54	3.43	2	1.00
4	30	20	1000	717	94.83	717	0	0.19	718.00	0.17	0	0.13	4	1.11
5	30	20	1500	1058	141.45	1046	956	0.51	1046.20	0.20	1.13	1.11	4	1.05
6	30	20	2000	1417	1166.71	1412	0	0.30	1412.62	0.25	0.35	0.30	4	1.05
7	20	20	1000	962	202.47	957	5463	2.87	957.19	0.23	0.51	0.50	6	2.00
8	20	20	1500	1306	585.36	1298	495	0.52	1298.19	0.25	0.61	0.59	5	1.82
9	20	20	2000	1909	473.17	1682	0	0.41	1682.64	0.25	11.89	11.85	5	1.54
10	10	20	1000	906	84.80	813	291125	75.88	813.47	0.20	10.26	10.21	8	3.33
11	10	20	1500	1287	286.52	1271	0	0.61	1272.03	0.30	1.24	1.16	8	3.33
12	10	20	2000	1928	852.76	1922	2499	7.54	1922.35	0.39	0.31	0.29	10	2.86
1	40	30	1000	309	110.18	280	0	0.20	280.84	0.16	9.38	9.11	3	1.00
2	40	30	1500	468	258.29	446	0	0.22	447.18	0.19	4.70	4.44	3	1.00
3	40	30	2000	790	425.44	766	0	0.34	766.48	0.22	3.03	2.97	4	1.00
4	30	30	1000	403	119.56	400	0	0.22	400.90	0.16	0.74	0.52	4	1.00
5	30	30	1500	599	310.58	584	0	0.23	584.42	0.19	2.50	2.43	4	1.07
6	30	30	2000	797	642.24	786	0	0.28	786.64	0.22	1.38	1.29	4	1.15
7	20	30	1000	591	283.56	546	0	0.27	547.15	0.19	7.61	7.41	6	1.43
8	20	30	1500	871	421.81	855	0	0.31	855.83	0.23	1.83	1.74	6	1.58
9	20	30	2000	1155	1000.63	1121	9878	4.12	1121.40	0.26	2.94	2.90	6	1.76
10	10	30	1000	961	3600	927	61128	75.83	927.81	0.27	3.53	3.45	13	3.00
11	10	30	1500	1256	3600	1226	0	0.53	1226.83	0.33	2.38	2.32	11	3.75
12	10	30	2000	2000	1303.82	1932	39180	287.52	1933.32	0.50	3.40	3.33	14	6.00

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