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Performance Analysis of Large CDMA Random Access Systems with Retransmission Diversity over Fading Channels

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Abstract

The random access systems, with retransmission diversity (RD) employment, over large random spreading code division multiple access (CDMA) channel subject to fading is investigated, under the assumption of infinite number of users and infinite spreading gain with their ratio converging to a constant. The low bound of the signal to interference and noise ratio (SINR) is shown to converge almost surely to a constant. The throughput, spectrum efficiency and energy efficiency in the dominating systems are obtained. The analytical results are confirmed by simulations. We find that in high traffic loads the throughput with fading is higher than that without. When the energy efficiency increases, the spectrum efficiency tends to two contrary values due to SNR increases or decreases. For the ordinary stable systems, the stability region is shown to shrink as the traffic increases and enlarge with RD employment.

Keywords: Code division multiple access (CDMA), random access, random spreading, fading channel, retransmission diversity

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1. Introduction

The random access code division multiple access (CDMA) network has received significant attention recently [1][2][3][4][5][6][7]. Unlike other random access networks [8][9], multipacket reception capability of random access CDMA networks can substantially improve the system performance [1][2][3]. Throughputs of several symmetric wireless multi-access systems where users have the same transmission rate and power are obtained in [6]. An analytical method to study the performance related parameters of the random access CDMA systems is presented under a realistic environment in [7].

Since the queues of all users are coupled by mutual interference, the system state is a high-order Markov chain in a finite-user CDMA network. It is very difficult to obtain engineering insights on the performance limits. One alternative approach is presented for large CDMA networks with deterministic access in [10], and it shows that as both the number of users and the spreading gain tend to infinity with their ratio converging to a constant, the signal to interference and noise ratio (SINR) of the linear receivers converges in probability to a constant. The same concept of large systems is extended to asynchronous systems [11] and multipath fading channels [12]. This suggests that the analysis in the random access system be feasible. The SINR of the large CDMA random access network is shown to converge almost surely to a constant [13][14]. The throughput and average delay of packet-switched large CDMA networks with linear multiuser detectors in multipath fading channels is analyzed in [15].

In random access CDMA systems, if a received packet contains errors, the packet is requested to be retransmitted. The random access CDMA systems with retransmission diversity (RD) employment over the additive white Gaussian noise (AWGN) channels is presented in [14][16] with application of a large system technique. It is shown that RD, if employed properly, can greatly improve the system performance in terms of network throughput, delay and spectrum efficiency, due to the fact that multiaccess interference, noise, and fading are random and time-variant.

In this paper, we extend the techniques of [14] to the fading channels and study the effect of fading on throughput, spectral efficiency and stability region. The SINR of large CDMA random access systems with match filter (MF) receivers and RD employment over fading channels is first shown to converge almost surely and be lower bounded. The throughput is then numerically evaluated by the Gaussian approximation to avoid the infinitely many integrals. The numerical evaluation is verified to be identical to the simulation result with a variety of traffic, spreading sequences and transmission probability. The analysis has also revealed a number of interesting new findings. It is obtained that the throughput in Rayleigh fading channels can be higher than that without fading. Although our finding is for random access system without coding, it confirms the result in [17] that the multiuser diversity over fading channels may increase spectral efficiency of a CDMA deterministic access system where the optimal code is applied. The spectrum and energy efficiencies are analyzed and both the analytical and simulation results show that the spectrum efficiency tends to two contrary values due to SNR increases or decreases. Since the multiple transmissions of the same packet with RD employment can be viewed as a channel coding with an adaptive code rate, the energy efficiency is not improved when the block code is employed. The results are also extended to ordinary stable systems. It is found that the stability region shrinks as the traffic increases and enlarges with RD employment.

The rest of this paper is organized as follows. Section 2 presents the system and signal models. Section 3 analyzes the limit SINR. Section 4 derives the throughput, efficiency, and stable region as well as block code performance. Simulation and numerical results are given in Section 5. Some conclusions are drawn in Section 6. Part of proofs is included in the Appendix.

2. System and Signal Models

2.1 System Model

Consider a large CDMA system where both the number of users K, and spreading gain N, tend to infinity while their ratio converges to a constant $K/N \to \alpha > 0$. Each user has a sufficiently large buffer to store arrival packets of length L bits, and randomly delivers the first coming packet in the buffer with probability $\theta(A) \in (0,1]$ to a base station through a bit-synchronous CDMA complex channel where A is the received signal amplitude. We suppose that the new packet arrival rate at each user is sufficiently high so that there is always a packet in the buffer waiting for transmission, then the random access system is considered as a dominating system with steady throughput equal to the maximum throughput of the random access system. Each packet is coded with an error detecting code so that the base station can detect all the error packets and inform the user the failure in detection through the reliable feedback channel which is assumed to be error-free. When the transmission fails, the user retransmits the same packet in the next slot with the same transmission probability until success. At the receiver, to employ RD, all the data collected from the initial transmission and retransmissions of the same packet is used to demodulate the packet by MF detector.

2.2 Signal Model

Without loss of generality, consider that user 1, the desired user, transmits a packet in slot n. The chip matched filter outputs a signal vector in a bit period as

$$\mathbf{r}(n) = A_1 h_1(n)\mathbf{s}_1(n)b_1(n) + \sum_{k=2}^{K} I_k(n)A_k h_k(n)\mathbf{s}_k(n)b_k(n) + \mathbf{w}(n) , \qquad (1)$$

where the bit indexes in a packet are dropped since the signal model for each bit is identical. A_1, $h_1(n)$, $\mathbf{s}_1(n)$ and $b_1(n)$ are the transmitted signal amplitude, attenuation coefficient of the fading channel, random spreading sequence and transmitted bit of the desired user and A_k, $h_k(n)$, $\mathbf{s}_k(n)$ and $b_k(n)$ are for interference users. $I_k(n)$ is the indicator function of packet transmission for user k. The user k is in the transmission state with probability $\Pr(I_k(n) = 1) = \theta(A_k)$ and in the idle state with probability $\Pr(I_k(n) = 0) = 1 - \theta(A_k)$. Since users transmit their packets independently, $E[I_i(j)I_k(j)] = \theta(A_i)\theta(A_k)$, $k \neq i$. Assume that as K tends to infinity, A_k's are upper bounded. $\mathbf{w}(n) \sim CN(0, \mathbf{I})$ is a complex noise vector with the standard circularly symmetric Gaussian distribution. That is, its real and imaginary components are mutually independent Gaussian variables with zero mean and variance ½. Since the signal power of user k is A_k^2 and noise power is unit, the SNR equals

$$SNR_k = A_k^2 . \qquad (2)$$

A spreading sequence can be written as

$$\mathbf{s}_k(n) = (s_{k1}(n), s_{k2}(n), \cdots, s_{kN}(n))^T ,$$

where each chip $s_{kj}(n) \in \left\{ -1/\sqrt{N}, +1/\sqrt{N} \right\}$ is chosen independently and equally likely. Each user independently and equiprobably selects a random sequence to spread all bits of each packet in each transmission no matter whether the packet is transmitted the first time or retransmitted.

The output of the MF $h_1^*(n)\mathbf{s}_1^T(n)$ where $h_1^*(n)$ is the complex conjugate of $h_1(n)$ is

$$y_1(n) = A_1 |h_1(n)|^2 b_1(n) + \sum_{k=2}^{K} I_k(n) A_k \left(h_1^*(n) h_k(n) \right) R_{1k}(n) b_k(n) + h_1^*(n) z(n), \qquad (3)$$

where $R_{1k}(n) = \mathbf{s}_1^T(n)\mathbf{s}_k(n)$ is the crosscorrelation between the spreading sequences of users 1 and k, and $z(n) \sim CN(0, 1)$ are mutually independent and identically distributed (i.i.d.) across slots. We denote that $z(n) = z_r(n) + iz_i(n)$ and $h_k(n) = h_{kr}(n) + ih_{ki}(n)$ where $z_r(n), z_i(n) \sim N(0, 1/2)$ are i.i.d.. Note that since the desired signal is real, we let

$$x_1(n) = \text{Re}\left(y_1(n) \right) = A_1 |h_1(n)|^2 b_1(n) + \sum_{k=2}^{K} I_k(n) A_k H_{1k}(n) R_{1k}(n) b_k(n)$$

$$+ \left(h_{1r}(n) z_r(n) + h_{1i}(n) z_i(n) \right), \qquad (4)$$

where

$$H_{1k}(n) = \text{Re}\left(h_1^*(n) h_k(n) \right) = h_{1r}(n) h_{kr}(n) + h_{1i}(n) h_{ki}(n). \qquad (5)$$

Without RD employment, the transmitted bit is decided by $\hat{b}_1(n) = \text{sgn}(x_1(n))$ where sgn($\bullet$) is the sign function.

2.3 Combiner output

Consider that at the end of the current slot n, user k has transmitted its current packet m_k times and the number of slots from the first transmission of the packet to the current slot is J_k, $k = 1,2,\ldots,K$. To detect the current packet of user 1, all the received signals during the slots $n-J_1+j$, $j = 1,\ldots,J_1$, are combined. Meanwhile, the receiver adopts interference canceling technique. For each user, the base station already knows all previous packets and does not know only the current packet. Thus the interference signals produced by the known packets can be reconstructed and eliminated from the received signals. By adding all the received desired signals and subtracting interference signals of known packets, we get a new decision variable

$$y_1'(m_1) = \sum_{j=0}^{J_1-1} I_1(n-j) A_1 |h_1(n-j)|^2 b_1(n)$$

$$+ \sum_{k=2}^{K} \sum_{j=0}^{J_k^\Delta-1} I_1(n-j) I_k(n-j) A_k \left(h_1^*(n-j) h_k(n-j) \right) R_{1k}(n-j) b_k(n)$$

$$+ \sum_{j=0}^{J_1-1} I_1(n-j) h_1^*(n-j) z(n-j), \qquad (6)$$

where $J_i^\Delta = \min(J_1, J_i)$. The three summation terms are the signal of the desired user, interference caused by the other users and noise, respectively. We can further get

$$x_1'(m_1) = \text{Re}(y_1'(m_1)) = \sum_{j=0}^{J_1-1} I_1(n-j) A_1 |h_1(n-j)|^2 b_1(n)$$

$$+ \sum_{k=2}^{K} \sum_{j=0}^{J_k^\Delta-1} I_1(n-j) I_k(n-j) A_k H_{1k}(n-j) R_{1k}(n-j) b_k(n)$$

$$+\sum_{j=0}^{J_1-1} I_1(n-j)\big(h_{1r}(n-j)z_r(n-j)+h_{1i}(n-j)z_i(n-j)\big). \qquad (7)$$

With the RD employment, the transmitted bit is decided by $\hat{b}_1(n)=\mathrm{sgn}(x_1^{'}(m_1))$.

3. Limit SINR

3.1 With RD Gain

With RD employment, we shall analyze the limit SINR of $x_1^{'}(m_1)$ with fixed m_1 as K and N tend to infinity. The power of desired signal equals $P_s = A_1^2\left(\sum_{i=1}^{m_1}|h_1(i)|^2\right)^2$ by noticing that $b_1(n)$ was transmitted m_1 times during the J_1 slots. With random spreading, the spreading sequence of a packet in each transmission is randomly selected, and then the interference of the same packet in different slots is different with the power equals to

$$
\begin{aligned}
P_{in} =\ & E\Bigg[\bigg(\sum_{k=2}^{K}\sum_{j=0}^{J_k^\Delta-1} I_1(n-j)I_k(n-j)A_k H_{1k}(n-j)R_{1k}(n-j)b_k(n) \\
& +\sum_{j=0}^{J_1-1} I_1(n-j)\big(h_{1r}(n-j)z_r(n-j)+h_{1i}(n-j)z_i(n-j)\big)\bigg)^2\Bigg] \\
=\ & \sum_{k=2}^{K} A_k^2\bigg(\sum_{j=0}^{J_k^\Delta-1} I_1(n-j)I_k(n-j)H_{1k}(n-j)R_{1k}(n-j)\bigg)^2 \\
& +E\Bigg[\bigg(\sum_{j=0}^{J_1-1} I_1(n-j)\big(h_{1r}(n-j)z_r(n-j)+h_{1i}(n-j)z_i(n-j)\big)\bigg)^2\Bigg],\ (8)
\end{aligned}
$$

Where expectation is taken with respect to transmitted bits and noise. The first term in (8) is the power of interference which we denote by P_i and the second term is the power of noise defined as P_n. The SINR is $\beta = P_s/P_{in}$.

Generally, the interference power in the large system limit is difficult to obtain. However, an upper bound of the limit interference power can be obtained and therefore a lower bound of the limit SINR can be obtained as given in the following theorem that is proved in the Appendix.

Theorem 1 (With Fading): For the random access CDMA system over Rayleigh fading where the distribution of fading coefficients $h_k(i)$ is bounded, the SINR of combiner output for a packet of m transmissions is almost surely lower bounded by

$$\beta_{fr}(A,h_1,...,h_m)=\frac{2A^2\sum_{i=1}^{m}|h_1(i)|^2}{1+\alpha E[A^2\theta(A)]}, \qquad (9)$$

where the expectation is taken with respect to A. The lower bound is equal to the true limit SINR for $m = 1$.

Over the AWGN channel, the channel gain $|h_k| = 1$ is constant for all k, of which the real and imaginary components equal $1/\sqrt{2}$. Then $H_{1k}(n) = 1$ and

$$x_1^{'}(m_1)=\sum_{j=0}^{J_1-1} I_1(n-i)A_1 b_1(n)+\sum_{k=2}^{K}\sum_{j=0}^{J_k^\Delta-1} I_1(n-j)I_k(n-j)A_k R_{1k}(n-j)b_k(n)$$

$$+\frac{1}{\sqrt{2}}\sum_{j=0}^{J_1-1}I_1(n-j)\big(z_r(n-j)+z_i(n-j)\big). \tag{10}$$

The power of interference plus noise conditioned with a fixed m_1 equals

$$P_{in}=E\left[\left(\sum_{k=2}^{K}\sum_{j=0}^{J_k^\Delta-1}I_1(n-j)I_k(n-j)A_kR_{1k}(n-j)b_k(n)+\frac{1}{\sqrt{2}}\sum_{j=0}^{J_1-1}I_1(n-j)\big(z_r(n-j)+z_i(n-j)\big)\right)^2\bigg|m_1\right]$$

$$=\sum_{k=2}^{K}A_k^2\left(\sum_{j=0}^{J_k^\Delta-1}I_1(n-j)I_k(n-j)R_{1k}(n-j)\right)^2+\frac{1}{2}E\left[\sum_{j=0}^{J_1-1}I_1(n-j)\big(z_r(n-j)+z_i(n-j)\big)^2\right]. \tag{11}$$

The first term is P_i and the second is P_n. Then the following corollary is obtained.

Colollary 1 (Without Fading): For the random access CDMA system in the complex Gaussian channel, the SINR of combiner output for a packet of m transmissions is almost surely lower bounded by

$$\beta_{gr}(A,m)=\frac{2mA^2}{1+2\alpha E[A^2\theta(A)]}. \tag{12}$$

Comparing (9) and (12), we can find that interference power in (9) is reduced by a factor of 2 since interference signals experience independent fading channels. This results is a little different with that in [14] due to that the complex channel is considered in this paper while in [14] a real AWGN channel is assumed.

3.2 Without RD Gain

Without RD employment, the power of the desired signal from (4) equals $P_s=A_1^2|h_1|^4$. The power of interference and noise equals

$$P_{in}=E\left[\left(\sum_{k=2}^{K}I_k(n)A_kH_{1k}(n)R_{1k}(n)b_k(n)+\big(h_{1r}(n)z_r(n)+h_{1i}(n)z_i(n)\big)\right)^2\right]$$

$$=\sum_{k=2}^{K}\big(I_k(n)A_kH_{1k}(n)R_{1k}(n)\big)^2+E\left[\big(h_{1r}(n)z_r(n)+h_{1i}(n)z_i(n)\big)^2\right], \tag{13}$$

where the first term is the power of interference and the second is of noise. Then the limit SINR is obtained directly from the **Theorem 1** with $m=1$.

Colollary 2 (With Fading): For the random access CDMA system in the Rayleigh fading channel without RD employment, the SINR converges almost surely to

$$\beta_{fw}(A,h)=\frac{2A^2|h|^2}{1+\alpha E[A^2\theta(A)]}. \tag{14}$$

Consider the equal power system with $P_i=P$ and $\theta(P_i)=\theta$. Then the SINR is equal to $2A^2|h|^2/(1+\alpha A^2\theta(A))$. Furthermore, for the deterministic access system, all users transmit their packets with probability 1 in each slot. Then $\theta(A)=1$, and the SINR can be obtained as $2A^2|h|^2/(1+\alpha A^2)$. Since SNR equals (2), the SINR can be rewritten as $2SNR|h|^2/(1+\alpha SNR)$, which is similar with the results in [17] with one difference of the coefficient 2 in the numerator. This is because the processing of extracting real part of the complex MF output deletes half of interference and noise.

Without fading, the channel gain is fixed and then $|h_k|=1$ for all k with which the real and imaginary components are $1/\sqrt{2}$. Therefore $H_{1k}(n)=1$ and

$$x_1(n) = A_1 b_1(n) + \sum_{k=2}^{K} I_k(n) A_k(n) R_{1k}(n) b_k(n) + \frac{1}{\sqrt{2}} (z_r(n) + z_i(n)) \,. \tag{15}$$

The signal power equals $P_s = A_1^2$ and the interference power equals

$$P_{in} = E\left[\left(\sum_{k=2}^{K} I_k(n) A_k(n) R_{1k}(n) b_k(n) + \frac{1}{\sqrt{2}} (z_r(n) + z_i(n)) \right)^2 \right]$$

$$= \sum_{k=2}^{K} \left(I_k(n) A_k(n) R_{1k}(n) \right)^2 + \frac{1}{2} E\left[(z_r(n) + z_i(n))^2 \right]. \tag{16}$$

Colollary 2 (Without Fading): For the random access CDMA system in the complex Gaussian channel without RD employment, the SINR converges almost surely to

$$\beta_{gw}(A) = \beta_{gr}(A,1) = \frac{2A^2}{1 + 2\alpha E[A^2 \theta(A)]} \,. \tag{17}$$

Similar to the case with RD employment, the interference in the large random access system with fading is lower than without.

4. System Performance

4.1 Throughput Performance

It is known that the interference of MF output for one transmission is asymptotically Gaussian in the large system limit [18]. Therefore the limit bit-error rate (BER) can be expressed by the Q function as $Q\left(\sqrt{\beta(A,m)} \right)$ in terms of the SINR defined as $\beta(A, m)$. Correspondingly, the probability of packet detection without error correct coding equals

$$q(A,m) = \left[1 - Q\left(\sqrt{\beta(A,m)} \right) \right]^L \,. \tag{18}$$

By [14], the throughput in the AWGN channel equals

$$T(A) = \begin{cases} \theta(A) q(A,1) & \text{without RD exploitation} \\ \dfrac{\theta(A)}{1 + \sum_{i=2}^{\infty} \prod_{j=1}^{i-1} (1 - q(A,j))} & \text{with RD exploitation} \end{cases} \tag{19}$$

In the fading channel, the expectation with respect to fading coefficients shall be taken. Correspondingly, the throughput without RD employment is equal to

$$T(A) = \int_0^{\infty} \theta(A) \left[1 - Q\left(\sqrt{\beta_{fw}(A,x)} \right) \right]^L f(x) dx \,, \tag{20}$$

where $f(x)$ is the probability density function of fading coefficient.

For the system with RD employment, the SINR is a function of m fading coefficients and then the throughput equals

$$T(A) = \underbrace{\int_0^{\infty} ... \int_0^{\infty}}_{\infty} \frac{\theta(A)}{1 + \sum_{i=2}^{\infty} \prod_{j=1}^{i-1} (1 - q_j(A, x_1, ..., x_j))} f(x_1)...f(x_{\infty}) dx_1...dx_{\infty} \,, \tag{21}$$

where $q_j(A, x_1, ..., x_j) = \left[1 - Q\left(\sqrt{\beta_{fr}(A, x_1, ..., x_j)} \right) \right]^L$. The exact value of throughput is difficult to calculate numerically because of the infinitely many integrals. Since the SINR is bounded, we use the Gaussian approximation to evaluate numerically the throughput.

In the stationary state, a user having amplitude A transmits its packet with probability $\theta(A)$

and the transmitted packet is supposed to be successfully detected after m transmissions. The probability distribution of m is equal to $q_m(A, x_1,\ldots,x_m)(1 - q_{m-1}(A, x_1,\ldots,x_{m-1})\ldots(1 - q_1(x_1)))$ which is the probability of $m - 1$ unsuccessful detections followed by a success. The packet delay $D(A)$ is defined as

$$D(A, x_1, x_2, \ldots) = E\left[\frac{m}{\theta(A)}\right] = \sum_{m=1}^{\infty} \frac{m}{\theta(A)} q_m(A, x_1, \ldots, x_m) \prod_{i=1}^{m-1}\left[1 - q_i(A, x_1, \ldots, x_i)\right], \quad (22)$$

where the product from $i = 1$ to $m - 1$ is equal to 1 when $m = 1$. Then the average packet delay can be obtained as

$$\begin{aligned}
D(A) = E\left[D(A, x_1, \ldots, x_\infty)\right] &= \sum_{m=1}^{\infty} \frac{m}{\theta(A)} E\left[q_m(A, x_1, \ldots, x_m) \prod_{i=1}^{m-1}\left[1 - q_i(A, x_1, \ldots, x_i)\right]\right] \\
&= \frac{1}{\theta(A)}\left\{E\left[q_1(A, x_1)\right] + 2E\left[q_2(A, x_1, x_2)\left[1 - q_1(A, x_1)\right]\right] + \right. \\
&\qquad 3E\left[q_3(A, x_1, x_2, x_3)\left[1 - q_1(A, x_1, x_2)\right]\left[1 - q_1(A, x_1)\right]\right] + \\
&\qquad \left. mE\left[q_m(A, x_1, \ldots, x_m) \prod_{i=1}^{m-1}\left[1 - q_i(A, x_1, \ldots, x_i)\right]\right] + \ldots\right\}, \quad (23)
\end{aligned}$$

where the expectation is taken with respect to fading coefficients $x_1, \ldots, x_\infty$. We denote

$$\begin{aligned}
p_m(A) &= E\left[q_m(A, x_1, \ldots, x_m) \prod_{i=1}^{m-1}\left[1 - q_i(A, x_1, \ldots, x_i)\right]\right] \\
&= \underbrace{\int_0^\infty \ldots \int_0^\infty}_{m} q_m(A, x_1, \ldots, x_m) \prod_{i=1}^{m-1}\left[1 - q_i(A, x_1, \ldots, x_i)\right] f(x_1) \ldots f(x_m) dx_1 \ldots dx_m . \quad (24)
\end{aligned}$$

So, (23) can be rewritten as

$$D(A) = \frac{1}{\theta(A)} \sum_{m=1}^{\infty} m p_m(A) . \quad (25)$$

When m is small, $p_m(A)$ can be obtained directly by multiple integrals. When m is larger than some positive value m_0, we can calculate $p_m(A)$ by Gaussian approximation instead of multiple integrals. The $q_m(A, x_1, \ldots x_m)$ in the expression of $p_m(A)$ can be estimated as

$$q_m(A, x_1, \ldots, x_m) \approx \left[1 - Q\left(\sqrt{\frac{2A_1^2 m E[x^2]}{1 + \alpha E[A^2 \theta(A)]}}\right)\right]^L = c_m \qquad m > m_0 . \quad (26)$$

Therefore $p_m(A)$ can be approximately estimated as

$$p_m(A) \approx c_m \sum_{i=m_0+1}^{m-1} (1 - c_i) \underbrace{\int_0^\infty \ldots \int_0^\infty}_{m_0} \prod_{i=1}^{m-1}\left[1 - q_i(A, x_1, \ldots, x_i)\right] f(x_1) \ldots f(x_{m_0}) dx_1 \ldots dx_{m_0} . \quad (27)$$

Since $T(A)$ packets are successfully transmitted on average over all the slots, the throughput is equal to the reciprocal of the packet delay $D(A)$, which is obtained as

$$T'(A) \approx \frac{1}{E\left[D(A, x_1, x_2, \ldots)\right]} = \frac{1}{D(A)} = \frac{\theta(A)}{\sum_{m=1}^{\infty} m p_m(A)} . \quad (28)$$

Then the throughput for the system with RD employment in fading channel can be approximately evaluated by (28).

4.2 Spectrum Efficiency

The spectrum efficiency is the total average number of bits successfully transmitted per second per hertz in the system. Consider a system with total bandwidth $W = N/T_b$ where T_b is the bit period. The spectral efficiency equals [14]

$$C = \alpha \int_0^\infty T(A)dF(A) \, , \tag{29}$$

bits/s/Hz where F is the distribution of amplitude A. For the system with RD employment over fading channel, the spectral efficiency equals

$$C = \alpha \int_0^\infty \frac{\theta(A)dF(A)}{\sum_{m=1}^\infty mp_m(A)} . \tag{30}$$

Consider that a user with power A^2 transmits one packet with spectrum efficiency C. For the system with throughput T, the minimum bit energy per unit noise level required for reliable communication is $\theta(A)A^2/T$ which we can write as

$$\frac{E_b}{N_0} = \frac{\theta(A)A^2}{T} = \frac{\theta(A)\mathsf{SNR}}{T} \, , \tag{31}$$

where the second equation is obtained from (2). Because the energy that each user spends per symbol relative to a noise spectral level $N_0 = 1$ is equal to SNR, the SINR in (9) can be written as

$$\beta_{fr}^{LB}(\mathsf{SNR}) = \frac{2\mathsf{SNR}\sum_{i=1}^m |h_1(i)|^2}{1 + \alpha E[\mathsf{SNR}\,\theta(\sqrt{\mathsf{SNR}})]} . \tag{32}$$

So the system with RD employment over Rayleigh fading channel that achieves total spectrum efficiency C given in bits per degree of freedom (b/s/Hz), has an energy per bit per noise level equal to

$$\frac{E_b}{N_0} = \mathsf{SNR} \underbrace{\int_0^\infty ... \int_0^\infty}_{\infty} \left(1 + \sum_{i=2}^\infty \prod_{j=1}^{i-1} \left(1 - \left(1 - Q\left(\sqrt{\frac{2\mathsf{SNR}\sum_{k=1}^j x_k^2}{1 + \alpha E[\mathsf{SNR}\theta]}} \right) \right)^L \right) \right) f(x_1)...f(x_\infty)dx_1...dx_\infty$$

$$\approx \frac{\mathsf{SNR}}{\theta} \sum_{m=1}^\infty mp_m(\mathsf{SNR}) \, , \tag{33}$$

where the second equation comes from the approximation in (28). Compared (30) and (33), increase of transmission probabilities θ or demanded traffic α can increase user's spectrum efficiency, but may also increase the energy cost.

4.3 Block code

When block code is adopted, some error packets can be corrected and the packet error probability will decrease. Considering the perfect code is adopted, the probability of error packets can be obtained as

$$q' = \sum_{m=r+1}^L \binom{L}{m} Q(\sqrt{\beta})^m \left[1 - Q(\sqrt{\beta}) \right]^{L-m} , \tag{34}$$

where β is the SINR and r is the minimum distance of the block code.

Meanwhile, the transmission efficiency decreases due to the code rate lower than one. Consider the code rate R. One user with power P_e transmits one packet when the throughput is up to T_c (packet/slot/user). So the power per information bit is P_e/R. The minimum bit energy

per noise level P_o required for reliable communication is

$$\left(\frac{E_b}{N_0}\right)_c = \frac{\theta_e P_e / R}{T_c P_o} = \frac{\theta \mathsf{SNR}}{T_c R} . \tag{35}$$

Compared with (31)

$$\frac{(E_b / N_0)_c}{(E_b / N_0)} = \frac{T_c R}{T} . \tag{36}$$

The block code can improve the power efficiency, only if the throughput ratio T_c/T is lager than $1/R$.

4.4 System Stability

The random access system in this paper is analyzed with the assumption that the packet length of user's buffer is not equal to zero so that there is always a new packet waiting for transmission. Each user has a queue of packets in its buffer. We suppose the new packet arrival rate to a user's buffer with signal amplitude A is $\lambda(A)$, and the service rate of the packet successfully transmitted is $u(A)$. A queue is stable if the arrival rate $\lambda(A)$ is less than the service rate $u(A)$ and then the user's buffer is empty with a nonzero probability. If all the queues in the system are stable, then the system is stable. The stability region is the collection of sets of all uses' arrival rates with which the system is stable. In our system, the throughput for the user with signal amplitude A equals to $T(A)$ packet per slot over fading channels when each user always has a packet in its buffer ready to transmit. While in the stable system, the probability that the queue is empty is greater than zero. So the total interference power to a user is not greater than that when all users always have packets ready to transmit, and the throughput in the stable system must be not less than $T(A)$. So if each user's arrival rate $\lambda(A)$ is less than its service rate $T(A)$, the system is stable and the stable region can be obtained.

5. Simulation and Numerical Results

In the following numerical and simulation results, four cases are considered: (a) Rayleigh fading channel with RD employment; (b) Gaussian channel with RD employment; (c) Rayleigh fading channel without RD employment; and (d) Gaussian channel without employment of RD.

We consider frequency-flat Rayleigh fading channel $h(n) \sim CN(0,1)$ with magnitude $|h(n)|$ distributed as [19]

$$f(r) = 2r \exp(-r^2) . \tag{37}$$

The fading coefficient $h(n)$'s do not change during one packet slot and do from one packet slot to another.

Consider an equal power system where all users have the same transmission probability and so they have the same steady throughput. Each user independently and equiprobably selected a sequence for all bits of each packet in each transmission. For simplicity, the throughput in simulation is averaged over all users. In all simulations, the first one thousand slots are excluded from the estimation of throughput. The total number of slots used in throughput computation equals four thousands. For numerical computation, four integral and Gaussian approximate estimation in (27) are adopted to calculate $p_m(A)$ when m is lager than four.

From **Fig. 1** we can find that the throughput in simulation converges to the analytical limit as the user number increases, which verifies the analytical result, especially for case (a) where the analytical limit result is processed by Gaussian approximate estimation. In case (a), the

throughput in Rayleigh fading is the highest among all cases in large θ, which is due to the interference reduction by a factor of two via extracting real part of the complex MF output. In the AWGN channel, because of low spreading gain, interference significantly affects SINR. Moreover, the uniformity of interferers' power makes combiner unable to reduce interference so that the throughput is lower than that in fading. This phenomenon is even more obvious in cases (c), (d). Comparing case (b), (d), RD provides significant gain which increases as θ increases.
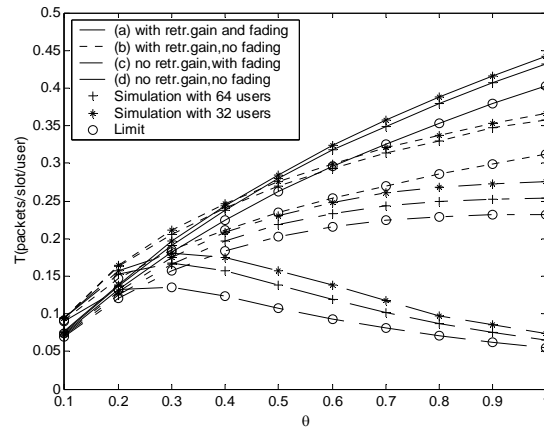

Fig. 1. Throughput versus transmission probability in an equal power system with $\alpha = 1$, SNR = 9dB, and packet size $L = 16$.

As shown in **Fig. 2** and **Fig. 3**, cases (b), (d) achieve much higher throughput and spectral efficiency than (a), (c) with small α. This is because the crosscorrelation between different sequences decreases to be very small as spreading gain N increases. In this situation, fading dominates the SINR. As α increase, the throughput and spectral efficiency in fading exceeds that in AWGN channels, due to multiuser diversity. In fading, a portion of users' can have a high channel gain and thus throughput does not drop as fast as traffic load increases. However, without fading all users are suffered from the increased interference as traffic load increases. With RD employment, the system performance can be significantly improved. Both with and without fading, higher throughput and spectral efficiency can be obtained by RD employment.
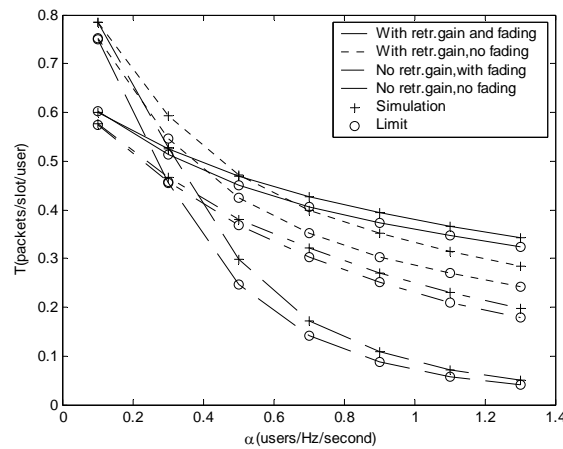

Fig. 2. Throughput versus demanded traffic α in an equal power system with SNR = 9dB, packet size L = 16, and θ = 0.8 in simulation K = 64.
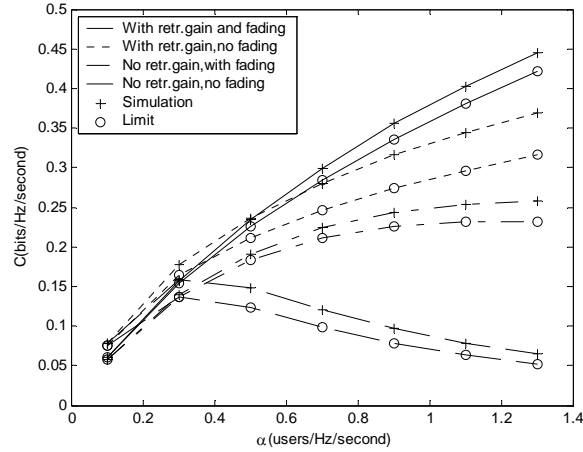
Fig. 3. Spectral efficiency vs. demanded traffic α in an equal power system with SNR = 9dB, packet size $L = 16$, and $\theta = 0.8$ in simulation $K = 64$.

Fig. 4 shows E_b/N_o versus SNR for a system with 64 users, $\alpha = 0.2$, $L = 40$ and $\theta = 0.8$. We can find that the energy per bit per noise level in simulation converges to the analytical limit no matter whether the RD is employed, which verifies mutually the simulation and the analytical results. The E_b/N_o is lower for system with RD than that without RD, which implies that RD employment can decrease the average transmission energy and improve the power efficiency. As SNR increases, the curve for system without RD converges to the one with RD, due to that when SNR is large enough, each packet can be transmitted successfully by one time, and then RD is no longer effective. When SNR is equal to about 3 dB, E_b/N_o for system over Rayleigh fading channel without RD employment reaches the minimum value, which is the optimal condition in terms of energy efficieny.
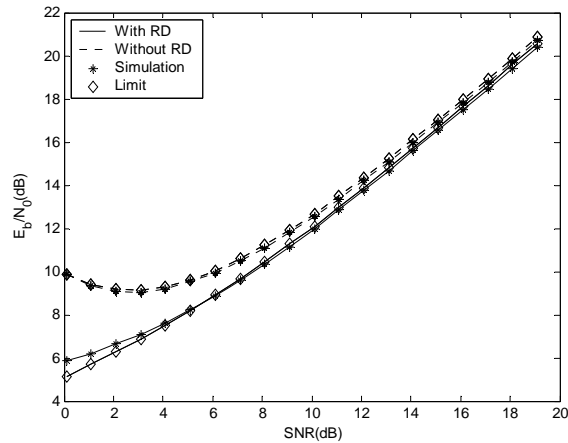

Fig. 4. E_b/N_o vs. SNR in an equal power system over Rayleigh fading channel with $\alpha = 0.2$, packet size $L = 40$, and $\theta = 0.8$ in simulation $K = 64$.

Fig. 5 shows the spectrum efficiency versus energy per bit per noise level. The system can achieve much higher spectrum efficiency with RD employment than that without RD. Observing the curve without RD, we can find an interesting phenomenon that for some E_b/N_o,

there are two spectrum efficiency values, due to different SNR value. Actually, when SNR tends to the positive infinity, the spectrum efficiency increases as E_b/N_o increases; but the spectrum efficiency decreases as E_b/N_o increases when SNR tends to the negative infinite.
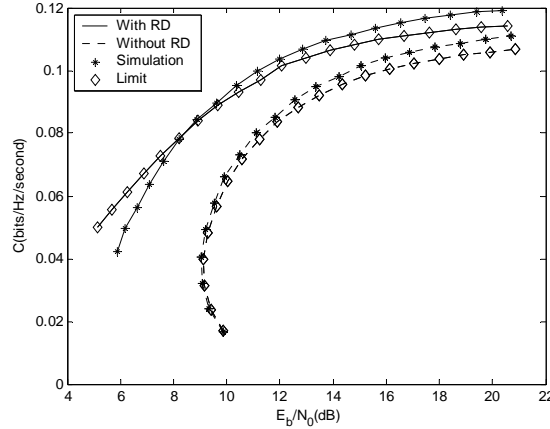

Fig. 5. Spectrum efficiency versus E_b/N_o in an equal power system over Rayleigh fading channel with α = 0.2, packet size L = 40, and θ = 0.8 in simulation K = 64.

Consider the linear (15, 11) Hamming code with code rate 11/15, which can correct one bit error. Since the Hamming code is perfect binary block code [20], the probability of packet detection without error is equal to

$$q' = \left[1 - Q\left(\sqrt{\beta}\right)\right]^L + \binom{15}{1}Q\left(\sqrt{\beta}\right)\left[1 - Q\left(\sqrt{\beta}\right)\right]^{L-1}$$

$$= \left(1 - Q\left(\sqrt{\beta}\right) + 15Q\left(\sqrt{\beta}\right)\right)\left[1 - Q\left(\sqrt{\beta}\right)\right]^{L-1}, \tag{38}$$

where β is the SINR.

The above two curves in the **Fig. 6** show the E_b/N_o in the system without RD employment with linear (15, 11) Hamming code and without, respectively. It is obviously that the system with Hamming code costs less energy than without, which is due to that the code can correct the one bit error packets and improve the transmission efficiency. However for the system with RD, there exist two different phenomena. When SNR is small, the E_b/N_o is lower with Hamming code than without but the gain is not very distinct. When SNR is larger than 5 dB, the result is converse. These phenomena are explainable that RD can improve the energy efficiency by itself. Under this condition, the benefit provided by Hamming code is lower than the cost that additional power is used for transmission the error correct bits. So the total power efficiency reduces, especially in the large SNR region.

Fig. 7 shows the stability regions of random access system with two classes of users over Rayleigh fading channels with different α and RD employment or without RD. The stable region is showed as the inner-bound constructed by the curve, λ_1 and λ_2 axes. It is obvious that the stable region of the system with RD is larger than that without RD. When α is equal to zero, the stable region is square and largest amongst all cases, due to that there is no mutual interference between the two classes of users, which means that any class of users can obtain equally largest arrival rate no matter what is the arrival rate for the other class of users when the system is stable. As α increases, the stable region will be made narrowed because the throughput decreases, which coincides with **Fig. 2**. Any point in the stable region represents that in order to achieve a stable system, we can always find two arrival rate values for the two

classes of users. While for the point out of the stable region, the system is unstable however the arrival rates are chosen.
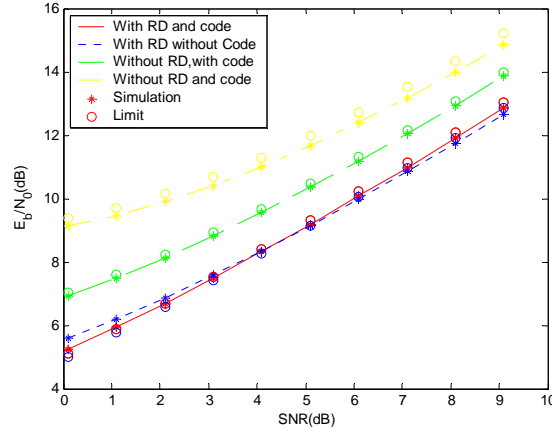

Fig. 6. E_b/N_o versus SNR in an equal power system over Rayleigh fading channel with α=1, packet size L=15, θ=1 and (15, 11) Hamming code in simulation K=64.
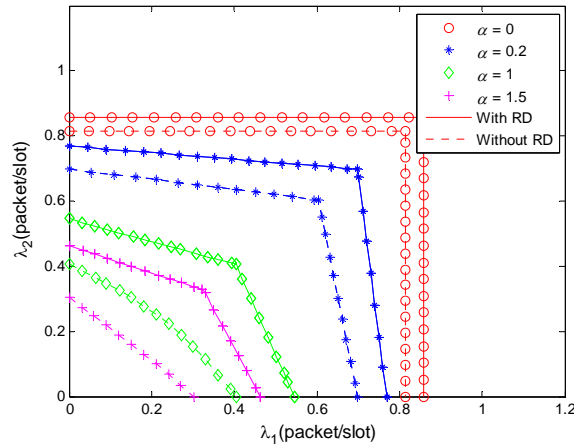

Fig. 7. Stability regions of random access system with two classes of users over Rayleigh fading channels with SNR = 9 dB, packet size L = 16.

6. Conclusions

In this paper, the performance of random access CDMA systems with RD employment over fading channels is investigated. With the assumption that both the number of users and spreading gain tend to infinity and their ratio converges to a constant, the SINR expression for the combined output of all the retransmission is firstly derived and shown to converge almost surely to a constant, which depends only on the traffic load, transmission probability, channel coefficient and distribution of transmission power. Then the throughput, spectral and power efficiency are analyzed for the dominating systems through the theoretical analysis and simulation, which shows that in addition to the random sequences, the fading can provide retransmission diversity and multiuser diversity gain. The latter is particularly useful when the

traffic load is high so that the throughput is higher with fading than without. RD can significantly increase the throughput both with and without fading and significantly improves on the spectrum efficiency and energy efficiency. The analytical result well predicts the simulation result. Furthermore, for the ordinary stable systems, it is demonstrated that the stability region shrinks as the traffic load increases and enlarges with RD employment.

Appendix

Lemma 1: Let $\{A_i\}$ be a positive and bounded sequence and its empirical distribution function converges to distribution function F, that is $\lim_{K\to\infty}(1/K)\sum_{i=1}^{K}I(A_i < x) = F(x)$. u_i is a sequence of independent random variables with probability distribution $\Pr(u_i = 1) = \theta(A_i)$ and $\Pr(u_i = 0) = 1-\theta(A_i)$. $\theta(A)$ is a function of A such that $E[A^2\theta(A)]$ exists where expectation is taken with respected to F. H_i is a sequence of i.i.d. random variables with up to eighth finite moment mean, which is independent of A_i and u_i. Then

$$\lim_{K\to\infty}\frac{1}{K}\sum_{i=1}^{K}A_i^2 H_i^2 u_i = \eta E[A^2\theta(A)], \qquad \text{almost surely} \qquad (39)$$

where η denotes the second moment mean of H_i.

Proof: Let $Y_K = (1/K)\sum_{i=1}^{K}A_i^2 H_i^2 u_i$. Then

$$E(Y_K) = E[H_i^2]E\left[1/K\sum_{i=1}^{K}A_i^2 u_i\right] = \eta/K\sum_{i=1}^{K}A_i^2\theta(A_i)$$

which converges to

$$\lim_{K\to\infty}\frac{\eta}{K}\sum_{i=1}^{K}A_i^2\theta(A_i) = \eta E[A^2\theta(A)]. \qquad (40)$$

Since

$$E\left[\left(Y_K - E(Y_K)\right)^4\right] = E\left[\left(\frac{1}{K}\sum_{i=1}^{K}A_i^2\left(H_i^2 u_i - \eta\theta(A_i)\right)\right)^4\right]$$

$$= E\left[\sum_{i=1}^{K}\frac{1}{K^4}A_i^8\left(H_i^2 u_i - \eta\theta(A_i)\right)^4 + \frac{6}{K^4}\sum_{i=1}^{K}\sum_{j=1,j\neq i}^{K}A_i^4 A_j^4\left(H_i^2 u_i - \eta\theta(A_i)\right)^2\left(H_j^2 u_j - \eta\theta(A_j)\right)^2 + \right.$$

$$\frac{4}{K^4}\sum_{i=1}^{K}\sum_{j=1,j\neq i}^{K}A_i^6 A_j^2\left(H_i^2 u_i - \eta\theta(A_i)\right)^3\left(H_j^2 u_j - \eta\theta(A_j)\right) +$$

$$\left.\frac{12}{K^4}\sum_{i=1}^{K}\sum_{j=1,j\neq i}^{K}\sum_{l=1,l\neq i,l\neq j}^{K}A_i^4 A_j^2 A_l^2\left(H_i^2 u_i - \eta\theta(A_i)\right)^2\left(H_j^2 u_j - \eta\theta(A_j)\right)\left(H_l^2 u_l - \eta\theta(A_l)\right)\right]. \quad (41)$$

Due to the mutual independence of all the random variables H_i's and u_i's,

$$E\left[H_j^2 u_j - \eta\theta(A_j)\right] = E\left[H_j^2\right]E\left[u_j\right] - \eta E\left[\theta(A_j)\right] = 0$$

and (41) can be rewritten as

$$E\left[\left(Y_K - E(Y_K)\right)^4\right] = \frac{1}{K^4}\sum_{i=1}^{K}A_i^8 E\left[\left(H_i^2 u_i - \eta\theta(A_i)\right)^4\right]$$

$$+\frac{6}{K^4}\sum_{i=1}^{K}\sum_{j=1,j\neq i}^{K}A_i^4 A_j^4 E\left[\left(H_i^2 u_i - \eta\theta(A_i)\right)^2\right]E\left[\left(H_j^2 u_j - \eta\theta(A_j)\right)^2\right]. \quad (42)$$

Since $E\left[\left(H_i^2 u_i - \eta\theta(A_i)\right)^4\right] = c_1$ and $E\left[\left(H_i^2 u_i - \eta\theta(A_i)\right)^2\right] = c_2$ for all i are constants irrelevant to K,

$$E\left[(Y_K - E(Y_K))^4\right] \leq \frac{A_{max}^8 c_1}{K^3} + \frac{A_{max}^8 (K-1)c_2}{K^3} = O\left(1/K^2\right). \tag{43}$$

By the Chebyshev inequality, for each $\varepsilon > 0$,

$$\Pr\left\{|Y_K - E(Y_K)| > \varepsilon\right\} \leq \frac{c}{\varepsilon^4 K^2}, \tag{44}$$

with a constant c, which leads to (39) by the Borel-Cantelli lemma.   □

*Proof of **Theorem 1**:*
In the fading channels, the noise power equals

$$P_n = E\left[\left(\sum_{j=0}^{J_1-1} I_1(n-j)\left(h_{1r}(n-j)z_r(n-j) + h_{1i}(n-j)z_i(n-j)\right)\right)^2\right]$$

$$= E\left[\sum_{j=0}^{m_1}\left(h_{1r}(n-j)z_r(n-j) + h_{1i}(n-j)z_i(n-j)\right)^2\right]$$

$$= E\left[\sum_{j=0}^{m_1}\left(h_{1r}^2(n-j)z_r^2(n-j) + 2h_{1i}(n-j)z_i(n-j)h_{1i}(n-j)z_i(n-j) + h_{1i}^2(n-j)z_i^2(n-j)\right)^2\right]$$

$$= \sum_{j=0}^{m_1}\left(h_{1r}^2(n-j)\cdot\frac{1}{2} + h_{1i}^2(n-j)\cdot\frac{1}{2}\right)$$

$$= \frac{1}{2}\sum_{j=0}^{m_1}|h_1(n-j)|^2. \tag{45}$$

Consider that the current packet of user 1 is transmitted m_1 times in the slots $n_1' = n - J_1 + 1$, n_2', ..., $n_{m_1}' = n$. Define

$$\tilde{\mathbf{s}}_1 = \frac{1}{\sqrt{m_1}}\left(\mathbf{s}_1^T(n_1'), \mathbf{s}_1^T(n_2'), ..., \mathbf{s}_1^T(n_{m_1}')\right)^T, \tag{46}$$

which is normalized to have length one, and

$$\tilde{\mathbf{s}}_k = \frac{1}{\sqrt{m_1}}\left(\mathbf{s}_k^T(n_1')H_{1k}(n_1')I(n_1')I(n_1' \geq n - J_k^\Delta + 1), \mathbf{s}_k^T(n_2')H_{1k}(n_2')I(n_2')I(n_2' \geq n - J_k^\Delta + 1),\right.$$

$$\left....,\mathbf{s}_k^T(n_{m_1}')H_{1k}(n_{m_1}')I(n_{m_1}')I(n_{m_1}' \geq n - J_k^\Delta + 1)\right)^T \qquad k=2,...,K, \tag{47}$$

where $I(\varphi)$ is the indicator function of event φ, which equals one if φ occurs and zero otherwise.

Let $\mathbf{S} = (\tilde{\mathbf{s}}_2,...,\tilde{\mathbf{s}}_k)$ and $\mathbf{A} = m_1\text{diag}(A_2, ..., A_K)$. The interference power in (8) can be rewritten as

$$P_i = \tilde{\mathbf{s}}_1^T(\mathbf{S}\mathbf{A}^2\mathbf{S}^T)\tilde{\mathbf{s}}_1. \tag{48}$$

Since $H_{1k}(n)$ is bounded, we denote $H_{1k}^{max} = \max(H_{1k}(n_1'), H_{1k}(n_2'),...,H_{1k}(n_{m_1}'))$. Then

$$\|\tilde{\mathbf{s}}_k\| \leq |H_{1k}^{max}|\|\tilde{\mathbf{s}}_k'\|, \tag{49}$$

where

$$\tilde{\mathbf{s}}'_k = \frac{1}{\sqrt{m_1}} \Big(\mathbf{s}_k^T(n_1')I(n_1')I(n_1' \geq n - J_k^\Delta + 1), \mathbf{s}_k^T(n_2')I(n_2')I(n_2' \geq n - J_k^\Delta + 1),$$

$$..., \mathbf{s}_k^T(n_{m_1}')I(n_{m_1}')I(n_{m_1}' \geq n - J_k^\Delta + 1) \Big)^T \qquad k=2,...,K , \qquad (50)$$

and $\|\tilde{\mathbf{s}}'_k\| \leq 1$. So the following inequation can be obtained.

$$P_i \leq \tilde{\mathbf{s}}_1^T (\mathbf{S}'\mathbf{A}^2\mathbf{H}^2\mathbf{S}'^T)\tilde{\mathbf{s}}_1 , \qquad (51)$$

where $\mathbf{H} = \mathrm{diag}(H_{12}^{max},...,H_{1k}^{max})$ and $\mathbf{S}' = (\tilde{\mathbf{s}}'_2,...,\tilde{\mathbf{s}}'_k)$. By [21], $\mathbf{S}'^T\mathbf{S}'$ is upper bounded as $\|\mathbf{S}'^T\mathbf{S}'\| \leq (1+\sqrt{\alpha/m_1})^2$ almost surely. Hence

$$\|\mathbf{S}'\mathbf{A}^2\mathbf{H}^2\mathbf{S}'^T\| \leq \|\mathbf{S}'\|\|\mathbf{A}^2\|\|\mathbf{H}^2\|\|\mathbf{S}'^T\| \leq (1+\sqrt{\alpha/m_1})^2 m_1^2 \max_{2 \leq k \leq K}\{A_k^2\} \max_{2 \leq k \leq K}\{H_{1k}^{max}\} , \qquad (52)$$

which implies that the spectral radius of $\mathbf{S}'\mathbf{A}^2\mathbf{S}'^T$ is bounded in K. By [22]

$$E\left\{ \left| P_i - \frac{1}{m_1 N} tr(\mathbf{S}\mathbf{A}^2\mathbf{S}^T) \right|^6 \right\} \leq \frac{c}{m_1^3 N^3} . \qquad (53)$$

In terms of the Chebyshev inequality, for each $\varepsilon > 0$

$$\mathrm{Pr}\left\{ \left| P_i - \frac{1}{m_1 N} tr(\mathbf{S}\mathbf{A}^2\mathbf{S}^T) \right| > \varepsilon \right\} \leq \frac{c}{\varepsilon^6 m_1^3 N^3} . \qquad (54)$$

It follows from the Borel-Cantelli lemma that P_i converges almost surely to $\lim_{k\to\infty} \frac{1}{m_1 N} tr(\mathbf{S}\mathbf{A}^2\mathbf{S}^T)$. So

$$\frac{1}{m_1 N} tr(\mathbf{S}\mathbf{A}^2\mathbf{S}^T) = \frac{1}{N} \sum_{k=2}^{K} A_k^2 (\mathbf{S}\mathbf{S}^T)_{kk}$$

$$= \frac{1}{N} \sum_{k=2}^{K} A_k^2 \sum_{j=0}^{J_k^\Delta-1} I_1(n-j)I_k(n-j)H_{1k}^2(n-j)$$

$$\leq \frac{1}{N} \sum_{k=2}^{K} A_k^2 \sum_{j=0}^{J_1-1} I_1(n-j)I_k(n-j)H_{1k}^2(n-j) \qquad (55)$$

$$= \frac{1}{N} \sum_{j=0}^{J_1-1} I_1(n-j) \sum_{k=2}^{K} A_k^2 I_k(n-j)H_{1k}^2(n-j) . \qquad (56)$$

By **Lemma 1**

$$\frac{1}{N} \sum_{k=2}^{K} A_k^2 I_k(n-j)H_{1k}^2(n-j) \to \alpha E[H_{1k}^2(n-j)]E[A^2\theta(A)] . \qquad (57)$$

Considering the Rayleigh fading channel, $h_r(n), h_i(n) \sim N(0, 1/2)$. So

$$E[H_{1k}^2(n-j)] = E\left[\big(h_{1r}(n-j)h_{kr}(n-j) + h_{1i}(n-j)h_{ki}(n-j)\big)^2 \right]$$

$$= E\left[h_{1r}^2(n-j)h_{kr}^2(n-j) + h_{1i}^2(n-j)h_{ki}^2(n-j) + 2h_{1r}(n-j)h_{kr}(n-j)h_{1i}(n-j)h_{ki}(n-j) \right]$$

$$= h_{1r}^2(n-j)\cdot\frac{1}{2} + h_{1i}^2(n-j)\cdot\frac{1}{2} = \frac{1}{2}|h_1(n-j)|^2 . \qquad (58)$$

Hence

$$P_i \leq \frac{1}{2}\alpha E[A^2\theta(A)] \sum_{j=0}^{J_1-1} I_1(n-j)|h_1(n-j)|^2 = \frac{1}{2}\alpha E[A^2\theta(A)] \sum_{j=0}^{m_1}|h_1(j)|^2 . \qquad (59)$$

And the limit SINR is lower bounded by (9).

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