(1) 1.1 of Ref 1.

(2) 1.2 of Ref 1.

(3) 1.5 of Ref 2.

(4) Show that, any ~~number~~ Integer which can be devided by 4 [without remainders] can also be devided by 2, i.e.,

a number $b \in \mathbb{Z}$ is divisable by 4 $\Longrightarrow$ $b \in \mathbb{Z}$ is divisible by 2

You may use either the direct method, proof by Contraposition or proof by Contradiction.

Moreover, show that the converse of the statement above is FALSE, i.e., show that the statement

$$\left(b \in \mathbb{Z} \text{ is divisable by } 2 \right) \Longrightarrow \left(b \in \mathbb{Z} \text{ is divisible by } 4 \right)$$

is FALSE.

Here, $\mathbb{Z}$ is the set of integers.

After completing this problem you should know the meaning of converse.

(5) Consider the following set of linear equations:

$$-x_1 + 3x_2 - 2x_3 = 1$$

$$-x_1 + 4x_2 - 3x_3 = 0$$

$$-x_1 + 5x_2 - 4x_3 = 0.$$

Put the equations above in matrix form

$Ax = b.$ where $x = [x_1\ x_2\ x_3]^T.$

A and b are to be accordingly defined. e.g; $A = [a_1\ a_2\ a_3]$, where $a_i \in \mathbb{R}^3$ for all i. In your answer you have to explicitely find a_i

— What can you tell about the statement

$$\text{rank}[A] = \text{rank}[A \mid b]$$

Is it true or false. ?

— Is $b \in \text{span}([a_1\ a_2\ a_3])$?

— Does the set of equation above has a solution ? Justify your answer.

(6) Consider the following equation:

$$-x_1 + 2x_2 = 5.$$

Put it in the matrix form $Ax = b$. $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

- What is rank(A) ?
- What is rank([A|b]) ?
- Comment on the number of solutions of $Ax = b$.

(7) Consider the following linear equations:

$$x_1 + x_2 = 2$$

$$2x_1 + x_2 = 3 \,.$$

Put the equations above in the Matrix form $Ax = b$, when $x = [x_1 \; x_2]^T$

- What is $\text{rank}(A)$?
- What is $\text{rank}([A \mid b])$?
- Is $b \in \text{Range}(A)$?

<u>Note:</u> Given a matrix $A = [a_1 \; a_2 \, .. \, a_k]$, when $a_i \in \mathbb{R}^n$,

$$\text{Range}(A) = \text{span}\,[a_1 \; a_2 \, .. \, a_k] \,.$$

More specifically,

$$\text{Range}(A) = \{ Ax \mid x \in \mathbb{R}^k \}$$

- Does the set of equations above has a solution. Is it unique?