

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/343623465>

# Decentralized Radio Resource Adaptation in D2D-U Networks

Article in IEEE Internet of Things Journal · August 2020

DOI: 10.1109/JIOT.2020.3016019

CITATIONS

9

READS

72

6 authors, including:



Rui Yin

Zhejiang University city college

93 PUBLICATIONS 1,737 CITATIONS

SEE PROFILE



Zheyi Wu

Zhejiang University

2 PUBLICATIONS 9 CITATIONS

SEE PROFILE



Shengli Liu

Zhejiang University

10 PUBLICATIONS 33 CITATIONS

SEE PROFILE



Celimuge Wu

The University of Electro-Communications

207 PUBLICATIONS 2,833 CITATIONS

SEE PROFILE

Some of the authors of this publication are also working on these related projects:



Distributed Vehicular computing and communications. [View project](#)



The 2nd IEEE International Workshop on Data Driven Intelligence for Networks and Systems (DDINS) [View project](#)

# Decentralized Radio Resource Adaptation in D2D-U Networks

Rui Yin, Zheyi Wu, Shengli Liu, Celimuge Wu, Jiantao Yuan, and Xianfu Chen

**Abstract**—Unlike the conventional *device-to-device* (D2D) networks, the *unlicensed D2D* (D2D-U) pairs can not only reuse the licensed channels with the *base station* (BS), but also share the unlicensed channels with the WiFi stations. One challenge arises from the fact that the co-channel interference on licensed channels and the collision probability on unlicensed channels may cause extra power consumption at the terminals. Accordingly, we first propose a channel access method for the D2D-U pairs on unlicensed channels. Then, a decentralized joint spectrum and power allocation scheme is designed to minimize the power consumption at D2D-U pairs. Different from the existing distributed schemes, the proposed scheme can guarantee the global minimization of power consumption across the D2D-U pairs. Simulation results validate the theoretical analysis and verify the performance from the proposed scheme.

**Index Terms**—D2D-U, new radio, radio resource management, decentralized algorithm.

## I. INTRODUCTION

During the past decades, energy consumption has been an imminent worldwide issue. The information and communication technologies are responsible for a significant proportion of global energy consumption [1]. With the development of the 5th generation (5G) wireless systems, the ubiquitous smart devices and diverse new applications, such as *augment reality* (AR), *virtual reality* (VR), and vehicular *Internet-of-Things* (IoT) [2], cause much more data exchange, which quickly drains the battery of devices. For the energy saving purpose, it becomes critical to reduce power consumption at the terminal devices in 5G wireless networks. An improper

power consumption strategy shortens the life of a device with limited battery capacity.

In addition to the power consumption, there are still many challenges for 5G systems [3]. Especially, various new applications bring the increase on the mobile data traffic unprecedentedly. It is estimated that the required data rate, in order to accommodate such amount of data traffic demand, will increase 1000x by 2020 [4]. Such a large increase on system capacity poses a great challenge to the existing mobile cellular networks.

To achieve high data rate, in general, there are two potential ways. One is to develop revolutionary techniques to improve the *spectrum efficiency* (SE), such as *massive multiple-input multiple-output* (M-MIMO), *orthogonal frequency-division multiple access* (OFDMA), and *device-to-device* (D2D) communications [5]. As a key technology for 5G *new radio* (NR) networks to improve SE, D2D enables two terminal devices to communicate directly without the assistance of a *base station* (BS) [6]. The other way is to exploit more available spectrum, such as the licensed-exempt (namely, unlicensed) spectrums and the millimeter wave band. The latest cellular networks, like the 4th generation (4G) *long term evolution* (LTE) and the 5G NR techniques, which have achieved very high SE, leave very little room for further effective amelioration on SE [7]. Therefore, exploiting more spectrums is believed to be feasible. The unlicensed spectrums (e.g., 2.4GHz and 5.8GHz), over which the *wireless local area network* (WLAN) systems operate, are widely underutilized. Applying spectrum-efficient technologies to unlicensed channels is an inevitable trend to enhance the system capacity [8].

In line with the above discussions, enabling D2D communications over unlicensed channels, namely, D2D-U underlaying 5G NR networks, is a promising approach to facilitate high data rate services and boost the system capacity. Although the D2D technology on licensed channels has been extensively investigated over the past years, it is still in its nascent stage and faces many technical challenges when operating over a unlicensed spectrum. Firstly, the fair coexistence needs to be guaranteed when D2D-U networks share the unlicensed spectrum with WiFi networks [9]. Secondly, due to the distributed structure of the D2D-U networks, a decentralized power and spectrum allocation scheme is required to avoid the overwhelming signaling overheads [10]. Thirdly, the co-channel interference between D2D-U and cellular links on licensed channels as well as the indeterminate feature of channel access with incumbent WiFi systems over unlicensed channels make the performance analysis of D2D-U communications extremely complicated [6]. Moreover, most developed schemes can

R. Yin is with the School of Information and Electrical Engineering, Zhejiang University City College, Hangzhou, 310018, China, and the School of Electrical, Electronic and Computer Engineering, KwaZulu-Natal University, Durban, 4041, South Africa (e-mail: yinrui@zucc.edu.cn).

Z. Wu, S. Liu, and J. Yuan are with the School of Information and Electronic Engineering, Zhejiang University, Hangzhou, 310027, China (e-mail: {21960194, victoryliu, yuanjiantao}@zju.edu.cn).

C. Wu is with the Graduate School of Informatics and Engineering, the University of Electro-Communications, Tokyo, 182-8585, Japan (e-mail: celimuge@uec.ac.jp).

X. Chen is with the VTT Technical Research Centre of Finland, Oulu, 90570, Finland (e-mail: xianfu.chen@vtt.fi).

This work was supported in part by the National Natural Science Foundation of China (Grant No. 61771429, 61703368), Zhejiang University City College Scientific Research Foundation (No. JZD18002), in part by the Zhejiang Lab's International Talent Fund for Young Professionals, in part by Aviation Key Laboratory of Science and Technology on Fault Diagnosis and Health Management (20183333001), in part by Key Laboratory of Urban Rail Transit Intelligent Operation and Maintenance Technology & Equipment of Zhejiang Province, and in part by Zhejiang Provincial Key Laboratory of Information Processing, Communication and Networking, Zhejiang, China.

Copyright (c) 2020 IEEE. Personal use of this material is permitted. However, permission to use this material for any other purposes must be obtained from the IEEE by sending a request to pubs-permissions@ieee.org.

not adapt to the varying wireless channel conditions without re-initiation and face intolerable computational complexity. Therefore, when studying D2D transmissions on unlicensed channels, the above needs carefully to be jointly addressed.

In the D2D-U enabled cellular networks, since the D2D-U terminals need to continuously sense the channel status in unlicensed channel, it becomes more important to reduce power consumption. Accordingly, we propose in this paper a joint spectrum and power allocation scheme to minimize the power consumption. The main technical contributions from this work are summarized as follows.

- 1) A novel channel access algorithm is derived for the D2D-U pairs to transmit on the unlicensed channels in addition to the licensed channels. Implementing the algorithm, the D2D-U pairs can decide the time fraction over the corresponding unlicensed channels under the fairness constraint.
- 2) Exploiting the distributed structure of a D2D-U enabled cellular network, a decentralized joint power and channel allocation scheme is proposed to minimize the overall power consumption across the D2D-U pairs. The proposed decentralized scheme is adaptive to the changing wireless environment, reducing both the computational complexity and the signaling overheads.
- 3) The convergence to the global optimality of the proposed scheme is theoretically proved. The corresponding computational complexity and signaling overheads are analyzed. Moreover, the simulation results are provided to verify the studies in this paper.

In next section, we review the related works. Section III introduces a D2D-U enabled 5G NR cellular network and develops a channel access algorithm for the D2D-U pairs on unlicensed channels for fair coexistence. In Section IV, we formally formulate an optimization problem of joint power and channel allocation with the aim of minimizing the power consumption across the D2D-U pairs in the system, for which a decentralized scheme is proposed. In section V, we prove the optimality of the proposed scheme and analyze the corresponding signaling overheads as well as computational complexity. Simulation results are presented in Section VI to demonstrate the effectiveness of the proposed scheme and verify the accuracy of the theoretical analysis. Finally, we draw conclusions in Section VII.

## II. RELATED WORK

For a D2D-U network, two obstacles should be addressed before starting resource allocation. The first one is the reasonable establishment of D2D links. Considering the increasing number of devices, [11] has studied how to establish dense D2D connections during peak time aided by users equipped limited cache. The second obstacle is the fair coexistence for D2D networks with the other incumbent systems, such as WiFi networks. To guarantee the fairness, two different channel access mechanisms, i.e., *duty cycle muting* (DCM) and *listen before talk* (LBT), have been proposed for *unlicensed LTE* (LTE-U) and *licensed assisted access* (LAA) networks [12]. The DCM mechanism has been first defined in [13], where

almost blank subframes in the LTE system are used to enable the WiFi transmission on unlicensed channels. In [14], a LBT based scheme has been proposed at the small cell networks to avoid the collisions, where the back-off window size can be adjusted adaptively based on the WiFi traffic load and the available spectrum.

As for resource allocation in 5G systems, different kinds of strategies are systematically investigated in [15]. Some have studied the decentralized resource allocation schemes [16], [17]. A blockchain-based decentralized resource management framework has been proposed in [16] to save the energy by the scheduler in cloud datacenters. In [17], a green resource allocation algorithm based on *Deep Reinforcement Learning* (DRL) has been presented. In contrast, a centralized joint mode selection, channel assignment and power allocation scheme has been proposed in [18] to maximize the system SE for D2D networks.

As far as the D2D enabled cellular network is concerned, many works have emphasized the importance of power consumption in D2D networks and studied the resource allocation to lower energy or power consumption. [19]–[22]. In [19], a green resource allocation scheme has been proposed for D2D communications underlying C-RAN to optimize system *Energy Efficient* (EE), and the problem has been modeled as a noncooperative game. An iterative power allocation algorithm has been proposed in [20], which is developed by exploiting nonlinear fractional programming and Lagrange dual decomposition. In [21], a distributed interference-aware energy-efficient resource allocation algorithm has been proposed where the non-cooperative game model has been applied to maximize the users' EE. By adopting a stable matching approach, the EE optimization problem has been addressed in [22] for the D2D enabled cellular networks, and the algorithm has been demonstrated by simulation that it can effectively improve the average EE.

Up to now, only a few studies considered the resource allocation for D2D-U based networks [23]–[26]. Since the D2D-U enabled networks are inherently decentralized, the pairing, spectrum management and power allocation scheme should be decentralized to cope with the resource management regulation rules on unlicensed channels [23], [24]. To reduce computational complexity, the DRL has been combined with the conventional convex optimization methods to allocate the power and spectrum to the D2D-U networks in a distributed way in [23]. In [24], a channel allocation and power control algorithm using particle swarm optimization has been proposed to manage the interference and improve the overall throughput of cellular and D2D users such that the minimum data rate requirement of WiFi users is guaranteed. Some works have proposed centralized schemes for the D2D-U networks, but may bring high computational complexity and huge signaling overheads [25], [26]. In [25], power allocation on licensed channels and spectrum allocation on unlicensed channels have been jointly optimized in a centralized way to maximize the system SE in D2D-U networks. A centralized subchannel allocation scheme has been proposed in [26] to maximize the sum data rates of cellular users and D2D-U pairs when only considering the unlicensed spectrum resource.

### III. SYSTEM MODEL

This paper considers a scenario where the 5G NR cellular network coexists with the WiFi network, as shown in Fig. 1. In the 5G NR network, the cellular users can either transmit via the BS, i.e., in *cellular mode* (CM), or communicate directly with each other, i.e., in D2D mode. When users working in the CM mode named as *CM users* (CU), different licensed channels will be allocated to different users to avoid the co-channel interference. When two users work in D2D mode, they can reuse the licensed uplink channels and share the unlicensed channels with the WiFi APs. Therefore, it is named as *D2D on unlicensed channels* (D2D-U), where the transmitter and receiver are denoted as D2DT and D2DR, respectively. Mathematically, the number of D2D-U pairs is represented by  $K$  and all the D2D-U pairs are denoted as a set  $\mathcal{K}$ . For a cellular user in the system, the work mode should be determined first. In this paper, we do not consider the mode selection and assume that the work mode of cellular users has already been determined before the channel access and resource allocation. Specifically, we assume that the work mode is decided by the BS based on the method proposed in [27].

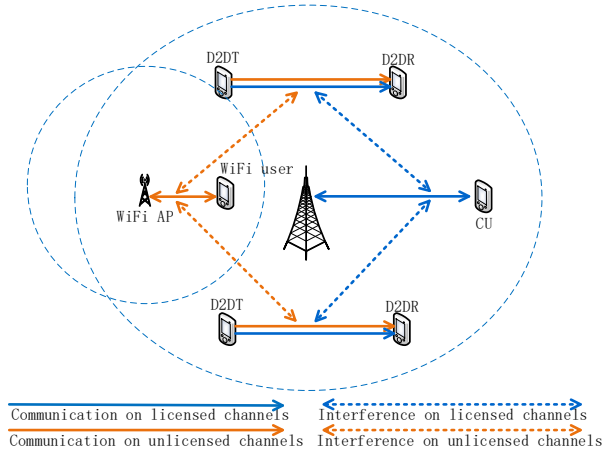


Fig. 1. The system model of the D2D-U enabled 5G NR network.

When using the licensed channel, the OFDMA technology [28] is adopted, where the licensed channel is divided into  $N$  subchannels. Then, the licensed uplink subchannels reused among D2D-U pairs are decided by the BS based on a centralized manner. Since the BS can obtain the geographic location, traffic type, and QoS requirements of all users, we assume that the interference among D2D-U pairs on licensed subchannels can be avoided. Mathematically, the set of licensed subchannels allocated to the D2D-U pair  $k$  is denoted as  $\mathcal{N}_k$ .

Similarly, the unlicensed band of the WiFi network is divided into  $W$  channels. We assume that there are  $W_k$  unlicensed channels available for D2D-U pair  $k$ , which is denoted as a set  $\mathcal{W}_k$ . To avoid the interference among D2D-U pairs on unlicensed channels, we assume that one unlicensed channel can only be used by one D2D-U pair. In the WiFi networks, the WiFi stations adopt binary exponential back-off mechanism to access the unlicensed channels [29]. Since the BS has no exclusive authority on the unlicensed channels

#### Algorithm 1 Channel Access for D2D-U Links on Unlicensed Channels.

- 1: Both the D2DT and D2DR sense the unlicensed channels and decide the sets of available unlicensed channel  $\mathcal{W}^{(T)}$  and  $\mathcal{W}^{(R)}$ , respectively.
- 2: The D2DR feeds back the set of available unlicensed channels,  $\mathcal{W}^{(R)}$ , to the D2DT via the allocated licensed channels.
- 3: If the available unlicensed channels at the D2DT,  $\mathcal{W}^{(T)}$ , has intersection with  $\mathcal{W}^{(R)}$ , the D2DT feeds back their common unlicensed channels  $\mathcal{W}^{(T)} \cap \mathcal{W}^{(R)}$  to the D2DR.
- 4: Both the D2DT and D2DR sense their common unlicensed channels and estimate the traffic load of the WiFi system.
- 5: The D2DR feeds back the estimated traffic load of the WiFi system to the D2DT.
- 6: The D2DT decides the time fraction on the unlicensed channels that can be used based on the maximum traffic load of the WiFi system sensed by the D2DT and D2DR.
- 7: The D2DT sends the synchronization signal to the D2DR via the control channel over licensed channel.
- 8: The D2DR starts to receive the data from the D2DT over the agreed unlicensed channels.

and the D2D-U pairs are randomly located in the coverage of the BS, the D2D-U pairs need to decide how to use the unlicensed channels individually, which will be introduced in the next section.

#### A. Channel Access Scheme on Unlicensed Channels

When D2D-U pairs communicate on unlicensed channels, centralized channel access scheme implemented at the BS can not be adopted due to the distributed structure of D2D-U networks. Therefore, a distributed channel access scheme should be designed for the D2D-U pairs.

According to the regularity rules of unlicensed channels, all users need to sense the state of the unlicensed channels before using it to avoid the collision. Therefore, when the D2D-U pairs share unlicensed channels with WiFi networks, both the D2DT and D2DR need to sense the unlicensed channels first. In the proposed channel access scheme, we define  $\mathcal{W}_k^{(T)}$  and  $\mathcal{W}_k^{(R)}$  as the sets of available unlicensed channels sensed by the D2DT <sub>$k$</sub>  and D2DR <sub>$k$</sub> , respectively. Then, for D2D-U pair  $k$ ,  $\mathcal{W}_k$  is the intersection of  $\mathcal{W}_k^{(T)}$  and  $\mathcal{W}_k^{(R)}$ , that is

$$\mathcal{W}_k = \mathcal{W}_k^{(T)} \cap \mathcal{W}_k^{(R)}. \quad (1)$$

When D2DT and D2DR of the same D2D-U pair have common available unlicensed channels, they can transmit over their common unlicensed channels. Therefore, if  $\mathcal{W}_k \neq \emptyset$ , the D2DT <sub>$k$</sub>  and D2DR <sub>$k$</sub>  have common available unlicensed channels and they can transmit over these unlicensed channels. Otherwise, they can not use the unlicensed channels. Besides, once the D2DT and D2DR decide their common available unlicensed channels, they need to decide the time fraction on the corresponding unlicensed channels that can be used.

In this paper, we assume that the DCM mechanism is adopted by the D2D-U pairs to compete for the corresponding

unlicensed channels with the WiFi users, while guaranteeing the fairness. Both the D2DT and D2DR sense their common available unlicensed channels and estimate the WiFi traffic load based on the method proposed in [30]. Then, the D2DR feeds back its estimated value on WiFi traffic load to the D2DT via the licensed subchannels. After the D2DT receives this information from the D2DR, it decides the time fraction that can be used on the corresponding unlicensed channels based on the maximum WiFi traffic load estimated by the D2DT and D2DR. Before the D2DT starts to transmit on unlicensed channels, the D2DR will receive the synchronization and other control signals from the D2DT on the licensed subchannels. Then, the D2DR starts to listen on the unlicensed channels while the D2DT starts the transmission. The detailed channel access scheme for D2D-U pairs can be summarized in Algorithm 1, and for convenience we ignore the subscript in the expression of the proposed scheme.

### B. Achievable Data Rate

When the D2D-U pair  $k$  reuses the licensed uplink subchannel  $n$ , the achievable data rate is given by

$$R_{k,n}^{(L)} = B^{(L)} \log \left( 1 + \frac{p_{k,n}^{(L)} h_{k,n}^{(L)}}{(I_{k,n} + N_0^{(L)}) B^{(L)}} \right), \quad (2)$$

where  $B^{(L)}$  is the bandwidth of the licensed subchannel,  $p_{k,n}^{(L)}$  is the transmission power of the D2DT <sub>$k$</sub>  on licensed subchannel  $n$ ,  $h_{k,n}^{(L)}$  stands for the channel gain on licensed subchannel  $n$  between the D2DT <sub>$k$</sub>  and D2DR <sub>$k$</sub> , and  $I_{k,n}$  represents the co-channel interference power spectrum experienced by the D2DR <sub>$k$</sub>  on subchannel  $n$ ,  $N_0^{(L)}$  is the noise power spectrum density on licensed subchannel  $n$ . To avoid the strong co-channel interference among different D2D-U pairs on licensed channels, the BS allocates different licensed subchannels to different D2D-U pairs when they are close. Therefore, the co-channel interference among D2D-U pairs is ignored in (2).

Therefore, the total data rates achieved on licensed channels at D2D-U pair  $k$  can be expressed as

$$R_k^{(L)} = \sum_{n \in \mathcal{N}_k} R_{k,n}^{(L)}. \quad (3)$$

On the other hand, when D2D-U pair  $k$  shares the unlicensed channel  $w$  with the WiFi users, its achievable data rate is expressed as

$$R_{k,w}^{(U)} = \rho_{k,w}^{(U)} B^{(U)} \log \left( 1 + \frac{p_{k,w}^{(U)} h_{k,w}^{(U)}}{B^{(U)} N_0^{(U)}} \right), \quad (4)$$

where  $\rho_{k,w}^{(U)}$  is the time fraction used by D2D-U pair  $k$  on the unlicensed channel  $w$ ,  $B^{(U)}$  is the bandwidth of the unlicensed channel,  $p_{k,w}^{(U)}$  is the transmission power adopted by the D2DT <sub>$k$</sub>  on unlicensed channel  $w$ ,  $h_{k,w}^{(U)}$  is the channel gain between D2DT <sub>$k$</sub>  and D2DR <sub>$k$</sub>  on unlicensed channel  $w$ , and  $N_0^{(U)}$  is the noise power spectrum density on unlicensed channel  $w$ . Accordingly, the total data rates achieved on unlicensed channels by D2D-U pair  $k$  is expressed as

$$R_k^{(U)} = \sum_{w \in \mathcal{W}_k} R_{k,w}^{(U)}. \quad (5)$$

Therefore, the total achievable data rates at D2D-U pair  $k$  is given by

$$R_k = R_k^{(L)} + R_k^{(U)}. \quad (6)$$

### C. Power Consumption

The power consumption of each D2D-U pair is composed of three parts. The first part is the power consumption on licensed subchannels, where the over-the-air transmission power can be expressed as

$$P_k^{(L)} = \sum_{n \in \mathcal{N}_k} \omega^{(L)} p_{k,n}^{(L)}, \quad (7)$$

where  $\omega^{(L)}$  is the inversion of *power amplifier efficiency* (PAE) on licensed subchannels and  $\omega^{(L)} > 1$ .

The second part is the power consumption on the unlicensed channels which is given by

$$P_k^{(U)} = \sum_{w \in \mathcal{W}_k} \rho_{k,w}^{(U)} \left( \omega^{(U)} p_{k,w}^{(U)} \right) + \sum_{w \in \mathcal{W}_k} \left( 1 - \rho_{k,w}^{(U)} \right) p_s^{(U)}, \quad (8)$$

where  $\omega^{(U)}$  is the inversion of PAE on the unlicensed channels and  $\omega^{(U)} \geq 1$ ,  $p_s^{(U)}$  is the power consumption of the D2D-U pair  $k$  on sensing the unlicensed channel  $w$  which is a constant. The first item of (8) is the over-the-air transmission power and the second item is the power consumption when D2D-U pair  $k$  senses the unlicensed channels.

The third part is the circuit power to sustain the running of the devices, which is denoted as  $p_k^{(C)}$ . Accordingly, the total power consumed at the D2D-U pair  $k$  is given by

$$P_k^{(tot)} = P_k^{(L)} + P_k^{(U)} + p_k^{(C)}. \quad (9)$$

Based on the above analysis, the overall power consumed by D2D-U pairs can be written as

$$P^{(tot)} = \sum_{k \in \mathcal{K}} P_k^{(tot)}. \quad (10)$$

### D. Co-channel Interference to the BS

When the D2D-U pair  $k$  reuses the licensed uplink subchannel,  $n$ , the co-channel interference brought to the BS is given by

$$I_{k,n}^{(L)} = p_{k,n}^{(L)} \cdot \tilde{h}_{k,n}^{(L)}, \quad \forall n \in \mathcal{N}, \quad (11)$$

where  $\tilde{h}_{k,n}^{(L)}$  is the interference channel power gain on licensed subchannel  $n$  from D2D-U pair  $k$  to the BS.

## IV. POWER CONSUMPTION OPTIMIZATION ALGORITHM DEVELOPMENT

In this section, we first formulate an optimization problem to minimize the overall power consumption of all D2D-U pairs, and then design a decentralized algorithm to obtain the global optimal power and spectrum allocation solution.

### A. Problem Formulation

Considering the constrained battery capacity of terminal devices in the D2D-U enabled NR systems, the power and channel allocation should be jointly optimized to minimize

the power consumption while restricting the co-channel interference to the BS and guaranteeing the fair coexistence with WiFi networks. Accordingly, the optimization problem can be formulated as

$$\min_{\mathbf{p}_k^{(L)}, \mathbf{p}_k^{(U)}, \rho_k^{(U)}, k \in \mathcal{K}} P^{(tot)} \quad (12)$$

subject to

$$R_k \geq \bar{r}_k, \forall k \in \mathcal{K}, \quad (12a)$$

$$\rho_{k,w}^{(U)} p_{k,w}^{(U)} \leq P_T^{(U)}, \forall w \in \mathcal{W}_k, \forall k \in \mathcal{K}, \quad (12b)$$

$$\rho_{k,w}^{(U)} \leq \bar{\beta}_w, \forall k \in \mathcal{K}, \forall w \in \mathcal{W}_k, \quad (12c)$$

$$\sum_{k \in \mathcal{K}} I_{k,n}^{(L)} \leq \bar{I}_n, \forall n \in \mathcal{N}_k, \quad (12d)$$

where  $\mathbf{p}_k^{(L)} = \{p_{k,i}^{(L)}\}_{i \in \mathcal{N}_k}$ ,  $\mathbf{p}_k^{(U)} = \{p_{k,i}^{(U)}\}_{i \in \mathcal{W}_k}$ ,  $\rho_k^{(U)} = \{\rho_{k,i}^{(U)}\}_{i \in \mathcal{W}_k}$  are the power allocation matrices on licensed subchannels, unlicensed channels and time fraction allocation matrix of unlicensed channels, the superscript upper case  $T$  stands for the transpose operation,  $P_T^{(U)}$  is the maximum power can be transmitted on an unlicensed channel,  $\bar{\beta}_w$  is the maximum time fraction which can be used by the D2D-U pair  $k$  on unlicensed channel  $w$ ,  $\bar{I}_n$  is the maximum co-channel interference can be accepted the BS, constraint (12a) is used to satisfy the minimum data rate requirements of D2D-U pairs, constraint (12b) is used to refine the transmission power on unlicensed channels due to the regulation requirement on unlicensed channels, constraint (12c) is used to guarantee the fair coexistence between the WiFi networks and the D2D-U transmission on unlicensed channels, constraint, and constraint (12d) is used to refine the co-channel interference to the BS on each licensed subchannel,  $n$ .

It is obvious that optimization problem (12) is a non-convex problem due to the product of  $\rho_{k,w}^{(U)} p_{k,w}^{(U)}$  in (12b). Fortunately, it can be converted into a convex optimization problem by replacing  $\rho_{k,w}^{(U)} p_{k,w}^{(U)}$  with  $\mu_{k,w}^{(U)}$ . By doing so, the objective function of problem (12) can be transformed into

$$\hat{P}^{(tot)} = \sum_{k \in \mathcal{K}} P_k^{(L)} + p_k^{(C)} + \hat{P}_k^{(U)}, \quad (13)$$

where

$$\hat{P}_k^{(U)} = \sum_{w \in \mathcal{W}_k} \omega^{(U)} \mu_{k,w}^{(U)} + \sum_{w \in \mathcal{W}_k} (1 - \rho_{k,w}^{(U)}) p_s^{(U)}. \quad (14)$$

Therefore, by replacing  $\rho_{k,w}^{(U)} p_{k,w}^{(U)}$  with  $\mu_{k,w}^{(U)}$  in optimization problem (12), it can be rewritten as

$$\min_{\mathbf{p}_k^{(L)}, \mu_k^{(U)}, \rho_k^{(U)}, k \in \mathcal{K}} \hat{P}^{(tot)} \quad (15)$$

subject to

$$\hat{R}_k \geq \bar{r}_k, \forall k \in \mathcal{K}, \quad (15a)$$

$$\mu_{k,w}^{(U)} \leq P_T^{(U)}, \forall w \in \mathcal{W}_k, \forall k \in \mathcal{K}, \quad (15b)$$

$$(12c) \text{ and } (12d), \quad (15c)$$

where

$$\hat{R}_k = R_k^{(L)} + \sum_{w \in \mathcal{W}_k} \rho_{k,w}^{(U)} B^{(U)} \log \left( 1 + \frac{\mu_{k,w}^{(U)} h_{k,w}^{(U)}}{B^{(U)} \rho_{k,w}^{(U)} N_0^{(U)}} \right), \quad (16)$$

and

$$\mu_k^{(U)} = \{\mu_{k,i}^{(U)}\}_{i \in \mathcal{W}_k}^T. \quad (17)$$

The convexity of problem (15) can be proved by the following theorem.

**Theorem 1.** Problem (15) is a convex optimization problem with respect to  $\mathbf{p}_k^{(L)}$ ,  $\mu_k^{(U)}$ ,  $\rho_k^{(U)}$ ,  $k \in \mathcal{K}$ .

*Proof:* Please see Appendix A. ■

**Remark 1.** For the convex optimization problem, methods such as the conventional interior point can be applied to obtain the optimal solution. Therefore, the optimization problem (15) can be directly solved by the existing methods. However, implementing the centralized joint power and spectrum allocation at the BS will cause large computational complexity and overwhelming signaling overheads, and thus they are not suitable to be implemented in the distributed D2D-U networks. Therefore, a decentralized energy saving scheme is proposed in sequel.

## B. Decentralized Power and Channel Allocation Scheme

To reduce computational complexity and signaling overhead, decentralized algorithm is proposed for each D2D-U pair to decide its own power and channel allocation strategy independently while reserving the global optimality in this subsection.

1) *Augmented Lagrangian duality:* It can be found from the optimization problem (15) that the D2D-U pairs are coupled with each other in the power and channel allocation strategies because of the constraint (12d). The remaining constraints are denoted as local constraints, which can be satisfied by each D2D-U pair independently. To decouple the resource allocation strategies at D2D-U pairs, the duality of problem (15) needs to be utilized, which is defined as

$$\begin{aligned} \max_{\lambda \in \mathbf{R}^N} f(\lambda) &= \max_{\lambda \in \mathbf{R}^N} \sum_{k \in \mathcal{K}} f_k(\lambda) \\ &= \max_{\lambda \in \mathbf{R}^N} \sum_{k \in \mathcal{K}} \inf_{\mathbf{x}_k \in \Omega_k} (P_k^{(tot)} - \lambda^T \mathbf{I}_{k,N} + \lambda^T \bar{\mathbf{I}}_N), \end{aligned} \quad (18)$$

where the Lagrangian multiplier  $\lambda$  is defined as  $\{\lambda_i\}_{i \in \mathcal{N}_k}$ ,  $\Omega_k$  is the set of power and spectrum allocation strategies satisfying all the local constraints (12a)-(12c) for D2D-U pair  $k$ ,  $\mathbf{I}_{k,N}$  is a  $N \times 1$  vector defined as  $\mathbf{I}_{k,N} = \{I_{k,i}^{(L)}\}_{i \in \mathcal{N}_k}^T$ , and  $\bar{\mathbf{I}}_N$  is a  $N \times 1$  vector defined as  $\bar{\mathbf{I}}_N = \{\bar{I}_i/K\}_{i \in \mathcal{N}_k}^T$ ,  $\mathbf{x}_k$  is  $\text{col}(\mathbf{p}_k^{(L)}, \mu_k^{(U)}, \rho_k^{(U)})$  with  $\text{col}(\mathbf{x}_1, \dots, \mathbf{x}_3) = [\mathbf{x}_1^T, \dots, \mathbf{x}_3^T]^T$  being defined as the column vector stacked with vectors  $\mathbf{x}_1, \dots, \mathbf{x}_3$ .

If the above duality is adopted directly, the subproblem needs to be solved to calculate the gradients at each D2D-U pair due to the global multiplier  $\lambda$ . Consequently, a constrained optimization problem with Laplacian matrix  $\mathbf{L}$  and local Lagrangian multiplier matrix defined as  $\Lambda = \text{col}(\lambda_1, \dots, \lambda_K) \in$

$\mathbf{R}^{KN}$  is formulated as

$$\max_{\mathbf{\Lambda}} F(\mathbf{\Lambda}) = \sum_{k \in \mathcal{K}} f_k(\boldsymbol{\lambda}_k) \quad (19)$$

subject to

$$(\mathbf{L} \otimes \mathbf{U}_N) \mathbf{\Lambda} = \mathbf{0}^{KN}, \quad (19a)$$

where  $\boldsymbol{\lambda}_k$  is a  $N \times 1$  vector, the constraint (19a) is used to guarantee the consensus on the Lagrangian multiplier among D2D-U pairs, i.e.,  $\boldsymbol{\lambda}_1 = \boldsymbol{\lambda}_2 = \dots = \boldsymbol{\lambda}_K$ .

Accordingly, the augmented Lagrangian duality of (19) with multipliers  $\boldsymbol{\chi} = \text{col}(\boldsymbol{\chi}_1, \dots, \boldsymbol{\chi}_K) \in \mathbf{R}^{KN}$  can be expressed as

$$\min_{\boldsymbol{\chi}} \max_{\mathbf{\Lambda}} Q(\mathbf{\Lambda}, \boldsymbol{\chi}) \\ = \sum_{k \in \mathcal{K}} q_k(\boldsymbol{\lambda}_k) - \boldsymbol{\chi}^T (\mathbf{L} \otimes \mathbf{U}_N) \mathbf{\Lambda} - \frac{1}{2} \mathbf{\Lambda}^T (\mathbf{L} \otimes \mathbf{U}_N) \mathbf{\Lambda}. \quad (20)$$

From (20), we can observe that the global Lagrangian multiplier  $\boldsymbol{\lambda}$  is converted into the local Lagrangian multipliers. That is, when D2D-U pairs reach the consensus on the multiplier, the optimal solution of (20) is equal to that of (15) [32].

2) *Decentralized Scheme*: The projected algorithm in [32] can be used to solve (20) in a decentralized way. According to the augmented Lagrangian duality derived in (20), two auxiliary variables for each D2D-U pair,  $k$ , are defined as  $\boldsymbol{\lambda}_k$  and  $\boldsymbol{\chi}_k$ , ( $k \in \mathcal{K}$ ). Based on the projected algorithm, the updating rule of the power and channel allocation strategy and the Lagrangian multiplier at each D2D-U pair  $k$  is given by

$$\begin{aligned} \mathbf{P}_k^{(L)}(t+1) &= \mathbf{P}_k^{(L)}(t) + \alpha \dot{\mathbf{P}}_k^{(L)}(t+1), \\ \boldsymbol{\mu}_k^{(U)}(t+1) &= \boldsymbol{\mu}_k^{(U)}(t) + \alpha \dot{\boldsymbol{\mu}}_k^{(U)}(t+1), \\ \boldsymbol{\rho}_k^{(U)}(t+1) &= \boldsymbol{\rho}_k^{(U)}(t) + \alpha \dot{\boldsymbol{\rho}}_k^{(U)}(t+1), \\ \boldsymbol{\lambda}_k(t+1) &= \boldsymbol{\lambda}_k(t) + \alpha \dot{\boldsymbol{\lambda}}_k(t+1), \\ \boldsymbol{\chi}_k(t+1) &= \boldsymbol{\chi}_k(t) + \alpha \dot{\boldsymbol{\chi}}_k(t+1), \end{aligned} \quad (21)$$

where  $t$  is the index of the updating step,  $\alpha$  represents updating stepsize,  $\dot{\mathbf{P}}_k^{(L)}(t+1)$ ,  $\dot{\boldsymbol{\mu}}_k^{(U)}(t+1)$ ,  $\dot{\boldsymbol{\rho}}_k^{(U)}(t+1)$ ,  $\dot{\boldsymbol{\lambda}}_k(t+1)$ , and  $\dot{\boldsymbol{\chi}}_k(t+1)$  are gradient flows calculated by

$$\begin{aligned} \dot{\mathbf{P}}_k^{(L)}(t+1) &= P_{\Omega_k}(\mathbf{P}_k^{(L)}(t) - \nabla f_k(\mathbf{P}_k^{(L)}(t)) - \boldsymbol{\lambda}_k(t) \cdot \tilde{\mathbf{H}}_k) - \mathbf{P}_k^{(L)}(t), \\ \dot{\boldsymbol{\mu}}_k^{(U)}(t+1) &= P_{\Omega_k}(\boldsymbol{\mu}_k^{(U)}(t) - \nabla f_k(\boldsymbol{\mu}_k^{(U)}(t))) - \boldsymbol{\mu}_k^{(U)}(t), \\ \dot{\boldsymbol{\rho}}_k^{(U)}(t+1) &= P_{\Omega_k}(\boldsymbol{\rho}_k^{(U)}(t) - \nabla f_k(\boldsymbol{\rho}_k^{(U)}(t))) - \boldsymbol{\rho}_k^{(U)}(t), \\ \dot{\boldsymbol{\lambda}}_k(t+1) &= - \sum_{j \in \mathcal{B}_k} (\boldsymbol{\lambda}_k(t) - \boldsymbol{\lambda}_j(t)) - \sum_{j \in \mathcal{B}_k} (\boldsymbol{\chi}_k(t) - \boldsymbol{\chi}_j(t)) \\ &\quad + (\bar{\mathbf{I}}_N/K - \mathbf{P}_k^{(L)}(t) \otimes \tilde{\mathbf{H}}_k), \\ \dot{\boldsymbol{\chi}}_k(t+1) &= \sum_{j \in \mathcal{B}_k} (\boldsymbol{\lambda}_k(t) - \boldsymbol{\lambda}_j(t)). \end{aligned} \quad (22)$$

Herein,  $\tilde{\mathbf{H}}_k$  is a  $N \times 1$  vector defined as  $\{\tilde{h}_{k,i}\}_{i \in \mathcal{N}_k}^T$ ,  $\nabla f_k(\mathbf{x})$  is the gradient of  $f_k(\mathbf{x})$  with respect to the vector  $\mathbf{x}$ , the set  $\mathcal{B}_k$  is to denote the neighbors of D2D-U pair  $k$ , which can share the information with it.  $P_{\Omega_k}(\mathbf{x})$  is the projection operation of  $\mathbf{x}$  into the feasible domain  $\Omega_k$  defined as the following optimization problem

$$P_{\Omega_k}(\mathbf{x}) = \arg \min_{\mathbf{y} \in \Omega_k} \|\mathbf{x} - \mathbf{y}\|. \quad (23)$$

**Algorithm 2** Update Scheme at D2D-U pair  $k$ ,  $k \in \mathcal{K}$ .

- 1: Initialize  $t = 0$ .
- 2: Initialize  $\mathbf{X}(0) = \{\mathbf{P}_k^{(L)}(0), \boldsymbol{\rho}_k^{(U)}(0), \boldsymbol{\mu}_k^{(U)}(0)\}, k \in \mathcal{K}$  with one step of projection operation.
- 3: Initialize  $\boldsymbol{\lambda}_k(0) = \bar{\mathbf{0}}$  and  $\boldsymbol{\chi}_k(0) = \bar{\mathbf{0}}$ .
- 4: **while**  $(\|\dot{\mathbf{X}}_t\|_2^2 + \|\dot{\boldsymbol{\lambda}}_t\|_2^2 + \|\dot{\boldsymbol{\chi}}_t\|_2^2) \geq \sigma$  **do**
- 5: Calculate the differential equations in (24), and find the solutions  $\bar{\mathbf{P}}_k^{(L)}(t+1)$ ,  $\bar{\boldsymbol{\rho}}_k^{(U)}(t+1)$ ,  $\bar{\boldsymbol{\mu}}_k^{(U)}(t+1)$ ,  $\bar{\boldsymbol{\lambda}}_k(t+1)$ , and  $\bar{\boldsymbol{\chi}}_k(t+1)$ .
- 6: Find the projection values by using interior-point method to solve the convex optimization problem formulated as (25).
- 7: Use the projection values to calculate gradient flows in (21), and update the value of  $\mathbf{P}_k^{(L)}(t+1)$ ,  $\boldsymbol{\rho}_k^{(U)}(t+1)$ ,  $\boldsymbol{\mu}_k^{(U)}(t+1)$ ,  $\boldsymbol{\lambda}_k(t+1)$ , and  $\boldsymbol{\chi}_k(t+1)$ .
- 8:  $t \leftarrow t + 1$ .
- 9: **end while**

It is noteworthy that in the last two equations in (22), each D2D-U pair needs to know the information on the Lagrangian dual multipliers of its neighbor D2D-U pairs. Mathematically, the information sharing among D2D-U pairs can be modeled as a graph,  $\mathcal{G} = (\mathcal{B}, \xi)$ , where  $\mathcal{B}$  is the set of D2D-U pairs and the edge set is defined as  $\xi \subset \mathcal{B} \times \mathcal{B}$ . If D2D-U pair  $k$  can receive information from D2D-U pair  $j$ , then D2D-U pair  $j$  is included into the D2D-U pair  $k$ 's neighbor set  $\mathcal{B}_k = \{j | (j, k) \in \xi\}$ .  $\mathcal{G}$  is defined as undirected when  $(k, j) \in \xi$  if and only if  $(j, k) \in \xi$  and  $\mathcal{G}$  is connected if there exists paths from one D2D-U pair to another D2D-U pair for any D2D-U pair.

To obtain the optimal power and spectrum allocation at each D2D-U pair in (21), the projection values in (22) need to be calculated first. To reach this goal, the following differential equations need to be solved

$$\begin{aligned} \bar{\mathbf{P}}_k^{(L)}(t+1) &= \mathbf{P}_k^{(L)}(t) - \nabla f_k(\mathbf{P}_k^{(L)}(t)) - \boldsymbol{\lambda}_k^{(U)}(t) \cdot \tilde{\mathbf{H}}_k, \\ \bar{\boldsymbol{\mu}}_k^{(U)}(t+1) &= \boldsymbol{\mu}_k^{(U)}(t) - \nabla f_k(\boldsymbol{\mu}_k^{(U)}(t)), \\ \bar{\boldsymbol{\rho}}_k^{(U)}(t+1) &= \boldsymbol{\rho}_k^{(U)}(t) - \nabla f_k(\boldsymbol{\rho}_k^{(U)}(t)). \end{aligned} \quad (24)$$

Then, based on (23), to find the projection values, the following convex optimization problem is formulated,

$$\begin{aligned} \min_{\mathbf{P}_k^{(L)}, \boldsymbol{\mu}_k^{(U)}, \boldsymbol{\rho}_k^{(U)}} & \left\| \mathbf{P}_k^{(L)} - \bar{\mathbf{P}}_k^{(L)}(t+1) \right\| \\ & + \left\| \boldsymbol{\mu}_k^{(U)} - \bar{\boldsymbol{\mu}}_k^{(U)}(t+1) \right\| + \left\| \boldsymbol{\rho}_k^{(U)} - \bar{\boldsymbol{\rho}}_k^{(U)}(t+1) \right\|, \end{aligned} \quad (25)$$

subject to (12a), (12b), and (12c).

By finding the optimal solution of (25), we can update  $\mathbf{P}_k^{(L)}(t+1)$ ,  $\boldsymbol{\mu}_k^{(U)}(t+1)$ ,  $\boldsymbol{\rho}_k^{(U)}(t+1)$  and the corresponding Lagrangian multipliers in (21). Accordingly, the updating scheme deployed at each D2D-U pair  $k$  can be concluded as Algorithm 2.

3) *Algorithm Implementation*: Each D2D-U pair has full knowledge on its own local power and spectrum constraints. The maximum available time fraction on the corresponding

---

**Algorithm 3** Algorithm Implementation at D2D-U Pairs
 

---

- 1: The BS broadcasts the co-channel interference constraint on the licensed subchannels to D2D-U pairs.
  - 2: Each D2D-U pair,  $k$ ,  $k \in \mathcal{K}$ , initializes the updating index  $t = 0$ , power and spectrum allocation matrix,  $\mathbf{X}_k(0)$ , Lagrangian dual multiplier vectors,  $\boldsymbol{\lambda}_k(0)$  and  $\boldsymbol{\chi}_k(0)$ .
  - 3: Each D2D-U pair applies the channel access scheme listed in Table I to decide the available time fraction on the corresponding unlicensed channels.
  - 4: **while**  $\left( \|\dot{\mathbf{X}}_t\|_2^2 + \|\dot{\mathbf{A}}_t\|_2^2 + \|\dot{\mathbf{X}}_t\|_2^2 \right) \geq \sigma$  **do**
  - 5:   Each D2D-U pair,  $k$ ,  $k \in \mathcal{K}$ , exchanges  $\boldsymbol{\lambda}_k(t)$  and  $\boldsymbol{\chi}_k(t)$  with its neighbor D2D-U pairs.
  - 6:   Each D2D-U pair,  $k$ ,  $k \in \mathcal{K}$ , updates its power and spectrum allocation strategy, achieves  $\mathbf{P}_k^{(L)}(t+1)$ ,  $\boldsymbol{\rho}_k^{(U)}(t+1)$ ,  $\boldsymbol{\mu}_k^{(U)}(t+1)$ ,  $\boldsymbol{\lambda}_k(t+1)$ , and  $\boldsymbol{\chi}_k(t+1)$ .
  - 7:   Let  $t \leftarrow t+1$ .
  - 8: **end while**
- 

unlicensed channels can be decided by each D2D-U pair based on the channel access scheme developed in Section II. A. Moreover, the information on the global co-channel interference constraint on each licensed subchannel,  $\bar{I}_n$ ,  $n \in \mathcal{N}$ , can be broadcasted by the BS. According to the decentralized scheme developed in last section, D2D-U pairs need to exchange the information on the Lagrangian dual multipliers during the implementation process. Since some D2D-U pairs may not be able to communicate directly, the BS can be used to assist the exchange on the Lagrangian multipliers among these D2D-U pairs. Based on above analysis, the implementation process of the proposed decentralized scheme is summarized in Algorithm 3.

## V. ANALYSIS ON THE PROPOSED ALGORITHM

Based on the developed scheme, the convergence to the optimal solution, the signaling overheads, and the computational complexity are analyzed in this section.

### A. Analysis on Optimality and Convergence

Based on [32], the projected method based decentralized algorithm converges to the optimal solutions when the objective function (15) and the system model satisfy three conditions which are described in the following theorem.

**Theorem 2.** *If the following three conditions are satisfied*

- 1) *The objective function is continuously differentiable convex functions with respect to the variables;*
- 2) *There exists a finite solution to the constraints of the optimization problem;*
- 3) *The information sharing graph  $\mathcal{G} = (\mathcal{B}, \xi)$  is undirected and connected. Therefore, the D2D-U pairs can exchange the information on the Lagrangian multipliers as demonstrated in (22),*

*the projected method used in Algorithm 1 converges to the optimal solution.*

Firstly, it is easy to verify that the objective function and constraints formulated in (15) satisfy the first condition in

Theorem 2. Secondly, the feasibility condition can be satisfied with the admission control policy used at the BS in the 5G NR networks. When it comes to the third condition, the BS is required to assist the sharing on Lagrangian dual multipliers among the D2D-U pairs since D2D-U pairs may be not able to communicate directly. In other words, the BS can relay the information on the Lagrangian dual multipliers among D2D-U pairs to realize the fully connection. Therefore, with the assistance of the BS, D2D-U pairs can be modeled as an undirected graph. Accordingly, the following corollary can be derived.

**Corollary 1.** *The proposed Algorithm 2 converges to the optimal solution of optimization problem (15).*

*Proof:* Please see Appendix B. ■

In addition, it is well-known that wireless communication system as well as the 5G NR network is a dynamic network due to the variable *channel state information* (CSI), WiFi traffic loads and so on. Based on [32], Algorithm 1 is an initial-free decentralized algorithm since the projection operation can guarantee the adaptation to the CSI variation. Even though the CSI changes, the proposed decentralized scheme can still converge to the new optimal resource management solution. Therefore, the proposed algorithm can be applied in the 5G NR network to achieve the optimal power and spectrum allocation adaptively.

### B. Signaling Overhead Analysis

In this section, the signaling overheads resulted from the proposed decentralized scheme are compared with that by the traditional centralized algorithm. If applying the centralized scheme at the BS, the data rate requirement  $\bar{r}_k$ , the available time fraction on unlicensed channels  $\beta_w$ , the channel power gain on each licensed subchannel  $h_{k,w}^{(L)}$ , and the channel power gain on each unlicensed channel  $h_{k,w}^{(U)}$ , need to be uploaded from each D2D-U pair, which would consume a lot of power and spectrum resource.

On the contrary, for the proposed decentralized algorithm, the signaling overheads are from the exchange of multipliers  $\boldsymbol{\lambda}_k$  and  $\boldsymbol{\chi}_k$  among D2D-U pairs with the help of the BS. Moreover, since the proposed scheme is adaptive to the changes of system state, such as the CSI and the available time fraction on unlicensed channels, without re-initiation, more resource can be saved to serve users instead of consuming for the signaling overheads. The required signaling overheads for centralized and decentralized algorithm are listed in table I on the following page.

It is noteworthy that the proposed decentralized algorithm is adaptive to the varying wireless radio environment. Therefore, it does not bring signaling overheads when the system state changes. However, the centralized algorithm needs to retransmitted parameters when the system state changes, which causes overwhelming signaling overheads.

### C. Algorithm Complexity Analysis

The classic interior-point methods presented in [31] can be applied to solve the centralized optimization problem



TABLE I  
SIGNALING OVERHEADS

Parameters	Centralized	Decentralized
Data rate requirements of D2D-U pairs, $\bar{r}_k$	✓	×
Maximum time fraction can be used on unlicensed channels, $\bar{\beta}_w$	✓	×
Channel power gain on licensed subchannels, $h_{k,n}^{(L)}$	✓	×
Channel power gain on unlicensed subchannels, $h_{k,n}^{(U)}$	✓	×
Co-channel interference D2D-U pairs brought to the BS, $\tilde{h}_{k,n}^{(L)}$	✓	×
$\lambda_k, k \in K$	×	✓
$\chi_k, k \in K$	×	✓

in (15). According to [31], the computational complexity of interior-point method is given by  $\mathcal{O}(\sqrt{n})$ , where  $n$  is the number of variables. When solving the problem (15),  $n$  is equal to  $\sum_{k \in K} (|\mathcal{N}_k| + 2|\mathcal{W}_k|)$ , where  $|\cdot|$  is the modular operation. Since the computational complexity is  $\mathcal{O}(\sqrt{\sum_{k \in K} (|\mathcal{N}_k| + 2|\mathcal{W}_k|)})$ . On the other hand, when applying the proposed decentralized scheme, the computational complexity is mainly from the projection operation. Since the projection operation can be treated as a convex optimization problem, the computational complexity is also given by  $\mathcal{O}(\sqrt{n})$ . Since each D2D-U pair can execute the scheme locally in parallel, the corresponding computational complexity at the D2D-U pair  $k$  can be written as  $\mathcal{O}(\sqrt{|\mathcal{N}_k| + 2|\mathcal{W}_k|})$ . Therefore, based on above analysis, we can observe that the computational complexity of the proposed decentralized scheme is much lower than that in centralized scheme.

## VI. SIMULATION RESULTS

In this section, the performance of the proposed scheme and the theoretical analysis are verified by the simulation results. In the simulation setup, we consider a NR network with one BS, 8 cellular users, 4 D2D-U pairs, and 16 licensed subchannels. When sharing the unlicensed channels with the WiFi networks, each D2D-U pair shares 4 unlicensed channels with its neighbor WiFi users. In addition, the D2D-U pairs are uniformly located in the service coverage of the BS and the distance between D2DT and D2DR within each D2D-U pair is uniformly distributed between 50 and 150 meters. We assume that the wireless channels on both licensed and unlicensed channels are block Rayleigh fading channels. In detail, the major parameters in the simulation are listed in Table II.

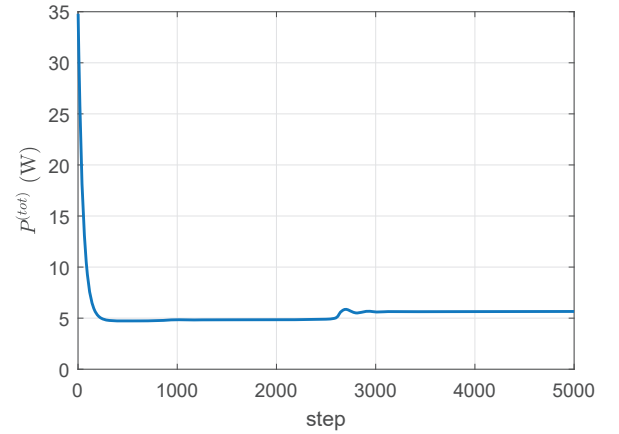
### A. Convergence Performance

First, the convergence process of the proposed scheme, including the total power consumption,  $P^{(tot)}$ , and Lagrangian multipliers,  $\lambda$  and  $\chi$ , is shown in Fig. 2. From Fig. 2, we can observe that the total power consumption of D2D-U pairs decreases and finally converges to a stable value. In addition,

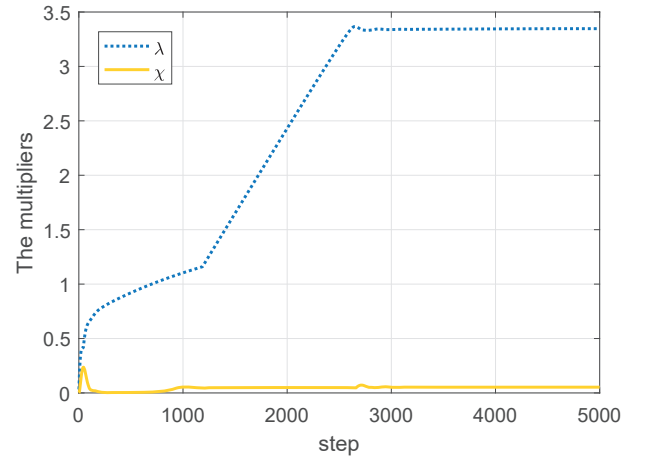
TABLE II  
SIMULATION PARAMETERS

Parameters	Value
Path loss model: Licensed: $a=3.75$ Unlicensed: $a=5$	$15 + a \log_{10}(d)$
Transmission power on unlicensed channel, $P_T^{(U)}$	23dBm
AWGN noise power spectrum density on the licensed and unlicensed channels, $N_0^{(L)}$ and $N_0^{(U)}$	-95dBm/Hz
Distance between the D2DT and D2DR	50m~150m
Bandwidth of the licensed and unlicensed channels, $B^{(L)}$ and $B^{(U)}$	20MHz

both Lagrangian multipliers,  $\lambda$  and  $\chi$ , converge to stable values as well. Therefore, each D2D-U pair can achieve a stable solution after implementing the proposed scheme via exchanging  $\lambda$  and  $\chi$  among D2D-U pairs in the system through the BS.



(a) The values of  $P^{(tot)}$  changed with iteration.



(b) The values of  $\lambda$  and  $\chi$  changed with iteration.

Fig. 2. The convergence of the proposed scheme.

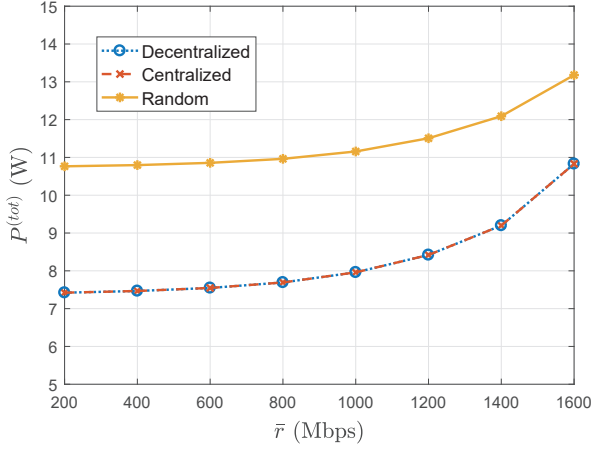


Fig. 3. The performance of decentralized scheme compared with centralized scheme and random scheme.

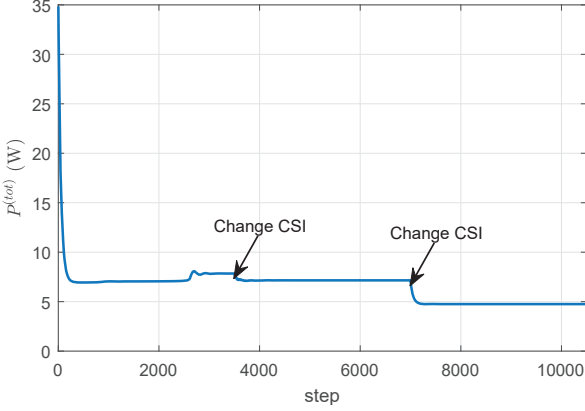


Fig. 4. The adaptiveness of the proposed algorithm.

### B. Optimized Performance from the Proposed Scheme

The performance comparison of the proposed decentralized scheme, centralized scheme, and the random scheme is presented in Fig. 3, respectively. Particularly, the result obtained by the centralized scheme is the global optimal solution to problem (12) achieved by the interior-point methods. And, in the random scheme, the transmission power of the D2D-U pairs on licensed subchannels are decided randomly first, then the spectrum and power on unlicensed channels are allocated by an optimization scheme. From the figure, we can observe that both schemes consume almost the same power (nearly fully coincident), which can prove the global optimality of the proposed decentralized scheme. In addition, the results of the decentralized scheme are much lower than that of the random scheme, indicating that this scheme effectively reduces the total power consumption of the D2D-U pair compared with the random scheme.

### C. Adaption to Varying CSI

To obtain further insights on the characteristics of the proposed scheme, in Fig. 4, we demonstrate its adaptive performance to the varying wireless channels. The CSI between the D2DT and D2DR within each D2D-U pair will change during the implementation process. From the figure, we can observe that the proposed decentralized scheme can reach to the new optimal resource allocation strategy without re-initialization

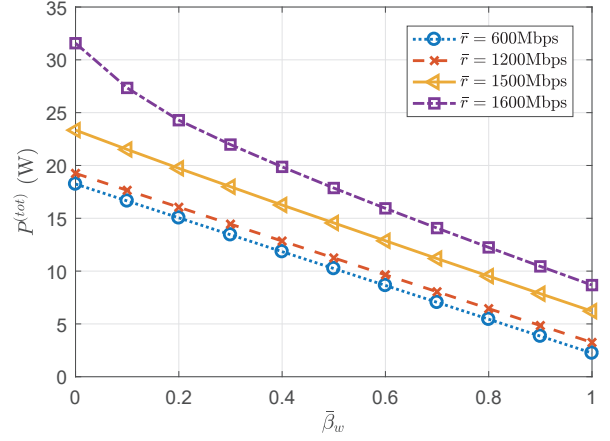


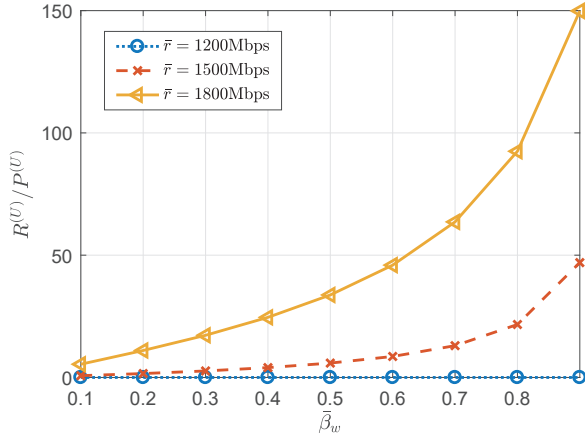
Fig. 5. The system power consumption with different data rate constraints and available time fractions on unlicensed channels.

after the CSI changes. It is indicated that the proposed scheme is adaptive to the variable system states.

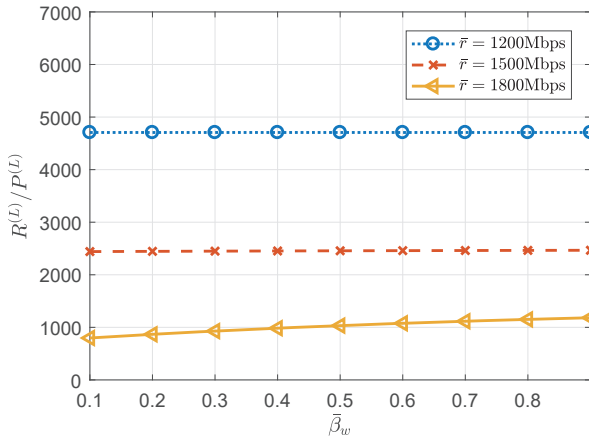
### D. Effect of the Time Fraction on Unlicensed Channels

Fig. 5 demonstrates the change on power consumption at D2D-U pairs with different data rate requirement constraints and available time fractions on unlicensed channels. From the figure, we can observe that the power consumption increases as the data rate requirements increase. It is obviously that when  $\bar{r}_k$  is small, data rate requirement constraint has less impact on the power consumption and less power is required to fulfill the data rate requirements. In addition, the power consumption at D2D-U pairs decreases when available unlicensed spectrum resources increase. Because with more available unlicensed spectrum, the D2D-U pairs have more freedom and opportunities to reduce the power consumption by jointly allocating the power and spectrum on licensed and unlicensed channels.

Fig. 6 presents how the EE, defined as the ratio of data rates and power consumption, on licensed and unlicensed channels changes with the available time fraction on unlicensed channel,  $\beta_w$ , with  $\bar{r} = 1200$  Mbits/s,  $\bar{r} = 1500$  Mbits/s, and  $\bar{r} = 1800$  Mbits/s, respectively. Herein,  $R^{(U)}$  is the total achievable data rates at D2D-U pairs on unlicensed channels, and  $R^{(L)}$  is the total achievable data rates at D2D-U pairs on licensed channels. As shown in Fig. 6 (a), when  $\bar{r} = 1200$  Mbits/s, the EE on the unlicensed channels is 0. On the other hand, as shown in Fig. 6 (b), the EE on the licensed channels have the same value with different  $\beta_w$ . Therefore, D2D pairs prefer to use the licensed subchannels due to the less power consumption on licensed subchannels, which does not need to sense the channel in advance. When  $\bar{r} = 1500/1800$  Mbits/s, the EE on the unlicensed channels increases with respect to  $\beta_w$ . In this case, the period to sense the unlicensed channels decreases, so more power is consumed to transmit on the unlicensed channels and the ratio,  $R^{(U)}/P^{(U)}$ , increases. In addition, it can be found that the EE of licensed channels is significantly higher than that of unlicensed channels, which is mainly caused by the channel conditions of licensed and unlicensed channels.



(a) The values of  $R^{(U)}/P^{(U)}$  changed with  $\bar{\beta}$ .



(b) The values of  $R^{(L)}/P^{(L)}$  changed with  $\bar{\beta}$ .

Fig. 6. The data transmission efficiency with different  $\bar{\beta}$ .

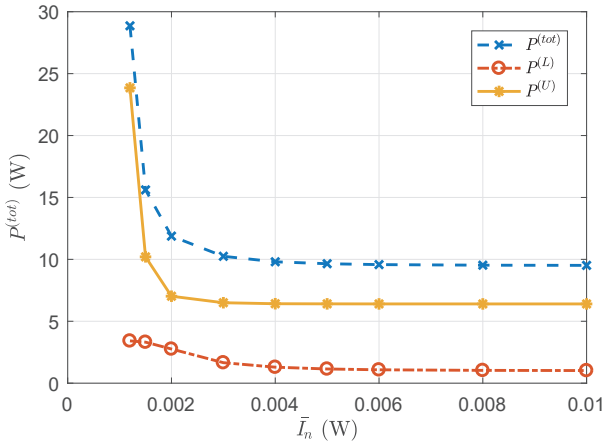


Fig. 7. The total power consumption in licensed and unlicensed channels with different co-channel interference.

#### E. Effect of the Co-Channel Interference

Fig. 7 depicts the impact of co-channel interference constraint,  $\bar{I}_n$ , on the power consumption at D2D-U pairs on licensed and unlicensed channels. From the figure, we can observe that the total power consumption decreases as co-channel interference constraint increasing. Particularly, when the co-channel interference constraint is strict, more power

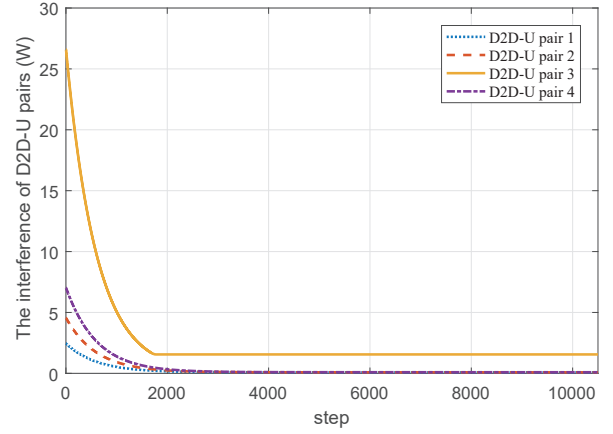


Fig. 8. System interference change with iteration

has to be allocated to the unlicensed channels to satisfy the data rate requirement constraint and less power is used on the licensed subchannels to fulfill the co-channel interference to the BS. On the other hand, when the co-channel interference bound,  $\bar{I}_n$ , increases, its impact on the power allocation of D2D-U pairs decreases. Therefore, the power allocation strategy hardly changes with  $\bar{I}_n$  since the constraint on the data rate requirements is more effective. Fig. 8 describes the convergence on the co-channel interference caused by four D2D-U pairs to the BS on one licensed subchannel. From the figure, we can observe that the co-channel interference decreases monotonically and finally meets the constraint. Therefore, with the proposed decentralized algorithm, the power and spectrum allocation at the D2D-U pairs always satisfies the global constraint in a decentralized manner.

## VII. CONCLUSIONS

In this paper, a channel access scheme to enable D2D-U transmissions on unlicensed channels is first proposed. Then, an optimization problem to minimize power consumption at D2D-U pairs is formulated, where the D2D-U pairs can not only reuse the licensed uplink channels, but also share unlicensed channels with the WiFi networks. Exploiting the distributed structure of the D2D-U enabled NR networks, we develop a decentralized power and spectrum allocation scheme, which can reduce not only the computational complexity, but also the signalling overheads. Moreover, the proposed scheme can be adaptive to the varying system states. The simulation results demonstrate the performance of the proposed decentralized scheme and validate the theoretical analysis.

## APPENDIX

### A. Proof of Theorem 1

In (13), we have already written  $P^{(tot)}$  with replacing  $p_{k,w}^{(U)}$  with  $\mu_{k,w}^{(U)}$ , that is,

$$P_k^{(tot)} = \sum_{n \in \mathcal{N}_k} w^{(L)} p_{k,w}^{(L)} + \sum_{w \in \mathcal{W}_k} w^{(U)} \cdot \mu_{k,w}^{(U)} + \sum_{w \in \mathcal{W}_k} (1 - \rho_{k,w}^{(U)}) p_s^{(U)} + P_k^{(C)}. \quad (26)$$

Therefore, the objective function in this optimization problem,  $P^{(tot)} = \sum_{k \in \mathcal{K}} P_k^{(tot)}$ , is liner and convex.

And in constrain (15a)  $\hat{R}_k \geq \bar{r}_k, (\forall k \in \mathcal{K})$  where

$$R_{k,n}^{(L)} = B^{(L)} \log \left( 1 + \frac{p_{k,n}^{(L)} h_{k,n}^{(L)}}{(I_{k,n} + N_0^{(L)}) B^{(L)}} \right), \quad (27)$$

and

$$\hat{R}_{k,w}^{(U)} = \rho_{k,w}^{(U)} B^{(U)} \log \left( 1 + \frac{\mu_{k,w}^{(U)} h_{k,w}^{(U)}}{B^{(U)} N_0^{(U)} \rho_{k,w}^{(U)}} \right). \quad (28)$$

We define a function

$$f(x) = \log(1+x), \quad (29)$$

which is a convex function.

Then,  $R_{k,n}^{(L)}$  can be written as

$$R_{k,n}^{(L)} = B^{(L)} f \left( \frac{p_{k,n}^{(L)} h_{k,n}^{(L)}}{(I_{k,n} + N_0^{(L)}) B^{(L)}} \right), \quad (30)$$

and  $R_{k,n}^{(L)}$  be proven to be convex.

To prove the convexity of constrain  $R_{k,w}^{(U)}$ , we define a function

$$h(x, y) = -x \log \left( 1 + \frac{y}{x} \right), \quad (31)$$

which has the Hessian matrix as

$$\mathbf{H} = \begin{bmatrix} \frac{y^2/x}{(x+y)^2} & -\frac{y}{(x+y)^2} \\ -\frac{y}{(x+y)^2} & \frac{x}{(x+y)^2} \end{bmatrix}. \quad (32)$$

So that the eigenvalues of the Hessian matrix is

$$\lambda_1 = 0 \text{ and } \lambda_2 = \frac{x^2 + y^2}{x^3 + 2x^2y + xy^2},$$

which are greater than or equal to zero when  $x \geq 0$ . Therefore, the function  $h(x, y)$  is a convex function when  $x \geq 0$ .

Then,  $R_{k,w}^{(U)}$  can be expressed as

$$\hat{R}_{k,w}^{(U)} = -f \left( \rho_{k,w}^{(U)} B^{(U)}, \frac{\mu_{k,w}^{(U)} h_{k,w}^{(U)}}{N_0} \right). \quad (33)$$

Since  $B^{(L)} > 0$  and  $\rho_{k,w}^{(U)} B^{(U)} > 0$ ,  $R_{k,w}^{(U)}$  are convex. So that constraint (15a)

$$\hat{R}_k = \sum_{n \in \mathcal{N}_k} R_{k,n}^{(L)} + \sum_{w \in \mathcal{W}_k} \hat{R}_{k,w}^{(U)}, \quad (34)$$

can be proven to be convex.

Furthermore other constraints in problem (15) are liner. Therefore, based on this replacement, the optimization problem (12) which is converted into optimization problem (15) can be proved to be a convex optimization problem.

### B. Proof of Corollary 1

The optimization problem (12a) can be converted to a general optimization problem, which is formulated as

$$\min_{x_k, k \in K} f(\mathbf{X}) = \sum_{k \in K} f_k(x_k), \quad (35)$$

subject to

$$\sum_{k \in K} w_k x_k = \sum_{k \in K} d_k, \quad (35a)$$

$$x_k \in \Omega_k, (k \in K), \quad (35b)$$

where (35a) is the global constraint (12d) in problem (12), (35b) corresponds to the local constraints (12a),(12b) and (12c).

Without loss of generality, we can assume that each agent has one variable, i.e.,  $x_k \in R$ . Define a new vector  $S = \text{col}(\mathbf{X}, \mathbf{A}, \mathbf{Z})$  and its feasible region is  $\bar{\Omega} = \Omega \times R^K \times R^K$ . Then, the updating rules of all the agents can be expressed as

$$\dot{S} = P_{\bar{\Omega}}(S - F(S)) - S, \quad (36)$$

where the vector function  $F(S)$  is denoted as

$$F(S) = \begin{pmatrix} \nabla f(\mathbf{X}) - \mathbf{W} \mathbf{A} \\ \mathbf{L} \mathbf{A} + \mathbf{L} \mathbf{Z} - (\mathbf{D} - \mathbf{W} \mathbf{X}) \\ -\mathbf{L} \mathbf{A} \end{pmatrix}. \quad (37)$$

To prove that algorithm can converge to its equilibrium point, the Lyapunov stability can be applied. Firstly, a Lyapunov function  $V_g$  is introduced to analyze the Lyapunov stability, which is given by

$$V_g = -\langle F(S), H(S) - S \rangle - \frac{1}{2} \|H(S) - S\|_2^2 + \frac{1}{2} \|S - S^*\|_2^2, \quad (38)$$

where  $H(S) = P_{\bar{\Omega}}(S - F(S))$ ,  $S^* = \text{col}(\mathbf{X}^*, \mathbf{A}^*, \mathbf{Z}^*)$ ,  $\mathbf{X}^*$  is the optimal solution to (35),  $\mathbf{A}^*$  and  $\mathbf{Z}^*$  are the optimal multipliers.

Due to

$$\begin{aligned} -\langle F(S), H(S) - S \rangle - \frac{1}{2} \langle H(S) - S, H(S) - S \rangle \\ \geq \frac{1}{2} \|S - H(S)\|_2^2, \end{aligned} \quad (39)$$

we can obtain that  $V_g \geq \frac{1}{2} \|S - H(S)\|_2^2 + \frac{1}{2} \|S - S^*\|_2^2 \geq 0$ , and  $V_g = 0$  if and only if  $S = S^*$ .

Since any asymmetric variational inequality can be converted to a differentiable optimization problem,  $\dot{V}_g$  can be expressed as

$$\dot{V}_g = (F(S) - (J_F(S) - \mathbf{I})(H(S) - S) + S - S^*) (H(S) - S), \quad (40)$$

where  $J_F(S)$  is the Jacobian matrix of  $F(S)$ , which is defined as

$$J_F(S) = \begin{pmatrix} \nabla^2 f(\mathbf{X}) & -\mathbf{W} & 0 \\ \mathbf{W} & \mathbf{L} & \mathbf{L} \\ 0 & -\mathbf{L} & 0 \end{pmatrix}. \quad (41)$$

According to the basic property of projection operation, it can be given that

$$\langle S - F(S) - H(S), H(S) - S^* \rangle \geq 0, \quad (42)$$

which is equal to

$$\langle S - F(S) - H(S), H(S) - S + S - S^* \rangle \geq 0, \quad (43)$$

Then, we can infer

$$\begin{aligned} \langle \mathbf{S} - \mathbf{S}^* + F(\mathbf{S}), H(\mathbf{S}) - \mathbf{S} \rangle + \|H(\mathbf{S}) - \mathbf{S}\|_2^2 \\ \leq -\langle F(\mathbf{S}), \mathbf{S} - \mathbf{S}^* \rangle. \end{aligned} \quad (44)$$

In addition, the following result can be obtained as well based on the assumptions in Section III-B.

$$\mathbf{S}^T J_F(\bar{\mathbf{S}}) \mathbf{S} = \mathbf{X}^T \nabla^2 f(\bar{\mathbf{X}}) \mathbf{X} + \Lambda^T \mathbf{L} \Lambda > 0, \forall \bar{\mathbf{S}} \in \bar{\Omega}. \quad (45)$$

Consequently, on the basis of (44) and (45),  $\dot{V}_g$  can be further analyzed as follows

$$\begin{aligned} \dot{V}_g = - (H(\mathbf{S}) - \mathbf{S})^T J_F(\mathbf{S}) (H(\mathbf{S}) - \mathbf{S}) + \|H(\mathbf{S}) - \mathbf{S}\|_2^2 \\ + \langle \mathbf{S} - \mathbf{S}^* + F(\mathbf{S}), H(\mathbf{S}) - \mathbf{S} \rangle \\ \leq -\langle F(\mathbf{S}), \mathbf{S} - \mathbf{S}^* \rangle. \end{aligned} \quad (46)$$

Since  $\langle F(\mathbf{S}), \mathbf{S} - \mathbf{S}^* \rangle \geq 0$  has been proved in [32],  $\dot{V}_g \leq 0$  can be proved and the equilibrium point is assumed to be  $\mathbf{S}^*$ . That means  $\dot{V}_g \leq 0$  and  $V_g = 0$  if and only if  $\mathbf{S} = \mathbf{S}^*$ . To ensure the Lyapunov stability of the equilibrium point,  $V_g \geq 0$  and  $\dot{V}_g \leq 0$  should be satisfied. Therefore, the equilibrium point of (35) is Lyapunov stable. In brief, the optimization problem (35) can converge to its equilibrium point, which ends the proof.

## REFERENCES

- [1] A. Mamta, R. Abhishek, and S. Navrati, "Next generation 5G wireless networks: a comprehensive survey," *IEEE Commun. Surv. & Tut.*, vol. 18, no. 3, pp. 1617-1655, Feb. 2016.
- [2] C. Wu, Z. Liu, F. Liu, T. Yoshinaga, Y. Ji, and J. Li, "Collaborative learning of communication routes in edge-enabled multi-access vehicular environment," *IEEE Trans. Cogn. Commun. & netw.*, vol. 99, no. 99, pp. 1-1, Jun. 2020.
- [3] H. Li, K. Ota, and M. Dong, "ECCN: orchestration of edge-centric computing and content-centric networking in the 5G radio access network," *IEEE Wireless Commun.*, vol. 25, no. 3, pp. 88-93, Jun. 2018.
- [4] Ericsson, "On the pulse of the networked society," *Ericsson mobility report*, Dec. 2016.
- [5] J. G. Andrews *et al.*, "What will 5G be?" *IEEE J. Sel. Areas Commun.*, vol. 32, no. 6, pp. 1065-1082, Jun. 2014.
- [6] H. Zhang, Y. Liao, and L. Song, "D2D-U: device-to-device communications in unlicensed bands for 5g system," *IEEE Trans. Wireless Commun.*, vol. 16, no. 6, pp. 3507-3519, Jun. 2017.
- [7] M. Shafi *et al.*, "5G: A tutorial overview of standards, trials, challenges, deployment, and practice," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 6, pp. 1201-1221, Jun. 2017.
- [8] *Feasibility Study on Licensed-Assisted Access to Unlicensed Spectrum (Release 13)*, TR 36.889 v13.0.0, 3GPP, Jul. 2015.
- [9] S. Zinno, G. Stasi, S. Avallone, and G. Vente, "On a fair coexistence of LTE and Wi-Fi in the unlicensed spectrum: a survey," *J. Computer Commun.*, vol. 115, pp. 35-50, Jan. 2018.
- [10] B. Chen, J. Chen, Y. Gao, and J. Zhang, "Coexistence of LTE-LAA and Wi-Fi on 5GHz with corresponding deployment scenarios: a survey," *IEEE Commun. Surv. & Tut.*, vol. 19, no. 1, pp. 7-32, 2017.
- [11] N. Zhao, X. Liu, Y. Chen, S. Zhang, Z. Li, B. Chen, and F. Alouini, "Caching D2D connections in small-cell networks," *IEEE Trans. Veh. Technol.*, vol. 67, no. 12, pp. 12326-12338, Dec. 2018.
- [12] B. Bojovic, L. Giupponi, Z. Ali, and M. Miozzo, "Evaluating unlicensed LTE technologies: LAA vs LTE-U," *IEEE Access*, vol. 7, pp. 89714-89751, Jul. 2019.
- [13] E. Almeida *et al.*, "Enabling LTE/Wi-Fi coexistence by LTE blank subframe allocation," in *Proc. IEEE ICC*, Budapest, Jun. 09-13. 2013.
- [14] R. Yin, G. Yu, A. Maaref, and G. Li, "LBT-based adaptive channel access for LTE-U systems," *IEEE Trans. on Wireless Commun.*, vol. 15, no. 10, pp. 6585-6597, Oct. 2016.
- [15] K. Wang, Q. Zhou, S. Guo, and J. Luo, "Cluster frameworks for efficient scheduling and resource allocation in data center networks: a survey," *IEEE Commun. Surv. & Tut.*, vol. 20, no. 4, pp. 3560-3580, 2018.
- [16] C. Xu, K. Wang, and M. Guo, "Intelligent resource management in blockchain-based cloud datacenters," *IEEE Cloud Comput.*, vol. 4, no. 6, pp. 50-59, Nov. 2017.
- [17] X. He, K. Wang, H. Huang, T. Miyazaki, Y. Wang, and S. Gou, "Green resource allocation based on deep reinforcement learning in content-centric IoT," *IEEE Trans. Emerg. Top Comput.*, Feb. 2018.
- [18] G. Yu, L. Xu, D. Feng, R. Yin, and G. Li, "Joint mode selection and resource allocation for device-to-device communications," *IEEE Trans. Commun.*, vol. 62, no. 11, pp. 3814-3824, Oct. 2014.
- [19] Z. Zhou, M. Dong, K. Ota, G. Wang, and L. Yang, "Energy-efficient resource allocation for D2D communications underlying cloud-RAN-based LTE-A networks," *IEEE IoT J.*, vol. 3, no. 3, pp. 428-438, Jun. 2016.
- [20] Z. Zhou, G. Ma, M. Dong, K. Ota, C. Xu, and Y. Jia, "Iterative energy-efficient stable matching approach for context-aware resource allocation in D2D communications," *IEEE Access*, vol. 4, pp. 6181-6196, Jul. 2016.
- [21] Z. Zhou, M. Dong, K. Ota, R. Shi, Z. Liu, and T. Sato, "Game-theoretic approach to energy-efficient resource allocation in device-to-device underlay communications," *IET Commun.*, vol. 9, no. 3, pp. 375-385, Feb. 2015.
- [22] Z. Zhou, K. Ota, M. Dong, and C. Xu, "Energy-Efficient matching for resource allocation in D2D enabled cellular networks," *IEEE Trans. Veh. Technol.*, vol. 66, no. 6, pp. 5256-5268, Jun. 2018.
- [23] Z. Zou, R. Yin, X. Chen, and Wu. C, "Deep reinforcement learning for D2D transmission in unlicensed bands," in *Proc. IEEE ICC*, China, ChangChun, Aug. 2019, pp. 42-47.
- [24] G. Girmay, Q. Pham, and W. Hwang, "Joint channel and Power Allocation for Device-to-Device Communication on Licensed and Unlicensed Band," *IEEE Access*, vol. 7, pp. 22196-22205, Feb. 2019.
- [25] R. Liu, G. Yu, F. Qu, and Z. Zhang, "Device-to-device communications in unlicensed spectrum: mode selection and resource allocation," *IEEE Access*, vol. 4, pp. 4720-4729, Sep. 2016.
- [26] H. Zhang, Y. Liao, and L. Song, "D2D-U: device-to-device communications in unlicensed bands for 5G system," *IEEE Trans. Wireless Commun.*, vol. 16, no. 6, pp. 3507-3519, Jun. 2017.
- [27] Y. Huang, A. Nasir, S. Durrani, and X. Zhou, "Mode selection, resource allocation and power control for D2D-enabled two-tier cellular network," *IEEE Trans. Commun.*, vol. 84, no. 8, pp. 3534-3547, Aug. 2016.
- [28] E. Dahlman, S. Parkvall, and J. Skold, "4G: LTE-advanced pro and the road to 5G," San Diego, CA, USA: Elsevier, Aug. 2016, pp. 285-307.
- [29] IEEE, Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications, IEEE Std. 802.11, Jan. 1997.
- [30] Y. Song, K.W. Sung, and Y. Han, "Coexistence of Wi-Fi and cellular with listen-before-talk in unlicensed spectrum," *IEEE Commun. Lett.*, vol. 20, no. 1, pp. 161-164, Jan. 2016.
- [31] F. Alizadeh, "Interior point methods in semidefinite programming with applications to combinatorial optimization," *SIAM J. Optimiz.*, vol. 5, no. 1, pp. 13-51, 1995.
- [32] Y. Peng, Y. Hong, and F. Liu, "Initialization-free distributed algorithms for optimal resource allocation with feasibility constraints and application to economic dispatch of power systems," *Automatica*, vol. 74, no. 10, pp. 259-269, Dec. 2016.