Unit-III

Coupled circuits

- Self inductance
- Mutual inductance
- Coefficient of coupling
- Analysis of coupled circuits
- Natural current
- Dot rule of coupled circuits
- Conductively coupled equivalent circuits
- problem solving.

Introduction:-

* A magnetic field can be produced by using a current-carrying conductor and the field can be made stronger by winding the conductor in to a coil. Even then, the magnetic field is relatively weak.

* By placing a piece of steel within the coil, the magnetic field becomes hundreds times stronger.

* Magnetic circuits play an important role in the performance of a number of devices.

* The two circuits are said to be coupled circuits if all or part of the electrical energy supplied to one circuit is transferred to the other circuit when one of the circuits is energised without having any electrical connection between them.

Eg: Transformer, gyrator

* There are many types of couplings like

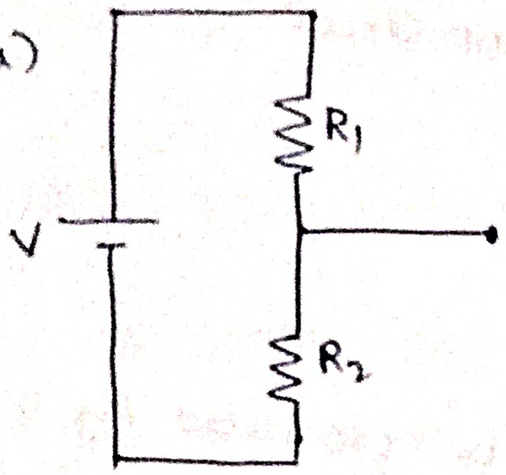
(a) Conductive coupling (potential divider)

(b) inductive or magnetic coupling by a two-winding transformer

(c) Conductive and inductive coupling by an auto transformer

(d) Two port network

(a)



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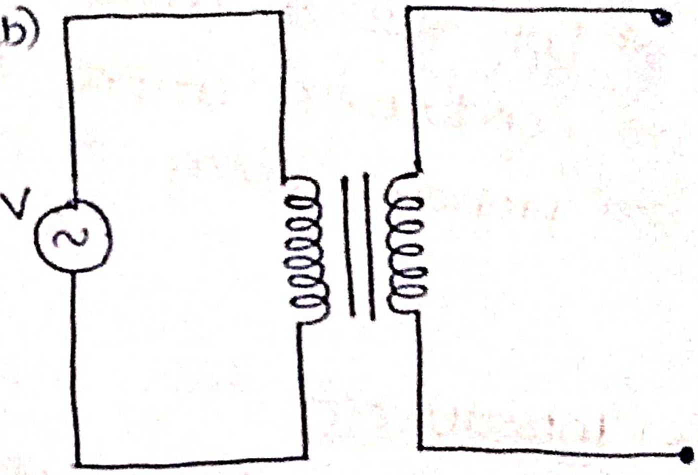

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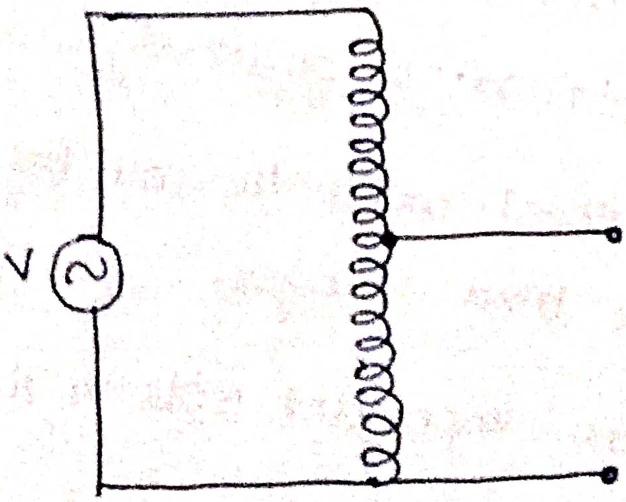

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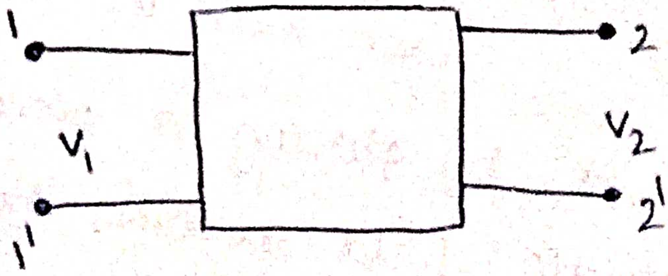


Magnetically coupled circuits:-

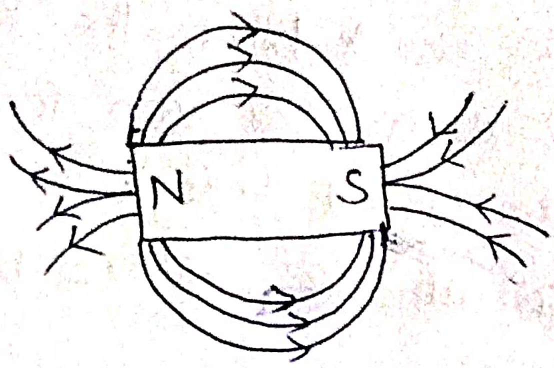
* When the two circuits are placed very close to each other such that a magnetic flux produced by one circuit links with both the circuits, then the two circuits are said to be magnetically coupled circuits.

* A wire of certain length, when twisted into coil becomes a basic Inductor. If a current is made to pass through an inductor, an electromagnetic field is developed. A change in the magnitude of the current, changes the electromagnetic field and hence induces a voltage in coil according to Faraday's law of electromagnetic induction.

* When two or more coils are placed very close to each other, then the current in one coil affects other coils by inducing voltage in them. Such coils are said to be mutually coupled coils. Such induced voltages in the coils are functions of the self inductances of the coils and mutual inductance between them.

iv) Some Basic Definitions:-

outside - North to South
inside - South to North



(I) Magnetic field:- The region around the magnet that is influenced by the magnetic lines of force is known as magnetic field.

(II) Magnetic lines of force:-

The imaginary lines that travel from North pole to South pole outside the magnet are known as magnetic lines of force.

(III) Magnetic flux (ϕ):-

The total number of lines of force passing through the magnetic material in a magnetic field and is denoted by ϕ. It is measured in weber.

Lines of magnetic flux have no physical existence.

The lines of magnetic flux have the following properties

(i) Each line of magnetic flux forms a closed loop.

(ii) Lines of magnetic flux never intersect.

(iii) the diagram of a line of magnetic flux at any point in a non-magnetic medium, such as air, is that of the north-seeking pole of a compass needle placed at that point.

(IV) Magnetic circuit:-

The closed path followed by the magnetic flux is called the magnetic circuit.

(V) Magnetic flux density (B):-

Magnetic flux density is defined as magnetic flux per unit area of the cross-section. It is denoted by B and is measured in Weber/m^2 or Tesla.

$$\text{Magnetic flux density (B)} = \frac{\text{Flux}}{\text{Area of cross-section}}$$

$\boxed{J = \frac{I}{A}}$ for electrical ckts

$$B = \frac{\phi}{A} \text{ Wb/m}^2 \text{ or Tesla.}$$

(VI) Magnetomotive force (MMF):-

MMF in a magnetic field is similar to electromotive force (emf) in an electric field. It is the basic quantity to determine the field magnitude.

The MMF is the work done in moving a unit north pole once around the magnetic circuit and its unit is Ampere-turns.

$$MMF = NI$$

where N = Number of turns of the coil

I = current flowing in the coil

(VII) Magnetic field strength (H):-

It is defined as a force experienced by a unit north pole at that point and is denoted by H. It is measured in Ampere turns per metre.

$$H = \frac{MMF}{l} = \frac{NI}{l} \text{ AT/m}$$

where l = length of the magnet

$\boxed{E = \frac{V}{d}}$

(VIII) Reluctance (S):-

The property of the material to oppose the establishment of magnetic flux is known as reluctance and is denoted by S. Its unit is AT/Wb.

$$S = \frac{MMF}{\phi} = \frac{NI}{\phi} = \frac{NI}{BA} = \frac{NI}{\mu_0 \mu_r H \cdot A} \quad (\because B = \mu H = \mu_0 \mu_r H)$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m and $\mu = \mu_0 \mu_r$. Thus

$$S = \frac{NI}{\mu_0 \mu_r (NI) \cdot A} \cdot l$$

$$S = \frac{l}{\mu_0 \mu_r A}$$

(IX) permeability: (μ):-

It is the ratio of the magnetic flux density to the magnetic field strength and its unit is Henry/m.

$$\mu = \frac{B}{H} = \mu_0 \mu_r$$

where μ_0 = absolute permeability of free space

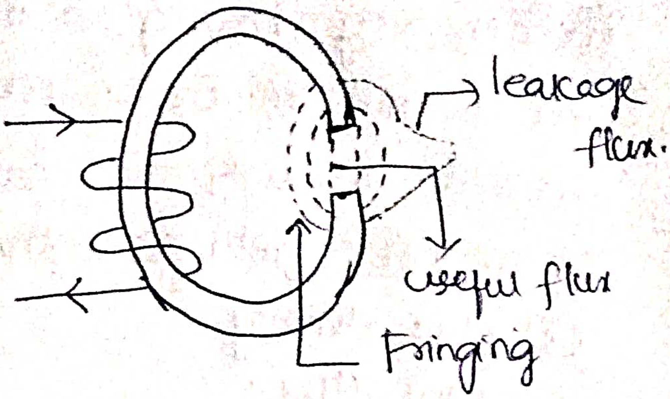
μ_r = relative permeability of the medium.

(X) Relative permeability:

It is the ratio of the permeability of a medium to the permeability of a free space.

$$\mu_r = \frac{\mu}{\mu_0}$$

(XI) Magnetic leakage:-



Consider a magnetic circuit formed through an iron ring with an air gap. The flux that passes through the air gap and forms a closed path through the iron ring is called as the useful flux. The flux that doesnot follow a closed path through the air gap is called the leakage flux.

(XII) coefficient of magnetic leakage: (λ):-

It is the ratio of the total flux to the useful flux.

$$\lambda = \frac{\phi_{Total}}{\phi_{useful}}$$

(XIII) Fringing:

The bulging of the flux in the air gap is known as fringing

(XIV) Retentivity:-

The ability of the magnetic material to retain magnetism even after removing the magnet is called the retentivity.

Flux in the iron = flux in the air gap + leakage flux

$$\phi_{iron} = \phi_{useful} + \phi_{leakage}$$

P A coil of 200 turns is wound uniformly over a wooden ring having a mean circumference of 600 mm and a uniform cross-sectional area of 500 mm^2. If the current through the coil is 4 A, calculate
(a) magnetic field strength
(b) flux density
(c) total flux.

Sol

$N = 200$

$l = 600\text{mm} = 600\times10^{-3} = 0.6\text{m}$

$A = 500\text{mm}^2 = 500\times10^{-6}\ \text{m}^2$

$I = 4A$

(a) $\boxed{H = \frac{NI}{l}} = \frac{200\times4}{0.6} = 1333\ \text{A/m}$

(b) $\boxed{B = \mu_0 \mu_r H}$ ($\because \mu_r = 1$ ie ring is wooden ring)

$= 4\pi\times10^{-7}\times1\times1333 = 0.001675\ \text{Wb/m}^2$ or T

(c) Total flux $\boxed{\phi = BA}$

$= 0.001675\times500\times10^{-6}$

$= 0.8375\ \mu\text{Wb}$

P Calculate the mmf required to produce a flux of 0.015 Wb across an air gap 2.5mm long, having an effective area of cross section 200 cm^2.

Sol

$\phi = 0.015\ \text{Wb}$

$l = 2.5\text{mm} = 2.5\times10^{-3} = 0.0025\text{m}$

$A = 200\text{cm}^2 = 200\times10^{-4} = 0.02\text{m}^2$

$\boxed{H = \frac{B}{\mu_0\mu_r}} = \frac{B}{\mu_0}$ ($\because \mu_r = 1$ for air) ie $\boxed{B = \frac{\phi}{A}} = \frac{0.015}{0.02} = 0.75\ Wb/m^2$

$\therefore H = \frac{B}{\mu_0} = \frac{0.75}{4\pi \times 10^{-7}} = 596831\ AT/m$

MMF required to send flux across the air gap

$$\boxed{H = \frac{NI}{l}}$$

$\Rightarrow NI = H \cdot l$

$= 596831 \overset{AT/m}{} \times 0.0025\ m$

$= 1492\ AT$

P An iron ring of mean length 50 cm has an air gap of 1mm and a winding of 200 turns. If the relative permeability of iron is 300, when a current 1A flows through the coil. Find the flux density.

Sol Iron ring length
$l = 50cm = 0.5m$

Air gap length $= 1mm = 1 \times 10^{-3}\ m$

$N = 200$

Iron relative permeability, $\mu_r = 300$

Reluctance of magnetic path, $\boxed{S_{Iron} = \frac{l}{\mu_0 \mu_r A}}$

$= \frac{0.5}{4\pi \times 10^{-7} \times 300 \times A} = \frac{1326}{A}$

Reluctance of Air gap, $S_{Air} = \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 1 \times A} = \frac{795}{A}$

$(\because \mu_r = 1$ for air$)$

Total reluctance, $S = \frac{1326}{A} + \frac{795}{A} = \frac{2121}{A}$

$$\boxed{\text{Flux} = \frac{\text{MMF}}{\text{Reluctance}}}$$

$$= \frac{NI}{S} = \frac{200 \times 1}{\frac{2121}{A}} = 94 \cdot 21 \times 10^{-3} \text{ A.Wb}$$

Flux density, $B = \frac{\phi}{A} = \frac{94 \cdot 21 \times 10^{-3} A}{A} = 94 \cdot 21 \times 10^{-3} \text{ Wb/m}^2$

⇝ Faradays laws of Electromagnetic Induction:

A method of obtaining an electric current with the aid of magnetic flux.

First law:

Faraday's first law states that when a conductor cuts the magnetic flux an e.m.f is induced in the conductor.

Second law:

According to the Faradays second law, the magnitude of induced e.m.f is equal to the rate of change of flux linkages.

flux linkage = No. of turns × flux linked with the coil.

Suppose a coil has N turns and the flux changes from an initial value of ϕ_1 weber to the final value of ϕ_2 weber in time 't' seconds

Initial flux linkages = $N\phi_1$

Final flux linkages = $N\phi_2$

Induced emf $(e) = \dfrac{N\phi_2 - N\phi_1}{t}$ Wb/sec or volts

$$e = \frac{N(\phi_2 - \phi_1)}{t}$$

In differential form, $e = N \cdot \dfrac{d\phi}{dt}$

i.e The magnitude of the generated emf is proportional to the rate at which the conductor cuts the magnetic flux. ~~the~~

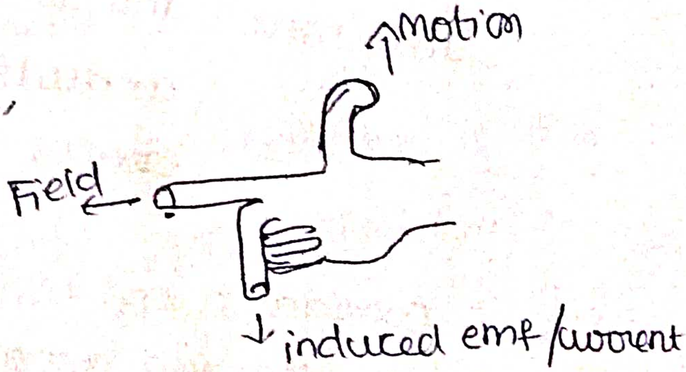
The direction of the induced or generated emf can be deduced by two methods.

(i) Fleming's right hand rule

(ii) Lenz's law.

Fleming's right hand rule:

According to this rule, the direction of the induced e.m.f can be determined by extending the thumb, fore finger and middle finger perpendicular to each other in such a manner that



(i) Thumb indicates the motion of conductor

(ii) Fore finger indicates the direction of field or flux.

(iii) Middle finger indicates the direction of induced emf

$$e = -N \frac{d\phi}{dt}$$

The minus sign on the right hand side of above equation represents the direction of induced emf by either Fleming's right hand rule or Lenz's law.

Lenz's law:-

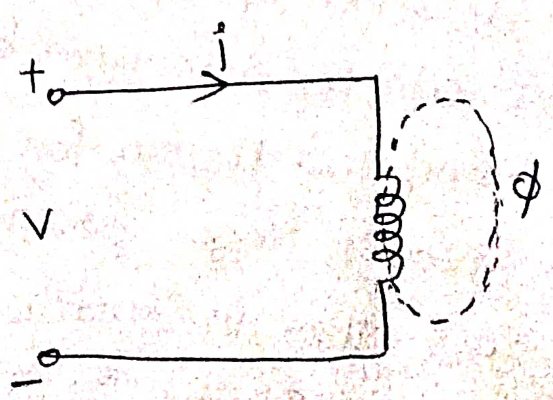
Statement:- The induced current will appear in such a direction that it opposes the change that produced in it.

Explaination:- Pushing a bar magnet towards the coil with its north pole facing the coil produces an induced current 'I' such that the forward motion of north pole is opposed.

Self inductance:-

Inductance:- This is a property of an electric conductor by which the change in current produces an electromotive force (emf)

Consider a coil having N turns carrying current i.



Due to the current flow, the flux ϕ is produced in the coil. The flux is measured in Wb(weber)

The flux produced by the coil links with the coil itself. Thus the total flux linkage of the coil will be NØ wb-turns.

If the current flowing through the coil changes, the flux produced in the coil also changes and hence flux linkage also changes.

According to Faraday's law, due to the rate of change of flux linkages, there will be induced e.m.f in the coil. This phenomenon is called self induction

The e.m.f or voltage induced in the coil due to the change of its own flux linked with it, is called self induced emf.

Note: The self induced e.m.f lasts till the current in coil is changing. The direction of such induced e.m.f is such that it opposes the cause producing it. i.e change in current

According to Lenz's law the direction of this induced e.m.f will be so as to oppose the cause producing it.

The cause is the current I hence the self induced e.m.f will try to set up a current which is in opposite direction to that of current I.

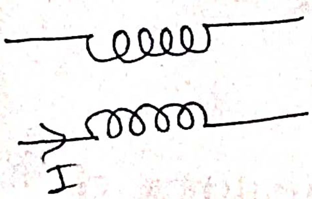
When current is increased, self induced e.m.f reduces the current tries to keep it to its original value.

If the current is decreased, self induced e.m.f increases the current and tries to maintain it back to its original value.

So any change in current through coil is opposed by the coil.

This property of the coil which opposes any change in the current passing through it is called Self Inductance or only Inductance.

rate of change of current produces an emf/voltage. If emf is produced in the same coil, then it is known as self inductance.



If the rate of change of current produces an emf in the nearby coil, it is known as mutual inductance.

$$V \propto \frac{di}{dt}$$

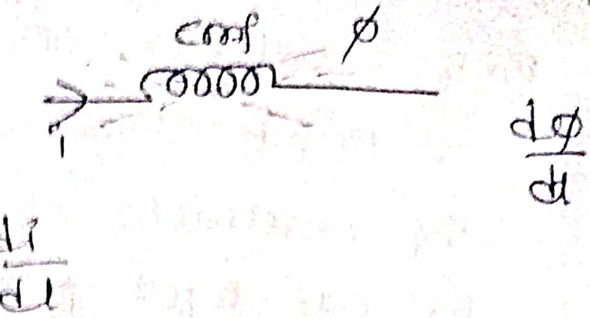
$$V = L\frac{di}{dt} \quad \text{—} (1)$$

L = self inductance, unit : henry

According to faraday's law

$$V = N\frac{d\phi}{dt} \quad \text{—} (2)$$

from eq(1) & eq(2)



$$L\frac{di}{dt} = N\frac{d\phi}{dt}$$

$$L\,di = N\,d\phi$$

$$LI = N\phi$$

$$\boxed{L = \frac{N\phi}{I}}$$

Here L is the coefficient of self inductance.

<u>Note</u>: A circuit possesses an inductance of 1 Henry when a current through coil is changing uniformly at the rate of one ampere per second inducing an opposing e.m.f 1 volt in it.

$$L = \frac{N\phi}{I}$$

but $\phi = \frac{M.M.F}{\text{Reluctance}} = \frac{NI}{S}$

$$\therefore L = \frac{N \cdot NI}{IS}$$

$$\boxed{L = \frac{N^2}{S} \text{ henrys}}$$

Now $S = \frac{l}{\mu a}$

$$\therefore L = \frac{N^2}{\frac{l}{\mu a}}$$

$$\boxed{L = \frac{N^2 \mu_0 \mu_r a}{l}}$$

where l = length of magnetic ckt

a = Area of cross-section of magnetic circuit through which flux is passing

Q If a coil has 500 turns is linked with a flux of 50 m Wb, when carrying a current of 125 A. calculate the inductance of the coil. If this current is reduced to zero uniformly in 0.1 sec, calculate the self induced e.m.f in the coil.

Sol The inductance is given by

$N = 500$

$\phi = 50\, mWb = 50 \times 10^{-3}\, Wb$

$I = 125\, A$

$t = 0.1\, sec.$

$$L = \frac{N\phi}{I}$$

$$L = \frac{500 \times 50 \times 10^{-3}}{125} = 0.2H$$

$$V = -L\frac{dI}{dt}$$

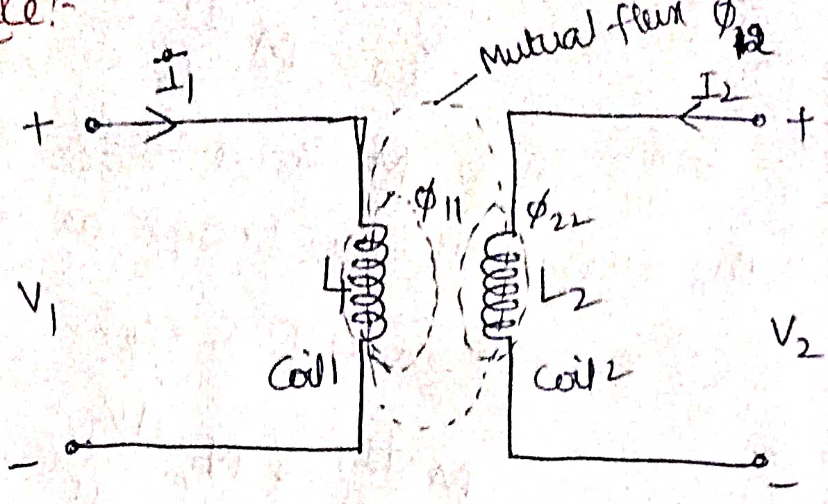
$$= -L\left[\frac{\text{Final value of } I - \text{Initial value of } I}{\text{Time}}\right]$$

$$V = -0.2 \times \left(\frac{0-125}{0.1}\right) = 250 \text{ volts}$$

This is positive because current is decreased. So this 'V' will try to oppose this decrease, means will try to increase current and will help the growth of the current.

Mutual Inductance:-

Mutual inductance is defined as the constant of proportionality between the rate of change in current in one circuit and the resulting e.m.f induced in another circuit.



$$\phi_{21} = \phi_{12}$$

A varying current in coil 1 produces a magnetic field. This magnetic field links with coil 2 and inducing an e.m.f between its ends.

Such an action in which a varying quantity in one circuit causes the development of a quantity in a different circuit is called mutual action or transfer action.

Two coils carrying currents I_1 and I_2: The coils will have leakage flux ϕ_{11} and ϕ_{22} for coils 1 and coil 2, respectively and a mutual flux ϕ_{12} where the flux of coil 1 links coil 2 or flux ϕ_{21} flux of coil 2 links coil 1.

The induced voltage of coil 2 may be written as

$$V_{L2} = N_2 \frac{d\phi_{12}}{dt} \quad \text{—} \ (1)$$

As ϕ_{12} is related to current of coil 1, the induced voltage is proportional to the rate of change of I_1.

$$\Rightarrow V_{L2} \propto \frac{dI_1}{dt}$$

$$V_{L2} = M \frac{dI_1}{dt} \quad —(2)$$

where M is constant of proportionality between the two coils, also known as mutual inductance.

from eq(1) & eq(2)

$$M \frac{dI_1}{dt} = N_2 \frac{d\phi_{12}}{dt}$$

$$\Rightarrow \boxed{M = N_2 \frac{d\phi_{12}}{dI_1}}$$

Similarly $\boxed{M = N_1 \frac{d\phi_{21}}{dI_2}}$

when the coils are linked with air as medium, the flux and current are linearly proportional to each other and the expressions of mutual inductance are written as

$$\boxed{M = N_2 \frac{\phi_{12}}{I_1}}$$

$$\boxed{M = N_1 \frac{\phi_{21}}{I_2}}$$

coefficient of mutual inductance (M) is defined as the property by which e.m.f gets induced in the second coil because of change in current through first coil.

Due to current I_1, the flux produced is ϕ_1. This flux ϕ_1 links with coil 1 and coil 2.

Similarly due to current I_2, the flux produced is ϕ_2. This flux ϕ_2 links with coil 2 as well as coil 1.

So in each coil there will be self induced e.m.f as well as mutually induced emf.

Let M be the mutual inductance between the two coils.

The magnitude of the self induced emf in coil 1 due to current I_1 is $L_1 \frac{dI_1}{dt}$. The magnitude of the mutually induced e.m.f in coil 1, due to current I_2 in coil 2 is $M \frac{dI_2}{dt}$.

$\therefore$ The magnitude of Total e.m.f induced in coil 1 is

$$V_1 = L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt}$$

I1y The magnitude of the self induced emf in coil 2 due to current I_2 is $L_2 \frac{dI_2}{dt}$. The magnitude of the ~~total e.m.f~~ mutually induced emf in coil 2, due to current I_1 in coil 1 is $M \frac{dI_1}{dt}$

$\therefore$ The magnitude of Total e.m.f induced in coil 2 is

$$V_2 = L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt}$$

(v) Coefficient of coupling (K):-

The amount of coupling between the inductively coupled coils is expressed in terms of the coefficient of coupling.

Coefficient of coupling is defined as the ratio of mutual inductance actually present between the two coils to the maximum possible value.

Consider two coils having self inductances L_1 and L_2 placed very close to each other. Let the number of turns of the two coils be N_1 and N_2 respectively. Let coil 1 carries current I_1 and coil 2 carries current I_2

For coil-1, self inductance $L_1 = \frac{N_1 \phi_1}{I_1}$

For coil-2, self inductance $L_2 = \frac{N_2 \phi_2}{I_2}$

Due to current I_1, the flux produced is ϕ_1 which links with both the coils. Then from the previous knowledge mutual inductance between two coils can be written as

$$M_{12} = \frac{N_2 \phi_{12}}{I_1}$$

where ϕ_{12} is the part of the flux ϕ_1 linking with coil 2. Hence we can write $\phi_{12} = K_1 \phi_1$

$$\therefore M_{12} = \frac{N_2 (K_1 \phi_1)}{I_1} \quad \text{——} (1)$$

Similarly due to current I_2, the flux produced is ϕ_2 which links with both the coils. Then the mutual inductance between two coils can be written as

$$M_{21} = \frac{N_1 \phi_{21}}{I_2}$$

where ϕ_{21} is the part of the flux ϕ_2 linking with coil 1. Hence we can write $\phi_{21} = K_2 \phi_2$.

$$M_{21} = \frac{N_1 (K_2 \phi_2)}{I_2} \quad \text{--- (2)}$$

Multiplying eq (1) & eq (2)

$$M_{12} \times M_{21} = \frac{N_2 (K_1 \phi_1)}{I_1} \cdot \frac{N_1 (K_2 \phi_2)}{I_2}$$

$$(\because M_{12} = M_{21} = M)$$

$$M \times M = K_1 K_2 \cdot \left(\frac{N_1 \phi_1}{I_1}\right)\left(\frac{N_2 \phi_2}{I_2}\right)$$

$$M^2 = K_1 K_2 L_1 L_2$$

$$M = \sqrt{K_1 K_2} \sqrt{L_1 L_2}$$

Let $K = \sqrt{K_1 K_2}$

$$M = K \sqrt{L_1 L_2}$$

where K is called coefficient of coupling

$$\boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$$

$0 < K < 1$

If $K=0$, there is no coupling

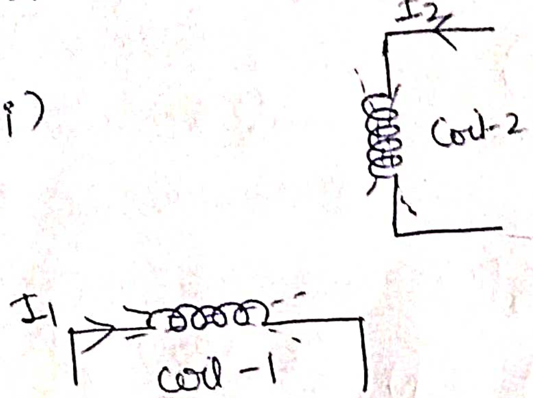
If $K=1$, this is called perfect coupling

K is non-negative fraction and has maximum value of unity

For iron core coupled circuits : $K = 0.99$

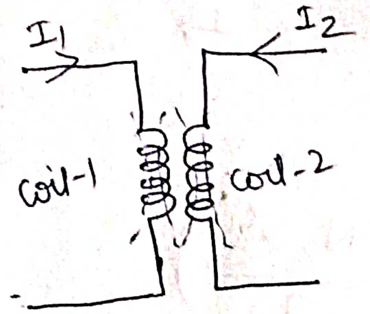
For Air core coupled circuits : $K = 0.4$ to 0.7

Eg:- (i)



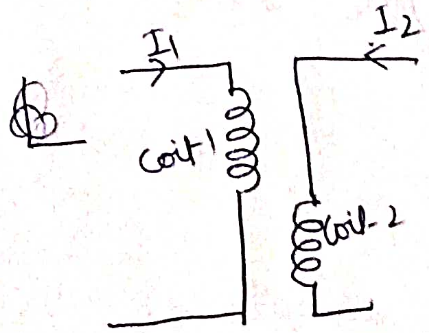
No coupling ($K=0$)

(ii)



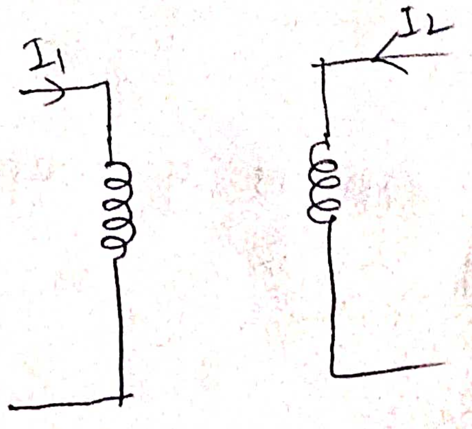
perfect coupling ($K=1$)

(iii)



Tight coupling ($K > 0.5$)

(iv)



Loose coupling ($K < 0.5$)

P The number of turns in two coupled coils are 600 and 1200 respectively. When a current of 4A flows in coil 1, the total flux in coil 1 is 0.5 m Wb and the flux linking coil 2 is 0.4 m Wb. Determine the self inductances of the coils and mutual inductance between them. Also calculate coefficient of coupling.

Sol: For coil 1,

$N_1 = 600$

$I_1 = 4A$

$\phi_1 = 0.5\,mWb$

$$L_1 = \frac{N_1 \phi_1}{I_1} = \frac{600 \times 0.5 \times 10^{-3}}{4}$$

$= 0.075H$

For coil 2

$N_2 = 1200$

$\phi_{12} = 0.4m$

here ϕ_2 and I_2 are not given.

The self inductance of a coil is directly proportional to the square of the number of turns ie $\boxed{L \propto N^2}$

$$\therefore \frac{L_1}{L_2} = \frac{N_1^2}{N_2^2}$$

$$\therefore L_2 = L_1 \times \frac{N_2^2}{N_1^2}$$

$$= 0.075 \times \frac{(1200)^2}{(600)^2}$$

$= 0.3H$

$$\therefore M_{12} = \frac{N_2 \phi_{12}}{I_1} \quad \text{or} \quad M_{21} = \frac{N_1 \phi_{21}}{I_2}$$

$$= \frac{1200 \times (0.4 \times 10^{-3})}{4}$$

$\because \phi_{21}$ & I_2 are not given

$M_{12} = 0.12\,H$

ie $M = 0.12\,H$

Hence the coefficient of coupling is given by,

$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.12}{\sqrt{(0.075)(0.3)}} = 0.8$$

P A ferromagnetic ring of cross-sectional area 800mm² and mean radius 170mm has two windings connected in series, one of 500 turns and one of 700 turns. If the relative permeability is 1200. Calculate the self inductance of each coil and the mutual inductance of each assuming that there is no flux leakage.

Sol $A = 800\,mm^2$

$r = 170\,mm \Rightarrow l = 2\pi r = 2\pi \times 170 \times 10^{-3}$

$N_1 = 500$

$N_2 = 700$

$\mu_r = 1200$

Reluctance, $\boxed{S = \frac{l}{\mu_0 \mu_r A}} = \frac{2\pi \times 170 \times 10^{-3}}{4\pi \times 10^{-7} \times 1200 \times 800 \times 10^{-6}}$

$= 8.85 \times 10^{5}$ AT/Wb

Self-inductance, $\boxed{L_1 = \frac{N_1^2}{S}} = \frac{500^2}{8.85 \times 10^5} = 0.283\,H$

$\left(\because L = \frac{N\phi}{I},\ \phi = \frac{NI}{L}\mu_0\mu_r A \quad \therefore L = \frac{N^2}{S}\right)$

Self-inductance, $L_2 = \frac{N_2^2}{S}$

$= \frac{700^2}{8.85 \times 10^5} = 0.552\,H$

Mutual inductance, $M = K\sqrt{L_1 L_2}$

Since there is no flux leakage, the coefficient of coupling, K is equal to 1.

$$\therefore M = 1\sqrt{0.283 \times 0.552}$$

$$\therefore M = 0.395\ H$$

Dot conventions:-

Dot convention is a technique, which gives the details about voltage polarity at the dotted terminal. This information is useful while writing KVL equations.

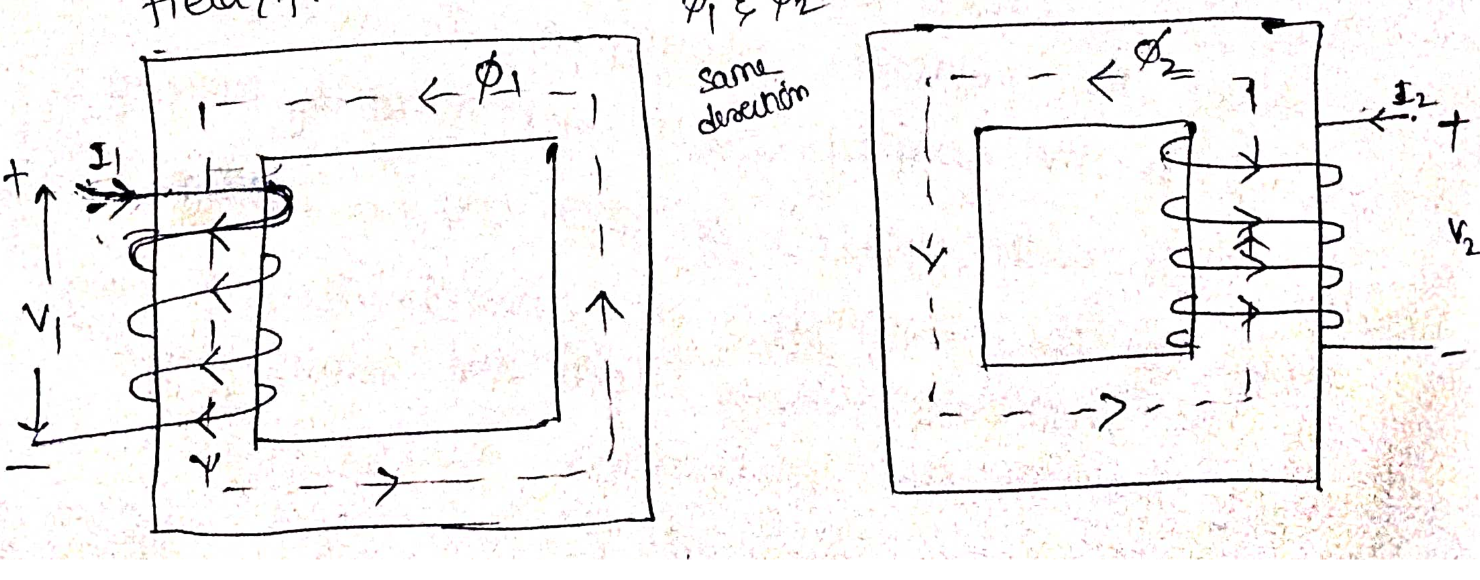
According to right hand thumb rule, the direction of thumb indicates the current, the direction of folded fingers indicates the direction of magnetic field (in case of straight conductors)

In case of coil, a small modification.

If the direction of folded fingers indicates the current, the direction of the thumb indicates the direction of magnetic field / flux.

ϕ_1 & ϕ_2

same direction

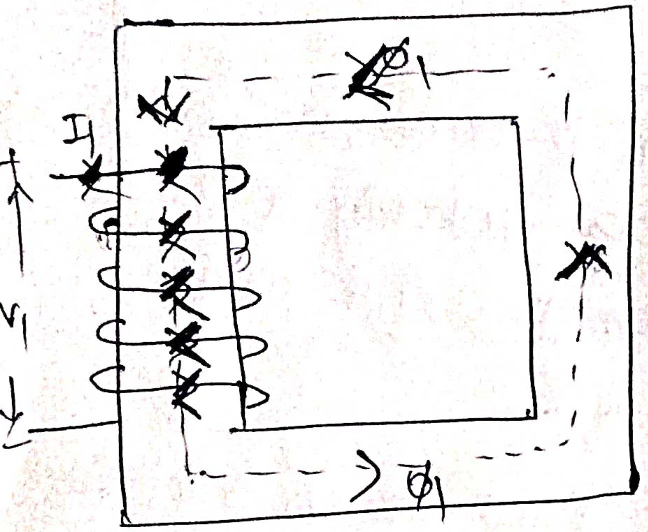


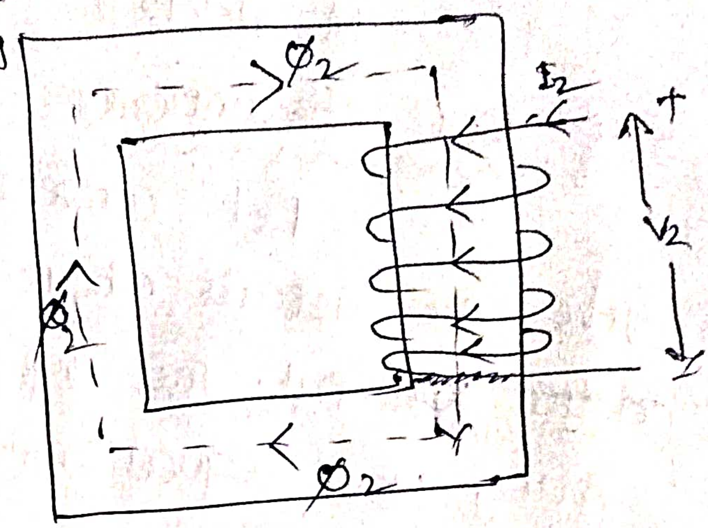
$V_1 = L\frac{dI_1}{dt} + M\frac{dI_2}{dt}$ $\qquad$ $V_2 = L\cdot\frac{dI_2}{dt} + M\cdot\frac{dI_1}{dt}$

$V_{12} = M\frac{dI_2}{dt}$ $\qquad$ (V_1 due to I_2)

$V_{21} = M\frac{dI_1}{dt}$ $\qquad$ (V_2 due to I_1)

ϕ_1 & ϕ_2 opposing





$V_{12} = -M\frac{dI_2}{dt}$ $\qquad$ $V_{21} = -M\frac{dI_1}{dt}$

$V_1 = L\cdot\frac{dI_1}{dt} - M\frac{dI_2}{dt}$ $\qquad$ $V_2 = L\frac{dI_2}{dt} - M\frac{dI_1}{dt}$

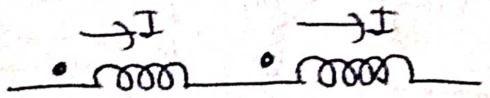
- To consolidate this process, dot convention is used. The relative polarity of the induced voltage in the coupled coil is determined by marking the coils with dots (•) on each coil, a dot is placed at the terminals. The currents through each of the mutually coupled coils are either going away from the dot (or) towards the dot.

(i) M is +ve if I is leaving (or) entering in the dot in both coils

(ii) M is −ve if I is entering the dot in one coil and leaving the dot in other coil & viceversa

Eg:

(1)



In both coils, 'I' is entering the dot, so M is positive.

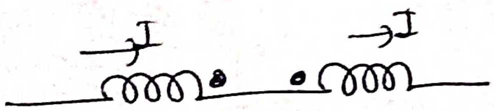
(2)

In both coils, 'I' is leaving the dot, so M is positive.

(3)

In 1st coil, I is entering the dot, in 2nd coil, I is leaving the dot. So 'M' is negative

(4)



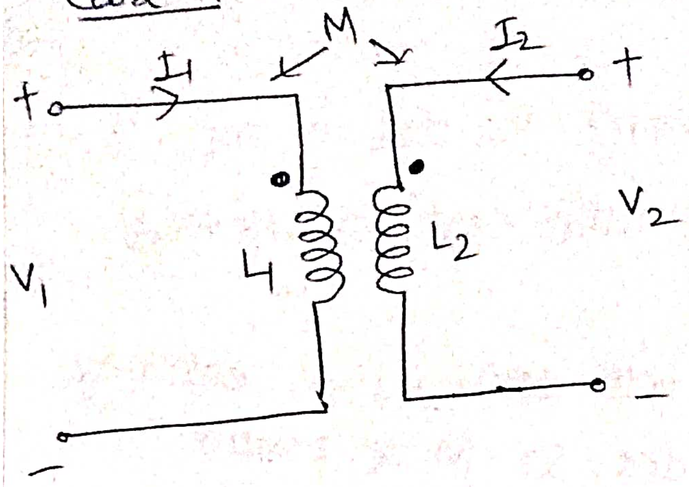
In 1st coil, I is leaving the dot, in 2nd coil, I is entering the dot, so M is negative.

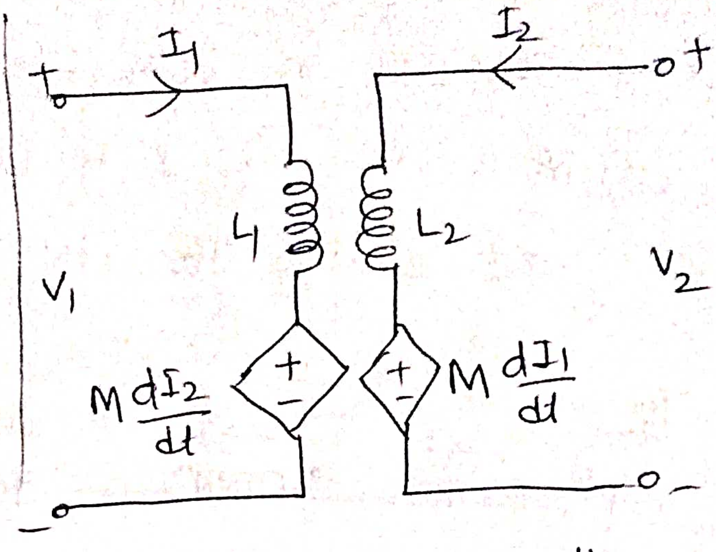
Consider two magnetically coupled coils L_1 and L_2 wound on same core.

Let current through coils L_1 and L_2 be I_1 & I_2 respectively.

The four possible combinations of the dot convention between the magnetically coupled coils and their equivalent circuits of dot convention are shown below.

Case 1:





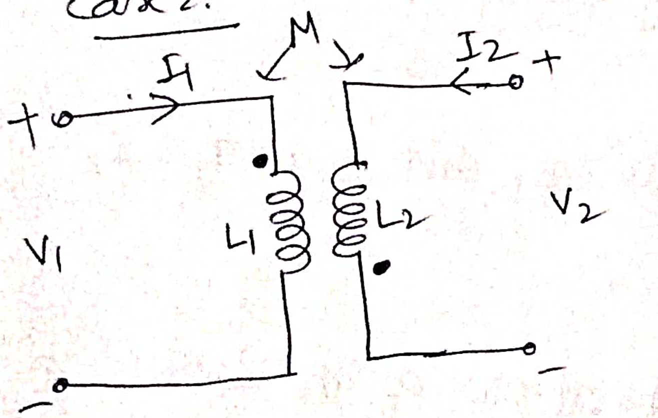
Both the currents I_1 and I_2 are entering the dotted terminals. Hence according to the dot convention, the mutually induced e.m.f in both the coils has the polarity same as self induced emf in respective coil.

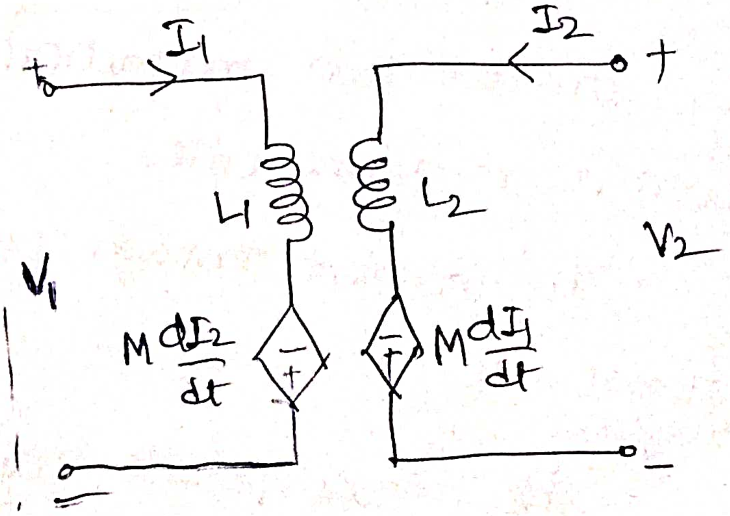
Applying KVL, the network equations of the equivalent circuit can be written as:

$$V_1 = L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt}$$

$$V_2 = L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt}$$

Case 2:

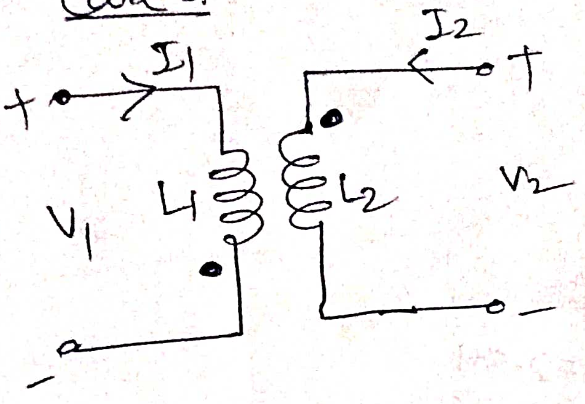


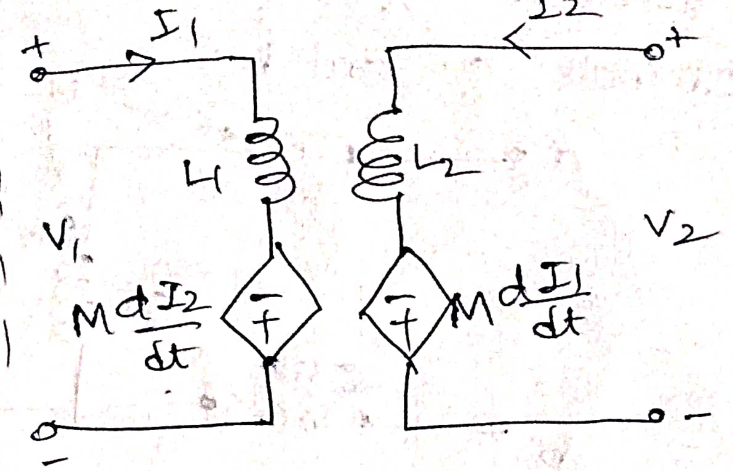


$$V_1 = L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt}$$

$$V_2 = L_2 \frac{dI_2}{dt} - M \frac{dI_1}{dt}$$

Case 3:

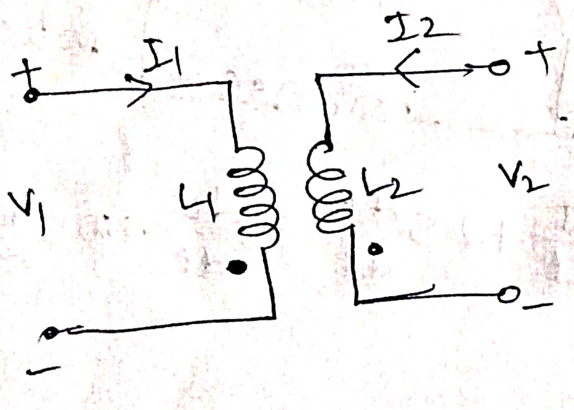


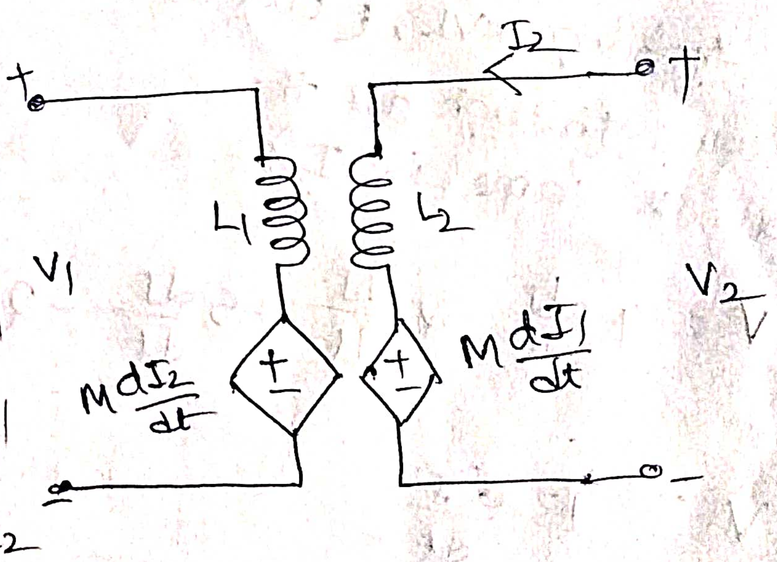


$$V_1 = L_1\frac{dI_1}{dt} - M\frac{dI_2}{dt}$$

$$V_2 = L_2\frac{dI_2}{dt} - M\frac{dI_1}{dt}$$

Case 4:

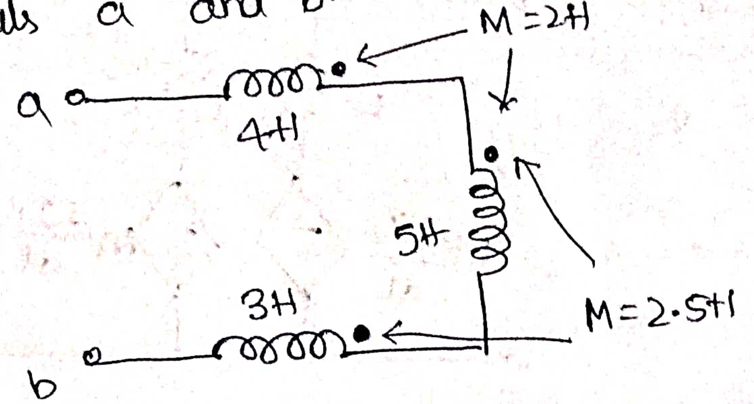




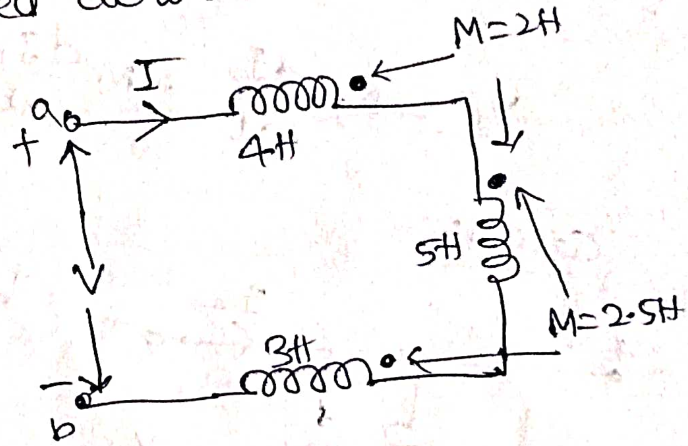
$$V_1 = L_1\frac{dI_1}{dt} + M\frac{dI_2}{dt}$$

$$V_2 = L_2\frac{dI_2}{dt} + M\frac{dI_1}{dt}$$

P Calculate effective inductance of the circuit across terminals a and b.



Sol: Assume that current I is flowing in series circuit and voltage developed across terminals a and b is as shown.



Applying KVL, (current flowing through all the coils is same i.e I).

$$V = 4\frac{dI}{dt} - 2\frac{dI}{dt} + 5\frac{dI}{dt} - 2\frac{dI}{dt} + 2.5\frac{dI}{dt} + 3\frac{dI}{dt} + 2.5\frac{dI}{dt}$$

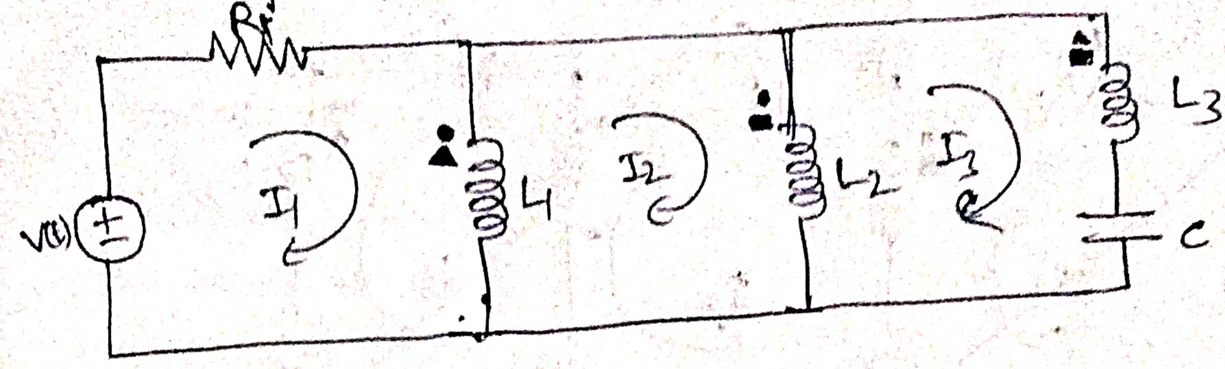
$$V = 13\frac{dI}{dt}$$

$$V = 13\frac{dI}{dt} = L_{eff}\frac{dI}{dt}$$

∴ Effective inductance across terminals a and b is L_{eff}

$L_{eff} = 13H$

P Write Loop equations for the circuit



Sol: Applying KVL to loop 1, we get

$$-I_1R_1 - L_1\frac{d(I_1-I_2)}{dt} - M_{12}\frac{d}{dt}(I_2-I_3) - M_{13}\frac{dI_3}{dt} + V(t) = 0$$

$$I_1R_1 + L_1\frac{d}{dt}(I_1-I_2) + M_{12}\frac{d}{dt}(I_2-I_3) + M_{13}\frac{dI_3}{dt} = V(t) \quad \text{— (1)}$$

Applying KVL to loop 2, we get

$$-L_1\frac{d}{dt}(I_2-I_1) - L_3\frac{d}{dt}(I_2-I_3) + M_{13}\frac{dI_3}{dt} - M_{23}\frac{dI_3}{dt} + M_{12}\frac{d}{dt}(I_2-I_3) + M_{21}\frac{d}{dt}(I_2-I_1) = 0$$

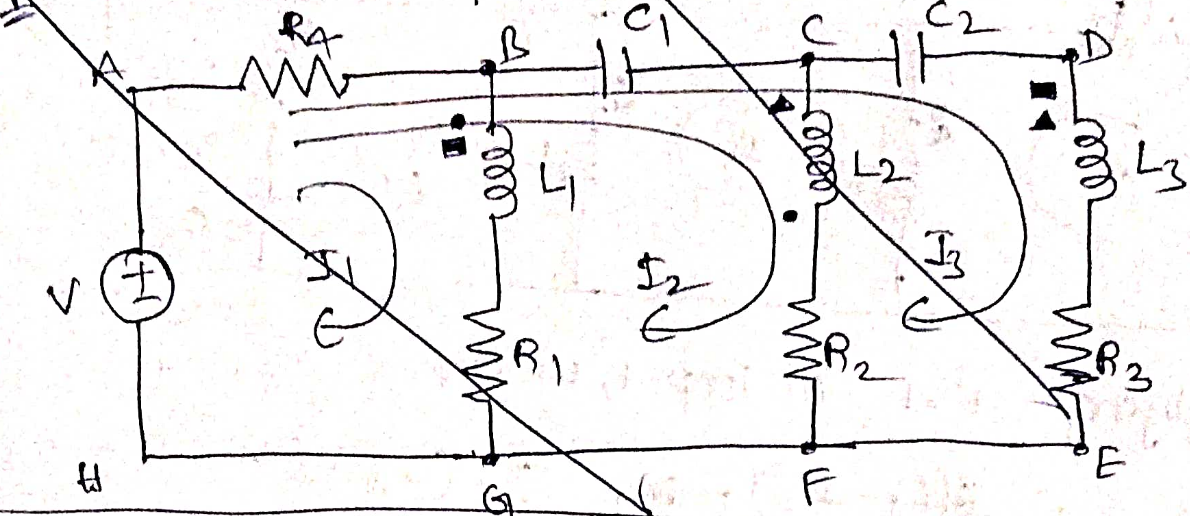
$$\therefore L_1\frac{d}{dt}(I_2-I_1) - M_{21}\frac{d}{dt}(I_2-I_1) + L_3\frac{d}{dt}(I_2-I_3) - M_{12}\frac{d}{dt}(I_2-I_3) + M_{23}\frac{dI_3}{dt} - M_{13}\frac{dI_3}{dt} = 0 \quad \text{— (2)}$$

Applying KVL to loop 3, we get,

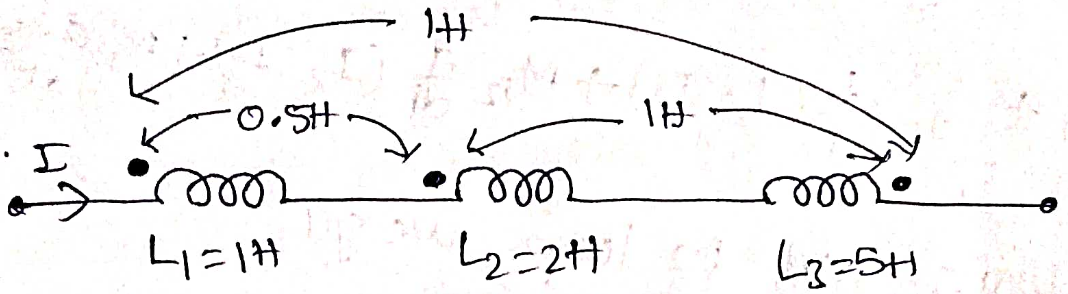
$$-L_3\frac{dI_3}{dt} - \frac{1}{C}\int_0^t I_3\,dt - L_2\frac{d}{dt}(I_3-I_2) - M_{31}\frac{d}{dt}(I_1-I_2) - M_{21}\frac{d}{dt}(I_1-I_2) + M_{23}\frac{dI_3}{dt} + M_{32}\frac{d}{dt}(I_3-I_2) = 0$$

$$\therefore L_3\frac{dI_3}{dt} - M_{23}\frac{dI_3}{dt} + \frac{1}{C}\int_0^t I_3\,dt + L_2\frac{d}{dt}(I_3-I_2) - M_{32}\frac{d}{dt}(I_3-I_2) + M_{31}\frac{d}{dt}(I_1-I_2) + M_{21}\frac{d}{dt}(I_1-I_2) = 0 \quad \text{— (3)}$$

Q Write mesh equations for the network shown.



Q Find total inductance of the series connected coupled coils as shown in figure.



Let us assume that current through circuit is i & voltage across is V.

Apply KVL to the ckt

$$V = 1\cdot\frac{dI}{dt} + 0.5\frac{dI}{dt} - 1\frac{dI}{dt} + 2\cdot\frac{dI}{dt} + 0.5\frac{dI}{dt} - 1\frac{dI}{dt}$$
$$+ 5\frac{dI}{dt} - 1\frac{dI}{dt} - 1\frac{dI}{dt}$$

$$V = 5\frac{dI}{dt}$$

$$\therefore V = L_{eq}\cdot\frac{dI}{dt}$$

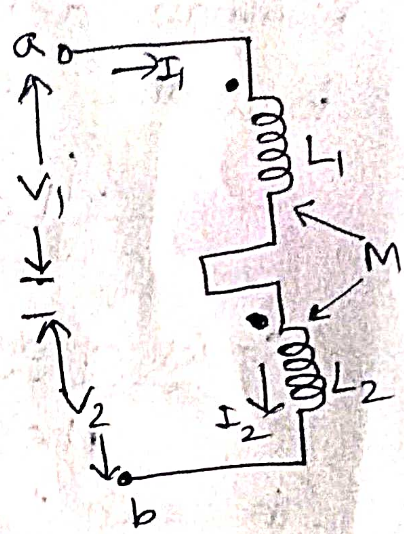
$$\therefore \boxed{L_{eq} = 5H}$$

iv) Electrically Joined coupled circuits:-

There are 4 ways in which coupled coils may be joined electrically.

1) Series aiding
2) series opposing
3) parallel aiding
4) parallel opposing

Series aiding:



Consider two coils which are inductively coupled and connected in series such that their induced fluxes or voltages are additive in nature.

Here currents I_1 and I_2 is nothing but current I which is entering dots for both the coils

Let L_1 and L_2 denote the self-inductances of the coils and M denote the mutual inductance.

Self induced voltage in coil 1 $= V_{L_1} = +L_1 \frac{dI}{dt}$

Self induced voltage in coil 2 $= V_{L_2} = +L_2 \frac{dI}{dt}$

Mutually induced voltage in coil 1 due to change in current in coil 2 $= V_1' = +M\frac{dI}{dt}$

Mutually induced voltage in coil 2 due to change in current in coil 1 $= V_2' = +M\frac{dI}{dt}$

According to KVL, $V = V_1 + V_2$

$V_1 = V_{L_1} + V_1' = L_1\frac{dI}{dt} + M\frac{dI}{dt}$ and $V_2 = V_{L_2} + V_2' = L_2\frac{dI}{dt} + M\frac{dI}{dt}$

$\therefore$ total induced voltage $= V_{L_1} + V_{L_2} + V_1' + V_2'$

$$= +\left(L_1 \frac{dI}{dt} + L_2 \frac{dI}{dt} + M\frac{dI}{dt} + M\frac{dI}{dt}\right)$$

$$V_{Tot} = +(L_1 + L_2 + 2M)\frac{dI}{dt}$$

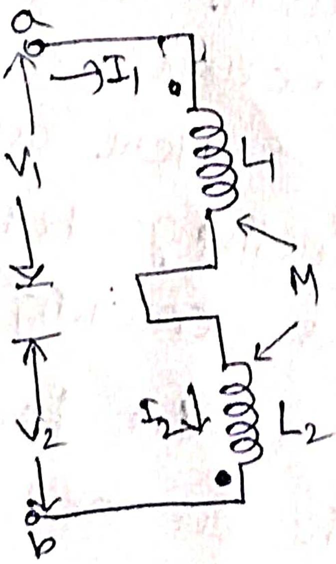
$$\cancel{V_{Tot} = L_{eq}\frac{dI}{dt}}$$

Here L_{eq} is the equivalent inductance across terminals a-b, then total induced voltage in single inductance would be equal to $+L_{eff}\frac{dI}{dt}$.

$$\therefore \boxed{L_{eff} = L_1 + L_2 + 2M}$$

Series opposing:-

The two coils are connected in such a way that the induced fluxes of voltages are of opposite polarities.



Here I_1 and I_2 are same series current I, which is entering dot for coil L_1 and leaving dot for coil L_2.

self induced voltage in coil 1 $= +L_1\frac{dI}{dt}$

self induced voltage in coil 2 $= +L_2\frac{dI}{dt}$

Mutually induced voltage in coil 1 due to change in current in coil 2 $= V_1' = -M\frac{dI}{dt}$

Mutually induced voltage in coil 2 due to change in current in coil 1 $= V_2' = -M\frac{dI}{dt}$

$\therefore$ Total induced Voltage $= V_1 + V_2 + V_1' + V_2'$

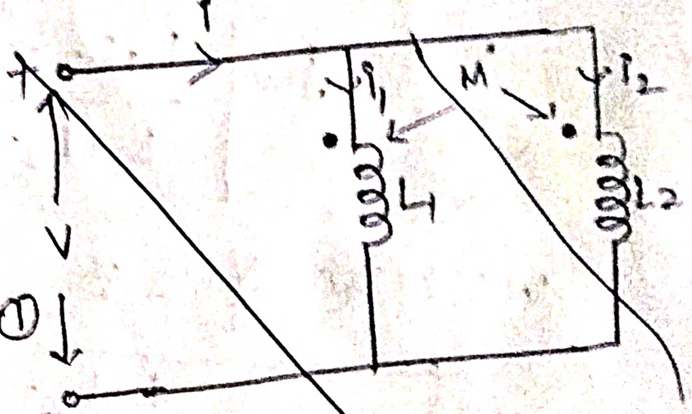
$$= +L_1 \frac{dI}{dt} + L_2 \frac{dI}{dt} - M\frac{dI}{dt} - M\frac{dI}{dt}$$

$$= +(L_1 + L_2 - 2M)\frac{dI}{dt}$$

If L is equivalent inductance across terminals a and b then total induced voltage in single inductance would be equal to $+L_{eff}\frac{dI}{dt}$.

$$\therefore \boxed{L_{eff} = L_1 + L_2 - 2M}$$

Parallel Aiding:



Applying KVL to loop 1,

$$V = j\omega L_1 (I_1 - I_2) - (j\omega M) I_2$$

$$= j\omega L_1 I_1 - j\omega (L_1 + M) I_2 \quad \text{—(1)}$$

Applying KVL to loop 2

$$0 = j\omega L_1 (I_2 - I_1) + j\omega L_2 I_2 - j\omega M I_2 - j\omega M (I_2 - I_1)$$

$$0 = -j\omega (L_1 + M) I_1 + j\omega (L_1 + L_2 - 2M) I_2$$

$$j\omega I_1 (L_1 + M) = j\omega (L_1 + L_2 - 2M) I_2$$

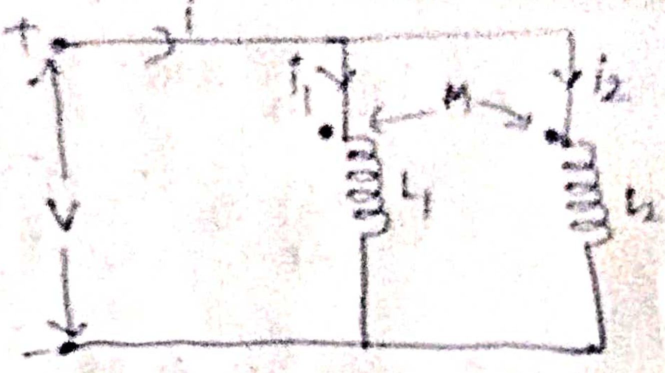
$$I_1 (L_1 + M) = (L_1 + L_2 - 2M) I_2$$

$$I_2 = \frac{L_1 + M}{L_1 + L_2 - 2M} I_1 \quad \text{—(2)}$$

Substitute eq (2) in eq (1)

Parallel Aiding:

Both currents are entering the dot. So, both these are in same direction and added to each other. So 'M' is +ve.



According to KCL

$$I = I_1 + I_2$$

differentiate w.r.t. t on both sides

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} \quad \text{——— (1)}$$

$$V = V_{L_1} = V_{L_2}$$

$$V_{L_1} = L_1 \frac{dI_1}{dt} + M\frac{dI_2}{dt} \qquad V_{L_2} = L_2 \frac{dI_2}{dt} + M\frac{dI_1}{dt}$$

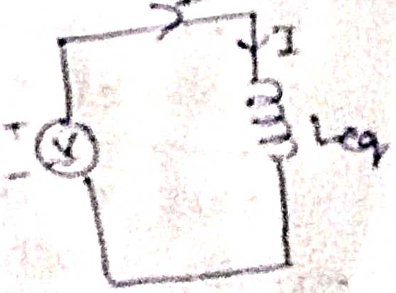
$$V_{L_1} = V_{L_2}$$

$$\therefore L_1 \frac{dI_1}{dt} + M\frac{dI_2}{dt} = L_2\frac{dI_2}{dt} + M\frac{dI_1}{dt}$$

$$(L_1 - M)\frac{dI_1}{dt} = (L_2 - M)\frac{dI_2}{dt}$$

$$\frac{dI_2}{dt} = \left(\frac{L_1 - M}{L_2 - M}\right)\frac{dI_1}{dt} \quad \text{——— (2)}$$

From the equivalent circuit $V = L_{eq}\frac{dI}{dt}$



$$V = L_{eq}\cdot\left(\frac{dI_1}{dt} + \frac{dI_2}{dt}\right)$$

$$= L_{eq}\left[\frac{dI_1}{dt} + \left(\frac{L_1 - M}{L_2 - M}\right)\frac{dI_1}{dt}\right]$$

$$= L_{eq}\frac{dI_1}{dt}\left[1 + \frac{L_1 - M}{L_2 - M}\right]$$

$$= L_{eq}\cdot\frac{dI_1}{dt}\left[\frac{L_2 - M + L_1 - M}{L_2 - M}\right] = L_{eq}\frac{dI_1}{dt}\left[\frac{L_1 + L_2 - 2M}{L_2 - M}\right] \text{— (3)}$$

we know $V = V_{L_1} = V_{L_2}$

$$V_{L_1} = L_1 \frac{dI_1}{dt} + M\frac{dI_2}{dt}$$

$$= L_1 \frac{dI_1}{dt} + \left(\frac{L_1 - M}{L_2 - M}\right)\frac{dI_1}{dt} \cdot M$$

$$= \frac{dI_1}{dt}\left[L_1 + M\left(\frac{L_1 - M}{L_2 - M}\right)\right]$$

$$= \frac{dI_1}{dt}\left[\frac{L_1L_2 - L_1M + ML_1 - M^2}{L_2 - M}\right]$$

$$V_{L_1} = \left(\frac{L_1L_2 - M^2}{L_2 - M}\right)\frac{dI_1}{dt} \quad \text{—— (4)}$$

By equating eq (3) & eq (4)

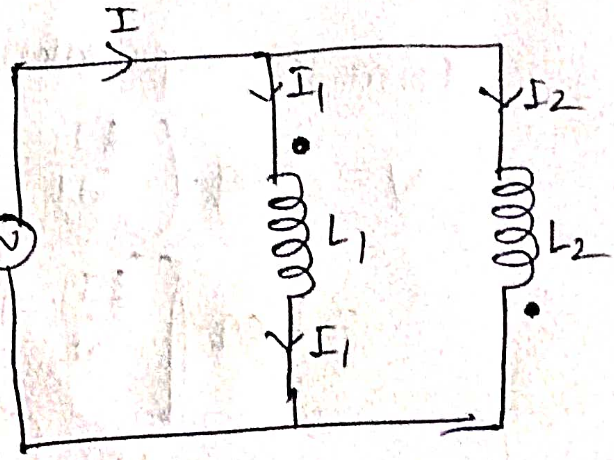
$$L_{eq}\frac{dI_1}{dt}\left[\frac{L_1 + L_2 - 2M}{L_2 - M}\right] = \left[\frac{L_1L_2 - M^2}{L_2 - M}\right]\frac{dI_1}{dt}$$

$$L_{eq} \cdot \left[\frac{L_1 + L_2 - 2M}{L_2 - M}\right] = \left[\frac{L_1L_2 - M^2}{L_2 - M}\right]$$

$$\boxed{L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}}$$

→ parallel opposing:-

flux in both inductors opposing each other.



According to KCL

$$I = I_1 + I_2$$

differentiate wrt t

$$\frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt} \quad \text{—— (1)}$$

$$V = V_{L_1} = V_{L_2}$$

... for the given figure.

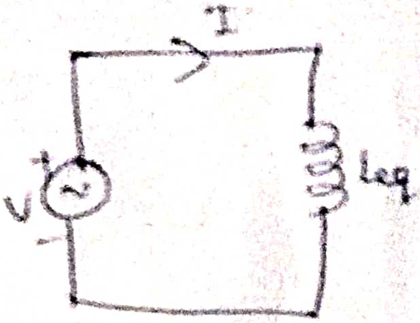
$$V_{L_1} = L_1 \frac{dI_1}{dt} - M\frac{dI_2}{dt} \qquad V_{L_2} = L_2 \frac{dI_2}{dt} - M\frac{dI_1}{dt}$$

$$L_1 \frac{dI_1}{dt} - M\frac{dI_2}{dt} = L_2\frac{dI_2}{dt} - M\frac{dI_1}{dt}$$

$$(L_1+M)\frac{dI_1}{dt} = (L_2+M)\frac{dI_2}{dt}$$

$$\frac{dI_2}{dt} = \left[\frac{L_1+M}{L_2+M}\right]\frac{dI_1}{dt} \quad \text{--- (2)}$$

from the equivalent circuit



$$V = L_{eq}\frac{dI}{dt}$$

$$= L_{eq}\left(\frac{dI_1}{dt} + \frac{dI_2}{dt}\right) \qquad (\because \text{from (1)})$$

$$= L_{eq}\left[\frac{dI_1}{dt} + \left(\frac{L_1+M}{L_2+M}\right)\frac{dI_1}{dt}\right]$$

$$= L_{eq}\frac{dI_1}{dt}\left[1 + \frac{L_1+M}{L_2+M}\right]$$

$$= L_{eq}\cdot\frac{dI_1}{dt}\left[\frac{L_2+M+L_1+M}{L_2+M}\right]$$

$$V = L_{eq}\frac{dI_1}{dt}\left[\frac{L_1+L_2+2M}{L_2+M}\right] \quad \text{--- (3)}$$

we know

$$V = V_{L_1} = V_{L_2}$$

$$V_{L_1} = L_1\frac{dI_1}{dt} - M\frac{dI_2}{dt}$$

$$= L_1\frac{dI_1}{dt} - M\left[\left(\frac{L_1+M}{L_2+M}\right)\frac{dI_1}{dt}\right]$$

$$= \frac{dI_1}{dt}\left[L_1 - \left(\frac{ML_1+M^2}{L_2+M}\right)\right]$$

$$= \frac{dI_1}{dt}\left[\frac{L_1L_2+L_1M-ML_1+M^2}{L_2+M}\right] = \frac{dI_1}{dt}\left[\frac{L_1L_2+M^2}{L_2+M}\right] \quad \text{--- (4)}$$

equating eq 3 & eq 4

$$L_{eq}\frac{dI_1}{dt}\left[\frac{L_1+L_2+2M}{L_2+M}\right] = \frac{dI_1}{dt}\left[\frac{L_1L_2-M^2}{L_2+M}\right]$$

$$\boxed{L_{eq} = \frac{L_1L_2-M^2}{L_1+L_2+2M}}$$

Q If a coil of 800 μH is magnetically coupled to another coil of 200 μH. The coefficient of coupling between two coils is 0.05. Calculate inductance if two coils are connected in

(i) Series aiding (ii) series opposing

(iii) parallel aiding (iv) parallel opposing

Sol: The mutual inductance between two coils is given by

$$M = K \cdot \sqrt{L_1 L_2}$$

$$= 0.05 \sqrt{800 \times 10^{-6} \times 200 \times 10^{-6}}$$

$$= 20 \mu H$$

(a) Let the effective inductance for magnetically coupled coil be L.

(i) series aiding $L = L_1 + L_2 + 2M$

$$= (800 \times 10^{-6}) + (200 \times 10^{-6}) + (2 \times 20 \times 10^{-6})$$

$$= 1040 \mu H$$

(ii) series opposing $L = L_1 + L_2 - 2M$

$$= (800 \times 10^{-6}) + (200 \times 10^{-6}) - (2 \times 20 \times 10^{-6})$$

$$= 960 \mu H$$

(iii) parallel aiding $L = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$

$$= 166.25 \mu H$$

(iv) parallel opposing $L = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$

$$= 153.46 \mu H$$

Energy in a pair of coupled coils:-

The energy stored in a single inductor L carrying current i is given by

$$W = \frac{1}{2} L i^2 \text{ Joules}$$

Consider a pair of coupled coils with self inductance L_1, L_2 and mutual inductance M.

The energy stored in pair of coupled coils is

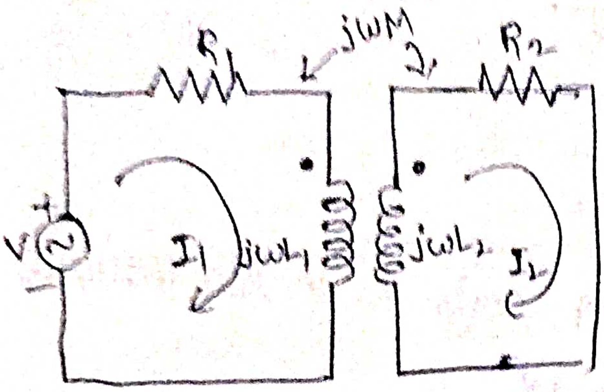
$$W = \begin{cases} \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2 & \text{— If } i_1 \text{ \& } i_2 \text{ either enter or leave dotted terminals of coils.} \\ \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 - M i_1 i_2 & \text{— If one current enters dotted terminal while other leaves dotted terminal.} \end{cases}$$

Conductively coupled Equivalent circuit:-

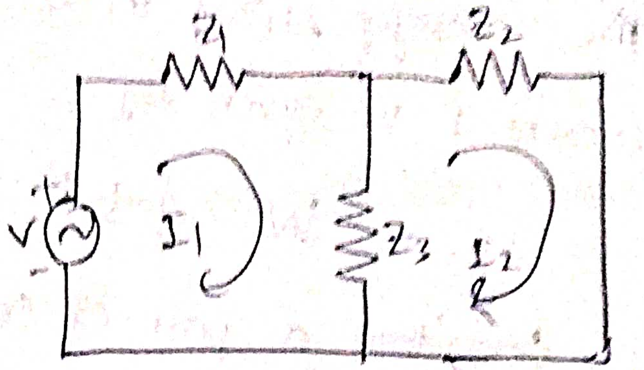
For easy circuit analysis, we replace magnetically coupled circuit with an equivalent network called conductively coupled circuit.

In conductively coupled circuit, there is no magnetic coupling involved, dot convention is also not needed and can be analyzed by mesh analysis and nodal analysis

Consider magnetically coupled circuit and its equivalent T section.



Magnetically coupled ckt



Equivalent T section of circuit

Applying KVL to loop1,

$$-R_1I_1 - j\omega L_1 I_1 + j\omega M I_2 + V = 0$$

$$-(R_1 + j\omega L_1)I_1 + (j\omega M)I_2 = -V$$

$$(R_1 + j\omega L_1)I_1 - (j\omega M)I_2 = V \quad (1)$$

Applying KVL to loop2

$$+j\omega M I_1 - R_2 I_2 - j\omega L_2 I_2 = 0$$

$$-(j\omega M)I_1 + (R_2 + j\omega L_2)I_2 = 0 \quad (2)$$

Writing eq (1) & eq (2) in matrix form.

$$\begin{bmatrix} R_1 + j\omega L_1 & -j\omega M \\ -j\omega M & R_2 + j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V \\ 0 \end{bmatrix}$$

Applying KVL to loop 1

$$-Z_1 I_1 - Z_3(I_1 - I_2) + V = 0$$

$$(Z_1 + Z_3)I_1 - (Z_3)I_2 = V \quad (3)$$

Applying KVL to loop2

$$-Z_2 I_2 - Z_3(I_2 - I_1) = 0$$

$$-Z_3 I_1 + (Z_2 + Z_3)I_2 = 0 \quad (4)$$

Writing eq (3) & eq (4) in matrix form

$$\begin{bmatrix} Z_1 + Z_3 & -Z_3 \\ -Z_3 & Z_2 + Z_3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V \\ 0 \end{bmatrix}$$

To have the circuits equivalent, impedance matrices must be equal.

$$\therefore \quad Z_1 + Z_3 = R_1 + j\omega L_1$$

$$Z_2 + Z_3 = R_2 + j\omega L_2$$

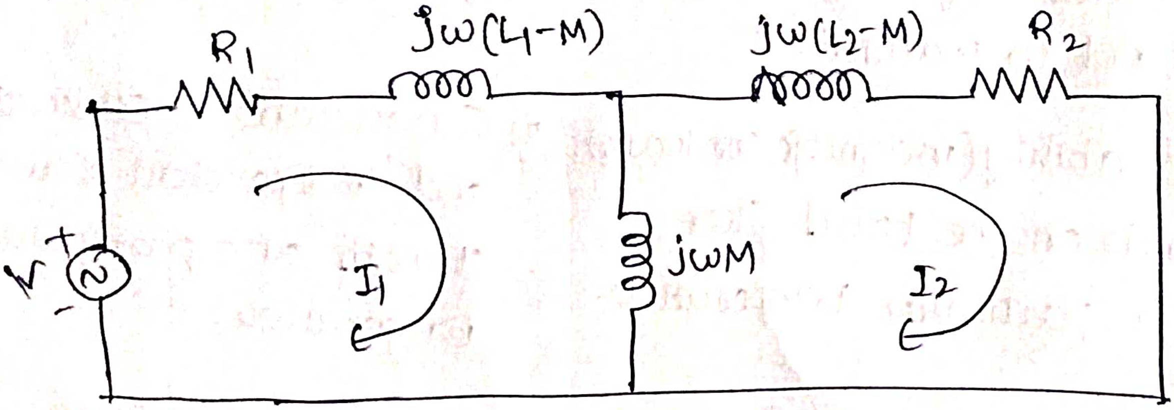
$$Z_3 = +j\omega M$$

$\therefore \quad Z_1 = R_1 + j\omega L_1 - j\omega M = R_1 + j\omega(L_1 - M)$

$Z_2 = R_2 + j\omega L_2 - j\omega M = R_2 + j\omega(L_2 - M)$

$Z_3 = j\omega M.$

Hence, the conductively coupled equivalent circuit of magnetically coupled circuit is



Comparision between Magnetic Circuits & Electrical Circuits:

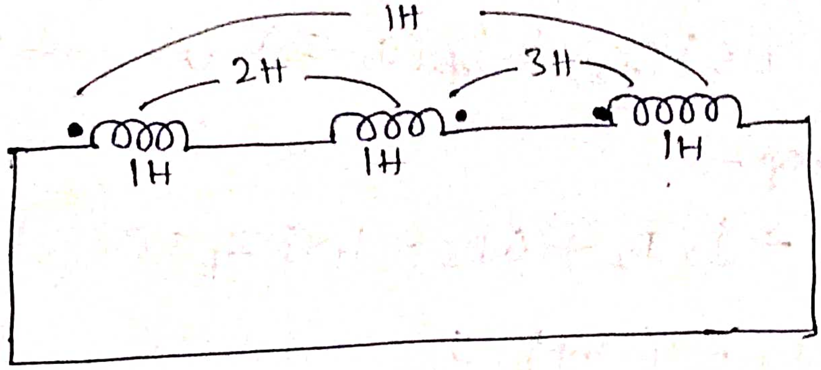
Similarities

Magnetic circuit	Electric Circuit
1) Flow is of Flux (ϕ). Response is flux in Webers	1) Flow is of current (I). Response is current in Amperes
2) Exciting force is MMF in Ampere turns.	2) Exciting force = emf in Volts
3) mmf drop = $S\phi$ AT	3) voltage drop = RI volts
4) Flux (ϕ) = $\frac{MMF}{S}$	4) current = $\frac{EMF}{R}$
5) Reluctance (S) = $\frac{1}{\mu}\frac{l}{a}$ AT/web	5) Resistance (R) = $\frac{\rho l}{a}$ ohm
6) permeability (μ) = $\frac{1}{S}$	6) conductivity (σ) = $\frac{1}{R}$
7) Flux density (B) is the no. of lines per sq. m. $B = \frac{\phi}{A}$ web/m^2	7) current density (J) is amperes per sq. m. $J = \frac{I}{a}$ Amp/m^2

Dissimilarities:-

Magnetic circuit	Electrical circuit
1) Flux does not actually flow in a magnetic circuit	1) current actually flows in an electric circuit.
2) Energy is initially needed to create the magnetic flux, but not to maintain it.	2) Energy is needed as long as current flows
3) permeability (magnetic conductance) depends on the total flux for a particular temperature.	3) Conductance is constant, and independent of current strength at a particular temperature
4) Magnetic resistance does not vary greatly from one substance to another,	4) Electrical resistance varies greatly from one substance to another.
5) It is very difficult to confine the flux into a desired path, and a certain amount of leakage of flux is always associated with magnetic materials	5) current can be easily confined into a desired path just by providing a conducting path for it.

P Find the equivalent Inductance for the given figure.



Sol:

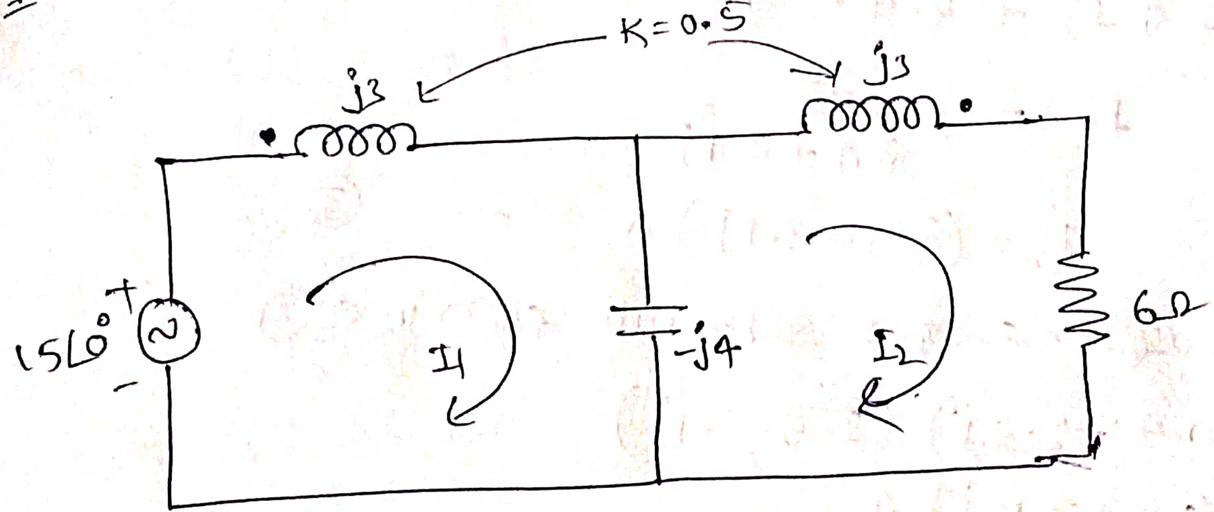
$L_{eq1} = 1 + 1 - 2(2)$
$= -2H$

$L_{eq2} = 1 + 1 - 2(3)$
$= -4H$

$L_{eq3} = 1 + 1 + 2(1)$
$= 4H$

$L_{eq} = L_{eq1} + L_{eq2} + L_{eq3}$
$= -2 - 4 + 4 = -2H$

P Find the loop currents for given circuit



Sol. Coefficient of coupling $K = \frac{M}{\sqrt{L_1 L_2}}$

$0.5 = \frac{M}{\sqrt{j3(j3)}}$

$0.5 = \frac{M}{j3}$

$\therefore M = 1.5j$

KVL for loop 1

$$-15\angle 0 + j3(I_1) - 1.5j(I_2) - j4(I_1 - I_2) = 0 \quad \text{--- (1)}$$

KVL for loop 2

$$-j4(I_2 - I_1) + j3 I_2 + 6 I_2 - 1.5j I_1 = 0$$

$$2.5j\, I_1 = (-6 + j)\, I_2$$

$$I_1 = \left(\frac{-6+j}{2.5j}\right) I_2$$

$$I_1 = (0.4 + 2.4j) I_2 \quad \text{--- (2)}$$

from eq (1)

$$3j I_1 - 4j I_1 - 1.5j I_2 + 4j I_2 = 15\angle 0^\circ$$

$$-j \cancel{I_1} \cancel{+} 2.5j\, I_2 = 15$$

$$-j(0.4 + 2.4j) I_2 + 2.5j\, I_2 = 15$$

$$-j0.4 I_2 + 2.4 I_2 + 2.5j I_2 = 15$$

$$I_2 = \frac{15}{2.4 + 2.1j}$$

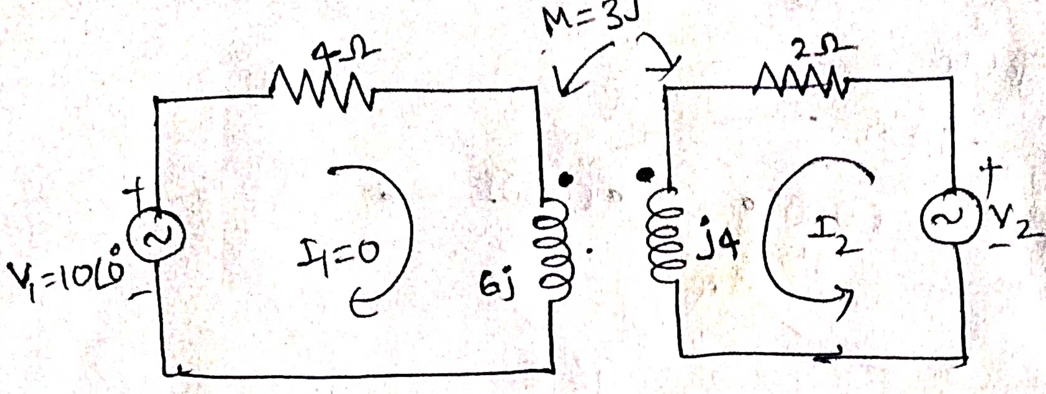
$$I_2 = (3.5 - 3.1j)\ A \quad \text{--- (3)}$$

∴ ~~from eq (2) & eq 3~~ substitute eq (3) in eq (2)

$$I_1 = (0.4 + 2.4j)(3.5 - 3.1j)$$

$$= (8.8 + 7.16j)\ A$$

P Find the V_2 such that the current in loop 1 is zero.



Sol for loop 1

$$-10\angle 0^\circ + 4I_1 + 6j\,I_1 + 3j\,I_2 = 0$$

since both are entering.

$(\because I_1 = 0)$

$$-10 + 3j\,I_2 = 0$$

$$3j\,I_2 = 10 \Rightarrow I_2 = \frac{10}{3j} = -3.3j$$

for loop 2

$$-V_2 + 2I_2 + j4\,I_2 + 3j\,I_1 = 0$$

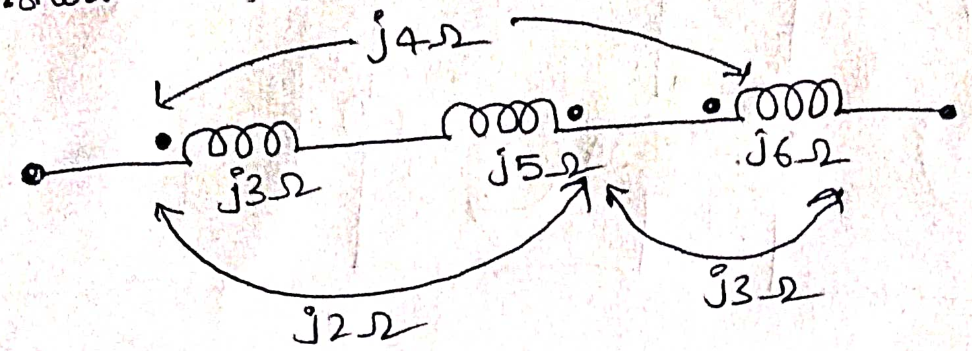
$$-V_2 + I_2(2+4j) = 0$$

$$V_2 = I_2(2+4j)$$

$$= -3.3j(2+4j)$$

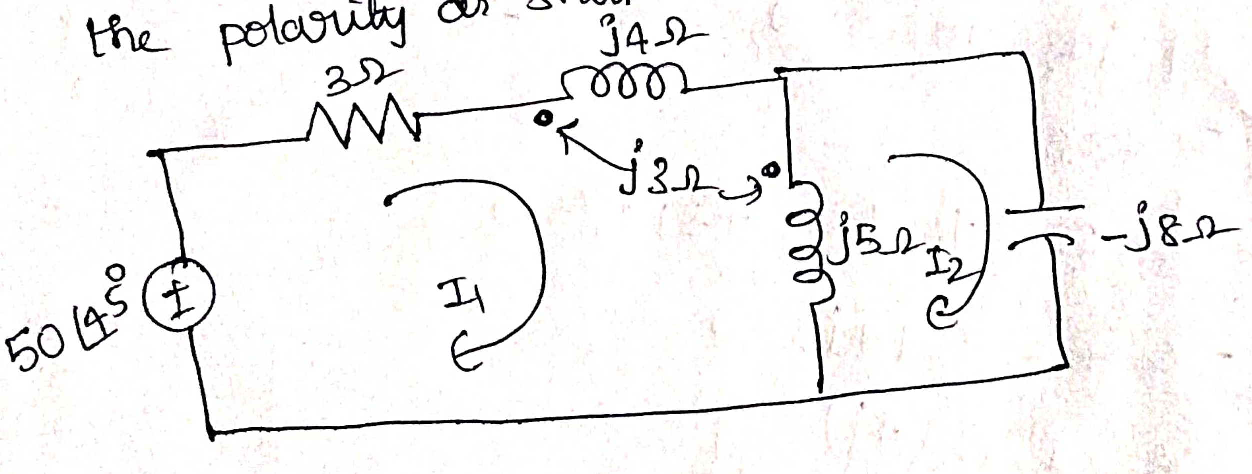
$$V_2 = (13.2 - 6.6j)\,V \quad \text{or} \quad V_2 = 14.7\angle -26^\circ.$$

P. using dot rules for coupled coils, determine the equivalent reactance of the coupled circuit shown in figure



sol:
$$Z_{eq} = j3 - j2 + j4 + j5 - j2 - j3 + j6 - j3 + j4$$
$$Z_{eq} = j12\ \Omega$$

P. Find the voltage across 5Ω reactance with the polarity as shown.



sol: Mesh-1:
$$50\angle 45^\circ = 3I_1 + j4I_1 + j3(I_1 - I_2) + j5(I_1 - I_2) + j3I_1$$

Mesh-2:
$$0 = -j8I_2 + j5(I_2 - I_1) - j3I_1$$