

→ CountAndSay(n) is the way you "say" CountAndSay(n-1)

Case(1) = 1

Case(4) = 1 2 1 1

Case(2) = 1 1

Case(5) = 1 1 1 2 2 1

Case(3) = 2 1

s = 3 3 2 2 2 5 1 1 1 1
j ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑
i ↑ ↑ ↑ ↑
ans = 2 3 3 2 1 5 4 1

⇒ Code :- ps String countAndSay(int n){

if(n==1) return "1";

String s = countAndSay(n-1) + "@";

// Now we have to modify s.

String ans = "";

int i = 0, j = 0;

while(j < s.length()){

if(s.charAt(i) == s.charAt(j)) j++;

else {

int len = j - i;

ans += len;

ans += s.charAt(i);

i = j;

}

}

return ans;

psvm {

int n = 4;

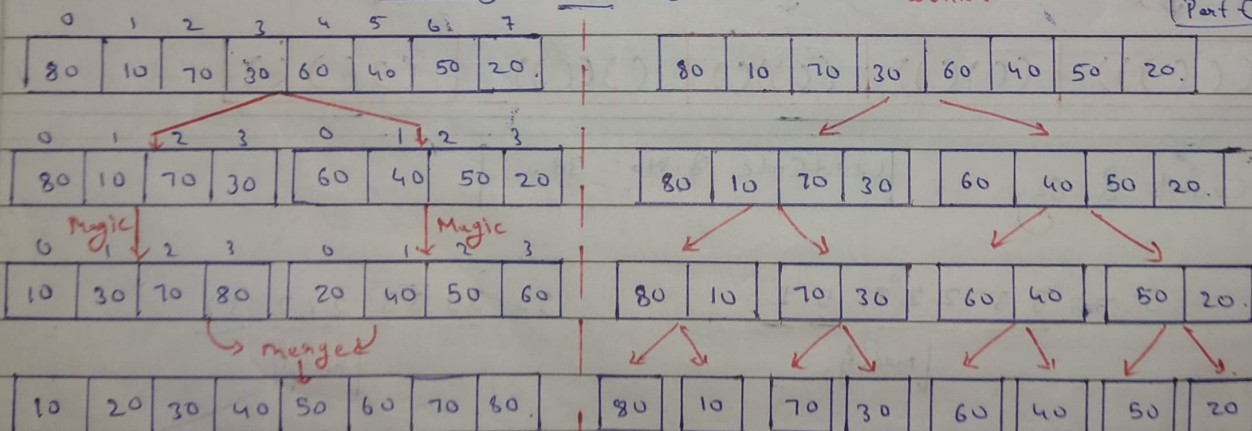
cout << countAndSay(n);

}

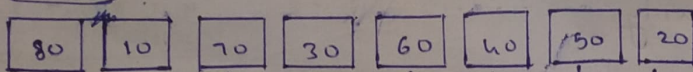
Merge Sort Algorithm

Work flow :-

Part (1)



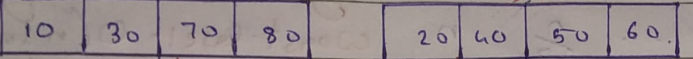
Part (2)



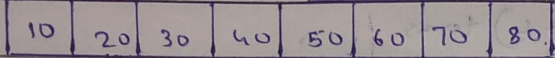
merge 2 Sorted Array.



merge 2 Sorted Array.



merge 2 Sorted Array.



Divide & Conquer

⇒ Code :- `psv mergesort(Int[] arr){`

`int n = arr.length;`

`if (n == 1) return; // Base Case.`

`// Create two arrays of n/2 size each...`

`int[] a = new int[n/2];`

`int[] b = new int[n-n/2];`

`// Copy Pasting...`

`for (int i = 0; i < n/2; i++){`

`a[i] = arr[i];`

`}`

`for (int i = 0; i < n-n/2; i++){`

`b[i] = arr[i + n/2];`

`}`

`// Magic...`

`mergesort(a);`

`mergesort(b);`

`// Merge these 'a' & 'b'...`

`merge(a, b, arr);`

`// Delete a & b...`

`a = null; b = null;`

`}`

`psv print(int[] arr){`

`for (int ele : arr){`

`cout << ele << " ";`

`}`

`cout << endl;`

`}`

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Copy pasting.

$$T(n) = \left[n + \left(\frac{n}{2} + \frac{n}{2} \right) + \left(\frac{n}{4} + \frac{n}{4} + \frac{n}{4} + \frac{n}{4} \right) + \left(\frac{n}{8} + \frac{n}{8} + \frac{n}{8} + \frac{n}{8} + \frac{n}{8} + \frac{n}{8} \right) + n \right]$$

merging

$$T(n) = 2[n + n + n]$$

$$T(n) = 2(1-1) \cdot n$$

`psv main {`

`int[] arr = {80, 30, 50, 20, 60, 10, 70, 40};`

`print(arr);`

`mergesort(arr);`

`print(arr);`

`}`

`psv merge(int[] a, int[] b, int[] c){`

`int i = 0, j = 0, k = 0;`

`while (i < a.length & j < b.length) {`

`if (a[i] <= b[j])`

`c[k++] = a[i++];`

`else`

`c[k++] = b[j++];`

`}`

`while (j < b.length)`

`c[k++] = b[j++];`

`while (i < a.length)`

`c[k++] = a[i++];`

`}`

Time And Space Complexity :-

n
 $n/2$
 $n/4$
 $n/8$
 $n/16$
 \vdots
 1

$n \quad \frac{n}{2} \quad \frac{n}{4} \quad \frac{n}{8} \quad \dots \quad 1$

or

$1 \quad 2 \quad 4 \quad \dots \quad \frac{n}{2} \quad n$

$(\log_2 n + 1)$ terms.

levels

Best Case :- $O(n \log n)$

Avg Case :- $O(n \log n)$

Worst Case :- $O(n \log n)$

$$T.N.O = 2(1-1).n = 2 \cdot (\log_2 n + 1 - 1) n$$

$$= 2 n \log n$$

$$T.C = O(n \log n)$$

In this basic code, the amount of space used = $(1-1).n$
 $= n \log_2 n$

$$S.C = O(n \log n)$$

we can improve it by deletion of a & b after merging into arr.

Best case :- $O(n)$

Worst Case :- $O(n)$

Avg case :- $O(n)$

$$\text{Extra Space Used} \therefore n + \frac{n}{2} + \frac{n}{4} + \dots = n \left(1 + \frac{1}{2} + \frac{1}{4} + \dots \right) < 2n$$

$$S.C = O(n)$$

Maths :-

$$1 + \frac{1}{2} + \frac{1}{4} + \dots < 2$$

$$n(1 + \frac{1}{2} + \frac{1}{4} + \dots) < 2 * n$$

Stability of Merge Sort :-

Yes. merge sort is stable.

arr = 5 1 2 5* 3

↓
Sorting.

1 2 3 5 5*

Make sure to code like this.

if(a[i] <= b[j])

c[k++] = a[i++];

if I use '<' instead of '<=' then it won't be stable.

→ Applications of Merge Sort

- ① $O(n \log n)$ worst case time complexity.
- ② Custom Sorting.
- ③ Sorting Linked Lists
- ④ Inversion Count & Related Problems

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[LeetCode 493]

Inversions Count Problem

→ You have to count the no. of inversions.

arr = { 8, 2, 5, 3, 1, 4 }

→ (8, 2), (8, 5), (8, 3), (8, 1), (8, 4)

→ (2, 1), (3, 1)

→ (5, 3), (5, 1), (5, 4)

[ans = 10]

$i < j$

arr[i] > arr[j]

Note :- Just need to modify the merge sort code.

Basic Approach [Brute Force]

→ Code :-

```
psv inversion(int[] a, int[] b){
```

```
    int i = 0, j = 0;
```

```
    while (i < a.length && j < b.length){
```

```
        if (a[i] > b[j]){
```

```
            count += (a.length - i);
```

```
            j++;
```

```
        }
```

```
        else i++;
```

```
    }
```

```
}
```

T.C $\Rightarrow O(n^2)$

S.C $\Rightarrow O(1)$

Not the Best

arr = { 8, 2, 5, 3, 1, 4 }

a = 2, 5, 8

b = 1, 3, 4

count = 3 + 2 + 2 = 7

arr[i] > arr[j]

ele from a

ele from b

* Merge sort

→ Create two array a & b of $n/2$ size each & copy paste.

→ Sort(a).

→ Sort(b).

(add the inversion function & call it)

→ Count inversions in a & b. [Extra]

→ Merge(a, b, arr). [also create count global variable]

M-2 → [just modify the merge function in mergesort code]

→ Code :- while (i < a.length && j < b.length){

```
    if (a[i] <= b[j])
```

```
        c[k++] = a[i++];
```

```
    else { // a[i] > b[j]
```

```
        count += (a.length - i); // Extra
```

```
        c[k++] = b[j++];
```

```
    }
```

[Only thing need to modify]

T.C \Rightarrow

$O(n \log n)$

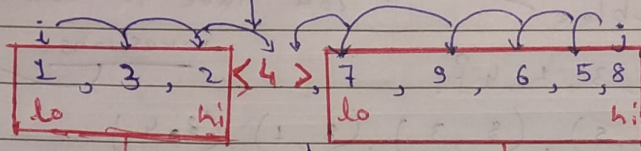
No need to create extra function

Quick Sort

arr = { 4, 9, 7, 1, 2, 3, 6, 5, 8 }

1, 9, 7, 4, 2, 3, 6, 5, 8

[Put pivot to its correct place]



[Arrange all the elements like this using pivot]

1, 3, 2

1, 3, 2

3, 2

2, 3

magic (Recursion)

7, 9, 6, 5, 8

6, 5, 7, 9, 8

⇒ Code :-

```
public static int partition(int[] arr, int lo, int hi) {
    int pivot = arr[lo], pivotIdx = lo;
    int smallerCount = 0;
    for (int i = lo + 1; i <= hi; i++) {
        if (arr[i] <= pivot) smallerCount++;
    }
    int connectIdx = pivotIdx + smallerCount;
    swap(arr, pivotIdx, connectIdx);
    // Partition
    int i = lo, j = hi;
    while (i < connectIdx & j > connectIdx) {
        if (arr[i] <= pivot) i++;
        else if (arr[j] > pivot) j--;
        else if (arr[i] > pivot & arr[j] <= pivot) {
            swap(arr, i, j);
        }
    }
    return connectIdx;
}
```

```
psvm {
    int[] arr = { 3, 2, 1, 5, 6, 4 };
    print(arr);
    int n = arr.length;
    quickSort(arr, 0, n-1);
    print(arr);
}
```

```
psv quickSort(int[] arr, int lo, int hi) {
    if (lo >= hi) return;
    int idx = partition(arr, lo, hi);
    quickSort(arr, lo, idx-1);
    quickSort(arr, idx+1, hi);
}
```


⇒ Time Space Complexity :-

→ Avg. Case = $O(n \log n)$

→ Space Complexity = Recursion call stack space → $O(\log n)$

⇒ Worst Case T.C. → arr = {1, 2, 3, 4, 5, 6, 7, 8, 9}

→ 1, 2, 3, 4, 5, 6, 7, 8, 9

the no. of levels here are not " $\log n$ " they are " n ".

No. of ops $\sim n + n-1 + n-2 + \dots + 1$

$= n \left(\frac{n+1}{2} \right) \sim O(n^2)$ T.C.

$O(n)$ S.C.

⇒ Randomized Pivot Point :-

→ Instead of choosing arr[lo] as pivot, we can choose arr[$\frac{lo+hi}{2}$] as pivot.

arr = {4, 9, 7, 1, 2, 3, 6, 5, 8}

Worst Case

↓

T.C. ⇒ $O(n \log n)$

S.C. ⇒ $O(\log n)$

1, 2, 7, 4, 9, 3, 6, 5, 8

3, 4, 9, 7, 6, 5, 8

4, 5, 6, 7, 9, 8

⇒ modified Code :-

```
ps int partition(int[] arr, int lo, int hi) {
    int mid = (lo + hi) / 2; // modified
    int pivot = arr[mid], pivotIdx = mid; // modified
    int smallerCount = 0;
    for (int i = lo; i <= hi; i++) {
        if (i == mid) continue; // modified
        if (arr[i] <= pivot) {
            smallerCount++;
        }
    }
```

int connectIdx = lo + smallerCount; // modified

swap(arr, pivotIdx, connectIdx);

// Partition

⇒ Merge Sort v/s Quick Sort

↓
→ Worst case it is Better

→ Stable

↓
→ More space Optimized

⇒ Just Need to modify partition function little bit

Quick Sort
→ Unstable

9. k^{th} Largest / Smallest Element in an Array

Smallest

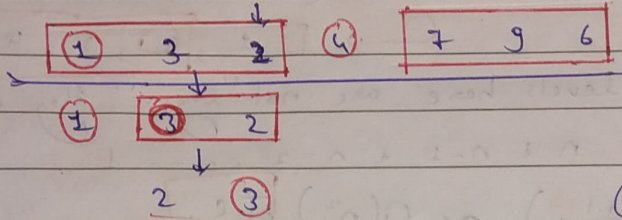
Using QuickSort.

arr = 4 9 7 1 2 3 6

1 9 7 4 2 3 6

T.C = $O(n)$ Best & Avg. Case.

k = 2



$O(n^2)$ Worst Case
 $O(n \log n)$ Worst Case.

Largest
Just need to modify the main method little bit.

⇒ Code :- static int ans;

```
ps int partition(int[] arr, int lo, int hi) {
    int pivot = arr[lo], pivotIdx = lo;
    int smallerCount = 0;
    for (int i = lo + 1; i <= hi; i++) {
        if (arr[i] <= pivot) smallerCount++;
    }
}
```

int correctIdx = pivotIdx + smallerCount;

swap(arr, pivotIdx, correctIdx);

// Partition.

int i = lo, j = hi;

while (i < correctIdx && j > correctIdx) {

if (arr[i] <= pivot) i++;

else if (arr[j] > pivot) j--;

else if (arr[i] > pivot && arr[j] <= pivot) {

swap(arr, i, j);

}

return correctIdx;

psv {

int[] arr = {4, 9, 1, 2, 6, 5, 8};

print(arr);

int k = 4;

ans = -1;

int n = arr.length;

quickselect(arr, 0, n-1, k);

print(arr);

cout << ans;

psv {

int[] arr = {4, 9, 1, 2, 6, 5, 8};

print(arr);

int k = 2;

ans = -1;

int n = arr.length;

quickselect(arr, 0, n-1, n-k+1);

print(arr);

cout << ans;

}

Change

psv quickselect(int[] arr, int lo, int hi, int k) {

if (lo > hi) return;

if (lo == hi) { // Not Really Required

if (lo == k-1) ans = arr[lo]; // NRR

return; // NRR

int idx = partition(arr, lo, hi);

if (idx == k-1) {

ans = arr[idx];

return;

if (k-1 < idx) {

quickselect(arr, lo, idx-1, k);

else

quickselect(arr, idx+1, hi, k);