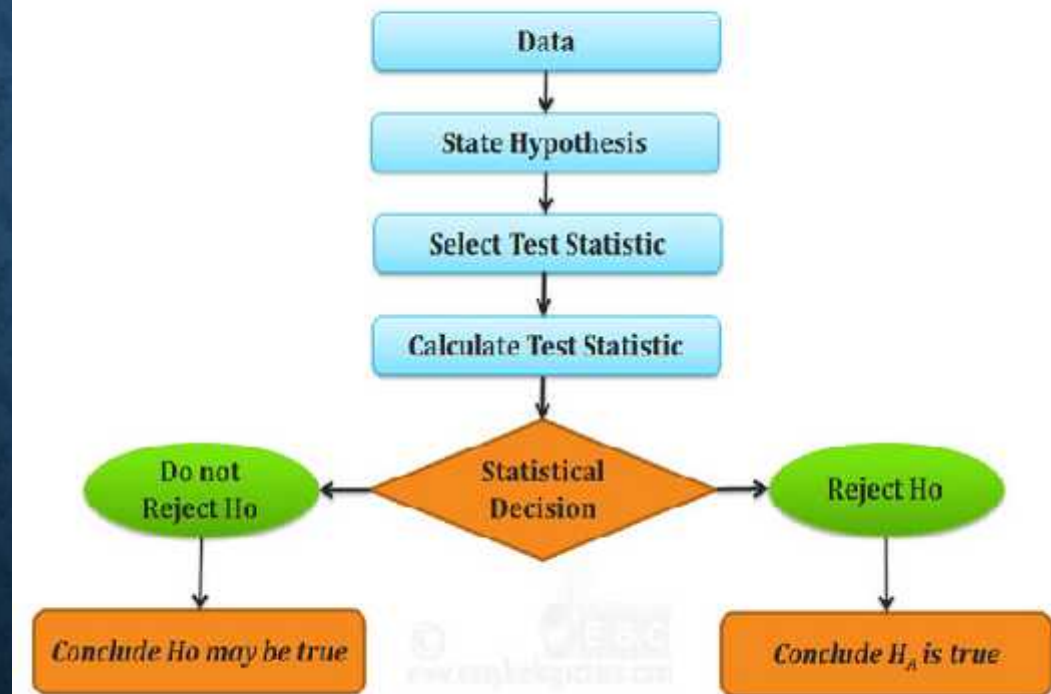# HYPOTHESIS TESTING

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HYPOTHESIS TESTING

- Hypothesis testing is a statistical method used to draw conclusions about population parameters based on sample data.

- It involves formulating two competing hypotheses: the null hypothesis (H_0), which represents no effect or no difference, and the alternative hypothesis (H_1), which states an effect or difference exists.

STEPS IN HYPOTHESIS TESTING

Data

State Hypothesis

Select Test Statistic

Calculate Test Statistic

Statistical Decision

Do not Reject Ho

Reject Ho

Conclude Ho may be true

Conclude H_A is true

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TYPES OF HYPOTHESES

Null Hypothesis (H_0):

- The null hypothesis represents the assumption of no effect, no difference, or no relationship in the population.

- The null hypothesis is often denoted as H_0 and is initially assumed to be true until evidence suggests otherwise.

- The goal of hypothesis testing is to assess the evidence against the null hypothesis and decide whether to reject it.

Alternative Hypothesis (H_1):

- The alternative hypothesis represents the opposite or alternative claim to the null hypothesis.

- It is denoted as H_1 and is supported when there is sufficient evidence to reject the null hypothesis.

- The alternative hypothesis can take different forms, depending on the research question and the specific claim being tested.
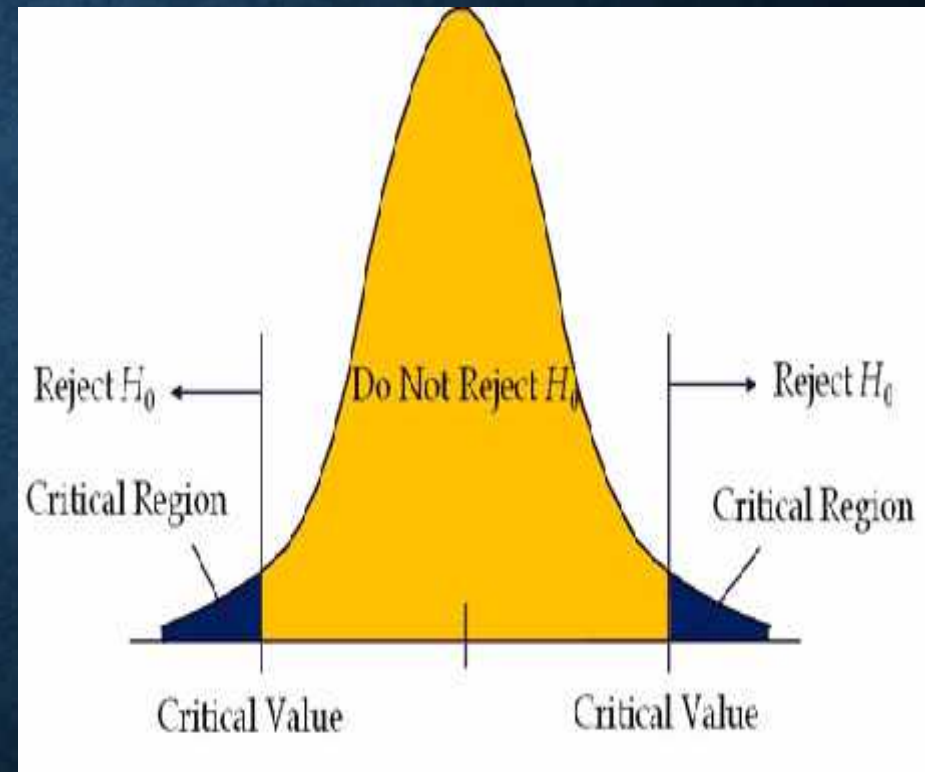
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TEST STATISTICS

- Test statistics quantify the discrepancy between the observed data and what would be expected under the null hypothesis.
- The choice of test statistic depends on the nature of the problem and the type of data being analyzed (e.g., t-statistic, z-score, chi-square statistic).
- The test statistic summarizes the information from the sample and provides a basis for comparing it to a reference distribution.

| Hypothesis testing	Test statistic	Rejection rule
Z-tests	Z-value	If Test statistic $\geq Z_{crit}$, then reject the null hypothesis. If Test statistic $\leq -Z_{crit}$, then reject the null hypothesis.
t-tests	t-value	If Test statistic $\geq t_{crit}$, then reject the null hypothesis H_0. If Test statistic $\leq -t_{crit}$, then reject the null hypothesis H_0.
ANOVA	F-value	If Test statistic $\geq F_{crit}$, then reject the null hypothesis H_0.
Chi-square	chi-square	If Test statistic $\geq \chi_{crit}$, then reject the null hypothesis H_0.

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CRITICAL REGIONS

- Critical regions, also known as rejection regions, are specific regions of the test statistic's distribution where the null hypothesis is rejected.

- These regions are determined based on the chosen significance level ( ) of the hypothesis test.

- The significance level represents the maximum probability of making a Type I error (incorrectly rejecting the null hypothesis when it is true).

- The critical region is typically located in the tail(s) of the test statistic's distribution.


Reject H_0 Do Not Reject H_0 Reject H_0

Critical Region   Critical Region

Critical Value   Critical Value

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SIGNIFICANCE LEVEL AND P-VALUE

- The significance level, denoted as (alpha), is a predetermined threshold set by the researcher before conducting the hypothesis test.

- It represents the maximum acceptable probability of making a Type I error, which is rejecting the null hypothesis when it is actually true.

- Commonly used significance levels include 0.05 (5%) and 0.01 (1%), but researchers can choose other levels based on the specific context and desired level of confidence.

- The significance level determines the critical region(s) of the test statistic's distribution, indicating where the null hypothesis will be rejected.

Significance level (α)

Fail to reject (not significant)   Reject (significant)

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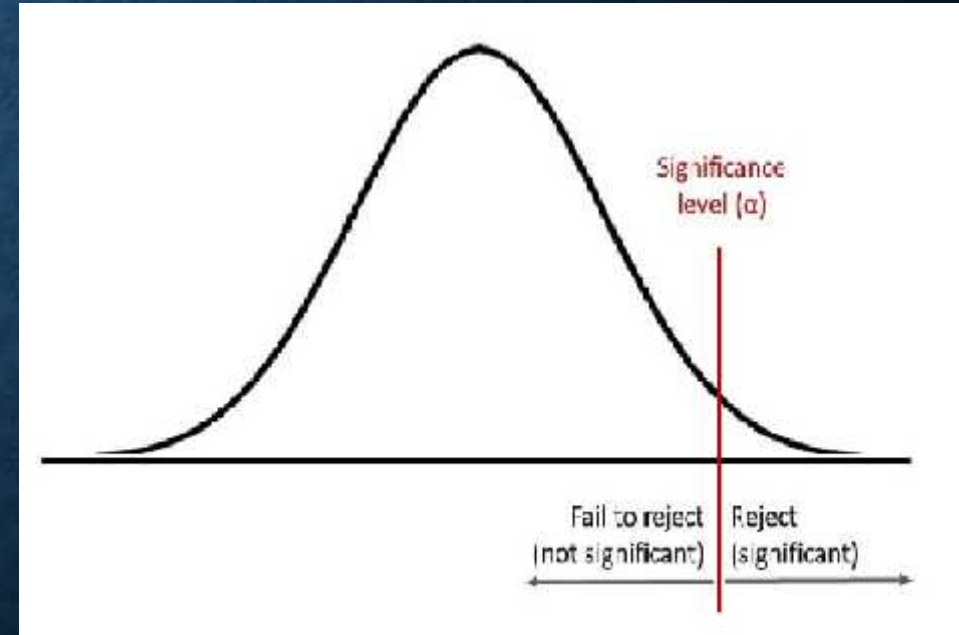
P-VALUE

- The p-value is a measure of the strength of evidence against the null hypothesis based on the observed data.

- If the p-value is less than the significance level (p-value <), the evidence is considered statistically significant.

- In this case, the null hypothesis is rejected in favor of the alternative hypothesis, suggesting that the observed data provides strong evidence against the null hypothesis.

- If the p-value is greater than or equal to the significance level (p-value), there is insufficient evidence to reject the null hypothesis, and it is not concluded that the alternative hypothesis is supported.

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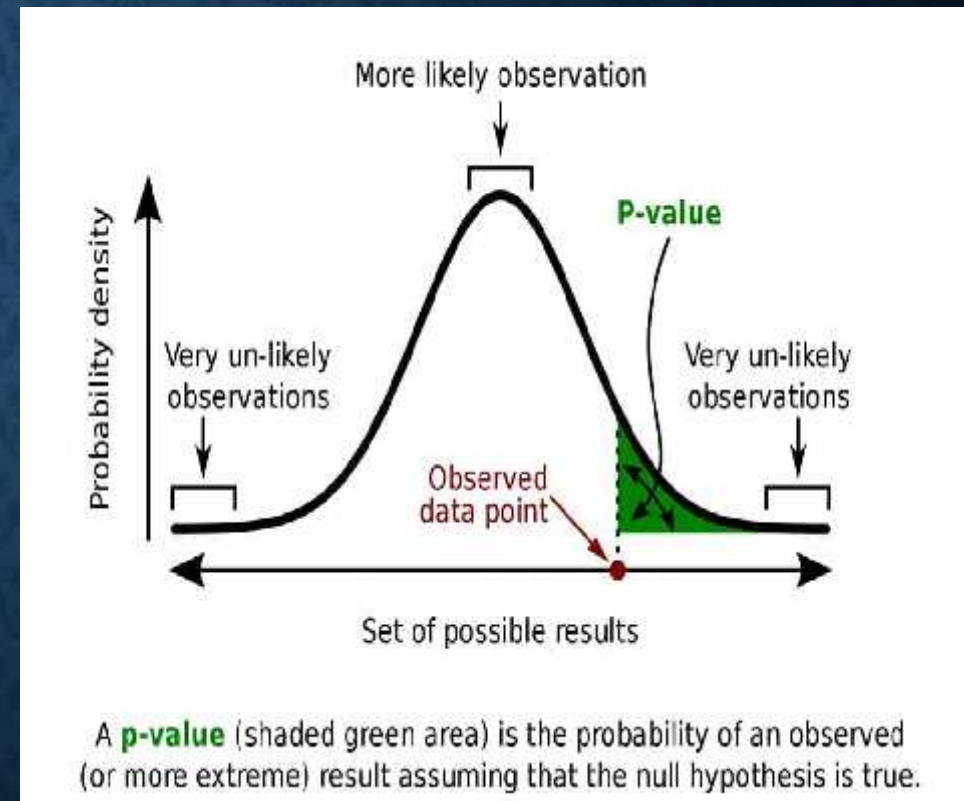
ONE-SAMPLE HYPOTHESIS TESTS

- One-sample hypothesis tests are statistical tests used to make inferences about a population parameter based on a single sample.

- One-sample hypothesis tests involve comparing a sample mean, proportion, or variance to a known population value or a hypothesized value.

- Examples include testing whether a new drug is effective compared to a placebo or assessing if a coin is fair.

Hypotheses

The null hypothesis for a 1-sample t-test is: $H_0: \mu = \mu_0$ where:

- μ = the population mean

- μ_0 = the hypothesized mean

You can choose any one of three alternative hypotheses:

$H_1: \mu > \mu_0$	One-tailed test
$H_1: \mu < \mu_0$	One-tailed test
$H_1: \mu \neq \mu_0$	Two-tailed test

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TWO-SAMPLE HYPOTHESIS TESTS

- Two-sample hypothesis tests compare two independent samples to examine differences in means, proportions, or variances between populations or groups.

- Examples include comparing the effectiveness of two different treatments or investigating gender-based salary differences.

Hypothesis Testing – Two Sample t-Test

- Comparing mean values from two small samples – two sample t-test

- H_1 states **'less than'** instead of simply **'unequal to'** – single-tailed t-test

- No reason variances should be the same – t-test assuming unequal variances

$$\nu = \frac{\left(\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}\right)^2}{\frac{(s_x^2/n_x)^2}{n_x - 1} + \frac{(s_y^2/n_y)^2}{n_y - 1}}$$

$$t = \frac{(\bar{y} - \bar{x}) - 0}{\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}}$$

x: **Forster**
y: **Lee**
s: Sample Standard Deviation
n: Number of replicates

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TYPE I AND TYPE II ERRORS

- Type I error occurs when the null hypothesis is rejected, but it is actually true (false positive).

- Type II error happens when the null hypothesis is not rejected, but it is actually false (false negative).

- Controlling the significance level ( ) helps minimize the risk of Type I errors, but it may increase the likelihood of Type II errors.

Type I and Type II Error		
Null hypothesis is ...	True	False
Rejected	Type I error False positive Probability = α	Correct decision True positive Probability = 1 - β
Not rejected	Correct decision True negative Probability = 1 - α	Type II error False negative Probability = β

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ASSUMPTIONS AND LIMITATIONS

- **Assumptions:** Hypothesis testing assumes that the data follows a specific distribution (e.g., normal distribution) and that the observations are independent and randomly sampled. Violations of these assumptions can affect the validity of the test results.

- **Limitations:** Hypothesis testing provides insights into statistical significance but does not guarantee practical or real-world significance. Additionally, hypothesis tests are influenced by sample size, effect size, and the chosen significance level, which may impact the interpretation of the results.

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Hypothesis testing provides a structured framework for evaluating claims, making decisions, and drawing conclusions based on sample data. By setting up hypotheses, calculating test statistics, and comparing results to critical values or p-values, researchers and analysts can assess the validity of statements, verify findings, and contribute to evidence-based decision making. It is important to understand assumptions, limitations, and the significance level to correctly interpret the results of hypothesis tests.

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THANK YOU

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