eg. In a die-throwing experiment, the discrete (finite) sample space is—

$$S = \{1, 2, 3, 4, 5, 6\}$$

eg. If the observe time (t) to failure of CRT (which is put on test), then the continuous sample space would be—

$$S = \{t : t \geq 0\}$$

(5) Impossible Event → The event which contains no elementary event at all, is known as an impossible event and it is denoted by ϕ. (null event)

eg. In a die-throwing experiment, the event— "obtaining an irrational no."

is an impossible event.

(6) Mutually Exclusive Events → Two or more events are said to be mutually exclusive (or disjoint) when no two of them can occure simultaneously. They can not have any elementary event in common.

ie Occurrence of one ⇒ non-occurrence of others.

eg. For a die-throwing experiment— events like—

(1) an odd number
(2) an even number

} ⇒ mutually exclusive

(7) Exhaustive Events → Two (or more) events are said to be exhaustive if they constitute the entire sample space.

ie The set of all possible outcomes of an experiment, gives an exhaustive set of events. Thus at least one of such events must occure.

eg. $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$, $C = \{2, 3, 5\}$

Here A & B together constitute an exhaustive set of events.

Prob: For a die throwing experiment, define the following events —:

A : event of getting an odd number.
B : event of getting an even number.
C : event of getting a prime number.
D : event of getting a no. more than 3.
E : event of getting 3 or its multiple.

i) Write down the sample space and events A, B, C, D and E.

Sample Space $(S) = \{1, 2, 3, 4, 5, 6\}$

Events are —.

$A = \{1, 3, 5\}$ $C = \{2, 3, 5\}$
$B = \{2, 4, 6\}$ $D = \{4, 5, 6\}$
$E = \{3, 6\}$.

ii) Obtain the following operations —.

Union: $A \cup B$, $D \cup E$, $A \cup C$

(*) $A = \{1, 3, 5\}$ $B = \{2, 4, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\} = S$.

where, A and B are exhaustive (an)

(*) $D = \{4, 5, 6\}$ $E = \{3, 6\}$

$D \cup E = \{3, 4, 5, 6\}$ (ans)

(*) $A = \{1, 3, 5\}$ $C = \{2, 3, 5\}$

$A \cup C = \{1, 2, 3, 5\}$ (ans)

Difference: A−B, D−E, C−D, A−C, C−A.

(*) $A = \{1, 3, 5\}$ $B = \{2, 4, 6\}$

$A - B = \{1, 3, 5\} = A$. (ans)

(*) $D = \{4, 5, 6\}$ $E = \{3, 6\}$

$D - E = \{4, 5\}$ (ans)

(*) $C = \{2, 3, 5\}$ $D = \{4, 5, 6\}$

$C - D = \{2, 3\}$ (ans)

(*) $A = \{1, 3, 5\}$ $C = \{2, 3, 5\}$

$$\left.\begin{aligned} A - C &= \{1\} \\ C - A &= \{2\} \end{aligned}\right\} \text{(ans)}$$

iii) Verify the following results —

(*) $C - E = C \cap E^c$

Given $C = \{2, 3, 5\}$ and $E = \{3, 6\}$

$\therefore E^c = \{1, 2, 4, 5\}$

L.H.S $C - E = \{2, 5\}$

R.H.S $C \cap E^c = \{2, 5\}$

$\therefore C - E = C \cap E^c$ (Proved)

Prob For tossing a single coin —

Event (E), Sample Space (S)

H = Obtaining Head.

T = Obtaining Tail

Therefore $S = \{H, T\}$

∴ The event points = 2. (ans)

If we toss a coin n times repeatedly, then there will be 2^n nos. of event points available.

For tossing two coins at a time —

Event (E), Sample Space (S)

H = Obtaining Head.

T = Obtaining Tail.

Therefore $S = \{HH, HT, TH, TT\}$.

∴ The event points $= 2^2 = 4$ (ans)

For tossing three coins at a time —

Event (E), Sample Space (S)

H = Obtaining Head.

T = Obtaining Tail.

Therefore, The event points $= 2^3 = 8$

H = 0
T = 1

∴ $S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$ (ans)

Let, partition the entire sample space (S) as —

A : Set of elements of ~~first~~ obtaining first head.

B : Set of elements of obtaining two heads.

∴ $A = \{HHH, HHT, HTH, HTT\}$

$B = \{HHH, HHT, HTH, THH\}$

∴ $A \cup B = \{HHH, HHT, HTH, HTT, THH\}$

$A \cap B = \{HHH, HHT, HTH\}$

Let, X: Set of elements of obtaining no heads.

$X = \{TTT\}$.

Therefore, if we want to find the set of elements where obtaining atleast one head.

$\therefore X^C = S - X = \{HHH, HHT, HTH, HTT, THH, THT, TTH\}$. (ans)

(8) Classical or Mathematical definition of Probability →

Suppose the sample space S of a random experiment E contains the finite no. of event points, say $n(S)$, all of which are known to be equally likely or mutually symmetrical. If $m(A)$ number of event-points of these $n(S)$ event points, in contained in the event A, connected with E, then the ratio $m(A)/n(S)$, is called of occurrence of the event A and is denoted by $P(A)$.

$$P(A) = \frac{m(A)}{n(S)}$$

$$= \frac{\text{no. of equally likely eventpoints contained in the event A.}}{\text{finite no. of equally likely event points contained in the sample space S of E.}}$$

Note:

(1) For the impossible event A connected with E, we have $\boxed{m(A) = 0}$

Hence, the probability of occurrence of an impossible event A is —

$$P(A) = \frac{0}{n(S)} = 0.$$

(2) For the certain event A connected with E, we have –

$$\boxed{m(A) = n(S)}$$

Hence, the probability of occurrence of a certain event A is —.

$$P(A) = \frac{n(S)}{n(S)} = 1.$$

(3) For any event A connected with E, we have —.

$$\boxed{0 \leqslant m(A) \leqslant n(S)}$$

or, $$\frac{0}{n(S)} \leqslant \frac{m(A)}{n(S)} \leqslant \frac{n(S)}{n(S)}.$$

or, $$\boxed{0 \leqslant P(A) \leqslant 1.}$$

Therefore, it is clearly evident that the probability of occurrence of an event is a Positive Proper Fraction.

(4) If an event A, connected with event E. that contains $m(A)$ number of equally likely event points then the event A^c (Complementary to A), contains. $\boxed{n(S) - m(A)}$ number of equally likely event points.

Therefore, by the classical definition —.

$$P(A^c) = \frac{n(S) - m(A)}{n(S)}$$

$$= 1 - \frac{m(A)}{n(S)}$$

i.e. $$\boxed{P(A^c) = 1 - P(A).}$$

i.e. $$\boxed{P(A) + P(A^c) = 1}$$

equally likely
[illegible]
[illegible]

⑤ If the probability of occurrence of an event A be (m/n).

Then $m:(n-m)$ = The odds in favour of the event A. (ঘটনার অনুকূলে সুযোগ)

$(n-m):m$ = The odds ~~in~~ against the event A. (ঘটনার প্রতিকূলে সুযোগ)

So, by the definition —

The odds in favour of the event A (A ঘটনার অনুকূলে সুযোগ) = no. of equally likely even points contained in A (A ঘটনার অন্তর্গত সমান সম্ভাব্য নমুনা বিন্দুর সংখ্যা) : no. of equally likely even points contained in A^c. (A^c ঘটনার অন্তর্গত সমান সম্ভাব্য নমুনা বিন্দুর সংখ্যা).

Similarly,

The odds against the event A (A ঘটনার প্রতিকূলে সুযোগ) = no. of equally likely even points contained in A^c (A^c ঘটনার অন্তর্গত সমান সম্ভাব্য নমুনা বিন্দুর সংখ্যা) : no. of equally likely even points contained in A. (A ঘটনার অন্তর্গত সমান সম্ভাব্য নমুনা বিন্দুর সংখ্যা)

④ If the odds in favour of an event A are $a:b$, then the probability of occurrence of A.

বা $\boxed{P(A) = \frac{a}{a+b}}$

④ If the odds against the event B are $a:b$, then the probability of occurrence of B.

বা $\boxed{P(B) = \frac{b}{a+b}}$

(9) Notations :—

Let S be the sample space of random experiment E and A and B are two events connected with E.

i) $P(A)$ = denotes the probability of occurrence of A.

ii) $P(A^c)$ = denotes the probability of non-occurrence of A.

iii) $P(A \cup B)$ = denotes the probability of occurrence of either A or B or both.

iv) $P(A \cap B)$ = denotes the probability of occurrence of simultaneous events A and B.

(10) Theorem of Total Probability :—

(Additive law of Probability).

The probability of occurrence of any one of n pairwise mutually exclusive events such as $A_1, A_2, \ldots, A_n$ is equal to the sum of probabilities of events.

$$P(A_1 \cup A_2 \cup \ldots \cup A_n) = P(A_1) + P(A_2) + \cdots + P(A_n).$$

[পরস্পর পৃথক n-সংখ্যক ঘটনা $A_1, A_2, \ldots, A_n$ -এর যেকোনো একটি ঘটার সম্ভাবনার সমান, প্রত্যেকটি ঘটনা পৃথকভাবে ঘটার সম্ভাবনাগুলির সমষ্টির সমান।]

NOTE : (1) If the events $A_1, A_2, \ldots, A_n$ are exhaustive then $(A_1 \cup A_2 \cup A_3 \cdots A_n)$ represents the sure event S.

i.e. $A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n = S$

$\therefore P(A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n) = P(S) = 1.$

or, $\boxed{P(A_1) + P(A_2) + \cdots + P(A_n) = 1}$

($\because$ the events are pairwise mutually exclusive).

(2) Let A be an event connected with E and A^c is the event complementary to A.

Then, $A \cup A^c = S$ where $S =$ sure event.

$\therefore P(A \cup A^c) = P(S) = 1. \quad (\because P(S) = 1)$

or, $\boxed{P(A) + P(A^c) = 1.}$

($\because$ A and A^c are mutually exclusive).

(ii) Axioms of Mathematical Probability :—.

Let, S be the sample space of a random experiment E and A be an any event connected with E.

if $\boxed{A \subseteq S}$

A real no. $P(A)$ associated with A, is called the probability of the event A if the following axioms are ~~sata~~ satisfied —.

Axiom-I : for any event A, $P(A) \geq 0$

Axiom-II : for the certain event S, $P(S) = 1$.

Axiom-III : for a finite or countably infinite number of pairwise mutually exclusive events $A_1, A_2, \cdots$ of S.

$P(A_1 \cup A_2 \cup \cdots \cancel{\text{whtn}}) = P(A_1) + P(A_2) + \cdots$