# 3. Fractions

## Learning Outcomes

**After studying this chapter, the students will be able to:**

- classify the fractions.
- compare the fractions.
- find the reciprocal of a fraction.
- add, subtract, multiply and divide the fractions.
- simplify the expressions involving fractions.

## FRACTION

The term **fraction** is derived from the latin word *fractus* meaning broken.

A fraction represents a part of a whole or a group. For example, $\frac{3}{5}$, $\frac{4}{7}$, $\frac{15}{8}$, etc.

In general, a fraction is a number of the form $\frac{a}{b}$, where $a$ and $b$ are whole numbers and $b \neq 0$.

Here, $a$ is called the **numerator** and $b$ is called the **denominator**.

## TYPES OF FRACTIONS

**Proper fraction:** A fraction whose numerator is less than the denominator is called a **proper fraction**. For example, $\frac{1}{2}$, $\frac{3}{5}$, $\frac{5}{12}$, etc.

**Improper fraction:** A fraction whose numerator is greater than or equal to the denominator is called an **improper fraction**.

For example, $\frac{7}{5}$, $\frac{12}{7}$, $\frac{28}{5}$, etc.

**Mixed fraction:** A mixed fraction consists of a natural number and a proper fraction. For example, $1\frac{2}{5} = 1 + \frac{2}{5}$, $1\frac{5}{7} = 1 + \frac{5}{7}$, $5\frac{3}{5} = 5 + \frac{3}{5}$, etc.

**Note**

Every mixed fraction can be written as an improper fraction and every improper fraction can be written as a mixed fraction.

Let us convert a mixed fraction into an improper fraction and an improper fraction into a mixed fraction.

a. $2\frac{1}{5} = 2 + \frac{1}{5} = \frac{2 \times 5 + 1}{5} = \frac{11}{5}$

b. $\frac{22}{5} = \frac{4 \times 5 + 2}{5} = 4 + \frac{2}{5} = 4\frac{2}{5}$

**Simple fraction:** A fraction whose numerator and denominator are both whole numbe called a **simple fraction** or **vulgar fraction**. For example, $\frac{5}{12}, \frac{13}{5}, \frac{7}{35}$, etc.

**Complex fraction: Complex fractions** are those fractions in which either numerator denominator or both are fractions. For example, $\frac{\frac{7}{5}}{2}, \frac{\frac{3}{2}}{\frac{1}{2}}, \frac{\frac{17}{13}}{\frac{10}{23}}$, etc.

**Decimal fraction:** A fraction whose denominator is a multiple of 10, i.e. of the form $10^n$ $n \in \mathbf{N}$ is called a **decimal fraction**. For example, $\frac{7}{10}, \frac{15}{100}, \frac{257}{1000}$, etc.

**Like fractions:** Fractions having the same denominator but different numerators are called **like fractions**. For example, $\frac{1}{9}, \frac{2}{9}, \frac{7}{9}$ are like fractions.

**Unlike fractions:** Fractions having different denominators are called **unlike fractions** For example, $\frac{3}{7}, \frac{5}{12}, \frac{7}{25}$ are unlike fractions.

**Equivalent fractions:** When numerator and denominator of a fraction are multiplied or divided by the same non-zero number, we get **equivalent fractions**. For example, $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}$ are equivalent fractions.

## SIMPLEST FORM OF A FRACTION

If a fraction is of the form $\frac{a}{b}$ in which $a$ and $b$ have no common factor other than 1 and $b \neq$ then the fraction is said to be in its simplest or lowest form. For example, $\frac{3}{5}, \frac{7}{11}, \frac{19}{35}$ are in simplest form as 3 and 5; 7 and 11; or 19 and 35 have no common factor other than 1.

**Example 1:** What fraction of an hour is 21 minutes?

**Solution:** There are 60 minutes in an hour.

$\therefore$ 21 minutes of an hour = $\frac{21}{60}$ or $\frac{7}{20}$

**Example 2:** Find the fraction equivalent to $\frac{17}{24}$ whose numerator is 51.

**Solution:** To get the numerator as 51, we multiply 17 by 3. So, denominator is also multiplied by 3.

$\Rightarrow \frac{17 \times 3}{24 \times 3} = \frac{51}{72}$

**Example 3:** Reduce $\frac{256}{400}$ to its simplest form.

**Solution:** The HCF of 256 and 400 is 16.

$$\therefore \frac{256}{400} = \frac{256 \div 16}{400 \div 16} = \frac{16}{25}$$

**Alternative method:**

Let us express both the numerator and denominator of a fraction as a product of their primes and then simplify.

$$\therefore \frac{256}{400} = \frac{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times 2 \times 2 \times 2 \times 2}{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times 5 \times 5} = \frac{16}{25}$$

## CONVERTING UNLIKE FRACTIONS INTO LIKE FRACTIONS

In order to convert unlike fractions into like fractions, the following steps must be performed:

1. Find the LCM of the denominators.
2. Convert each fraction into an equivalent fraction having denominator equal to the LCM obtained in step 1.

**Example:** Convert the fractions $\frac{5}{12}, \frac{7}{14}, \frac{3}{9}$ into like fractions.

**Solution:** LCM of 12, 14 and 9 is 252.

Equivalent fractions with denominator 252 are:

$$\frac{5}{12} = \frac{5 \times 21}{12 \times 21} = \frac{105}{252}; \frac{7}{14} = \frac{7 \times 18}{14 \times 18} = \frac{126}{252} \text{ and } \frac{3}{9} = \frac{3 \times 28}{9 \times 28} = \frac{84}{252}$$

Thus, the given fractions are converted into equivalent like fractions $\frac{105}{252}, \frac{126}{252}, \frac{84}{252}$ respectively.

## COMPARISON OF FRACTIONS

While comparing like fractions, we simply compare their numerators as the denominators are the same. For example, consider $\frac{4}{15}$ and $\frac{22}{15}$.

Since $22 > 4$, $\therefore \frac{22}{15} > \frac{4}{15}$.

**Note**

When two fractions having the same denominator are compared, then the fraction with the greater numerator will be the greater fraction.

If the fractions are unlike, then we first convert unlike fractions into like fractions and then compare. For example, consider $\frac{3}{10}$ and $\frac{7}{15}$.

Now, let us convert the unlike fractions into like fractions.

Here, LCM of 10 and 15 is 30.

$\therefore \frac{3}{10} = \frac{3 \times 3}{10 \times 3} = \frac{9}{30}$ and $\frac{7}{15} = \frac{7 \times 2}{15 \times 2} = \frac{14}{30}$

Since $14 > 9$, $\therefore \frac{14}{30} > \frac{9}{30} \Rightarrow \frac{7}{15} > \frac{3}{10}$.

**Example:** Write the following fractions in ascending order: $\frac{7}{12}, \frac{18}{15}, \frac{20}{26}$.

**Solution:** LCM of 12, 15 and 26 is 780.

$\frac{7}{12} = \frac{7 \times 65}{12 \times 65} = \frac{455}{780}$; $\frac{18}{15} = \frac{18 \times 52}{15 \times 52} = \frac{936}{780}$ and $\frac{20}{26} = \frac{20 \times 30}{26 \times 30} = \frac{600}{780}$

Since $455 < 600 < 936$, $\therefore \frac{455}{780} < \frac{600}{780} < \frac{936}{780}$ or $\frac{7}{12} < \frac{20}{26} < \frac{18}{15}$.

Hence, the given fractions arranged in ascending order are $\frac{7}{12}, \frac{20}{26}, \frac{18}{15}$.

## INSERTING A FRACTION BETWEEN TWO FRACTIONS

If $\frac{a}{b}$ and $\frac{c}{d}$ are two fractions, then $\frac{a+c}{b+d}$ lies between these two fractions.

We observe that the numerator of the required fraction is the sum of the numerators of the given fraction and the denominator is the sum of the denominators of the given fractions.

Thus, if $\frac{a}{b} < \frac{c}{d}$, then $\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$

Similarly, we can again find a fraction lying between $\frac{a}{b}$ and $\frac{a+c}{b+d}$ by this process. Also, we can find a fraction lying between $\frac{a+c}{b+d}$ and $\frac{c}{d}$. Thus, there is no end to it. Hence, the number of fractions lying between any two given fractions is infinite.

**Example:** Insert three fractions between $\frac{4}{11}$ and $\frac{9}{16}$.

**Solution:** Fractions lying between $\frac{4}{11}$ and $\frac{9}{16}$ is $\frac{4+9}{11+16} = \frac{13}{27}$

Again, fractions lying between $\frac{4}{11}$ and $\frac{13}{27}$ is $\frac{4+13}{11+27} = \frac{17}{38}$

and fractions lying between $\frac{13}{27}$ and $\frac{9}{16}$ is $\frac{13+9}{27+16} = \frac{22}{43}$

Hence, the fractions lying between $\frac{4}{11}$ and $\frac{9}{16}$ are $\frac{13}{27}, \frac{17}{38}, \frac{22}{43}$.

**Try This**

Instead of performing successive additions, we can also insert a number of fractions by converting the given fractions into like fractions, i.e.

$\frac{4}{11} = \frac{4 \times 16}{11 \times 16} = \frac{64}{176}$ and $\frac{9}{16} = \frac{9 \times 11}{16 \times 11} = \frac{99}{176}$, then we can write many fractions lying between $\frac{64}{176}$ and $\frac{99}{176}$ such as $\frac{65}{176}, \frac{66}{176}, \frac{67}{176}$ and so on.

Now, insert 5 fractions lying between the following:

1. $\frac{1}{3}$ and $\frac{7}{6}$  2. $\frac{12}{25}$ and $\frac{17}{5}$  3. $\frac{5}{12}$ and $\frac{7}{9}$  4. $\frac{27}{11}$ and $\frac{35}{12}$

## Exercise 3.1

1. Identify the following fractions as mixed, vulgar, complex and decimal fractions.

a. $\frac{17}{100}$ b. $5\frac{3}{7}$ c. $\frac{112}{17}$ d. $\frac{\frac{5}{17}}{2}$ e. $12\frac{1}{7}$

f. $\frac{\frac{3}{1}}{7}$ g. $\frac{\frac{3}{12}}{5}$ h. $\frac{10}{27}$ i. $\frac{50}{1000}$ j. $\frac{110}{1000}$

2. Write the even numbers between 5 and 50. What fraction of them are multiples of 3?

3. What fraction of an hour is 30 minutes?

4. Convert the following fractions into mixed fractions.

a. $\frac{120}{26}$ b. $\frac{37}{7}$ c. $\frac{125}{24}$ d. $\frac{117}{19}$ e. $\frac{67}{30}$ f. $\frac{74}{16}$

5. Convert the following mixed fractions into improper fractions.

a. $5\frac{3}{7}$ b. $17\frac{2}{9}$ c. $10\frac{1}{17}$ d. $2\frac{3}{115}$ e. $3\frac{2}{5}$ f. $2\frac{4}{15}$

6. Find five equivalent fractions of $\frac{8}{9}$.

7. Find the equivalent fraction of $\frac{56}{72}$ whose

a. numerator is 14. b. denominator is 9.

8. Reduce the following fractions to their simplest form.

a. $\frac{72}{144}$ b. $\frac{166}{360}$ c. $\frac{175}{340}$ d. $\frac{135}{50}$ e. $\frac{76}{12}$ f. $\frac{46}{100}$

9. Convert the following unlike fractions into equivalent like fractions.

a. $\frac{3}{16}, \frac{7}{12}, \frac{9}{4}$ b. $\frac{8}{26}, \frac{7}{13}, \frac{12}{51}$ c. $\frac{9}{15}, \frac{17}{75}, \frac{10}{60}$

10. Arrange the given fractions in descending order.

a. $\frac{5}{7}, \frac{6}{12}, \frac{9}{24}, \frac{8}{13}$ b. $\frac{5}{12}, \frac{7}{13}, \frac{17}{24}, \frac{9}{14}$

11. Arrange the given fractions in ascending order.

a. $\frac{7}{6}, \frac{10}{12}, \frac{13}{21}, \frac{8}{3}$ b. $\frac{17}{30}, \frac{5}{17}, \frac{4}{5}, \frac{3}{5}$

12. Insert two fractions lying between:

a. $\frac{5}{12}$ and $3\frac{1}{4}$ b. $\frac{17}{12}$ and $\frac{5}{19}$ c. $\frac{9}{7}$ and $1\frac{2}{3}$

# OPERATIONS ON FRACTIONS

## Addition of Fractions

In order to add fractions, follow these steps:

1. Convert the mixed fractions (if any) into improper fractions.
2. Convert the fractions into like fractions and then add the numerators, keeping the same denominator.

**Example:** Add the following fractions.

a. $\frac{13}{15} + \frac{14}{15}$

b. $\frac{17}{13} + \frac{9}{52}$

c. $\frac{9}{21} + 7\frac{3}{11} + 3\frac{5}{7}$

**Solution:**

a. $\frac{13}{15} + \frac{14}{15} = \frac{13+14}{15} = \frac{27}{15} = \frac{9}{5}$

b. $\frac{17}{13} + \frac{9}{52} = \frac{68+9}{52} = \frac{77}{52}$ (∵ LCM of 13 and 52 is 52.)

c. $\frac{9}{21} + 7\frac{3}{11} + 3\frac{5}{7} = \frac{9}{21} + \frac{80}{11} + \frac{26}{7} = \frac{99+1680+858}{231}$ (∵ LCM of 21, 11 and 7 is 231)

$= \frac{2637}{231} = \frac{879}{77} = 11\frac{32}{77}$

## Subtraction of Fractions

In order to subtract fractions, follow these steps:

1. Convert the mixed fractions (if any) into improper fractions.
2. Convert the fractions into like fractions and then subtract the numerators, keeping the same denominator.

**Example 1:** Subtract the following fractions.

a. $\frac{113}{21} - \frac{107}{21}$

b. $\frac{31}{17} - \frac{13}{9}$

**Solution:**

a. $\frac{113}{21} - \frac{107}{21} = \frac{113-107}{21} = \frac{6}{21} = \frac{2}{7}$

b. $\frac{31}{17} - \frac{13}{9} = \frac{279-221}{153} = \frac{58}{153}$

**Example 2:** Simplify the following.

a. $7\frac{2}{3} + 3\frac{5}{7} - 2\frac{1}{4}$

b. $3\frac{5}{8} + \frac{6}{16} - \frac{5}{24} + 3\frac{1}{2}$

**Solution:**

a. $7\frac{2}{3} + 3\frac{5}{7} - 2\frac{1}{4} = \frac{23}{3} + \frac{26}{7} - \frac{9}{4}$

$= \frac{644+312-189}{84}$ (∵ LCM of 3, 7 and 4 is 84)

$= \frac{767}{84} = 9\frac{11}{84}$

**Alternative method:**

$7\frac{2}{3} + 3\frac{5}{7} - 2\frac{1}{4} = \frac{23}{3} + \frac{26}{7} - \frac{9}{4} = \frac{23 \times 28}{3 \times 28} + \frac{26 \times 12}{7 \times 12} - \frac{9 \times 21}{4 \times 21}$ (∵ LCM of 3, 7 and 4 is 84.)

$= \frac{644}{84} + \frac{312}{84} - \frac{189}{84} = \frac{644 + 312 - 189}{84} = \frac{767}{84} = 9\frac{11}{84}$

b. $3\frac{5}{8} + \frac{6}{16} - \frac{5}{24} + 3\frac{1}{2} = \frac{29}{8} + \frac{6}{16} - \frac{5}{24} + \frac{7}{2} = \frac{174 + 18 - 10 + 168}{48}$

(∵ LCM of 8, 16, 24 and 2 is 48)

$= \frac{350}{48} = \frac{175}{24} = 7\frac{7}{24}$

**Example 3:** What should be added to $5\frac{3}{5}$ to get $8\frac{3}{7}$?

**Solution:** The number that should be added to $5\frac{3}{5}$ to get $8\frac{3}{7}$ is obtained by simplifying

$8\frac{3}{7} - 5\frac{3}{5}$.

$\Rightarrow 8\frac{3}{7} - 5\frac{3}{5} = \frac{59}{7} - \frac{28}{5} = \frac{295 - 196}{35}$ (∵ LCM of 7 and 5 is 35)

$= \frac{99}{35} = 2\frac{29}{35}$

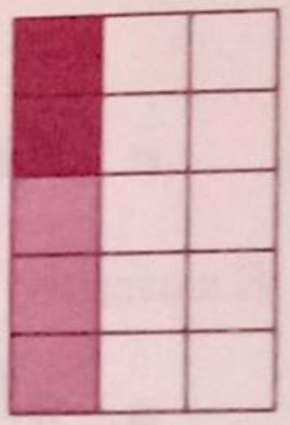
## Multiplication of Fractions

Let us illustrate $\frac{2}{5}$ of $\frac{1}{3}$ which means $\frac{2}{5} \times \frac{1}{3}$ pictorially.

Let the rectangle represent one unit. It is divided into 3 equal parts. Here, one part (shaded portion) represents $\frac{1}{3}$.

Now, this part is further divided into 5 equal parts and we take two parts out of it to represent $\frac{2}{5}$ (double shaded region). The whole rectangle is divided into 15 parts and 2 parts are taken out of it.

Thus, $\frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$



From above, we conclude that when fractions are multiplied, the numerator of the resultant is the product of numerators and the denominator of the resultant is the product of the denominators.

Thus, if $\frac{a}{b}$ and $\frac{c}{d}$ are two fractions, then $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

Hence, in order to multiply fractions, follow these steps:

1. Change the mixed fraction into an improper fraction.
2. The product = $\frac{\text{Numerator} \times \text{Numerator}}{\text{Denominator} \times \text{Denominator}}$.
3. Reduce the fraction to the lowest form.
4. Convert the fraction into a mixed fraction in case the product is an improper fraction.

b. $\dfrac{\left(\frac{2}{3} - \frac{3}{5}\right) \text{ of } \left(3\frac{1}{2} - 2\frac{1}{6}\right)}{\left(3 - \frac{5}{9}\right)} = \left(\frac{10-9}{15}\right) \text{ of } \left(\frac{7}{2} - \frac{13}{6}\right) \div \left(\frac{27-5}{9}\right)$

$= \frac{1}{15} \text{ of } \left(\frac{21-13}{6}\right) \div \frac{22}{9}$

$= \frac{1}{15} \times \frac{\cancel{4}^{2}}{\cancel{6}_{\cancel{2}}} \times \frac{\cancel{9}^{3}}{\cancel{22}_{11}} = \frac{1}{\cancel{15}_{5}} \times \frac{\cancel{2}^{2}}{11} = \frac{2}{55}$

**Example 2:** Simplify: $1\frac{4}{7} \div \left|\left\{2\frac{1}{3} - \overline{(5 + 2 - 3)}\right\} + 3\frac{1}{2}\right|$

**Solution:** $1\frac{4}{7} \div \left|\left\{2\frac{1}{3} - \overline{(5 + 2 - 3)}\right\} + 3\frac{1}{2}\right| = 1\frac{4}{7} \div \left|\left\{2\frac{1}{3} - (5 - 1)\right\} + 3\frac{1}{2}\right|$

$= \frac{11}{7} \div \left|\left\{2\frac{1}{3} - 4\right\} + 3\frac{1}{2}\right| = \frac{11}{7} \div \left|\left\{\frac{7}{3} - 4\right\} + 3\frac{1}{2}\right| = \frac{11}{7} \div \left|\left\{\frac{7-12}{3}\right\} + 3\frac{1}{2}\right| = \frac{11}{7} \div \left|\frac{-5}{3} + 3\frac{1}{2}\right|$

$= \frac{11}{7} \div \left|\frac{-5}{3} + \frac{7}{2}\right| = \frac{11}{7} \div \left|\frac{-10+21}{6}\right| = \frac{11}{7} \div \frac{11}{6} = \frac{\cancel{11}}{7} \times \frac{6}{\cancel{11}} = \frac{6}{7}$

## Exercise 3.2

1. Add the following fractions.

a. $\frac{1}{8} + \frac{13}{8}$ b. $\frac{20}{17} + \frac{14}{17}$ c. $1\frac{3}{5} + \frac{6}{7}$ d. $3\frac{3}{7} + 5\frac{1}{4} + 2\frac{1}{3}$

e. $\frac{1}{5} + \frac{2}{3} + 1\frac{1}{5}$ f. $\frac{17}{12} + \frac{13}{6}$ g. $\frac{12}{5} + \frac{19}{100}$ h. $\frac{23}{4} + \frac{1}{6}$

2. Subtract:

a. $\frac{9}{22} - \frac{3}{22}$ b. $3\frac{1}{3} - 1\frac{1}{2}$ c. $5\frac{6}{7} - 1\frac{3}{14}$ d. $11\frac{1}{2} - 5\frac{1}{8}$

e. $\frac{73}{7} - \frac{13}{21}$ f. $6\frac{1}{3} - \frac{5}{6}$ g. $3\frac{1}{8} - \frac{1}{5}$ h. $12\frac{1}{2} - 3\frac{1}{4}$

3. Multiply:

a. $\frac{1}{3} \times 21$ b. $\frac{8}{9} \times \frac{6}{7}$ c. $1\frac{5}{7} \times \frac{14}{25}$ d. $2\frac{3}{11} \times \frac{11}{15}$

e. $\frac{1}{2} \times \frac{2}{5}$ f. $\frac{3}{11} \times \frac{55}{24}$ g. $6\frac{2}{5} \times \frac{5}{16}$ h. $4\frac{1}{2} \times 3\frac{2}{9}$

i. $1\frac{1}{2} \times 3\frac{2}{3}$ j. $4\frac{2}{3} \times \frac{15}{28}$ k. $1\frac{1}{2} \times 3\frac{5}{9}$ l. $1\frac{1}{4} \times 2\frac{3}{5}$

4. Find the reciprocal of each of the following fractions and classify the reciprocals as proper and improper fractions.

a. $\frac{7}{8}$ b. $\frac{9}{5}$ c. $2\frac{1}{5}$ d. $1\frac{2}{3}$ e. $6\frac{3}{7}$ f. $1\frac{1}{2}$

5. Simplify:

a. $\frac{2}{3} \div \frac{7}{6}$ b. $\frac{7}{9} \div \frac{3}{2}$ c. $\frac{3}{10} \div \frac{21}{25}$ d. $\frac{9}{11} \div \frac{3}{22}$ e. $\frac{31}{33} \div \frac{62}{12}$ f. $2\frac{1}{5} \div 4\frac{2}{5}$

6. Convert the following complex fractions into simple fractions.

a. $\frac{\frac{5}{7}}{12}$ b. $\frac{\frac{1}{5}}{\frac{2}{25}}$ c. $\frac{\frac{2}{3}}{6}$ d. $\frac{\frac{7}{8}}{\frac{56}{64}}$ e. $\frac{1\frac{3}{4}}{\frac{7}{12}}$ f. $\frac{\frac{9}{10}}{\frac{60}{44}}$

7. Simplify:

a. $3\frac{1}{4} + 7\frac{2}{9} - 1\frac{1}{4}$ b. $13\frac{2}{3} + 3 - 13\frac{1}{2} + 3\frac{3}{4}$ c. $\frac{4}{5} \times 1\frac{3}{5} \div 5\frac{3}{25}$ d. $\frac{12}{5} \times \frac{75}{64} \div \frac{1}{9}$

8. Simplify:

a. $1\frac{3}{7}$ of $5 \div \frac{3}{4} + \frac{2}{3} \times \frac{3}{4} - \frac{2}{3}$

b. $1\frac{2}{3}\left\{\left(\overline{\frac{7}{8} + \frac{3}{4}} \times \frac{2}{3}\right) \div 1\frac{3}{4} + \frac{5}{8}\right\}$

c. $5\frac{1}{3} - \left\{4\frac{1}{3} - \left(3\frac{1}{3} - \overline{2\frac{1}{3} - \frac{1}{3}}\right)\right\}$

d. $\frac{\left(1\frac{2}{5} + 1\frac{2}{3} \div 2\frac{1}{3}\right)}{\left(1\frac{1}{3} \text{ of } 1\frac{4}{5} \div \frac{2}{5}\right)}$

9. Simplify the following.

a. $\left(\frac{4}{5} + \frac{5}{8}\right)$ of $\frac{15}{19} \div \frac{8}{9} + \frac{1}{2} \times \frac{3}{4} \times \frac{5}{12} + 2\frac{1}{8}$

b. $5\frac{1}{2} - \left[2\frac{1}{3} - \left\{3\frac{1}{4} - \left(2\frac{1}{6} - \overline{4\frac{1}{3} + 1\frac{2}{3}}\right)\right\}\right]$

c. $\frac{3}{12}$ of $\frac{\left(\frac{2}{5} + \frac{4}{15}\right)}{\left(\frac{2}{5} - \frac{3}{5}\right)}$

d. $\frac{3}{4} + \left\{\frac{3}{4} + \frac{3}{4} \div \left(\frac{3}{4} + \frac{3}{4}\right)\right\}$

10. Classify the following as true or false.

a. The reciprocal of a proper fraction is a proper fraction.
b. The reciprocal of an improper fraction is a proper fraction.
c. The reciprocal of a mixed fraction is a proper fraction.
d. The reciprocal of zero is zero.
e. When 1 is divided by a fraction, then the quotient is its reciprocal.

11. What must be added to $5\frac{1}{4}$ to get $11\frac{1}{2}$?

12. Find the least number that should be added to $5\frac{1}{2}$ to get $12\frac{3}{7}$.

13. Multiply $4\frac{3}{7}$ by $3\frac{1}{2}$ and divide the result by $4\frac{1}{7}$?

14. Find the least number which should be added to $6\frac{5}{11}$ to make it an integer.

## APPLICATIONS OF FRACTIONS IN DAILY LIFE

Fractions are used in real life. Some of the problems are discussed below.

**Example 1:** Savita has 27 marbles and Neelu has $\frac{4}{9}$ of the number of marbles Savita has. How many marbles does Neelu have?

**Solution:** Neelu has $\frac{4}{9}$ of 27 marbles, i.e. $\frac{4}{9} \times 27 = 4 \times 3 = 12$.

Therefore, Neelu has 12 marbles.

**Example 2:** In a class of 42 students, $\frac{3}{7}$ of them are boys. How many girls are there in the class?

**Solution:** Total number of students in the class = 42

Number of boys in the class = $\frac{3}{7}$ of $42 = \frac{3}{\cancel{7}} \times \overset{6}{\cancel{42}} = 18$

Therefore, the number of girls in the class is $42 - 18 = 24$.

**Example 3:** As per the will of a person, $\frac{1}{3}$ of his savings went to his wife, $\frac{5}{12}$ of what is left to each of his two sons and the rest to his daughter. If the daughter received ₹ 50,000, then find the total worth of all the assets.

**Solution:** Let the total savings of the person be ₹ $x$.

Therefore, wife's share = ₹ $\frac{1}{3}x$

Remaining share = ₹ $\left(1 - \frac{1}{3}\right)x$ = ₹ $\left(\frac{3-1}{3}\right)x$ = ₹ $\frac{2}{3}x$

$\Rightarrow$ Each son's share = $\frac{5}{12}$ of ₹ $\frac{2}{3}x = \left(\frac{5}{12} \times \frac{2}{3}\right)x$ = ₹ $\frac{5}{18}x$

$\therefore$ Daughter's share = $\left(\frac{2}{3}x - \frac{2 \times 5x}{18}\right) = \left(\frac{2}{3} - \frac{5}{9}\right)x = \left(\frac{6-5}{9}\right)x$ = ₹ $\frac{1}{9}x$

But, $\frac{1}{9}x$ = ₹ 50,000

$\therefore$ Total amount $x$ = ₹ 50,000 $\div \frac{1}{9}$ = ₹ 50,000 × 9 = ₹ 4,50,000.

Hence, the total worth of all the assets is ₹ 4,50,000.

**Example 4:** $\frac{3}{5}$ of a milk container is full. If 20 litres of milk is drawn from it, then $\frac{7}{12}$ of the container is full. Find the capacity of the container.

**Solution:** Here, $\frac{3}{5}$ of the capacity of the container $-\frac{7}{12}$ of the capacity of the container = 20 L

i.e. $\left(\frac{3}{5}-\frac{7}{12}\right)$ of the capacity of the container = 20 L

$\Rightarrow \left(\frac{36-35}{60}=\frac{1}{60}\right)$ of the capacity of the container = 20 L

$\therefore$ Capacity of the container = $(20 \times 60)$ L = 1200 L.

**Example 5:** Madhu travelled 28 km and found that $\frac{5}{7}$ of her journey was still left. What was the length of the total journey?

**Solution:** Let the total journey be 1, then journey covered = $1-\frac{5}{7}=\frac{7-5}{7}=\frac{2}{7}$.

$\Rightarrow \frac{2}{7}$ of the journey = 28 km

$\therefore$ Total journey = $28 \div \frac{2}{7}=\frac{28 \times 7}{2}$ = 98 km

Hence, the length of the total journey was 98 km.

**Example 6:** The perimeter of a rectangular field is $\frac{17}{20}$ m. If the measure of its length is $\frac{2}{5}$ m, find the measure of its breadth.

**Solution:** We know that the perimeter of rectangle = $2(l+b)$.

$\Rightarrow \frac{17}{20}=2\left(\frac{2}{5}+b\right) \Rightarrow \frac{17}{40}=\frac{2}{5}+b \Rightarrow b=\frac{17}{40}-\frac{2}{5}=\frac{17-16}{40}=\frac{1}{40}$ m

Thus, the measure of its breadth is $\frac{1}{40}$ m.

## Exercise 3.3

1. In a class of 44 students, $\frac{5}{11}$ of them are girls. How many boys are there in the class?
2. In a class of 45 students, $\frac{1}{5}$ opt to play cricket, $\frac{2}{3}$ of the total opt to play football and the rest opt to play hockey.
   a. Find the number of students who opt to play cricket, football and hockey.
   b. What fraction of the total number of students opt to play hockey?
3. Students of class VII were asked to choose their subject. $\frac{2}{3}$ of the class chose mathematics and the rest computer science. If there are 48 students in the class, how many students chose computer science?

4. In a party, $\frac{5}{9}$ of the persons were men and the rest were women. If the total gathering included 1260 persons, find the number of women present.

5. Shyam plants 6 saplings in a row in a garden. The distance between two adjacent saplings is $\frac{3}{4}$ m. Find the distance between the first and the last saplings.

6. $3\frac{3}{4}$ cups of flour are required for making a loaf of bread. How many cups of flour are required to make 32 loaves?

7. A man gave half of his money to his wife, three-fourths of what was left to his son and the rest to his daughter. If the daughter receives ₹ 25000, what was the total sum?

8. Ram spends $\frac{4}{5}$ of his income and saves the rest. If his total income is ₹ 36000, what are his savings?

9. A milk container can contain 14 litres of milk. How many glasses of milk, each having 350 mL capacity, can be poured from the container?

10. From a piece of 33 m long cloth, how many shirts can be stitched if 1 shirt requires $2\frac{1}{5}$ m cloth?

11. $\frac{1}{8}$ of the passengers of a train were children, $\frac{2}{5}$ of the passengers were men and the rest were women. If the number of women travelling in the train is 380, find the total number of passengers in the train.

12. Divide $8\frac{3}{4}$ m of a ribbon equally among 14 girls. How much will each girl get?

13. Ashok bought a second-hand car for ₹ 90,000. He paid $\frac{1}{6}$ of the price in cash and the rest in 10 equal monthly instalments. Find the amount he had to pay every month.

14. After covering $\frac{2}{5}$ of my journey, I find that I have covered 16 km. How much of the journey is still left?

15. A book has 465 pages. If $\frac{4}{5}$ of the pages have pictures on them and $\frac{2}{3}$ of the rest have text, then find the number of pages that have:

    a. pictures on them.

    b. text on them.

16. The area of a rectangle is $12\frac{7}{4}$ $m^2$. If the measure of its length is $\frac{7}{12}$ m, then find its breadth. Also, find its perimeter.

17. Priya read $\frac{3}{8}$ of a book in one day and $\frac{2}{5}$ of the remaining the next day. If 156 pages of the book were still left unread, how many pages did the book contain?

## Maths Around Us

*In ancient Rome, fractions were only written using words to describe part of the whole. They were based on the unit of weight which was called the as. One as had twelve parts known as uncia (from which we have the word "ounce").*

*For example,* $\frac{1}{12}$ *was called uncia;* $\frac{6}{12}$ *was called semis;* $\frac{1}{24}$ *was called semuncia;* $\frac{1}{144}$ *was called scripulum.*

## Mental Maths

1. *A shopkeeper sells milkshakes in packs of two sizes. The small milkshake pack contains 350 mL and the large milkshake pack contains* $\frac{3}{5}$ *more than the small milkshake pack. Compute the following:*
   a. *How much does the large milkshake pack contain?*
   b. *If Raj drinks* $\frac{2}{3}$ *of a small milkshake and his sister Ria drinks* $\frac{1}{2}$ *of a large milkshake, then who drinks more?*
2. *A sum of money was distributed among three children Sonal, Priya and Prachi, respectively. Sonal and Priya received* $\frac{1}{6}$ *and* $\frac{1}{8}$ *of the sum and the remaining amount was given to Prachi. If Prachi received ₹ 1950 more than Sonal, then how much should Prachi give to Priya so that Prachi has twice as much as Priya?*

## Activity

*Let us find the product of fractions using grids.*

*Consider* $4\frac{1}{3} \times 3\frac{1}{7}$, *then*

| × | 4 | $\frac{1}{3}$ |
|---|---|---|
| 3 | 12 | 1 |
| $\frac{1}{7}$ | $\frac{4}{7}$ | $\frac{1}{21}$ |

$= 12 + 1 + \frac{4}{7} + \frac{1}{21} = 13 + \frac{12+1}{21} = 13 + \frac{13}{21} = 13\frac{13}{21}$

*Now, try to find the product using the above method.*

1. $4\frac{2}{7} \times 3\frac{1}{4}$
2. $7\frac{1}{2} \times 3\frac{6}{7}$
3. $5\frac{1}{2} \times 3\frac{4}{7}$

## Revision Time

1. How many natural numbers are there between 4 and 20? What fraction of them are prime numbers, even numbers and odd numbers? Also find the product of these fractions.

2. Reduce the following fractions to the lowest form.

   a. $\frac{64}{154}$ b. $\frac{70}{126}$ c. $\frac{147}{252}$ d. $\frac{51}{187}$ e. $\frac{363}{968}$ f. $\frac{105}{345}$

3. Arrange the following fractions in ascending order.

   a. $\frac{3}{5}, \frac{2}{7}, \frac{5}{12}, \frac{13}{17}$ b. $\frac{11}{15}, \frac{9}{11}, \frac{5}{6}, \frac{3}{4}$

4. Insert four fractions lying between $\frac{2}{3}$ and $\frac{6}{7}$.

5. Simplify:

   a. $\frac{\left(1\frac{2}{5} + 1\frac{2}{3} \div \frac{5}{3}\right)}{\left(1\frac{1}{3} \text{ of } 1\frac{4}{5} \div \frac{2}{5}\right)}$ b. $8\frac{3}{4} - \left\{11 - \left(3 - \overline{1\frac{1}{4} - \frac{5}{8}}\right)\right\}$

6. The cost of a piece of cloth measuring $1\frac{3}{4}$ m is ₹ 248.50. Charu purchases 9 such pieces of cloth. Find the amount she has to pay and the total length of the cloth purchased by her?

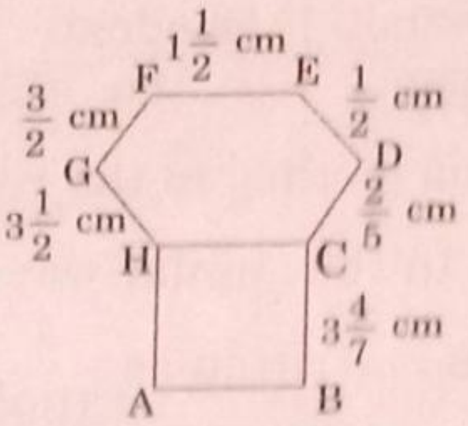
7. A man spends $\frac{7}{9}$ of his income and saves the rest. If he saves ₹ 6426 per month, find his monthly income.

8. $\frac{2}{5}$ of a tub is full of water. If 10 litres of water is taken out from it, then $\frac{3}{10}$ of the tub is full. What is the capacity of the tub?

9. In the adjoining figure, find the perimeter of:

   a. hexagon CDEFGH b. square ABCH

   Whose perimeter is greater and by how much?




10. $\frac{1}{4}$ of the passengers of a train are children, $\frac{2}{5}$ of the passengers are men and the rest are women. If the number of women travelling in the train is 420, find the total number of passengers in the train.

11. Priya worked as an intern for 5 days, working $3\frac{1}{4}$ hours, $5\frac{1}{2}$ hours, $4\frac{1}{6}$ hours, $7\frac{1}{3}$ hours and $5\frac{1}{3}$ hours on the respective days. How much did she earn in 5 days at the rate of ₹ 24 per hour?