3

SQUARES AND SQUARE ROOTS

INTRODUCTION

In earlier classes you have learnt that area of a square = side × side i.e. area of a square = $(\text{side})^2$ (i.e. square of side).

If the side of a square is given a units, then

area of square = $(a \times a)$ sq. units = a^2 sq. units

So, square of a number is the product of the number by itself.

For example:

$$4^2 = 4 \times 4,\ 9^2 = 9 \times 9,\ \left(\frac{2}{5}\right)^2 = \frac{2}{5} \times \frac{2}{5}.$$

In this chapter, we will study about square numbers or perfect squares, properties and patterns of square numbers, Pythagorean triplets, square roots and various methods to find square roots of natural numbers, square roots of decimals and fractions.

SQUARE NUMBERS OR PERFECT SQUARES

Consider the following table:

Table 1

Natural number	*Square*
1	$1^2 = 1 \times 1 = 1$
2	$2^2 = 2 \times 2 = 4$
3	$3^2 = 3 \times 3 = 9$
4	$4^2 = 4 \times 4 = 16$
5	$5^2 = 5 \times 5 = 25$
6	$6^2 = 6 \times 6 = 36$
7	$7^2 = 7 \times 7 = 49$
8	$8^2 = 8 \times 8 = 64$
9	$9^2 = 9 \times 9 = 81$
10	$10^2 = 10 \times 10 = 100$

You can see that 1, 4, 9, 16, 25, ... are the natural numbers which are the squares of natural numbers. Such numbers are called *square numbers* or *perfect squares*.

Squares of natural numbers are called square numbers or perfect squares.

In general, if a natural number m can be expressed as n^2, where n is also a natural number, then m is a square number or perfect square.

From table 1, you can write all the square numbers between 1 and 100. You will find that rest of the natural numbers are not perfect squares.

Hence, we can say that

All the natural numbers are not perfect squares.

How do we know, whether a given natural number is a perfect square?

Observe the following:

(*i*) Let us consider a square number 16.

It can be expressed as

$$16 = \underbrace{2 \times 2}_{} \times \underbrace{2 \times 2}_{}$$

Observation: 16 can be expressed as the product of pairs of equal prime factors.

Prime factorisation

2	16
2	8
2	4
2	2
	1

(*ii*) Let us consider another square number 144.

It can be expressed as

$$144 = \underbrace{2 \times 2}_{} \times \underbrace{2 \times 2}_{} \times \underbrace{3 \times 3}_{}$$

Observation: 144 can also be expressed as the product of pairs of equal prime factors.

Prime factorisation

2	144
2	72
2	36
2	18
3	9
3	3
	1

(*iii*) Now consider a non-square number 48.

It can be expressed as

$$48 = \underbrace{2 \times 2}_{} \times \underbrace{2 \times 2}_{} \times 3$$

Observation: 48 cannot be expressed as the product of pairs of equal prime factors (3 cannot be paired).

Prime factorisation

2	48
2	24
2	12
2	6
3	3
	1

Thus, we conclude that

A perfect square can always be expressed as the product of pairs of equal prime factors.

Example 1. *Is 324 a perfect square?*

Solution. Given number is 324.

It can be expressed as

$$324 = \underbrace{2 \times 2}_{} \times \underbrace{3 \times 3}_{} \times \underbrace{3 \times 3}_{}$$

Since 324 can be expressed as the product of pairs of equal prime factors.

Hence, 324 is a perfect square.

Prime factorisation

2	324
2	162
3	81
3	27
3	9
3	3
	1

Example 2. *Is 1152 a perfect square?*

Solution. Given number is 1152.

It can be expressed as

$$1152 = 2 \times \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{3 \times 3}$$

Since one 2 is left unpaired,

$\therefore$ 1152 cannot be expressed as the product of pairs of equal prime factors.

Hence, 1152 is not a perfect square.

Prime factorisation

2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

Example 3. *Show that 676 is a perfect square. Find the number whose square is 676.*

Solution. Given number is 676.

It can be expressed as

$$676 = \underbrace{2 \times 2} \times \underbrace{13 \times 13}$$

Since 676 can be expressed as the product of pairs of equal prime factors.

Hence, 676 is a perfect square.

Also $676 = (2)^2 \times (13)^2$

$= (2 \times 13)^2 = (26)^2$

Hence, 26 is the number whose square is 676.

Prime factorisation

2	676
2	338
13	169
13	13
	1

Example 4. *Find the smallest natural number by which 720 should be multiplied to make it a perfect square.*

Solution. Given number is 720.

It can be expressed as

$$720 = \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{3 \times 3} \times 5$$

Since 5 is left unpaired, so to make 720 a perfect square 5 should be paired.

So, the given number should be multiplied by 5.

Hence, the smallest natural number by which 720 be multiplied to make it a perfect square is 5.

Prime factorisation

2	720
2	360
2	180
2	90
3	45
3	15
5	5
	1

Example 5. *Find the smallest natural number by which 2527 should be divided to make it a perfect square.*

Solution. Given number is 2527.

It can be expressed as

$$2527 = 7 \times \underbrace{19 \times 19}$$

Since 7 is left unpaired, so to make 2527 a perfect square it should be divided by 7.

Hence, the smallest natural number by which 2527 be divided to make it a perfect square is 7.

Prime factorisation

7	2527
19	361
19	19
	1

Exercise 3.1

1. Which of the following natural numbers are perfect squares? Give reasons in support of your answer.
   (*i*) 729 (*ii*) 5488 (*iii*) 1024 (*iv*) 243
2. Show that each of the following numbers is a perfect square. Also find the number whose square is the given number.
   (*i*) 1296 (*ii*) 1764 (*iii*) 3025 (*iv*) 3969
3. Find the smallest natural number by which 1008 should be multiplied to make it a perfect square.
4. Find the smallest natural number by which 5808 should be divided to make it a perfect square. Also, find the number whose square is the resulting number.

PROPERTIES OF SQUARE NUMBERS

Property 1

Consider the following table:

Table 2

Natural number	*Square*	*Natural number*	*Square*	*Natural number*	*Square*
1	1	11	121	21	441
2	4	12	144	22	484
3	9	13	169	23	529
4	16	14	196	24	576
5	25	15	225	25	625
6	36	16	256	26	676
7	49	17	289	27	729
8	64	18	324	28	784
9	81	19	361	29	841
10	100	20	400	30	900

From the above table, we can observe that unit digits (*i.e.* digit in one's place) of square numbers are 0, 1, 4, 5, 6 or 9. None of these ends with 2, 3, 7 or 8 at unit's place.

Can we say that if a number ends in 0, 1, 4, 5, 6 or 9, then it must be a square number?

Answer is no, since the numbers 30, 21, 34, 45, 76 or 89 are not square numbers.

Hence, we can conclude that:

A square number always ends with 0, 1, 4, 5, 6 or 9 at unit's place but converse is not true.

Property. ***A number having 2, 3, 7 or 8 at its unit place is never a square number.***

Example 1. *Can we say whether the following numbers are perfect squares? Give reasons.*
(*i*) 2037 (*ii*) 2353 (*iii*) 9278 (*iv*) 67302

Solution. (*i*) 2037 is not a perfect square, since it ends with 7 at unit's place.
(*ii*) 2353 is not a perfect square, since it ends with 3 at unit's place.
(*iii*) 9278 is not a perfect square, since it ends with 8 at unit's place.
(*iv*) 67302 is not a perfect square, since it ends with 2 at unit's place.

Property 2

Observe the following:

$$\left.\begin{aligned} 10^2 &= 100 \\ 20^2 &= 400 \end{aligned}\right\} \rightarrow \text{Two zeros at the end}$$

$$\left.\begin{aligned} 100^2 &= 10000 \\ 200^2 &= 40000 \end{aligned}\right\} \rightarrow \text{Four zeros at the end}$$

$$\left.\begin{aligned} 1000^2 &= 1000000 \\ 2000^2 &= 4000000 \end{aligned}\right\} \rightarrow \text{Six zeros at the end}$$

We can see that square numbers can only have even number of zeros at the end.

However, the converse may not be true. For example, the numbers 300, 60000 and 8000000 end with even number of zeros but these are not square numbers.

So we can conclude that:

Property. ***A number ending in an odd number of zeros is never a perfect square.***

For example:

The numbers 30, 5000, 700000 end in one, three and five zeros respectively. Hence, none of the numbers is a perfect square.

Property 3

Observe the following:

$$\left.\begin{aligned} 2^2 &= 4 \\ 4^2 &= 16 \\ 6^2 &= 36 \\ 8^2 &= 64 \end{aligned}\right\} \rightarrow \text{square of an even natural number is even}$$

and

$$\left.\begin{aligned} 1^2 &= 1 \\ 3^2 &= 9 \\ 5^2 &= 25 \\ 7^2 &= 49 \end{aligned}\right\} \rightarrow \text{square of an odd natural number is odd}$$

Property. ***Squares of even natural numbers are even and squares of odd natural numbers are odd.***

Example 2. *The square of which of the following numbers would be an odd number/an even number? Why?*

(*i*) 997 (*ii*) 238 (*iii*) 579

Solution. (*i*) $(997)^2$ is odd because 997 is an odd number.

(*ii*) $(238)^2$ is even because 238 is an even number.

(*iii*) $(579)^2$ is odd because 579 is an odd number.

Property 4

Study the table 2 and observe the unit place in number and its square. You will see that

(*i*) If a number has **1 or 9 in the unit's place, then its square ends in 1.**

(*ii*) If a number has **4 or 6 in the unit's place, then its square ends in 6.**

(*iii*) If a number has **2 or 8 in the unit's place, then its square ends in 4.**

(*iv*) **If a number has 3 or 7 in the unit's place, then its square ends in 9.**
(*v*) **If a number has 5 in the unit's place, then its square ends in 5.**
(*vi*) **If a number has 0 in the unit's place, then its square ends in 0.**

SOME INTERESTING PATTERNS

1. Numbers between consecutive square numbers

Observe the following:

$$1 \ (= 1^2)$$
$$\underline{2, 3,}\ 4 \ (= 2^2)$$
$$\underline{5, 6, 7, 8,}\ 9 \ (= 3^2)$$
$$\underline{10, 11, 12, 13, 14, 15,}\ 16 \ (= 4^2)$$

From the above, we observe that

(*i*) $2^2 - 1^2 = 3$ and non-square numbers between 1^2 and 2^2 are two (*i.e.* 2×1) which is 1 less than the difference of two squares and the numbers are 2, 3.

(*ii*) $3^2 - 2^2 = 5$ and non-square numbers between 2^2 and 3^2 are four (*i.e.* 2×2) which is 1 less than the difference of two squares and the numbers are 5, 6, 7, 8.

In the same way,

non-square numbers between 3^2 and 4^2 are $(4^2 - 3^2) - 1 = (16 - 9) - 1$
$= 6 = 2 \times 3$

and the numbers are 10, 11, 12, 13, 14, 15.

In general, we can say that:

There are 2n non-square numbers between the squares of two consecutive numbers n and (n + 1).

Example 3. *How many natural numbers lie between 9^2 and 10^2?*

Solution. We know that there are $2n$ non-square numbers between n^2 and $(n + 1)^2$

$\therefore$ natural numbers between 9^2 and $(9 + 1)^2$ are $2 \times 9 = 18$

Hence, there are 18 natural numbers between 9^2 and 10^2.

2. Adding odd numbers

Observe the following:

$$1 = 1 = 1^2$$
$$1 + 3 = 4 = 2^2$$
$$1 + 3 + 5 = 9 = 3^2$$
$$1 + 3 + 5 + 7 = 16 = 4^2$$
$$1 + 3 + 5 + 7 + 9 = 25 = 5^2$$

We observe that sum of first n odd natural numbers is n^2.

Now consider the natural numbers which are not perfect squares. Check whether we can express them as the sum of consecutive odd natural numbers starting from 1.

Consider the number 24. Successively subtract 1, 3, 5, 7, ... from it.

(*i*) $24 - 1 = 23$ (*ii*) $23 - 3 = 20$ (*iii*) $20 - 5 = 15$
(*iv*) $15 - 7 = 8$ (*v*) $8 - 9 = -1$.

Thus we are not able to express 24 as the sum of consecutive odd natural numbers starting from 1.

Hence, we can say that:

If a natural number cannot be expressed as a sum of successive odd natural numbers starting with 1, then it is not a perfect square.

Example 4. *Find whether each of the following numbers is a perfect square or not?*

(*i*) 81 (*ii*) 59.

Solution. (*i*) Given number is 81. Successively subtract 1, 3, 5, 7, ... from it.

$81 - 1 = 80;$ $80 - 3 = 77;$ $77 - 5 = 72;$ $72 - 7 = 65;$

$65 - 9 = 56;$ $56 - 11 = 45;$ $45 - 13 = 32;$ $32 - 15 = 17;$

$17 - 17 = 0$

Thus, $81 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17$.

Since 81 can be expressed as the sum of consecutive odd natural numbers starting with 1.

Hence, 81 is a perfect square.

(*ii*) Given number is 59. Successively subtract 1, 3, 5, 7, ... from it.

$59 - 1 = 58;$ $58 - 3 = 55;$ $55 - 5 = 50;$ $50 - 7 = 43;$

$43 - 9 = 34;$ $34 - 11 = 23;$ $23 - 12 = 11;$ $11 - 13 = -2$

Since 59 cannot be expressed as the sum of consecutive odd natural numbers starting with 1.

Hence, 59 is not a perfect square.

3. Square of odd numbers as a sum of consecutive numbers

Observe the following:

$3^2 = 9 = 4 + 5$ $\left(4 = \frac{3^2 - 1}{2} \text{ and } 5 = \frac{3^2 + 1}{2}\right)$

$5^2 = 25 = 12 + 13$ $\left(12 = \frac{5^2 - 1}{2} \text{ and } 13 = \frac{5^2 + 1}{2}\right)$

$7^2 = 49 = 24 + 25$ $\left(24 = \frac{7^2 - 1}{2} \text{ and } 25 = \frac{7^2 + 1}{2}\right)$

$9^2 = 81 = 40 + 41$ $\left(40 = \frac{9^2 - 1}{2} \text{ and } 41 = \frac{9^2 + 1}{2}\right)$

$11^2 = 121 = 60 + 61$ $\left(60 = \frac{11^2 - 1}{2} \text{ and } 61 = \frac{11^2 + 1}{2}\right)$

Thus, we can say that:

We can express the square of any odd number greater than 1 as the sum of two consecutive natural numbers.

Converse may not be true *i.e.* the sum of any two consecutive positive integers may not be a perfect square number. For example, $6 + 7 = 13$, which is not a perfect square number.

Example 5. *Express the following as the sum of two consecutive integers:*

(*i*) 21^2 (*ii*) 29^2

Solution. (*i*) Given number is $21^2 = 441$

So first number is $\frac{21^2 - 1}{2} = \frac{441 - 1}{2} = \frac{440}{2} = 220$

and second number is $\frac{21^2 + 1}{2} = \frac{441 + 1}{2} = \frac{442}{2} = 221$

$\therefore \quad 21^2 = 220 + 221$

(ii) Given number is $29^2 = 841$

So first number is $\frac{29^2 - 1}{2} = \frac{841-1}{2} = \frac{840}{2} = 420$

and second number is $\frac{29^2+1}{2} = \frac{841+1}{2} = \frac{842}{2} = 421$

$\therefore 29^2 = 420 + 421.$

4. Square of numbers with unit digit 5

Observe the following:

$25^2 = 625 = (2 \times 3)$ hundreds $+ 25$

$35^2 = 1225 = (3 \times 4)$ hundreds $+ 25$

$125^2 = 15625 = (12 \times 13)$ hundreds $+ 25$

Thus, we can find the square of numbers with unit digit 5, by using the formula

$(a5)^2 = (10a + 5)^2 = (10a + 5)(10a + 5) = 10a \times 10a + 10a \times 5 + 5 \times 10a + 5 \times 5$

$= 100a^2 + 100a + 25 = 100a(a + 1) + 25$

$$(a5)^2 = a(a + 1) \text{ hundreds} + 25$$

Example 6. *Find the square of 85.*

Solution. Given number is 85,

comparing it with $a5$, we have $a = 8$.

We know that

$(a5)^2 = a(a + 1)$ hundreds $+ 25$

$\therefore \quad (85)^2 = 8(8 + 1)$ hundreds $+ 25$

$= (8 \times 9)$ hundreds $+ 25 = 7225.$

Pythagorean triplets

Observe the following:

(i) $3^2 + 4^2 = 9 + 16 = 25 = 5^2$

$\therefore 3^2 + 4^2 = 5^2$

The collection of numbers (3, 4, 5) is known as *Pythagorean triplet*.

(ii) $6^2 + 8^2 = 36 + 64 = 100 = 10^2$

$\therefore 6^2 + 8^2 = 10^2$

$\Rightarrow$ (6, 8, 10) is also a Pythagorean triplet.

Thus we can say that

Three natural numbers m, n, p are said to form a **Pythagorean triplet** (m, n, p) if $m^2 + n^2 = p^2$.

For any natural number $m > 1$, we have

$$(2m)^2 + (m^2 - 1)^2 = (m^2 + 1)^2$$

So $(2m, m^2 - 1, m^2 + 1)$ forms a Pythagorean triplet.

Example 7. *Write a Pythagorean triplet whose smallest number is 8.*

Solution. Given number is 8.

We know that, we can find Pythagorean triplet by using $(2m, m^2 - 1, m^2 + 1)$

Let us first assume $m^2 - 1 = 8 \Rightarrow m^2 = 9 \Rightarrow m = 3$

Therefore, $2m = 6$ and $m^2 + 1 = 10$

$\therefore$ The triplet is 6, 8, 10. But 8 is not the smallest number of this triplet.

So, let us assume $2m = 8 \Rightarrow m = 4$

$\therefore m^2 - 1 = 4^2 - 1 = 15$ and $m^2 + 1 = 4^2 + 1 = 17$

$\therefore$ The required triplet is (8, 15, 17) with smallest number 8.

Example 8. *Find a Pythagorean triplet whose one number is 12.*

Solution. Given number is 12.

Let us first assume, $m^2 - 1 = 12 \Rightarrow m^2 = 13$

$\Rightarrow$ value of m will not be a natural number.

Now, we assume $m^2 + 1 = 12 \Rightarrow m^2 = 11$

$\Rightarrow$ value of m will not be a natural number.

Now, let us assume $2m = 12 \Rightarrow m = 6$

$\therefore m^2 - 1 = 6^2 - 1 = 35$ and $m^2 + 1 = 6^2 + 1 = 37$

$\therefore$ The required triplet is (12, 35, 37) with one number 12.

REMARK

All Pythagorean triplets may not be obtained using this form. For example, another triplet with one number 12 is (5, 12, 13).

Exercise 3.2

1. Write five numbers which you can decide by looking at their one's digit that they are not square numbers.
2. What will be the unit digit of the squares of the following numbers?
   (*i*) 951 (*ii*) 502 (*iii*) 329 (*iv*) 643 (*v*) 5124
   (*vi*) 7625 (*vii*) 68327 (*viii*) 95628 (*ix*) 99880 (*x*) 12796
3. The following numbers are obviously not perfect squares. Give reason.
   (*i*) 567 (*ii*) 2453 (*iii*) 5298 (*iv*) 46292 (*v*) 74000
4. The square of which of the following numbers would be an odd number or an even number? Why?
   (*i*) 573 (*ii*) 4096 (*iii*) 8267 (*iv*) 37916
5. How many natural numbers lie between square of the following numbers?
   (*i*) 12 and 13 (*ii*) 90 and 91
6. Without adding, find the sum.
   (*i*) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15$
   (*ii*) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 + 27 + 29$
7. (*i*) Express 64 as the sum of 8 odd numbers.
   (*ii*) Express 121 as the sum of 11 odd numbers.
8. Express the following as the sum of two consecutive integers:
   (*i*) 19^2 (*ii*) 33^2 (*iii*) 47^2
9. Find the squares of the following numbers containing 5 in unit's place:
   (*i*) 45 (*ii*) 305 (*iii*) 525
10. Write a Pythagorean triplet whose one number is
   (*i*) 8 (*ii*) 15 (*iii*) 63 (*iv*) 80

SQUARE ROOTS

Answer the following question:
If the area of a square is 169 m^2. What is the length of side of the square?
We know that area of square = $(\text{side})^2$
Let the length of side of the square be x m
then $169 = x^2$
To find the length of side, we need to find a number whose square is 169.
Hence, finding a number whose square is known, is called finding the **square root**.
Thus, we can say that:

The square root of a number n is that number which when multiplied by itself gives n as the product.

Thus, if m is the square root of number n, then
$m \times m = n$ or $m^2 = n$.
The square root of a number n is denoted by $\sqrt{n}$

$\therefore \quad m = \sqrt{n}$

Finding square roots

As we know that the subtraction is the inverse (opposite) operation of addition and division is the inverse operation of multiplication. Similarly, finding the *square root is the inverse operation of finding the square.*

Thus, we have $2^2 = 4 \Rightarrow$ square root of 4 is 2 (*i.e.* $\sqrt{4} = 2$)
$3^2 = 9 \Rightarrow$ square root of 9 is 3 (*i.e.* $\sqrt{9} = 3$)
$7^2 = 49 \Rightarrow$ square root of 49 is 7 (*i.e.* $\sqrt{49} = 7$)

REMARK

Since $3^2 = 9$ and $(-3)^2 = 9$. Hence, we can say that square roots of 9 are 3 and -3. Actually we have two integral roots of a perfect square number, but in this chapter, we shall take up only positive square root of a natural number. Thus, we have $\sqrt{9} = 3$ (not -3).

Finding the square roots through prime factorisation

In the beginning of this chapter we have discussed that:

A square number can always be expressed as the product of pairs of equal prime factors.

Method to find the square root of a number by resolving it into prime factors:

(*i*) *Express the given number as the product of primes.*
(*ii*) *Make groups in pairs of the same prime.*
(*iii*) *Take one factor from each pair of primes. Multiply them together, the product so obtained is the required square root of the given number.*

REMARKS

* Instead of writing the prime factors in pairs, we can write them in the exponential notation. Then for finding the square root of the number, take half of each index value.
* Square root of a fraction = $\dfrac{\text{square root of its numerator}}{\text{square root of its denominator}}$.

Example 1. *Find the square root of 324 by prime factorisation.*

Solution. Given number is 324.

Prime factorisation

2	324
2	162
3	81
3	27
3	9
3	3
	1

Expressing 324 into prime factors, we have

$$324 = \underbrace{2\times 2}\times\underbrace{3\times 3}\times\underbrace{3\times 3}$$

$\Rightarrow \quad \sqrt{324} = 2\times 3\times 3 = 18$ (Taking one prime number from each pair)

$\Rightarrow \quad \sqrt{324} = 18$

Hence, the square root of 324 is 18.

Example 2. *Find the square root of 6400 by prime factorisation.*

Solution. Given number is 6400.

Expressing 6400 into the prime factors, we have

$$6400 = \underbrace{2\times 2}\times\underbrace{2\times 2}\times\underbrace{2\times 2}\times\underbrace{2\times 2}\times\underbrace{5\times 5}$$

$$\sqrt{6400} = 2\times 2\times 2\times 2\times 5 = 80$$

Hence, the square root of 6400 is 80.

2	6400
2	3200
2	1600
2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

Example 3. *Find the square root of the following numbers by prime factorisation method:*

(*i*) 7056 (*ii*) $10\frac{86}{121}$ (*iii*) 42.25

Solution. (*i*) $7056 = 2\times 2\times 2\times 2\times 3\times 3\times 7\times 7$

$= 2^2\times 2^2\times 3^2\times 7^2$

$\therefore \quad \sqrt{7056} = 2\times 2\times 3\times 7 = 84.$

2	7056
2	3528
2	1764
2	882
3	441
3	147
7	49
	7

(*ii*) $10\frac{86}{121} = \frac{1296}{121}$

Converting mixed fraction to improper fraction

$= \frac{2\times 2\times 2\times 2\times 3\times 3\times 3\times 3}{11\times 11} = \frac{2^2\times 2^2\times 3^2\times 3^2}{(11)^2}$

$\therefore \quad \sqrt{10\frac{86}{121}} = \frac{2\times 2\times 3\times 3}{11} = \frac{36}{11} = 3\frac{3}{11}.$

(*iii*) $42.25 = \frac{4225}{100}$

$= \frac{169}{4}$

Converting decimal to fraction

$= \frac{13\times 13}{2\times 2} = \frac{13^2}{2^2}$

Expressing in lowest terms

$\therefore \quad \sqrt{42.25} = \frac{13}{2} = \frac{13\times 5}{2\times 5} = \frac{65}{10} = 6.5$

Example 4. *The students of class VIII of a school donated ₹2401 in all, for Prime Minister's National Relief Fund. Each student donated as many rupees as the number of students in the class. Find the number of students in the class.*

Solution. Given that amount donated by the students of class VIII = ₹2401.

Let the number of students in class VIII = x

then, the amount donated by each student = ₹x

∴ Total amount donated for Prime Minister's Relief Fund = $x \times x$

According to given $x \times x = 2401$

$\Rightarrow \quad x^2 = 2401$

$\Rightarrow \quad x^2 = 7 \times 7 \times 7 \times 7 = 7^4$

$\Rightarrow \quad x = 7^2 = 7 \times 7 = 49.$

Hence, the number of students in the class is 49.

Prime factorisation

7	2401
7	343
7	49
7	7
	1

Example 5. *Is 2352 a perfect square? If not, find the smallest multiple of 2352 which is a perfect square. Find the square root of the new number.*

Solution. Given number is 2352.

Expressing it into prime factors, we have

$$2352 = \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times 3 \times \underbrace{7 \times 7}$$

We can see that 3 is left unpaired.

So 2352 cannot be expressed as the product of pairs of equal prime factors.

Hence, 2352 is not a perfect square.

In order to make 2352 a perfect square, 3 should be paired. So, the smallest number by which 2352 is multiplied to make it a perfect square is 3.

Prime factorisation

2	2352
2	1176
2	588
2	294
3	147
7	49
7	7
	1

$$\therefore \quad 2352 \times 3 = \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{7 \times 7} \times \underbrace{3 \times 3}$$

Thus, $2352 \times 3 = 7056$ is a perfect square.

Hence, the smallest multiple of 2352 which is a perfect square is 7056

and $\sqrt{7056} = \sqrt{\underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{7 \times 7} \times \underbrace{3 \times 3}}$

$= 2 \times 2 \times 7 \times 3 = 84$

Hence, the square root of new number is 84.

Example 6. *Find the smallest natural number by which 9408 must be divided so that the quotient is a perfect square. Find the square root of the quotient.*

Solution. Given number is 9408.

Expressing it into prime factors, we have

$$9408 = \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times 3 \times \underbrace{7 \times 7}$$

Since 3 is left unpaired, so to make 9408 a perfect square, it should be divided by 3.

Now $9408 \div 3 = 3136 = \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{7 \times 7}$

We can see that quotient 3136 can be expressed as the product of pairs of equal prime factors, so 3136 is a perfect square. Hence, the smallest number by which 9408 must be divided so that the quotient is a perfect square is 3.

Prime factorisation

2	9408
2	4704
2	2352
2	1176
2	588
2	294
3	147
7	49
7	7
	1

$$3136 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7$$
$$= 2^6 \times 7^2$$
$$\Rightarrow \sqrt{3136} = 2^3 \times 7^1 = 8 \times 7 = 56.$$

Hence, the square root of the quotient is 56.

Example 7. *Find the smallest square number which is divisible by each of the numbers 6, 9 and 15.*

Solution. Given numbers are 6, 9 and 15.

LCM of 6, 9 and 15 = $3 \times 2 \times 3 \times 5 = 90$

$\therefore$ Smallest number which is divisible by each one of 6, 9 and 15 = 90

Prime factorisation

$$\begin{array}{r|l} 3 & 6, 9, 15 \\ \hline & 2, 3, 5 \end{array}$$

Prime factorisation of 90 = $2 \times \underbrace{3 \times 3} \times 5$.

We can see that 90 cannot be expressed as the product of pairs of equal prime factors, so 90 is not a square number.

In order to make 90 a square number, we need to multiply it by 2×5 *i.e.* by 10.

$\therefore$ The required square number = $90 \times 10 = 900$

Hence, the smallest square number which is divisible by 6, 9, 15 is 900.

Exercise 3.3

1. Find the square roots of the following numbers by prime factorisation method:
   (*i*) 784 (*ii*) 441 (*iii*) 1849 (*iv*) 4356 (*v*) 6241
   (*vi*) 8836 (*vii*) 8281 (*viii*) 9025
2. Find the square roots of the following numbers by prime factorisation method:
   (*i*) $9\frac{67}{121}$ (*ii*) $17\frac{13}{36}$ (*iii*) 1.96 (*iv*) 0.0064
3. For each of the following numbers, find the smallest natural number by which it should be multiplied so as to get a perfect square. Also find the square root of the square number so obtained:
   (*i*) 588 (*ii*) 720 (*iii*) 2178 (*iv*) 3042 (*v*) 6300
4. For each of the following numbers, find the smallest natural number by which it should be divided so that this quotient is a perfect square. Also find the square root of the square number so obtained:
   (*i*) 1872 (*ii*) 2592 (*iii*) 3380 (*iv*) 16244 (*v*) 61347
5. Find the smallest square number that is divisible by each of the following numbers:
   (*i*) 3, 6, 10, 15 (*ii*) 6, 9, 27, 36 (*iii*) 4, 7, 8, 16
6. 4225 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows and the number of plants in each row.
7. The area of a rectangle is 1936 sq. m. If the length of the rectangle is 4 times its breadth, find the dimensions of the rectangle.
8. In a school a P.T. teacher wants to arrange 2000 students in the form of rows and columns for P.T. display. If the number of rows is equal to number of columns and

64 students could not be accommodated in this arrangement. Find the number of rows.

9. In a school, the students of class VIII collected ₹2304 for a picnic. Each student contributed as many rupees as the number of students in the class. Find the number of students in the class.
10. The product of two numbers is 7260. If one number is 15 times the other number, find the numbers.
11. Find three positive numbers in the ratio 2 : 3 : 5, the sum of whose squares is 950.
12. The perimeters of two squares are 60 metres and 144 metres respectively. Find the perimeter of another square equal in area to the sum of the first two squares.

Finding square root of a number by division method

Finding the square roots by prime factorisation is useful for small numbers, but when numbers are large and factorisation is not easy, then finding the square roots by this method becomes lengthy and difficult. We find the square root of large numbers by **long division method**. This method is explained with the help of few examples given below.

Example 1. *Find the square root of the following numbers:*

(*i*) 729 (*ii*) 7056 (*iii*) 55696 (*iv*) 288369

Solution. (*i*) Steps

1. Place a bar (or arrow) over every pair of digits from right to left (←) *i.e.* starting from unit's digit. If the number of digits is odd, then the left most digit too will have a bar. Each pair of digits and then remaining one digit (if any) on the extreme left is called **period**.
2. Take the first pair of digits or the single digit as the case may be. In this case, it is the digit 7. Find the greatest number whose square is less than or equal to 7. Such a number is 2. Write 2 on the top in the quotient and also in the divisor. Subtract 2^2 *i.e.* 4 from 7. The remainder is 3.

Double
$$\begin{array}{r|l} & 2\ 7 \\ \hline 2 & \overline{7}\ \overline{29} \\ & -4 \\ \hline 47 & 3\ 29 \\ & -3\ 29 \\ \hline & 0 \end{array}$$

3. Bring down the pair of digits under the next bar (*i.e.* 29 in this case) to the right of the remainder. So the new dividend is 329.
4. Double the quotient (*i.e.* 2 in this case) to get 4 and enter it with a blank on its right at the place of new divisor.
5. Find the largest possible digit to fill the blank which will also become the new digit in the quotient, such that when the new divisor is multiplied by the new digit in the quotient the product is less than or equal to the dividend.
   In this case $47 \times 7 = 329$. So we choose the new digit as 7. Place 329 under 329. Subtract and get remainder 0.

$\therefore \quad \sqrt{729} = 27$

(*ii*) Steps

1. Place a bar over every pair of digits from right to left (←).
2. Take the first pair of digits. In this case, it is 70. Find the greatest number whose square is less than or equal to 70. Such a number is 8. Write 8 on the top in the quotient and also in the divisor. Subtract 8^2 *i.e.* 64 from 70. The remainder is 6.
3. Bring down the pair of digits under next bar (*i.e.* 56 in this case) to the right of the remainder. So the new dividend is 656.

Double
$$\begin{array}{r|l} & 8\ 4 \\ \hline 8 & \overline{70}\ \overline{56} \\ & -64 \\ \hline 164 & 6\ 56 \\ & -6\ 56 \\ \hline & 0 \end{array}$$

4. Double the quotient (*i.e.* 8 in this case) to get 16 and enter it with a blank at the place of new divisor.
5. Find the largest possible digit to fill the blank which will also become the new digit in the quotient, such that when the new divisor is multiplied by this new digit in the quotient the product is less than or equal to the dividend. In this case $164 \times 4 = 656$. So we choose new digit as 4. Place 656 under 656. Subtract and get remainder 0.

$\therefore \sqrt{7056} = 84$

(iii) Steps

1. Place a bar (or arrow) over every pair of digits from right to left (←).
2. Take the first pair of digits or the single digit as the case may be. In this case, it is the digit 5. Find the greatest number whose square is 5 or less than 5. Such a number is 2. Write 2 on the top in the quotient and also in the divisor. Subtract 2^2 *i.e.* 4 from 5. The remainder is 1.
3. Bring down the pair of digits under the next bar (*i.e.* 56 in this case) to the right of the remainder. So the new dividend is 156.
4. Double the quotient (*i.e.* 2 in this case) to get 4 and enter it with a blank on its right at the place of new divisor.
5. Find the largest possible digit to fill the blank which will also become the new digit in the quotient, such that when the new divisor is multiplied by the new digit in the quotient the product is less than or equal to the dividend.
   In this case $43 \times 3 = 129$, so we choose the new digit as 3. Place 129 under 156. Subtract and get the remainder 27.
6. Bring down the pair of digits under the next bar (*i.e.* 96 in this case) to the right of the remainder. So the new dividend is 2796.
7. Double the quotient (*i.e.* 23 in this case) to get 46 and enter it with a blank on its right at the place of new divisor.
8. Find the largest possible digit to fill the blank which will also become the new digit in the quotient, such that when the new divisor is multiplied by this new digit in the quotient the product is less than or equal to the dividend.
   In this case $466 \times 6 = 2796$. So we choose the new digit as 6. Place 2796 under 2796. Subtract and get the remainder 0.

Double

	2 3 6
2	5 56 96
	4
43	1 56
	1 29
466	27 96
	27 96
	0

$\therefore \sqrt{55696} = 236.$

(iv) Steps

1. Place a bar over every pair of digits from right to left (←).
2. Take the first pair of digits. In this case, it is 28. Find the greatest number whose square is 28 or less than 28. Such a number is 5. Write 5 on the top in the quotient and also in the divisor. Subtract 5^2 *i.e.* 25 from 28. The remainder is 3.
3. Bring down the pair of digits under the next bar (*i.e.* 83 in this case) to the right of the remainder. So the new dividend is 383.
4. Double the quotient (*i.e.* 5 in this case) to get 10 and enter it with a blank at the place of new divisor.

	5 3 7
5	28 83 69
	25
103	3 83
	3 09
1067	74 69
	74 69
	0

5. Find the largest possible digit to fill the blank which will also become the new digit in the quotient, such that when the new divisor is multiplied by this new digit in the quotient the product is less than or equal to dividend. In this case $103 \times 3 = 309$, so we choose the new digit as 3. Place 309 under 383 and get the remainder 74.
6. Repeat the process of steps 3, 4 and 5. Remainder is 0.

$\therefore \sqrt{288369} = 537$

Example 2. *Find the least number that must be subtracted from 1989 so as to get a perfect square. Also find the square root of the perfect square.*

Solution. Given number is 1989

Let us try to find its square root by long division method:

	44
4	$\overline{19}\,\overline{89}$
	−16
84	3 89
	−3 36
	53

We get the remainder = 53.

It means $(44)^2$ is less than 1989 by 53.

$\therefore$ If we subtract 53 from 1989, we get $(44)^2$ which is a perfect square number.

Hence, the least number that must be subtracted from 1989 so as to make it a perfect square is 53.

$\therefore$ Required perfect square number $= 1989 - 53 = 1936$ and $\sqrt{1936} = 44$.

Example 3. *Find the greatest 4-digit number, which is a perfect square.*

Solution. Greatest 4-digit number = 9999

Let us try to find its square root by long division method:

	99
9	$\overline{99}\,\overline{99}$
	−81
189	18 99
	−17 01
	198

We get the remainder = 198.

It means $(99)^2$ is less than 9999 by 198.

$\therefore$ If we subtract 198 from 9999, we get $(99)^2$ which is a perfect square number.

$\therefore$ Required perfect square number $= 9999 - 198 = 9801$.

Hence, the greatest 4-digit number, which is a perfect square is 9801.

Example 4. *Find the least number that must be added to 6412 so as to get a perfect square.*

Solution. Given number is 6412

Let us try to find its square root by long division method:

	80
8	$\overline{64}\,\overline{12}$
	−64
160	012

We get the remainder = 12.

This shows that $(80)^2 < 6412$

Next perfect square number is $(81)^2 = 6561$

$\therefore$ Required number $= (81)^2 - 6412 = 6561 - 6412 = 149$.

Hence, the least number that must be added to 6412 so as to make it a perfect square is 149.

Activity 2

Square roots of 2 and 3

To verify by activity method that

(*i*) $\sqrt{2} = 1.4$ (approx) (*ii*) $\sqrt{3} = 1.7$ (approx).

Materials required

(*i*) A white chart paper (*ii*) Geometry box (*iii*) Pencil.

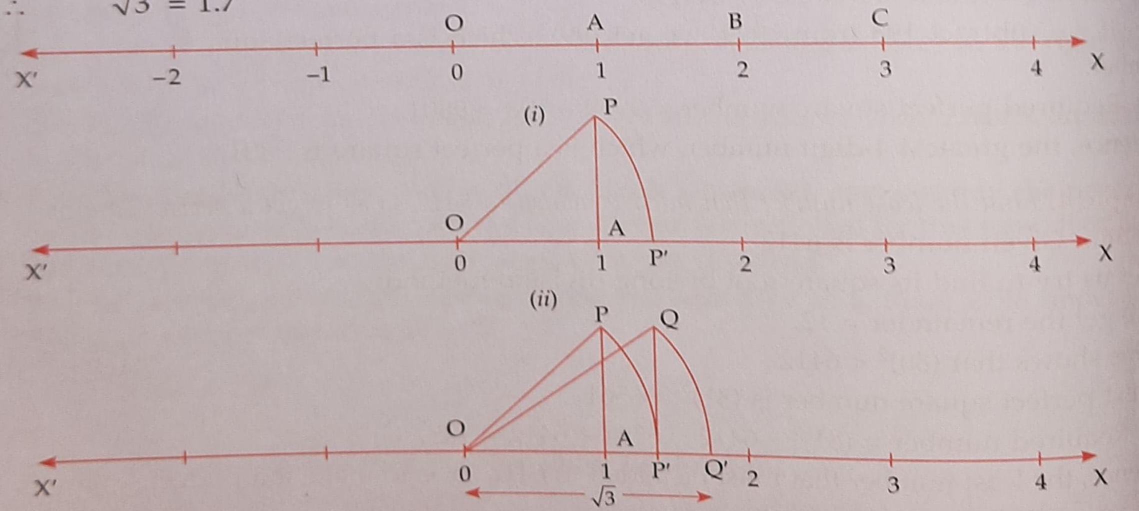
Steps

(a) To find $\sqrt{2}$, proceed as under:

1. Draw a number line XOX′ (shown in fig. (*i*)).
2. Mark points on the number line starting from O such that distance between two consecutive points is 1 unit.
3. Label these points as A, B, C, ..., then the points A, B, C, ... will represent the numbers 1, 2, 3, ... respectively.
4. Draw a perpendicular line segment AP of unit length to X′OX at the point A.
5. Join OP. By using Pythagoras theorem in ΔOPA, we get
$$OP^2 = OA^2 + AP^2 = 1^2 + 1^2 = 1 + 1 = 2 \Rightarrow OP = \sqrt{2}.$$
Thus, length of segment OP = $\sqrt{2}$ units.
6. With O as centre and radius OP, draw an arc cutting the number line at P′ (as shown in fig. (*ii*)).
7. Measure OP′, OP′ = 1.4 units (approximately)
$\therefore \sqrt{2} = 1.4$

(b) To find $\sqrt{3}$, proceed as under:

1. Draw $P'Q \perp OP'$ such that P′Q = 1 unit (as shown in fig. (*iii*)).
2. Join OQ. By using Pythagoras theorem in $\Delta OP'Q$, we get
$$OQ^2 = OP'^2 + P'Q^2 = (\sqrt{2})^2 + 1^2 = 2 + 1 = 3$$
$$\Rightarrow \quad OQ = \sqrt{3}.$$ Thus, length of segment OQ = $\sqrt{3}$.
3. With O as centre and radius OQ, draw an arc cutting the number line at Q′ (as shown in fig. (*iii*)).
4. Measure OQ′, OQ′ = 1.7 units (approximately)
$\therefore \quad \sqrt{3} = 1.7$



Square roots of decimals

Long division method is also useful to find the square roots of decimals.

To learn and understand the method of finding the square roots of decimals observe the following example:

Example 5. *Find the square root of the following numbers:*

(*i*) 12.0409 (*ii*) 0.00064516

Solution. (*i*) Steps

1. Starting from decimal point, put arrows (or bars) on the pairs of integers from right to left (←) as usual and on the decimal part from left to right (→).

2. Take the first pair of digits. In this case, it is 12. Find the greatest number whose square is 12 or less than 12. Such a number is 3. Write 3 on the top in the quotient and also in the divisor. Subtract 3^2 *i.e.* 9 from 12. The remainder is 3.
3. Since the next pair of digits is after decimal point, therefore, write the decimal point in the quotient.
4. Bring down the pair of digits below the next bar (*i.e.* 04, in this case) to the right of the remainder. So the new dividend is 304.
5. Double the quotient (*i.e.* 3) to get 6 and enter it with a blank on its right at the place of new divisor.
6. Find the largest possible digit to fill the blank which will also become the new digit in the quotient, such that when the new divisor is multiplied by the new digit in the quotient the product is less than or equal to dividend. In this case, 64 × 4 = 256, so we choose the new digit as 4. Place 256 under 304. Subtract and get the remainder 48.

Repeat the process of steps 4, 5 and 6. Remainder is 0.

	3.47
3	12.04 09
	9
64	3 04
	2 56
687	48 09
	48 09
	0

$\therefore \quad \sqrt{12.0409} = 3.47$

(*ii*) Steps

1. Integral part is zero, so start from decimal point and put bars (or arrows) on the pair of integers from left to right (→).
2. Since the first pairs of digits is after the decimal point, therefore, write the decimal point in the quotient.
3. As the first pair of digits after decimal point consists of both zero. Write 0 in the quotient to the right of the decimal point.
4. As the next pair of digits is 06. Find the greatest number whose square is 6 or less than 6. Such a number is 2. Write 2 in the quotient to the right of 0 and also write 2 in the divisor. Subtract 2^2 *i.e.* 4 from 6. The remainder is 2. Bring down the pair of digits below the next bar and proceed as in part (*i*).

	.0254
2	0.00 06 45 16
	4
45	245
	225
504	20 16
	20 16
	0

$\therefore \sqrt{0.00064516} = 0.0254$

NOTE

When the square root of a number is not exact, to find its approximate value correct to a certain place of decimal, we obtain the square root of the given number to one more place and then round off to the desired place.

Example 6. *Find the square root of 9.81 correct to 2 decimal places.*

Solution. Since the addition of zero (zeros) at the end of a decimal number does not change its value, therefore, write 9.81 as 9.810000.

Now proceed as in example 5 part (*i*) to find the square root.

= 3.132...

= 3.13 correct to 2 decimal places.

	3.132
3	9.81 00 00
	9
61	81
	61
623	20 00
	18 69
6262	1 31 00
	1 25 24
	5 76

Example 7. *Find the square root of 5 correct to 2 decimal places. Hence, find the value of $3-\sqrt{5}$.*

Solution. $\sqrt{5} = 2.236...$

$= 2.24$ correct to 2 decimal places.

$\therefore\ 3-\sqrt{5} = 3-2.24$

$= 0.76$

	2.236
2	5.00 00 00
	4
42	1 00
	84
443	16 00
	13 29
4466	271 00
	267 96
	3 04

Square root of a fraction when numerator or denominator or both are not perfect squares:

Method: First divide the numerator by the denominator and convert the given fraction into a decimal number, then find the square root of the decimal number.

Example 8. *Find the square root of the following fractions correct to 2 decimal places.*

(*i*) $6\frac{5}{8}$ (*ii*) $20\frac{2}{3}$.

Solution. (*i*) $6\frac{5}{8} = 6+\frac{5}{8} = 6+\frac{5\times125}{8\times125}$

$= 6+\frac{625}{1000} = 6.625$

$\sqrt{6.625} = 2.573...$

$= 2.57$ correct to decimal places

$\therefore\ \sqrt{6\frac{5}{8}} = 2.57$ correct to 2 decimal places.

	2.573
2	6 . 62 50 00
	4
45	2 62
	2 25
507	37 50
	35 49
5143	201 00
	154 29
	46 71

(*ii*) $20\frac{2}{3} = 20+\frac{2}{3} = 20 + 0.666666....$

$= 20.666666...$

$\sqrt{20.666666} = 4.546...$

$\therefore\ \sqrt{20\frac{2}{3}} = 4.55$ correct to 2 decimal places.

	4.546
4	20 . 66 66 66
	16
85	4 66
	4 25
904	41 66
	3616
9086	550 66
	545 16
	5 50

Exercise 3.4

1. Find the square root of each of the following by division method:

(*i*) 2401 (*ii*) 4489 (*iii*) 106929

(*iv*) 167281 (*v*) 53824 (*vi*) 213444

2. Find the square root of the following decimal numbers by division method:
(*i*) 51.84 (*ii*) 42.25 (*iii*) 18.4041 (*iv*) 5.774409
3. Find the square root of the following numbers correct to two decimal places:
(*i*) 645.8 (*ii*) 107.45 (*iii*) 5.462 (*iv*) 2 (*v*) 3
4. Find the square root of the following fractions correct to two decimal places:
(*i*) $11\frac{3}{8}$ (*ii*) $5\frac{5}{11}$ (*iii*) $7\frac{1}{3}$
5. Find the least number which must be subtracted from each of the following numbers to make them a perfect square. Also find the square root of the perfect square number so obtained:
(*i*) 2000 (*ii*) 984 (*iii*) 8934 (*iv*) 11021
6. Find the least number which must be added to each of the following numbers to make them a perfect square. Also find the square root of the perfect square number so obtained:
(*i*) 1750 (*ii*) 6412 (*iii*) 6598 (*iv*) 8000
7. Find the smallest four digit number which is a perfect square.
8. Find the greatest number of six digits which is a perfect square.
9. In a right triangle ABC, $\angle B = 90°$.
(*i*) If AB = 14 cm, BC = 48 cm, find AC (*ii*) If AC = 37 cm, BC = 35 cm, find AB.
10. A gardener has 1400 plants. He wants to plant these in such a way that the number of rows and number of columns remains same. Find the minimum number of plants he needs more for this.
11. There are 1000 children in a school. For a P.T. drill they have to stand in such a way that the number of rows is equal to number of columns. How many children would be left out in this arrangement?
12. Amit walks 16 m south from his house and turns east to walk 63 m to reach his friend's house. While returning, he walks diagonally from his friend's house to reach back to his house. What distance did he walk while returning?
13. A ladder 6 m long leaned against a wall. The ladder reaches the wall to a height of 4.8 m. Find the distance between the wall and the foot of the ladder.

Objective Type Questions

MENTAL MATHS

1. Fill in the blanks:
(*i*) A number ending in ..... is never a perfect square.
(*ii*) If a number has digits ..... in the unit's place, then its square ends in 1.
(*iii*) Sum of first 10 odd natural numbers is .....
(*iv*) Number of non-square numbers between 11^2 and 12^2 is .....
(*v*) Number of zeros in the end of the square of 400 is .....
(*vi*) Square of any ..... number can be expressed as the sum of two consecutive natural numbers.
(*vii*) For a natural number $m > 1$, $(2m, m^2 - 1, m^2 + 1)$ is called .....

2. State whether the following statements are true (T) or false (F):
   (*i*) All natural numbers are not perfect squares.
   (*ii*) A perfect square can never be expressed as the product of pairs of equal prime factors.
   (*iii*) A number having 2, 3, 7 or 8 at its unit place is never a square number.
   (*iv*) A number having 0, 1, 4, 5, 6 or 9 at its unit place is always a square number.
   (*v*) A number ending in an even number of zeros is always a perfect square.
   (*vi*) Square of an odd number is always an odd number.
   (*vii*) There are $2n$ non-square numbers between the squares of consecutive numbers n and $(n + 1)$.
   (*viii*) (4, 6, 8) is a Pythagorean triplet.

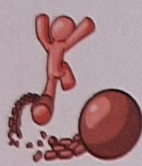
MULTIPLE CHOICE QUESTIONS

Choose the correct answer from the given four options (3 to 14):

3. How many natural numbers lie between 25^2 and 26^2?
   (*a*) 49 (*b*) 50 (*c*) 51 (*d*) 52
4. Square of an even number is always
   (*a*) even (*b*) odd (*c*) even or odd (*d*) none of these
5. $1 + 3 + 5 + 7 + \ldots$ up to n terms is equal to
   (*a*) $n^2 - 1$ (*b*) $(n + 1)^2$ (*c*) $n^2 + 1$ (*d*) n^2
6. $\sqrt{208 + \sqrt{2304}}$ is equal to
   (*a*) 18 (*b*) 16 (*c*) 14 (*d*) 22
7. $\sqrt{0.0016}$ is equal to
   (*a*) 0.04 (*b*) 0.004 (*c*) 0.4 (*d*) none of these
8. The smallest number by which 75 should be divided to make it a perfect square is
   (*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4
9. $\sqrt{3\frac{6}{25}}$ is equal to
   (*a*) $\frac{5}{9}$ (*b*) $\frac{4}{5}$ (*c*) $\frac{9}{5}$ (*d*) $\frac{5}{4}$
10. The smallest number by which 162 should be multiplied to make it a perfect square is
    (*a*) 4 (*b*) 3 (*c*) 2 (*d*) 1
11. If the area of a square field is 961 unit^2, then the length of its side is
    (*a*) 29 units (*b*) 41 units (*c*) 31 units (*d*) 39 units
12. The smallest number that should be subtracted from 300 to make it a perfect square is
    (*a*) 11 (*b*) 12 (*c*) 13 (*d*) 14
13. If one number of Pythagorean triplet is 6, then the triplet is
    (*a*) (4, 5, 6) (*b*) (5, 6, 7) (*c*) (6, 7, 8) (*d*) (6, 8, 10)
14. Given that $\sqrt{1521} = 39$, the value of $\sqrt{0.1521} + \sqrt{15.21}$ is
    (*a*) 42.9 (*b*) 4.29 (*c*) 3.51 (*d*) 35.1

Higher Order Thinking Skills (HOTS)

1. A square field is to be ploughed. Ramu get it ploughed in ₹34560 at the rate of ₹15 per sq. m. Find the length of side of square field.
2. If $\sqrt{2025} + \sqrt{0.0612 + x} = 45.25$, find the value of x.
3. Find the square root of 5, correct to 2 decimal places. Hence, find the value of $\sqrt{\dfrac{3+\sqrt{5}}{3-\sqrt{5}}}$.
4. Lalit has some chocolates. He distributed these chocolates among 13 children in such a way that he gave one chocolate to first child, 3 chocolates to second child, 5 chocolates to third and so on. Find the number of chocolates Lalit had.



Summary

★ Square of a number is the product of number by itself.

★ Squares of natural numbers are called *square numbers* or *perfect squares*.

★ All natural numbers are not perfect squares.

★ A perfect square (or square number) can always be expressed as the product of pairs of equal prime factors.

★ A square number always ends with 0, 1, 4, 5, 6 or 9 at unit's place. But converse is not true.

★ A number having 2, 3, 7 or 8 at its unit place is never a square number.

★ A number ending in an odd numbers of zeros is never a perfect square.

★ Squares of even natural numbers are even and squares of odd natural numbers are odd.

★ There are $2n$ non-square numbers between the squares of the consecutive numbers n and $(n + 1)$.

★ Sum of first n odd natural numbers is n^2.

★ If a natural number cannot be expressed as a sum of successive odd natural numbers starting with 1, then it is not a perfect square.

★ Square of any odd natural number greater than 1 can be expressed as the sum of two consecutive natural numbers.

★ Square of numbers with unit digit 5 can be found using $(a5)^2 = a\,(a + 1)$ hundreds + 25

★ Three natural numbers m, n, p are said to form a Pythagorean triplet (m, n, p) if $m^2 + n^2 = p^2$.

★ For any natural number $m > 1$, $(2m, m^2 - 1, m^2 + 1)$ is called a Pythagorean triplet.

★ The square root of a number n is that number which when multiplied but itself gives n as the product.

★ Square root is the inverse operation of finding square.

★ There are two integral square roots of a perfect square number, positive square root of a number is denoted by the symbol $\sqrt{\ }$

e.g. $3^2 = 9 \Rightarrow \sqrt{9} = 3$.

Check Your Progress

1. Show that 1089 is a perfect square. Also find the number whose square is 1089.
2. Find the smallest number which should be multiplied by 3675 to make it a perfect square. Also find the square root of this perfect square.
3. Express 144 as the sum of 12 odd numbers.
4. How many numbers lie between 99^2 and 100^2?
5. Write a Pythagorean triplet whose one number is 17.
6. Find the smallest square number which is divisible by each of the numbers 6, 8, 9.
7. In an auditorium the number of rows is equal to number of chairs in each row. If the capacity of the auditorium is 1764. Find the number of chairs in each row.
8. Find the length of diagonal of a rectangle whose length and breadth are 12 m and 5 m respectively.
9. Find the square root of following numbers by prime factorisation:
   (*i*) 5625 (*ii*) 1521
10. Find the square root of following numbers by long division method:
    (*i*) 21904 (*ii*) 108241
11. Find the square root of following decimal numbers:
    (*i*) 17.64 (*ii*) 13.3225
12. Find the square root of following fractions:
    (*i*) $1\frac{25}{144}$ (*ii*) $11\frac{225}{576}$
13. Find the least number which must be subtracted from 2311 to make it a perfect square.
14. Find the least number which must be added to 520 to make it a perfect square.
15. Find the greatest number of 5 digits which is a perfect square.