Multicapacity Process

Two Non-interacting Tanks

Non-interacting capacities always result in an overdamped / critically damped response. Never underdamped.

For n = no. of capacities in series, (1st order systems)

$$\boxed{T.F = \frac{K_{p1} \cdot K_{p2} \cdot K_{p3} \cdots K_{pn}}{(\tau_{p1} s+1)(\tau_{p2} s+1)(\tau_{p3} s+1)\cdots(\tau_{pn} s+1)}}$$

Interacting Tanks

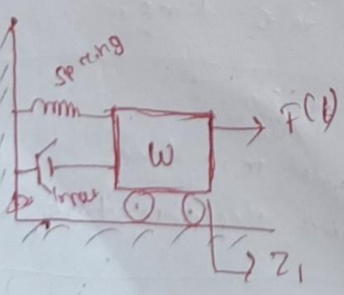
→ Poles are always distinct and real poles.

→ Two interacting capacities can be viewed as non-interacting capacities but with modified effective time constants.

→ Effect of interaction is to change the ratio of effective time constants for the two tanks. (One tank becomes faster, the other becomes slower).

→ The slower tank becomes controlling, and overall response becomes more sluggish due to interaction.

INHERENTLY SECOND ORDER SYSTEMS



(1) Spring (Towards left) → $-Ky$

K = Hooke's Constant

(2) Viscous friction (←) $-C\frac{dy}{dt}$

C = Damping Coefficient

(3) External force $(F(t))$ →

By Newton's law of motion, rate of change of momentum = Sum of the forces.

$$\omega \frac{d^2y}{dt^2} = -ky - c\frac{dy}{dt} + F(t)$$

ω = mass $\quad \frac{d^2y}{dt^2}$ = Acceleration

Dong

$$\frac{\omega}{k}\frac{d^2y}{dt^2} + \frac{c}{k}\frac{dy}{dt} + y = \frac{F(t)}{k}$$

$$\tau^2 \frac{d^2y}{dt^2} + 2\xi\tau\frac{dy}{dt} + y = x(t)$$

$$\boxed{\tau = \sqrt{\frac{\omega}{k}}} \qquad 2\times\xi\times\sqrt{\frac{\omega}{k}} = \frac{c}{k}$$

$$\Rightarrow \boxed{\xi = \frac{c}{2\sqrt{\omega k}}}$$

$$\tau^2 s^2 y(s) + 2\xi\tau s y(s) + y(s) = x(s)$$

with $y(0) = 0$, $y'(0) = 0$

$$\Rightarrow \boxed{\frac{y(s)}{x(s)} = \frac{1}{\tau^2 s^2 + 2\xi\tau s + 1}}$$

$$G(s) = \frac{1/\tau^2}{\left(s + \frac{\xi + \sqrt{\xi^2-1}}{\tau}\right)\left(s + \frac{\xi - \sqrt{\xi^2-1}}{\tau}\right)}$$

Step Input

For a step change in response

$$y(s) = \frac{1/\tau^2}{s\left(s + \frac{\xi + \sqrt{\xi^2-1}}{\tau}\right)\left(s + \frac{\xi - \sqrt{\xi^2-1}}{\tau}\right)}$$

For $\xi < 1$,

$$y(t) = 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi t/\tau} \sin\left(\sqrt{1-\xi^2}\,\frac{t}{\tau} + \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

$$y(t) = 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi t/\tau} \sin(\omega t + \phi)$$

For $\xi = 1$, $$y(t) = 1 - \left(1 + \frac{t}{\tau}\right) e^{-t/\tau}$$

For $\xi > 1$, $$y(t) = 1 - e^{-\frac{\xi t}{\tau}}\left(\cosh\sqrt{\xi^2-1}\,\frac{t}{\tau} + \frac{\xi}{\sqrt{\xi^2-1}} \sinh\sqrt{\xi^2-1}\,\frac{t}{\tau}\right)$$

Impulse Input

For $\xi < 1$,

$$y(t) = \frac{1}{\tau} \times \frac{1}{\sqrt{1-\xi^2}} e^{-\frac{\xi t}{\tau}} \sin\sqrt{1-\xi^2}\,\frac{t}{\tau}$$

For $\xi = 1$, $$y(t) = \frac{1}{\tau^2}\, t\, e^{-t/\tau}$$

For $\xi > 1$,

$$y(t) = \frac{1}{\tau} \times \frac{1}{\sqrt{1-\xi^2}} e^{-\frac{\xi t}{\tau}} \sinh\sqrt{\xi^2-1}\; t/\tau$$

Sinusoidal Input

For $t \to \infty$,

$$\boxed{y(t) = \frac{A}{\sqrt{(1-\omega^2\tau^2)^2 + (2\xi\tau\omega)^2}} \sin(\omega t + \phi)}$$

$$\boxed{\phi = \tan^{-1}\left(-\frac{2\xi\tau\omega}{1-\omega^2\tau^2}\right)}$$

Q5. The transfer function $G(s) = \frac{1}{s^2+s+1}$.

Determine the following parameters for a step change of magnitude 5, find

(a) % overshoot (b) Max. value of response

(c) ultimate value of response. (d) Decay Ratio

(e) Rise time (f) Period of oscillation

Ans

$$G(s) = \frac{1}{s^2+s+1}$$

$\tau^2 = 1 \Rightarrow \tau = 1$

$2 \times \xi \times 1 = 1$

$\Rightarrow \xi = 0.5$

$$\% \text{ O.S.} = e^{\frac{-\pi \times 0.5}{\sqrt{1-\xi^2}}} = 0.163 = 16.3\%$$

1

$$\frac{Y(s)}{5/s} = \frac{1}{s^2+s+1} \Rightarrow Y(s) = \frac{5}{s(s^2+s+1)}$$

$$\frac{5}{s(s^2+s+1)} = \frac{A}{s} + \frac{Bs}{s^2+s+1} + \frac{C}{s^2+s+1}$$

$5 = A(s^2+s+1) + Bs^2 + Cs(s^2+s+1)$

$s=0$ $A(1) = 5 \Rightarrow A = 5$

$s=1$ $5(3) + B + C(3) = 5 \Rightarrow B + 3C = -10$

$s=-1$ $5($

~~TRANSFER FUNCTIONS AND INPUT OUTPUT MODELS~~

$$D.R. = e^{-\frac{2\pi\xi}{\sqrt{1-\xi^2}}} = (O.S)^2 = 0.0265$$
$$= 2.65\%$$

$$U.V = \lim_{s \to 0} s\,Y(s) \quad \text{(Final value theorem)}$$

$$= \lim_{s \to 0} \frac{5s}{s(s^2+s+1)} = 5$$

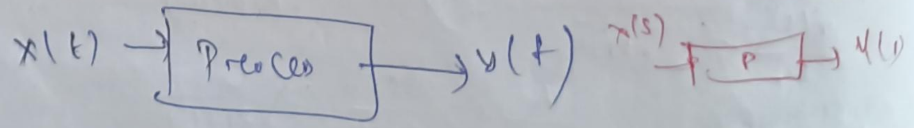
Max. value :- $5 \times (O.S + 1)$
$$= 5 \times 1.163$$
$$= 5.815$$

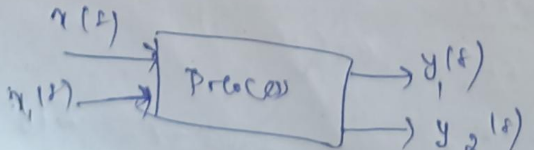
$$T = \frac{2\pi\tau}{\sqrt{1-\xi^2}} = \frac{2\pi \times 1}{\sqrt{1-0.5^2}} = 7.2552\, s$$

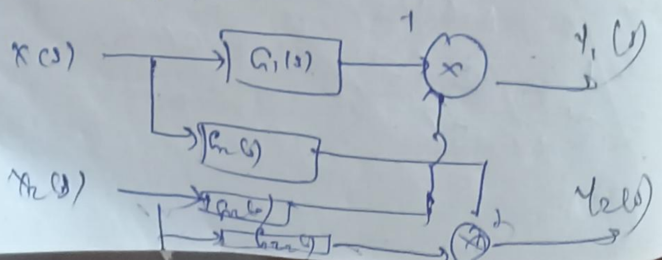
$t_R =$

U-tube manometer, Pneumatic control valve

TRANSFER FUNCTIONS AND INPUT OUTPUT MODELS







$G(s)$ = Transfer function that relates output to input.

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix}$$

This representation is called block diagram representation,

$G(s)$ is the transfer function that relates the input to the output, and the process diagram is known as block diagram

Output in Deviation

$$T.F = \frac{\text{Laplace Transform of Output in deviation form}}{\text{Laplace Transform of Input in Deviation form}}$$

The transfer function describes completely the dynamic behaviour of the output when the corresponding input changes are given

Eg.

$$Y_1(s) = G_{11}(s)\,X_1(s) + G_{12}(s)\,X_2(s)$$

$$Y_2(s) = G_{21}(s)\,X_1(s) + G_{22}(s)\,X_2(s)$$

Process with m inputs and n outputs, we have $m \times n$ matrix

POLES AND ZEROS OF A TRANSFER FUNCTION

$$\frac{Y(s)}{X(s)} = G(s)$$

Transfer function is a ratio of polynomials in s. ~~The~~

General notation for N polynomial :- Q(s)

General notation for D Polynomial :- P(s)

Normally Order of $Q(s) < P(s)$ and system with time delays, the numerator contain ~~polynomial~~ exp(Z).

- The roots of Q(s) are zeroes. ~~The roots of P(s) are poles~~ At zeroes, the transfer functions becomes zero. (The roots of numerator polynomial).
- The roots of P(s) are poles. At poles, the transfer functions becomes infinite.
- To invert this equation to time domain, all common inputs can be represented as ratio of polynomials.
- To invert Y(s) to time domain, we should know the roots P(s) and q(s). Thus the terms resulting from inverting by partial fraction are uniquely characterised by the poles and roots of q(s).

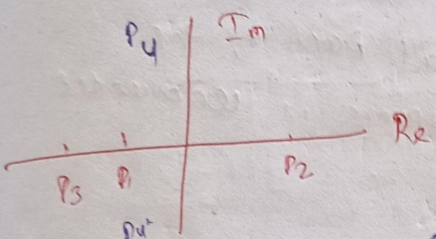
$$Y(s) = \frac{Q(s)}{P(s)} \times \frac{A(s)}{q(s)}$$

- If we know where the poles of a system are located in the complex s plane, we can ~~qualitatively~~ determine qualitatively the characteristic of the system response to a particular input.

without additional computations

$$G(s) = \frac{Q(s)}{P(s)} = \frac{Q(s)}{(s-P_1)(s-P_2)(s-P_3)^m(s-P_4)(s-P_4^*)(s-P_5)}$$

$$= \frac{C_1}{s-P_1} + \frac{C_2}{s-P_2} + \left[\frac{C_3}{s-P_3} + \frac{C_4}{(s-P_3)} + \frac{C_5}{(s-P_3)^m}\right] + \dots$$



, P_1 and P_2 on the real axis on inversion gives exponential terms such as $C_1 e^{P_1 t}$, $C_2 e^{P_2 t}$.

As $P_1 < 0$, $C_1 e^{P_1 t}$ decays exponentially to zero whereas $C_2 e^{P_2 t}$ grows exponentially to infinity.

P_1 and P_2 are real and distinct pole.

- Multiple real poles

$$\left(C_3 + \frac{C_{3}t}{1!} + \frac{C_3 t^2}{2!} \dots\right] e^{P_3 t}$$

If $P_3 > 0$, then, $e^{P_3 t} \to \infty$ as $t \to \infty$

If $P_3 < 0$, then $e^{P_3 t} \to 0$ as $t \to \infty$.

If $P_3 = 0$, then $e^{P_3 t} = 1$.

A real multiple pole gives rise to terms which either grows to infinity if the poles are positive or zero. or decay to zero

- Complex conjugate poles (P_4, P_4^*)

If $P_4 = \alpha + j\beta$ and $P_4^* = \alpha - j\beta$, then $e^{\alpha t}\sin(\beta t + \theta)$. It is a periodic oscillatory function

While ~~$e^{\alpha t}$~~ αt behaviour depends on real part "α".

For $\alpha > 0$, $e^{\alpha t} \to \infty$ as $t \to \infty$

For $\alpha < 0$, $e^{\alpha t} \to 0$ as $t \to \infty$

For $\alpha = 0$, $e^{\alpha t} = 1$.

Poles at Origin

Gives a constant after inversion.

For poles having positive real part, the system is unstable as the response is unbounded one.

N CAPACITIES IN SERIES

$$G(s) = \frac{K_{P1} K_{P2} K_{P3} \cdots K_{PN}}{(\tau_{P1}s+1)(\tau_{P2}s+1)\cdots(\tau_{PN}s+1)}$$

$$Y_N = Y_0 e^{-t/\tau}\left[\frac{t^{N-1}}{\tau^{N-1}(N-1)!} + \frac{t^{N-2}}{\tau^{N-2}(N-2)!} + \cdots + \frac{t}{\tau} + 1\right].$$

The denominator of transfer function will be nth order polynomial in s.

$$a_N s^N + a_{N-1} s^{N-1} + \cdots + a_0 s_1 + a_0$$

$$G(s) = G_1(s) \times G_2(s) \cdots \times G_N(s)$$

- For n non-interacting capacities in series, the response of the characteristics of an overdamped system (S shaped) and sluggish.
- Increase in no. of capacities increases the sluggishness.

For n interacting capacities in series, interaction increases the sluggishness of the overall response

- For such systems, Controller is necessary to improve the speed of response and keep output at desired value.

Ex :- Jacketed ~~water~~ batch cooler

(a) Heat capacity of the mixture in the tank
(b) Heat capacity of the tank's wall.
~~(c) the~~ (c) Heat capacity of the coolant in the jacket.

For jacketed continuous flow cooler, same heat supply is there,

(a) Total material capacity of tank,
(b) Tank's capacity for component A.
(c) Heat capacity of tank's content, tank wall and coolant in jacket.

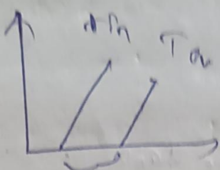
~~DISTILLA~~
Distillation or Gas Absorption Columns

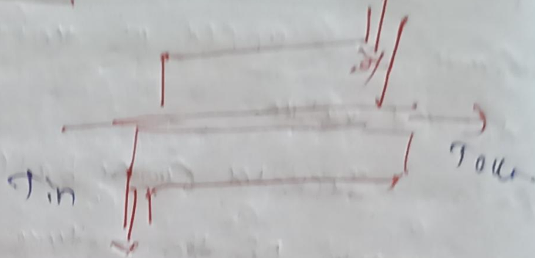
→ In tray column, each tray has two capacities.
(→ mass and heat.)

→ n no. of trays are there → n

Syst

SYSTEM WITH DEAD TIME





t_d → Dead Time

1st order

$f(t)$ → [1st order Process] → $y(t)$ → [Dead Time] → $y(t-t_d)$

$$f(s) \rightarrow \boxed{\frac{K_p}{\tau_p s+1}} \rightarrow \boxed{e^{-t_d s}} \rightarrow L\{y(t-t_d)\}$$

$$\frac{L\{y(t-t_d)\}}{Lf(t)} = \frac{K_p e^{-t_d s}}{\tau_p s+1}$$

2nd Order

$$f(s) \rightarrow \boxed{\frac{K_p}{\tau^2 s^2 + 2\xi\tau s+1}} \rightarrow \boxed{e^{-t_d s}} \rightarrow \frac{K_p e^{-t_d s}}{\tau^2 s^2 + 2\xi\tau s+1}$$

$$e^{-t_d s} \approx \frac{1-\frac{t_d}{2}s}{1+\frac{t_d}{2}s} \quad \text{(1st order approximation)}$$

$$e^{-t_d s} \approx \frac{t_d^2 s^2 - 6t_d s + 12}{t_d^2 s^2 + 6t_d s + 12} \quad \text{(2nd order approximation)}$$

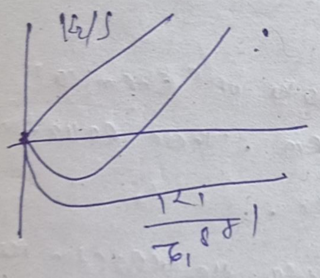
With dead time, order of system increases, Process with dead time are difficult to control and the output knows no information about current events.

SYSTEM WITH INVERSE RESPONSE

Initially the response is in opposite direction where it ends. Such behaviour is called inverse response or non-minimum phase response.

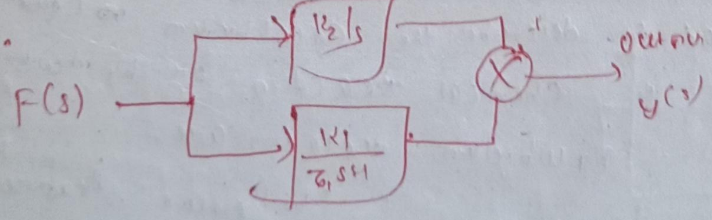
The volume of decreases due to decrease in volume of entraped vapour. The constant heat supply, once again, the level starts rising that leads to a pure capacity response. The result of two opposing effects is represented by:-

$$\frac{K_2}{s} + \frac{K_1}{\tau_1 s + 1} = \frac{(K_2 \tau_1 - K_1) s + K_2}{s(\tau_1 s + 1)}$$



When $K_2 \tau_1 < K_1$, $\frac{-K_1}{\tau_1 s + 1}$ Dominating

At point 3, $\frac{-K_2}{K_2 \tau_1 + K_1} > 0$



ADDITIONAL ELEMENTS OF MATHEMATICAL MODELS

In addition to balance equations, we need other relationships to express thermodynamic equilibria, reaction rates, transport rates for mass, momentum and heat and equation of state.

Transport rate equation

These are required to describe the rate of mass, energy and momentum transfer between a system and its surrounding.

Ex- In tank heater, heat supplied by the steam to the liquid is given by :

$$q = UA(T_{st} - T)$$

Kinetic rate equation

Needed to describe the rates of chemical reactions taking place in a system. These equations are based on chemical kinetics.

Ex- For 1st order reaction in CSTR, the reaction rate

$$r = -k_1 C_A$$

$$k_1 = k_0 e^{-\frac{E}{RT}}$$

Reaction and phase equilibria equations

Needed to describe the equilibrium situations reached during a chemical reaction or by two or more phases. Equilibrium relations are -

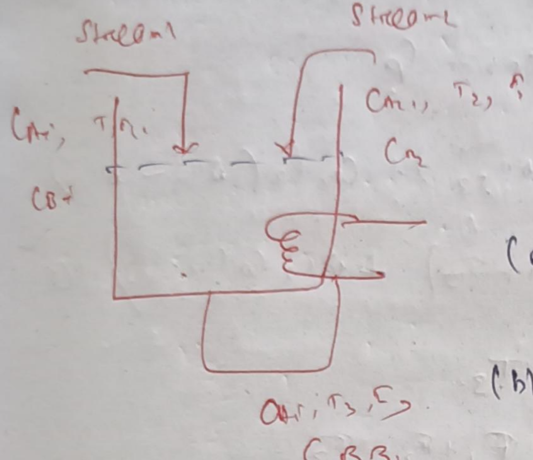
(a) $T_{vapour} = T_{pressure}$ (b) $P_{liquid} = P_{vapour}$

(c) $\mu_{i,l} = \mu_{i,g}$

Equation of State

Needed to describe the relationship among the intensive variables of a system describing the thermodynamic state.

Ex- Ideal gas law, Van der Waal equation



Fundamental quantities needed to developing process are :-

(a) Total mass of tank including A and B in the tank.

(b) Total energy

$$(\rho_1 F_1 + \rho_2 F_2) - \rho_3 F_3 = \frac{d}{dt}(\rho_3 V)$$

$V = Ah$

$\rho_3 = f(C_{A3}, C_{B3}, T_3)$ $\quad \rho_2 = f(C_{A2}, C_{B2}, T_2)$

$\rho_1 = f(C_{A1}, C_{B1}, T_1)$

$\rho_1 = \rho_2 = \rho_3 = \rho$

$$\frac{d}{dt}(V C_A) = V\frac{dC_A}{dt} + C_A\frac{dV}{dt} = (C_{A1}F_1 + C_{A2}F_2) - C_{A3}F_3$$

$V \frac{dC_A}{dt} =$

$$\frac{d}{dt}(\rho V h_3) = \rho(F_1 h_1 + F_2 h_2) - \rho F_3 h_3 \pm Q$$

$h_1(T_1) = h_1(T_0) + C_{p1}(T_1 - T_0)$

$h_2(T_2)$

$h_3(T_3) = h_3(T_0)$

$\rho h_3(T_0) = C_{A3} \tilde{H}_A + C_{B3} \tilde{H}_B + C_{A3} \Delta H_{R3} (T_0)$

$$\rho C_{P3} V \frac{dT_3}{dt} = C_{A1} F_1 (\Delta \tilde{H}_{R1} - \Delta \tilde{H}_{R3}) + C_{A2} F_2 (\Delta \tilde{H}_{R2} - \Delta \tilde{H}_{R3})$$
$$+ \rho F_1 [C_{P1}(T_1 - T_0) - C_{P3}(T_3 - T_0)]$$
$$+ \rho F_2 [C_{P2}(T_2 - T_0) - C_{P3}(T_3 - T_0)] \pm Q$$

Assuming $C_{P1} = C_{P2} = C_{P3} = C_p$

$$\rho C_p V \frac{dT_3}{dt} = C_{A1} F_1 (\Delta \tilde{H}_{S1} - \Delta \tilde{H}_{S3}) + C_{A2} F_2 (\Delta \tilde{H}_{S2} - \Delta \tilde{H}_{S3})$$
$$+ \rho_1 F_1 C_p (T_1 - T_3) + \rho F_2 C_p (T_2 - T_3) \pm Q$$

State variables :- V, C_{A3}, T_3.

State equations :- $\frac{dV}{dt} = F_1 + F_2 - F_3$

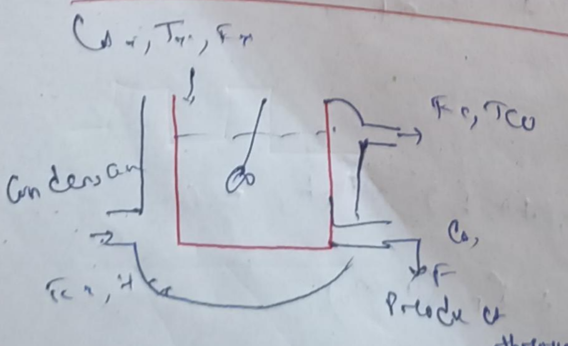
$$V \frac{dC_{A3}}{dt} = (C_{A1} - C_{A3}) F_1 + (C_{A2} - C_{A3}) F_2$$

Input variables :- $F_1, C_{A1}, T_1, F_2, C_{A2}, T_2, F_3$

Output variables :- V or h, C_{A3}, T_3.

Parameters :- $\rho, C_p, \Delta \tilde{H}_{S1}, \Delta \tilde{H}_{S2}, \Delta \tilde{H}_{S3}$.

MATHEMATICAL MODEL OF CSTR



Exothermic reaction $A \to B$ is taking place in the reactor. Reactor is cooled by coolant flow through the jacket. The fundamental dependent quantities are -

(a) Total mass of reacting mixture in the tank.

(b) Mass of chemical A in the reacting mixture.

(c) Total energy.

~~Total~~ Total mass balance :-

$$\frac{d}{dt}(\rho V) = \rho_i F_i - \rho F \pm 0$$

$$\frac{d(N_A)}{dt} = \frac{d(C_A V)}{dt} = C_{Ai} F_i - C_A F_i - r V$$

$$\frac{dH}{dt} = \rho_i F_i h_i (T_i) - \rho F h(T) - Q$$

~~Mass of~~ Mass of component B can be found from total mass and mass of A.

r = Rate of Reaction.

N_A = No. of moles of A.

~~Energy~~ $E = U + K + P$ $\qquad \frac{dK}{dt} = 0 = \frac{dP}{dt}$

$$\frac{dE}{dt} = \frac{dU}{dt} \approx \frac{dH}{dt}$$

To represent balance equations in better form for process control, identification of appropriate state variables is necessary.

There is weak dependency of density on C_A, C_B and T.

Then $\rho_i = \rho$ $\qquad \frac{d}{dt}(\rho V) = \rho \frac{dV}{dt}$

$$\frac{dV}{dt} = F_i - F \qquad V = \text{state variable.}$$

$$\frac{d}{dt}(C_A v) = C_A \frac{dv}{dt} + v\frac{dC_A}{dt}$$

$$v\frac{dC_A}{dt} = -C_A(F_f - F) + C_{Af}F_f - C_A F - k_0 e^{-E/RT} C_A v$$

$$r = k_0 e^{-E/RT} C_A$$

C_A and v are state variables.

$$\frac{dH}{dt} = \frac{\partial H}{\partial T}\cdot\frac{dT}{dt} + \frac{\partial H}{\partial n_A}\frac{\partial n_A}{\partial t} + \frac{\partial H}{\partial n_B}\frac{dn_B}{dt}$$

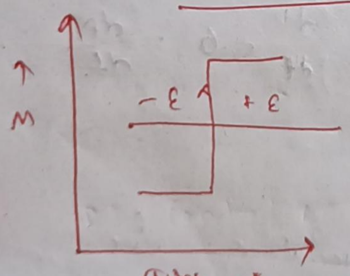
$$\frac{dn_B}{dt} = \frac{d}{dt}(C_B v) = 0 - C_B F + r v$$

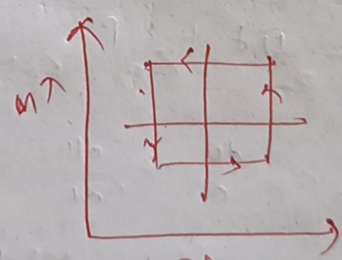
State variables :- v, C_A, T

State eqn :- $\frac{dv}{dt} = (F_f - F)$

The manipulated variables are Q and F.

ON-OFF CONTROL





In this control action, manipulated variable is either maximum or minimum depending upon the controlled variable is greater or less than the set point.

A hysteresis loop is developed. A differential gap is generated due to time lag. The manipulated variable maintain its previous value.

On/off controller is cheapest and most widely used controller.

For this control action, there is a continuous linear relation between the deviation values and manipulated variable.

PROPORTIONAL CONTROLLER

Output $\propto$ Error,

$$C_t = K_c\,\mathcal{E}(t) + C_s$$

where K_c = proportional gain or sensitivity.

C_s = Constant controller via signal.

K_c = Constant parameter expressed by proportional band PB.

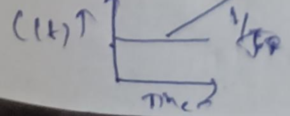
Higher the value of K_c, higher is the sensitivity of controller.

$$C(t) - C_s = K_c\,\mathcal{E}(t)$$

$$\Rightarrow C'(s) = K_c\,\mathcal{E}(s)$$

$$\Rightarrow \boxed{\frac{C'(s)}{\mathcal{E}(s)} = G_c = K_c}$$

INTEGRAL CONTROL ACTION



Integral control action changes the manipulated variable at a rate proportional to the deviation

$$\frac{d(t)}{dt} = \frac{\varepsilon(t)}{\tau_I}$$

$$C'(s) = \frac{\varepsilon(s)}{\tau_I s}$$

$$\Rightarrow G_c = \frac{C'(s)}{\varepsilon(s)} = \frac{1}{\tau_I s}$$

DERIVATIVE CONTROL ACTION

It is the control action in which the manipulated variable is proportional to rate of change of deviation

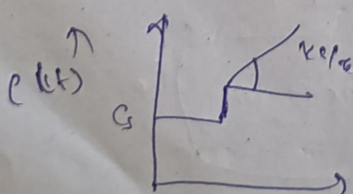
$$C(t) = K_c \tau_D \frac{d\varepsilon(t)}{dt}$$

The derivative action anticipates the effect of large load changes or error, and reduces the maximum error.

This control action is suitable for systems where error changes rapidly. For step change in error the output of derivative control action is

PI Control Action

$$C(t) = K_c \varepsilon(t) + \frac{K_c}{\tau_I} \int_0 \varepsilon(t)\,dt + C_s$$



Reset time is an adjustable parameter referred as minutes/repeat. Usually it varies in the range 0.1 to 50 minute

Some time the controller is calculated as $\frac{1}{\tau_I}$ (Repeats/min). Known as reset rate.

For unit step change in error at $t=0$, contribution of integral time is 0. The output of controller will be $K_c e(t) + \frac{K_c}{\tau_I} \int_0^{\tau_I} \varepsilon(t)\,dt$

The integral action repeated the response of proportional action. So reset time is the time needed by controller to repeat the initial proportional action change on its output. The integral action changes the manipulate variable until error is eliminated in the process output, thus eliminate error in the process output.

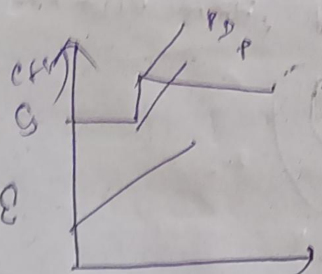
$$\boxed{G(s) = K_c\left(1 + \frac{1}{\tau_I s}\right)}$$

Q. And causes large overshoots.

The model represented by PDE and the model is a distributed parameter model. In lumped parameter model, lengths not a independent variable.

PROPORTIONAL DERIVATIVE (PD) Control Action

$$C(t) = K_c \mathcal{E}(t) + K_c \tau_D \frac{d\mathcal{E}(t)}{dt} + C_s$$



$$C(t) = A K_c t + A K_c \tau_D + C_s.$$

It is the additive effect of proportional and derivative actions. Mathematically,

$$C(t) = K_c \mathcal{E}(t) + K_c \tau_D \frac{d\mathcal{E}(t)}{dt} + C_s.$$

The derivative action also called as rate control or anticipatory control.

For linear change in error, $\mathcal{E}(t) = At$

$$C(t) = \frac{C'(s)}{\mathcal{E}(s)} = K_c(1 + \tau_D s)$$

C(t) then changes linearly at a rate $A K_c$. The derivative action anticipates the linear change in error and adds a additional output δ on to the proportional action

$$C(s) = K_C\,\mathcal{E}(s) + K_C \tau_D\, s\,\mathcal{E}(s) + C_s$$

$$\Rightarrow C(s) - C_s = K_C(1 + \tau_D s)\,\mathcal{E}(s)$$

$$\Rightarrow \boxed{\frac{C'(s)}{\mathcal{E}(s)} = K_C(1 + \tau_D s)}$$

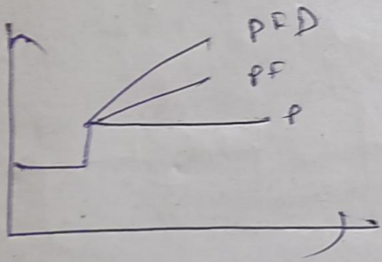
For noisy response this controller compute large derivative and yields large control action although it is not needed.

PID Controller

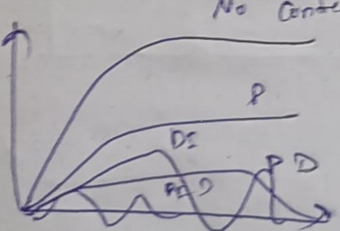
Proportional + reset + rate controller.

$$C(t) = K_C\,\mathcal{E}(t) + \frac{K_C}{\tau_I}\int_0^t \mathcal{E}(t)\,dt + K_C \tau_D \frac{d\mathcal{E}(t)}{dt} + C_s$$

$$\boxed{C(s) = \frac{C'(s)}{\mathcal{E}(s)} = K_C\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)}$$



For step change in error, the output of 3 controllers P, PI and PID are shown



PID is the most costly controller. The deviation is almost 0.

The controller selection depends upon the problem characteristic. The most general observations are:-

1. Proportional action :- There is offset $\neq 0$. Large oscillation and maximum action.

→ Smallest maximum error. Stabilising Time is also less. Offset is less than the proportional action. (Almost half)

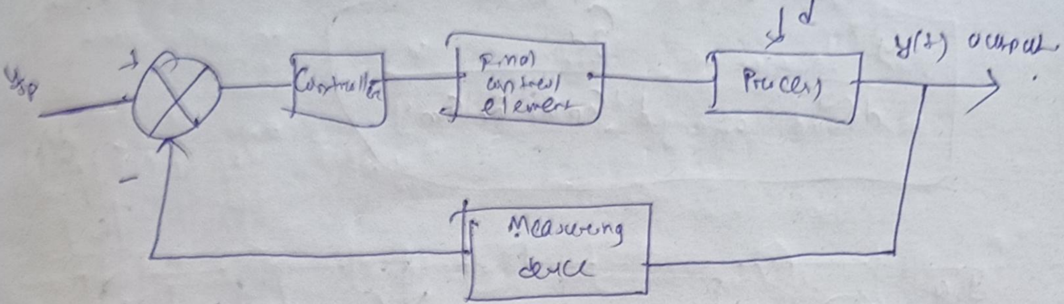
PI Controller

It has the next ~~maximum~~ smallest maximum deviation. Offset → 0. Stability time increases by integral action.

Offset = Set value − Ultimate value of Response

⑥

CLOSED LOOP RESPONSE



G_p = Process transfer function

G_m = Measuring Device transfer function

G_c = Controller Transfer function

G_f = Final control element transfer function

G_d = Disturbance transfer function.

Process :- $y(s) = G_p(s)\, G_m(s) + G_d(s)\, d'(s)$

Measuring device :- $y_m'(s) = G_m(s)\, y'(s)$

Comparator :- $\varepsilon(s) = y'_{set}(s) - y_m'(s)$

Final control element $G_m(s) = G_c(s)\, d(s)$

$$[1 + G_p(s)\, G_c(s)\, G_m(s)\, G_f(s)]\, y'(s) = G_f(s)\, G_p(s)\, G_c(s)\, y_{set}'(s) + G_d(s)\, d'(s)$$

$$\boxed{y'(s) = \frac{G_p G_c G_f\, y_{set}(s)}{1 + G_p G_c G_f G_m} + \frac{G_d}{1 + G_p G_f G_c G_m} d'(s)}$$

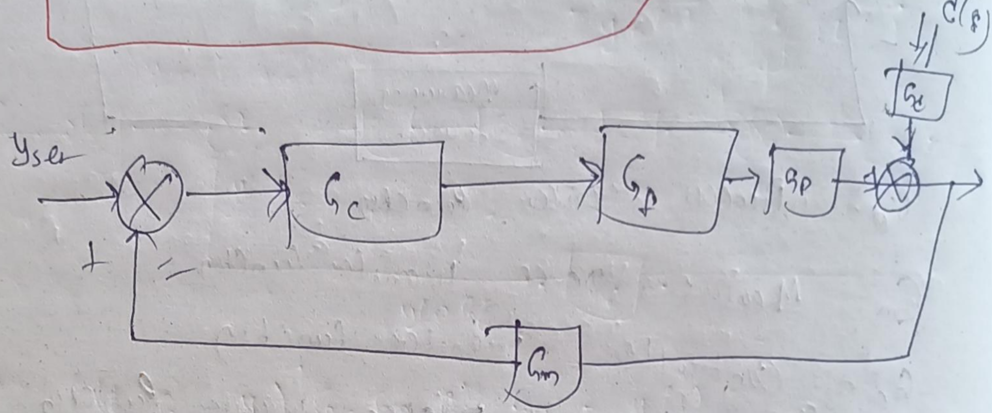
For change in set point

$$\frac{y'(s)}{y_{set}(s)} = \frac{G_p G_f G_c}{1 + G_p G_f G_c G_m} = G_{sp}(s)$$

→ Called as Servo problem

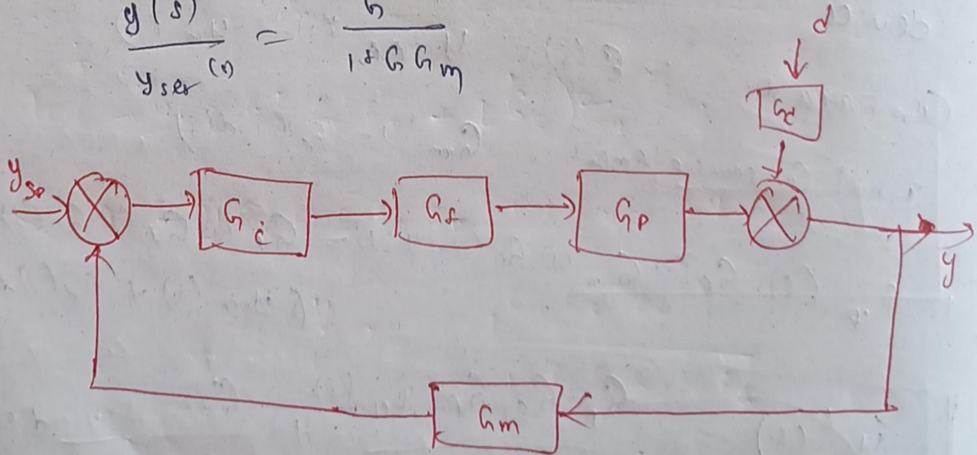
Q. For change in disturbance, it is regulatory problem

$$\boxed{\frac{y'(s)}{d(s)} = \frac{G_d}{1+G_p G_f G_c G_m}}$$



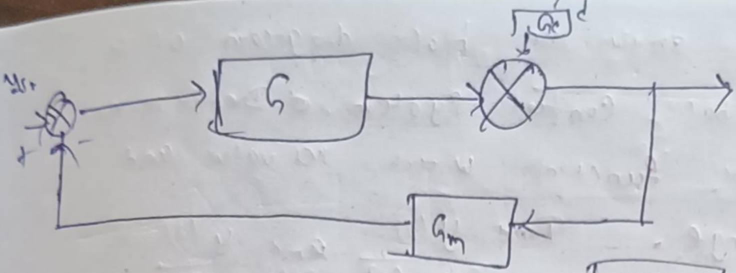
For The transfer function of the servo problem is the ratio of forward path transfer functions upon (1 + product of forward path transfer function and backward path transfer function)

$$\frac{y(s)}{y_{set}(s)} = \frac{G}{1+G G_m}$$

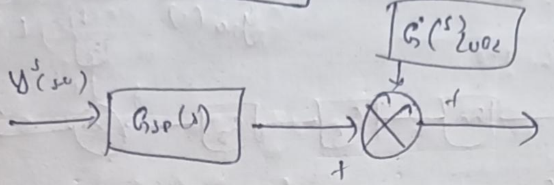


$$y'(s) = \frac{G_p G_f G_c}{1+G_p G_f G_c G_m} y'_{sp}(s) + \frac{G_d}{1+G_p G_f G_c G_m} d'(s)$$

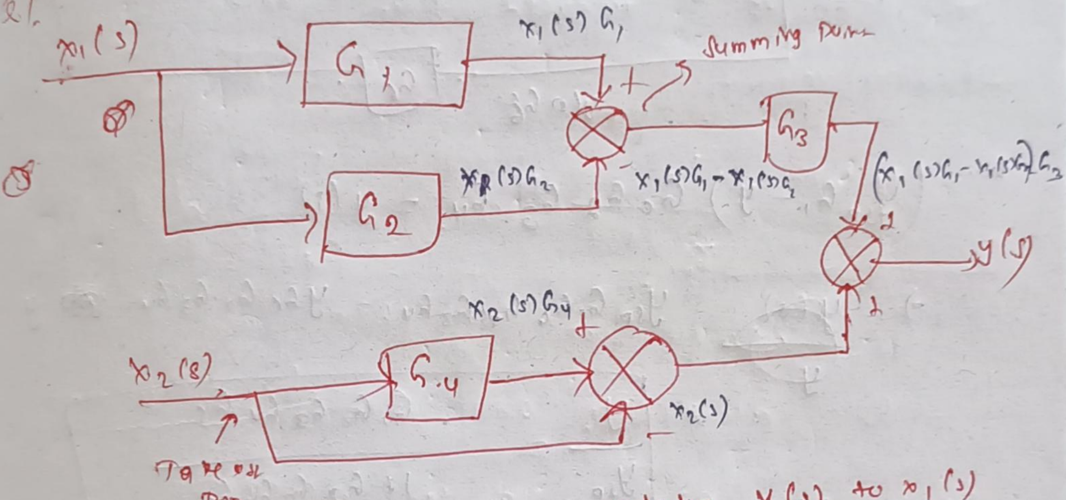
$$G_{sp}(s) = \frac{G}{1+G G_m} \qquad G_{load}(s) = \frac{G_d}{1+G G_m}$$



Servo
Regulation



Q.



Determine the transfer function relating $Y(s)$ to $X_1(s)$ and $X_2(s)$ from the block diagram shown in the figure.

Ans.

$$(X_1(s)G_1 - X_2(s)G_2)G_3 + (X_2(s)G_4 - X_2(s)) = Y(s)$$

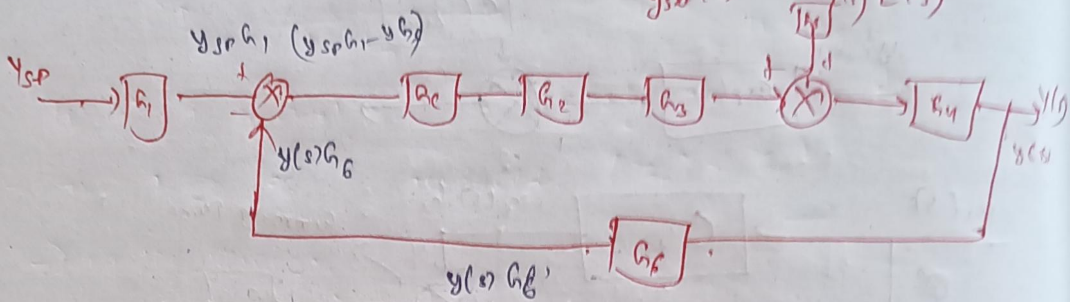
$$\Rightarrow X_1(s)G_1G_3 - X_2(s)G_2G_3 + X_2(s)G_4 - X_2(s) = Y(s)$$

For $X_2(s) = 0$,

$$\frac{Y(s)}{X_1(s)} = G_1G_3 - G_2G_3$$

For $X_1(s) = 0$, $\frac{Y(s)}{X_2(s)} = G_4 - 1$

Q. 2. The figure shows the block diagram of a typical feed back control system. Determine the transfer functions w.r.t. set value and the load change. Find $\frac{y(s)}{y_{sp}(s)}$ and $\frac{y(s)}{L(s)}$.



Ans. $(y_{sp}G_1 - y)\, G_c G_2 G_3 G_4 = y(s)$

$\Rightarrow y_{sp} G_1 G_c G_2 G_3 G_4 - y G_c G_2 G_3 G_4 = y$

$$\Rightarrow \boxed{\frac{y}{y_{sp}} = \frac{G_1 G_2 G_3 G_4 G_c}{1 + G_2 G_3 G_4 G_c}}$$

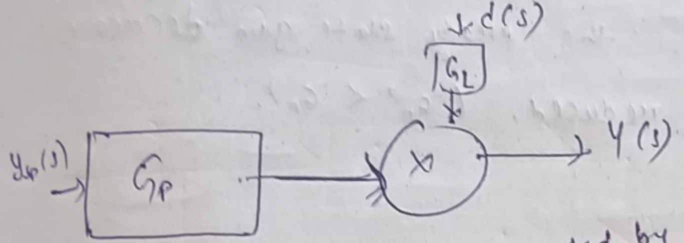
$(-yG_6 G_c G_2 G_3 + L G_5) G_4 = y$

$\Rightarrow -y G_6 G_c G_2 G_3 G_4 + L G_4 G_5 = y$

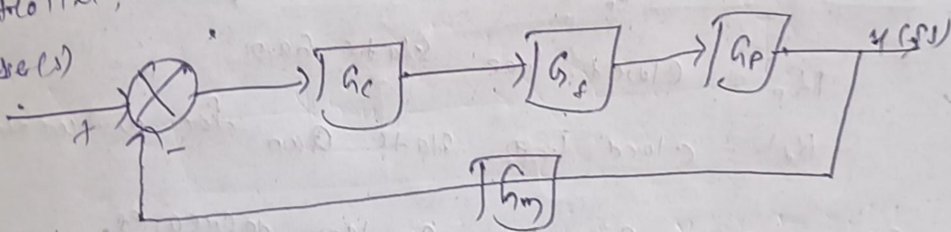
$$\Rightarrow \frac{y}{L} = \frac{G_4 G_5}{1 + G_c G_2 G_3 G_4 G_6}$$

EFFECT OF PROPORTIONAL CONTROLLER ON FIRST ORDER SYSTEM

$$\boxed{G_p(s) = \frac{K_p}{\tau_p s + 1}} \qquad G_L(s) = \frac{K_d}{\tau_p s + 1}$$



If this system is controlled by a proportional controller,



For servo problem

$$\frac{y(s)}{y_{sp}(s)} = \frac{G_c G_f G_p}{1 + G_c G_f G_p G_m}$$

Assuming $G_f = G_m = 1$ $\quad G_p = \frac{K_p}{\tau_p s + 1}$, $\quad G_c = K_c$

$$\frac{y(s)}{y_{sp}(s)} = \frac{\dfrac{K_p K_c}{\tau_p s + 1}}{1 + \dfrac{K_p K_c}{\tau_p s + 1}}$$

$$\Rightarrow \frac{y(s)}{y_{sp}(s)} = \frac{K_p K_c}{\tau_p s + 1 + K_p K_c}$$

$$\Rightarrow \boxed{\frac{y(s)}{y_{sp}(s)} = \frac{\dfrac{K_p K_c}{1 + K_p K_c}}{\left(\dfrac{\tau_p}{1 + K_p K_c}\right) s + 1}}$$

$$\boxed{\frac{y(s)}{y_{sp}} = \frac{K_p'}{\tau_p' s + 1}}$$

$$K_p' = \frac{K_p K_c}{1 + K_p K_c} \qquad \tau_p' = \frac{\tau_p}{1 + K_p K_c}$$

With P controller, the steady state gain decreases.

The ~~term~~ τ is reduced. ($\tau_p' < \tau_p$).

→ Faster response.

$$y'(s) = \frac{K_p'}{\tau_p' s + 1} y_{set}(s) + \frac{K_d'}{\tau_p' s + 1} d(s)$$

K_p' = Closed loop state gain

K_d' = Closed loop state gain for disturbance

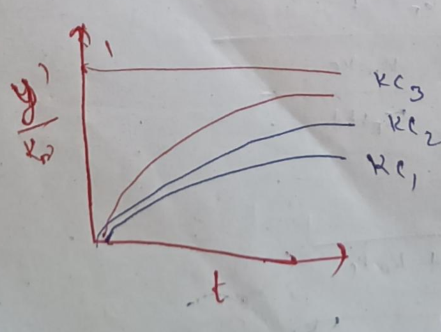
→ State gain and time constant decreases.

→ First order system remains first order.

For servo problem, let $y_{sp}(s) = \frac{1}{s}$.

$$y'(s) = \frac{K_p'}{(\tau_p' s + 1) s}$$

$$\boxed{y'(t) = K_p' \left(1 - e^{-t/\tau_p'}\right)}$$



The ultimate value of response after $t \to \infty$ will be K_p'.

$K_p' < K_p$

Response never reaches the desired set point. There will always ~~offset~~ offset with P controller.

Offset = New value of set Point − Ultimate value of Response

$$= 1 - K_p' = 1 - \frac{K_p K_c}{1 + K_p K_c}$$

$$= \frac{K_p K_c - K_p K_c + 1}{1 + K_p K_c}$$

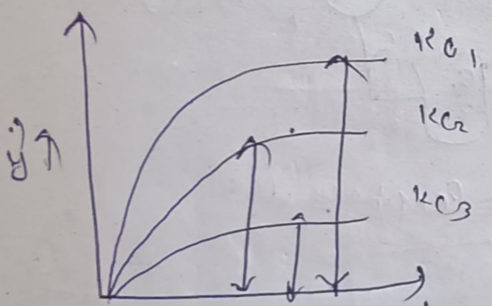
$$= \frac{1}{1 + K_p K_c}$$

Hence, $\boxed{\text{Offset} = \frac{1}{1 + K_p K_c}}$

$K_{c1} > K_{c2} > K_{c3}$ (offset for)

Gain :- $K_{c3} > K_{c2} > K_{c1}$

For regulator problem,



Regulator Problem

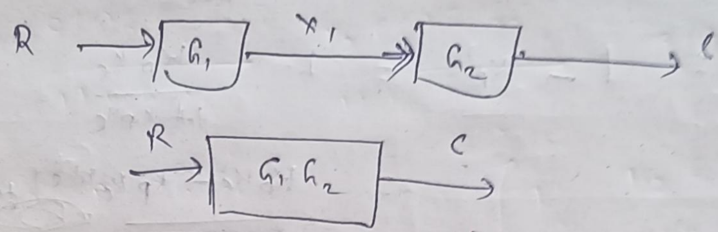
$K_{c1} < K_{c2} < K_{c3}$

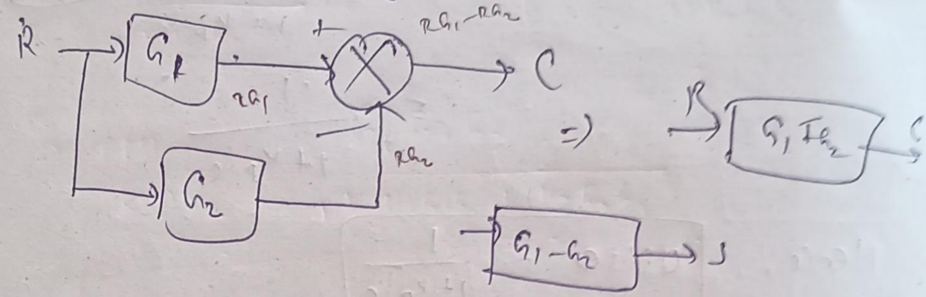
↓

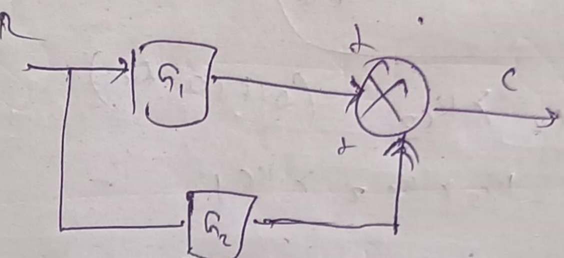
Offset

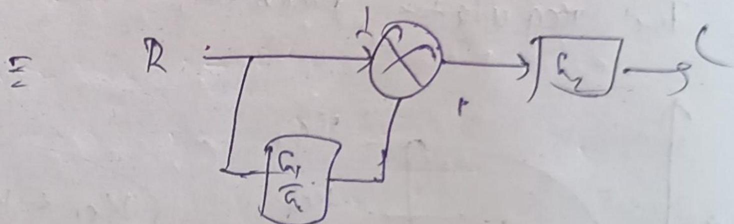
Extreme high values of K_c causes instability

BLOCK DIAGRAM REDUCTION

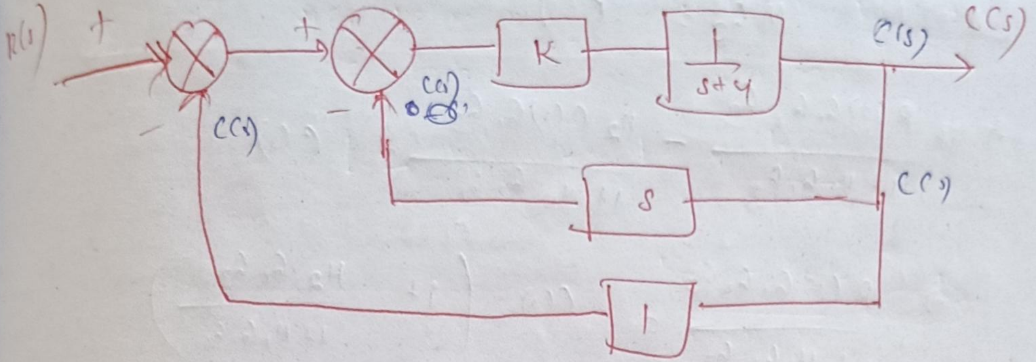
1. 

2. 

3. 



Q ~~Rearrangement of Feedback path~~



$$\left(R(s) - C(s) - C(s)s\right) K \times \frac{1}{s+4} = C(s)$$

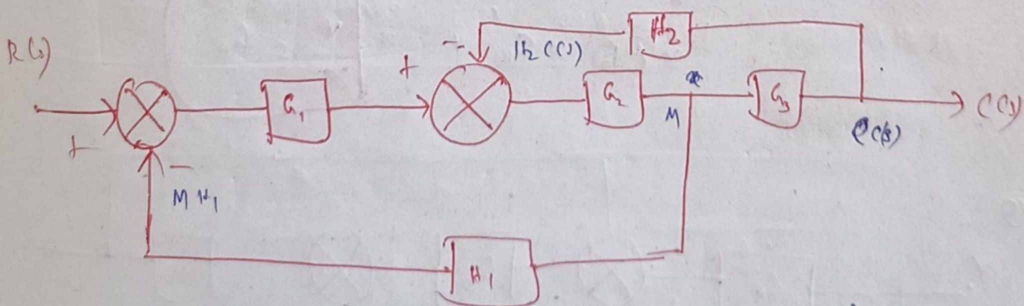
$$\Rightarrow \left(R(s) - C(s) - C(s)s\right)\frac{K}{s+4} = C(s)$$

$$\Rightarrow \frac{R(s)K}{s+4} - \frac{C(s)K}{s+4} - \frac{C(s)sK}{s+4} = C(s)$$

$$\Rightarrow \frac{R(s)K}{s+4} = C(s)\left(1 + \frac{Ks}{s+4} + \frac{K}{s+4}\right)$$

$$\Rightarrow \cancel{\frac{C(s)}{R(s)} = \frac{K}{s+4+2K}}$$

$$\Rightarrow \boxed{\frac{C(s)}{R(s)} = \frac{K}{s+4+Ks+K}}$$



$$\left((R(s) - MH_1)\cdot G_1 - H_2\, C(s)\right) G_2 = M$$

$$\Rightarrow \left(R(s)G_1 - MH_1G_1 - H_2\, C(s)\right) G_2 = M$$

$$\Rightarrow R(s)G_1G_2 - MH_1G_1G_2 - H_2\, C(s)G_2 = M$$

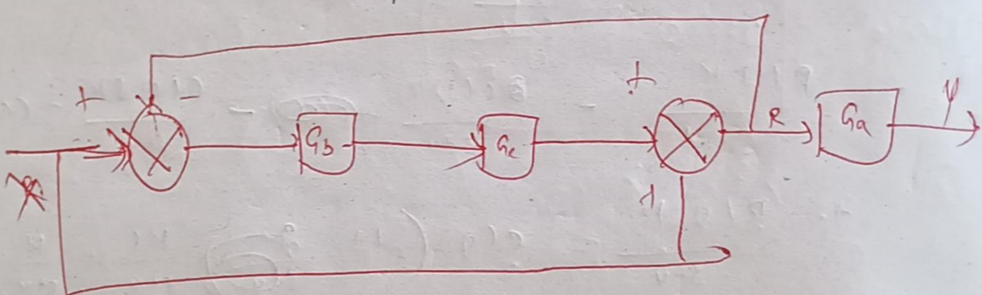
$$\Rightarrow M = \frac{R(s)G_1G_2 - H_2\, C(s)G_2}{1+H_1G_1G_2}$$

$$M G_B = C(s)$$

$$\Rightarrow \frac{R(s) G_1 G_2 G_3}{1 + H_1 G_1 G_2} - \frac{H_2 C(s) G_2 G_3}{1 + H_1 G_1 G_2} = C(s)$$

$$\Rightarrow \frac{R(s) G_1 G_2 G_3}{1 + H_1 G_1 G_2} = C(s)\left(1 + \frac{H_2 G_2 G_3}{1 + H_1 G_1 G_2}\right)$$

$$\Rightarrow \frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3}{1 + H_1 G_1 G_2 + H_2 G_2 G_3}$$



$$(X - R) G_b G_c + X = R$$

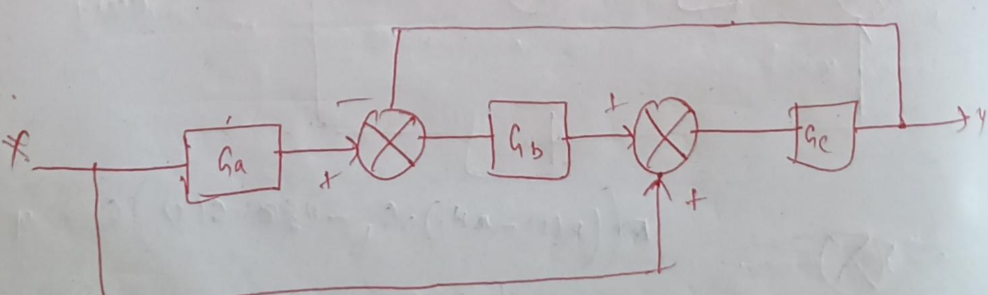
$$\Rightarrow X G_b G_c - R G_b G_c + X = R$$

$$\Rightarrow R = \frac{X(G_b G_c + 1)}{G_b G_c + 1} = X$$

$$R G_a = Y$$

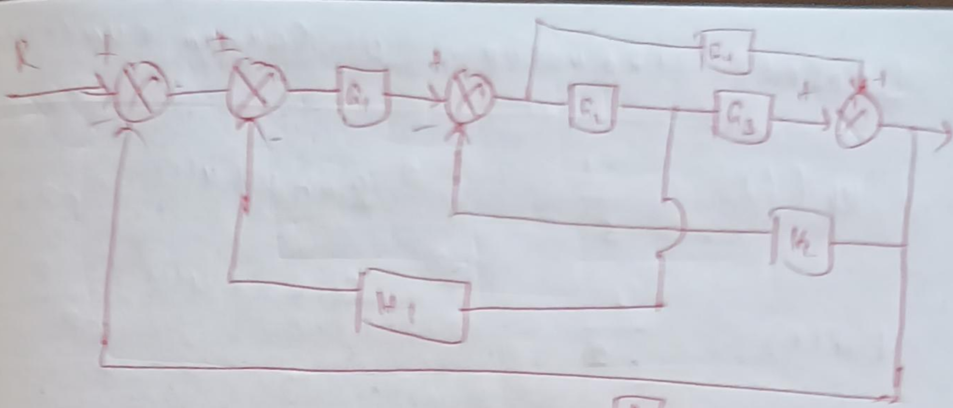
$$\Rightarrow X G_a = Y \Rightarrow \boxed{\frac{Y}{X} = G_a}$$

Q.

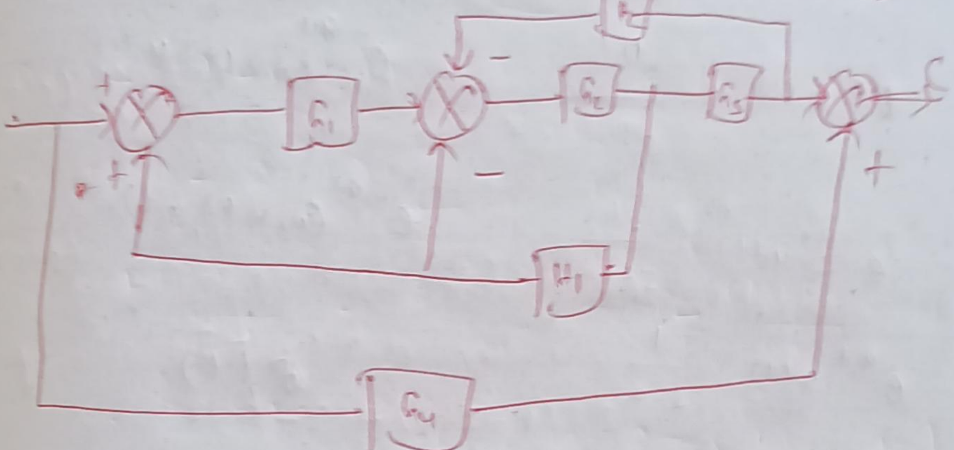


Determine $\frac{Y(s)}{X(s)}$.

Q



Q.



LAPLACE TRANSFORM OF COMMON FUNCTIONS

$u(t)$	$\frac{1}{s}$	$\sinh kt\, u(t)$	$\frac{k}{s^2-k^2}$
$t\,u(t)$	$\frac{1}{s^2}$	$\cosh kt\, u(t)$	$\frac{s}{s^2-k^2}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$	$e^{-at}\sin k(t)\, u(t)$	$\frac{k}{(s+a)^2+k^2}$
$e^{-at} u(t)$	$\frac{1}{s+a}$	$e^{-at}\cos k(t)\, u(t)$	$\frac{s+a}{(s+a)^2+k^2}$
$e^{at} u(t)$	$\frac{1}{s-a}$	$\delta(t)$	1
$t^n e^{-at} u(t)$	$\frac{n!}{(s+a)^{n+1}}$		
$\sin kt\, u(t)$	$\frac{k}{s^2+k^2}$		
$\cos kt\, u(t)$	$\frac{s}{s^2+k^2}$		

INVERSE LAPLACE OF SOME SELECTED EXPRESSIONS

$$\frac{1}{(s+a)(s+b)} = \frac{e^{-at}-e^{-bt}}{b-a}$$

$$\frac{1}{(s+a)(s+b)(s+c)} = \frac{e^{-at}}{(b-a)(c-a)} + \frac{e^{-bt}}{(c-b)(a-b)} + \frac{e^{-ct}}{(a-c)(b-c)}$$

$$\frac{a}{(s+b)^2} = a t e^{-bt}$$

$$\frac{a}{(s+b)^3} = \frac{at^2}{2} e^{-bt}$$

$$\frac{a}{(s+b)^{n+1}} = \frac{a}{n!} t^n e^{-bt}$$

$$\frac{1}{s(as+1)} = 1 - e^{-t/a}$$

$$\frac{s}{(s^2+\omega^2)^2} \quad \frac{1}{2\omega} t \cos(\omega t)$$

LAPLACE TRANSFORM OF DERIVATIVE AND INTEGRAL FUNCTIONS

$$L\left\{\frac{df(t)}{dt}\right\} = sF(s) - f(0)$$

$$L\left\{\frac{d^2 f(t)}{dt^2}\right\} = s^2 F(s) - sf(0) - f'(0)$$

$$L\left\{\frac{d^n f(t)}{dt^n}\right\} = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) \ldots s^0 f^{n-1}(0)$$

Initial Value Theorem

$$\lim_{t \to 0} f(t) = \lim_{s \to \infty} sF(s)$$

Final value Theorem

$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

Q. $\frac{dx}{dt} + x = 1$ $\quad$ $x(0) = x'(0) = 0$

Ans $\quad sX(s) + X(s) = \frac{1}{s}$

$\Rightarrow X(s)(1+s) = \frac{1}{s} \Rightarrow X(s) = \frac{1}{s(s+1)}$

$\Rightarrow X(s) = \frac{1}{s} - \frac{1}{s+1} \Rightarrow x(t) = 1 - e^{-t}$

Q. $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 2$, $x(0) = x'(0) = 0$

Ans: $s^2 x(s) + 2s\,x(s) + 2x(s) = \frac{2}{s}$

$\Rightarrow x(s) = \frac{2}{s(s^2+2s+2)}$

$\frac{2}{s(s^2+2s+2)} = \frac{A}{s} + \frac{Bs}{s^2+2s+2} + \frac{C}{s^2+2s+2}$

$2 = A(s^2+2s+2) + Bs^2 + Cs$

$s=0$ $\quad A = 1$

$s=1$ $\quad 1(5) + B + C = 2$

$\Rightarrow B + C = -3$

$s=-1$ $\quad 1(1-2+2) + B - C = 2$

$\Rightarrow B - C = 1$

$2B = -4 \Rightarrow B = -2$

$C = B - 1 = -2 - 1 = -3.$

$\frac{1}{s} - \frac{2s}{s^2+2s+2} - \frac{3}{s^2+2s+2}$

$x(s) = \frac{2}{s(s^2+2s+2)} = \frac{2}{s(s+1-j)(s+1+j)}$

$\frac{A}{s}$ $\quad 2 = A(s+1-j)(s+1+j) + Bs(s+1+j) + Cs(s+1-j)$

$s=0,$ $\quad A(1-j)(1+j) = 2$

$\Rightarrow A(1-j^2) = 2$

$\Rightarrow A = 1$

$s = -1-j$ $\quad B(-1-j) = 2 \Rightarrow B = \frac{-2}{1+j}$

$s = -1+j$

$s = -1 + j \qquad B(-1+j)\,2j = 2$

$$\Rightarrow B = \frac{j}{j-1} = \frac{j(j+1)}{j^2-1} = \frac{j^2+j}{-2} = \frac{-1+j}{-2} = \frac{1-j}{2}$$

$s = -1 - j \qquad C(-1-j)(-2j) = 2$

$$\Rightarrow C = \frac{j}{j+1} = \frac{j(j-1)}{j^2-1} = \frac{j^2-j}{-2} = \frac{-1-j}{-2} = \frac{1+j}{2}$$

$A = 1 \qquad B = \frac{1-j}{2} \qquad C = \frac{1+j}{2}$

$$x(s) = 2\left(\frac{1}{s} + \frac{\frac{1-j}{2}}{s+1-j} + \frac{\frac{1+j}{2}}{s+1+j}\right)$$

$$= 2\left(1 + \frac{1-j}{2}e^{-t(1-j)} + \frac{1+j}{2}e^{-t(1+j)}\right)$$

Q. Solve the function $x(t)$ whose laplace transform

i) $$X(s) = \frac{s^2-s-6}{s^3-2s^2-s+2}$$

Ans

$$X(s) = \frac{s^2-s-6}{s^3-2s^2-s+2} = \frac{s^2-3s+2s-6}{s^2(s-2)-1(s-2)}$$

$$= \frac{s(s-3)+2(s-3)}{(s-2)(s^2-1)}$$

$$= \frac{(s-3)(s+2)}{(s-2)(s-1)(s+1)}$$

$(s-3)(s+2) = A(s-1)(s+1) + B(s-2)(s+1) + C(s-2)(s-1)$

Q. The laplace transform $x(s) = \frac{s+1}{s^2-2s+5}$.

$$x(s) = \frac{1}{(s+1)^3(s+2)}$$

$$= \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)^3} + \frac{D}{s+2}$$

EFFECT OF INTEGRAL CONTROL ACTION

SERVO PROBLEM

$$\boxed{G(s) = \frac{G_p G_f G_c}{1 + G_p G_f G_c G_m}}$$

For any common feedback control system.

Assuming $G_m = G_f = 1$.

$$G(s) = \frac{G_p G_c}{1 + G_p G_c}$$

Considering 1st order system, $G_p = \frac{K_p}{\tau_p s + 1}$

Using integral controller, $G_c = \frac{K_c}{\tau_I s}$

$$G(s) = \frac{\left(\frac{K_p}{\tau_p s+1}\right)\left(\frac{K_c}{\tau_I s}\right)}{1 + \left(\frac{K_p}{\tau_p s+1}\right)\left(\frac{K_c}{\tau_I s}\right)}$$

$$\Rightarrow G(s) = \frac{K_p K_c}{(\tau_p s+1)(\tau_I s) + K_p K_c}$$

$$\Rightarrow \boxed{G(s) = \frac{K_p K_c}{\tau_p \tau_I s^2 + \tau_I s + K_p K_c}}$$

Hence, $G(s) = \dfrac{K_p K_c}{\tau_p \tau_I s^2 + \tau_I s + K_p K_c}$

~~Taking~~ Comparing it with general 2nd order system, $K_p K_c = 1$

$$\tau_p \frac{\tau_I}{K_p K_c} = \tau^2 \qquad 2\xi\tau = \frac{\tau_I}{K_p K_c}$$

$$\tau = \sqrt{\frac{\tau_p \tau_I}{K_p K_c}}$$

$$\cancel{2 \times \xi \times \sqrt{\tau_p \tau_I} = \tau_I}$$

$$\rightarrow \boxed{\cancel{\xi = \frac{1}{2}\sqrt{\frac{\tau_I}{\tau_p}}}}$$

$$\xi = \frac{1}{2}\sqrt{\frac{\tau_I}{\tau_p K_p K_c}}$$

→ Integral action (a) increases the order of dynamics by 1

(b) By increasing the order, the response becomes more sluggish.

(c) For a unit step change in the set point.

$$Y(s) = \frac{1}{s} \times \frac{1}{\tau^2 s^2 + 2\xi\tau s + 1}$$

Nature of the response depends on the value of ξ.

$$U.V.R = \lim_{s \to 0} s\,Y(s)$$

$$= \lim_{s \to 0} \frac{1}{\tau^2 s^2 + 2\xi\tau s + 1} = 1$$

Offset $= 1 - 1 = 0$ (No offset)

Integral action eliminates any offset.

~~Response can be~~

→ ε depends upon the value of K_c and τ_I

→ So tuning of the controller is important.

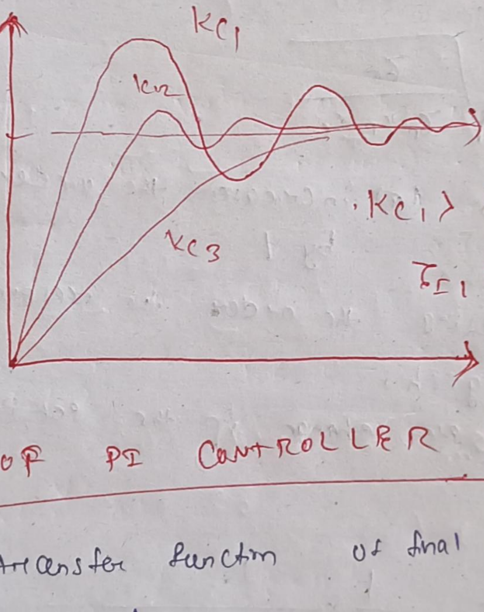
→ As $K_c \uparrow \Rightarrow \varepsilon \downarrow$, the consequences are.

(i) Response moves from sluggish overdamped

(ii) Overshoot and decay ratio increases.

(iii) Thus we can improve speed of closed loop response and larger at the expense of higher deviation and oscillation.

~~(iv) Same effect~~



EFFECT OF PI CONTROLLER ON A FIRST ORDER PROCESS

→ Assume transfer function of final control element and measurement = 1.

For servo problem, $G(s) = \dfrac{G_p G_c}{1 + G_p G_c}$

$G_p = \dfrac{K_p}{\tau_p s + 1}$ $\quad G_c = K_c\left(1 + \dfrac{1}{\tau_I s}\right)$

$$G(s) = \frac{\left(\dfrac{K_p}{\tau_p s + 1}\right) K_c \left(1 + \dfrac{1}{\tau_I s}\right)}{1 + \left(\dfrac{K_p}{\tau_p s + 1}\right) K_c \left(1 + \dfrac{1}{\tau_I s}\right)}$$

$$\Rightarrow G(s) = \frac{K_p K_c(\tau_D s+1)}{(\tau_p s+1)(\tau_I s) + K_p K_c(\tau_D s+1)}$$

$$\Rightarrow G(s) = \frac{K_p K_c(\tau_I s+1)}{\tau_p \tau_I s^2 + \tau_I s + K_c K_c \tau_I s + K_p K_c}$$

$$\Rightarrow G(s) = \frac{\tau_I s+1}{\frac{\tau_p \tau_I s^2}{K_p K_c} + s\left(\frac{\tau_I + K_p K_c}{K_p K_c}\right) + 1}$$

$$Y(s) = \frac{1}{s} \times \frac{\tau_I s+1}{\frac{\tau_p \tau_I s^2}{K_p K_c} + s\left(\frac{\tau_I + K_p K_c}{K_p K_c}\right) + 1}$$

$$= \underbrace{\frac{\tau_I}{\tau^2 s^2 + 2\xi\tau s + 1}}_{\text{Impulse}} + \underbrace{\frac{1}{s(\tau^2 s^2 + 2\xi\tau s + 1)}}_{\text{step}}$$

$$Y(t) = \frac{\tau_I}{\tau\sqrt{1-\xi^2}} e^{-\xi t/\tau} \sin\frac{\sqrt{1-\xi^2}\,t}{\tau}$$

$$+ 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi t/\tau} \sin\left(\frac{\sqrt{1-\xi^2}}{\tau}t + \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right)$$

$$\text{U.V.R} = \lim_{s\to 0} sY(s)$$

$$= 0 + 1 = 1$$

$$\text{Offset} = 1 - 1 = 0$$

→ The order of the response increases by 1.

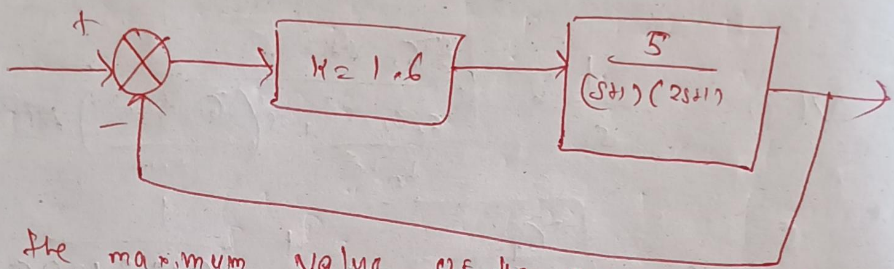
→ The offset is eliminated.

→ As $K_c \uparrow$ ⇒ Response is faster and oscillatory.

Large value of K_c create a very sensitive response.

→ As $\tau_c \downarrow$ ⇒ Same phenomena appears.

Q. The set point of the control system shown in figure given a step change of 0.1 unit. Determine



(a) the maximum value of the output and the time at which it occurs.

(b) Offset

(c) Period of oscillation.

From the block diagram algebra develop the overall transfer function for closed system.

Ans.

$$G(s) = \frac{1.6 \times 5}{(s+1)(2s+1)} = \frac{8}{(s+1)(2s+1)}$$

$$X(s) = \frac{0.1}{s}$$

$$G(s) = \frac{8 \times 0.1}{s(s+1)(2s+1)} = \frac{0.8}{s(s+1)(2s+1)}$$

$$\frac{1}{s(s+1)(2s+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{2s+1}$$

$$1 = A(s+1)(2s+1) + Bs(2s+1) + Cs(s+1)$$

$s = 0$ $\quad A = 1$

$s = -\frac{1}{2}$ $\quad C\left(-\frac{1}{2}\right)\left(\frac{1}{2}\right) = 1 \Rightarrow C = -4$

$s = -1$ $\quad B(-1)(-1) = 1 \Rightarrow B = 1$

$$Y(s) = \frac{0.8}{s} + \frac{0.8}{s+1} - \frac{4\times 0.8}{2s+1}$$

$$\Rightarrow Y_1(s) = \frac{0.8}{s} + \frac{0.8}{s+1} - \frac{1.6}{s+0.5}$$

$$\Rightarrow Y(t) = 0.8 + 0.8e^{-t} - 1.6e^{-0.5t}$$

$$\frac{dY(t)}{dt} = 0$$

$$\Rightarrow -0.8e^{-t} + 0.8e^{-0.5t} = 0$$

$$\Rightarrow 0.8e^{-t} = 0.8e^{-0.5t}$$

$$\Rightarrow e^{0.5t} = 1 \Rightarrow 0.5t = 0$$

$$\Rightarrow t = 0$$

$~~Y(t) = 0.8~~$ $Y(t) = 0.8$.

$$Y(s) = \frac{0.8}{s(s+1)(2s+1)}$$

$$(s+1)(2s+1) = 2s^2 + s + 2s + 1 = 2s^2 + 3s + 1$$

$$\tau = \sqrt{2} \qquad 2 \times \sqrt{2} \times \xi = 3$$

$$\Rightarrow \xi = \frac{3}{2\sqrt{2}} = 1.06$$

$$O.S. = e^{-\frac{\pi \times \frac{3}{2\sqrt{2}}}{\sqrt{1-\frac{9}{8}}}} =$$

~~t_{max}~~ $= 0.0112$ = Offset

~~Offset~~ = $t_{max} = 1.574$

$r = 3.157$

Q. A pneumatic PI Controller has an output pressure of 10 psi. When the set point and pen point are together. This set point and pen points are suddenly displaced by 0.5 inch. Then the following data are obtained.

Time (s)	Psi
0^-	10
0^+	8
20	7
60	5
90	3.5

Determine the gain of controller and integral time.

$$\cancel{P = P_s + K_c \varepsilon(t) + \frac{K_c}{\tau_I s}}$$

$$P = P_s + K_c \varepsilon(t) + \frac{K_c}{\tau_I} \int_0^t \varepsilon(t)\, dt$$

$$\varepsilon(t) = 0.5\, u(t)$$

$K_c = -4$ psi. $\tau_I = 40$ s

EFFECT OF D- CONTROLLER ACTION

$$\cancel{\frac{Y(s)}{X(s)} = \frac{K_p K_c}{1 + K_p K_c}} \quad (\text{Taking } \cancel{H_p = K_c})$$

$$G(s) = \frac{G_p G_c}{1 + G_p G_c} \quad (\text{Taking } G_m = G_f = 1)$$

$$G_c = K_c \tau_D s \qquad G_p = \frac{K_p}{\tau_p s + 1}$$

$$\frac{Y(s)}{\frac{1}{s}} = \frac{(K_c \tau_D s)\left(\frac{K_p}{\tau_p s + 1}\right)}{1 + (K_c \tau_D s)\left(\frac{K_p}{\tau_p s + 1}\right)}$$

$$\Rightarrow \frac{Y(s)}{\frac{1}{s}} = \frac{K_c K_p \tau_D s}{\tau_p s + 1 + K_p K_c \tau_D s}$$

$$\Rightarrow \boxed{Y(s) = \frac{K_c K_p \tau_D s}{s\{(\tau_p s+1) + K_p K_c \tau_D s\}}}$$

→ the D-Control action does not change the order of the response.
(Process 1st order)

→ The modified time constant is $\tau_p' = \tau_p + K_p K_c \tau_D$

Effective time constant $\tau_p' > \tau_p$.

The response of the control process is slower than that of the original first order process.

Furthermore, as $K_c \uparrow \Rightarrow \tau_p' \uparrow$ increases, and the response becomes progressively slower.

Second Order Process

$$G_p = \frac{K}{\tau^2 s^2 + 2\xi \tau s + 1}$$

Determine the CLTF of the 2nd order process with D-Controller.

Ans.

$$G(s) = \frac{K_c \tau_D s \times \frac{K}{\tau^2 s^2 + 2\xi \tau s + 1}}{1 + K_c \tau_D s \times \frac{K}{\tau^2 s^2 + 2\xi \tau s + 1}}$$

$$\Rightarrow G(s) = \frac{K K_c \tau_D s}{\tau^2 s^2 + s(2\xi\tau + K K_c \tau_D) + 1}$$

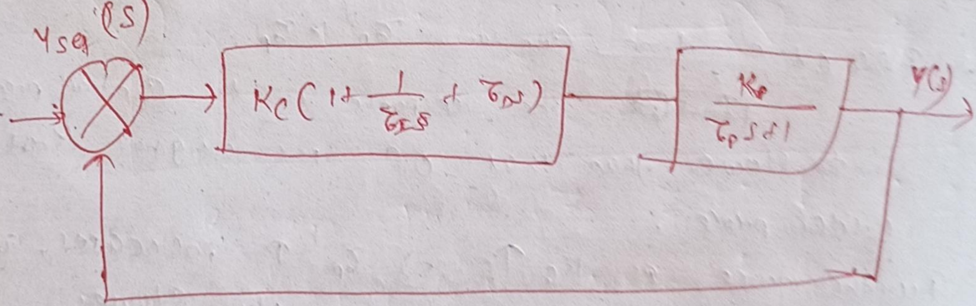
→ The natural period of oscillation of closed loop remains the same as open loop process.

→ ξ' (modified value of damping factor) $> \xi$.
The closed loop response is more damped and damping increases as K_c and τ_D increases.

→ The decrease in speed and increase in damping demonstrates that the derivative control action produces more robust behaviour.

EFFECT OF PID CONTROL ACTION

$$\boxed{G_c = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)}$$



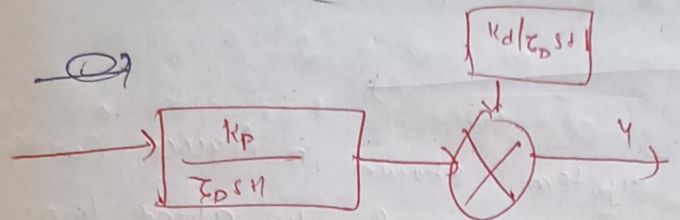
$$\frac{y(s)}{y_{sp}(s)} = \frac{\dfrac{K_p K_c\,(1 + \tau_I s + \tau_D \tau_I s)}{\tau_I s\,(\tau_p s + 1)}}{\dfrac{\tau_I s\,(\tau_p s + 1) + K_p K_c\,(1 + \tau_I s + \tau_D \tau_I s)}{\tau_I s\,(\tau_p s + 1)}}$$

$$\Rightarrow \frac{y(s)}{y_{sp}(s)} = \frac{\tau_D \tau_I s^2 + \tau_I s + 1}{\dfrac{\tau_p \tau_I}{K_p K_c} s^2 + \tau_I\left(\dfrac{1 + K_p K_c}{K_p K_c}\right) s + 1}$$

$$\Rightarrow \frac{y(s)}{y_{sp}(s)} = \frac{\tau_D \tau_I s^2 + \tau_I s + 1}{\tau^2 s^2 + 2\xi \tau s + 1}$$

$$\tau = \sqrt{\left(\tau_D + \frac{\tau_P}{K_P K_C}\right)\tau_I}$$

$$\xi = \frac{1}{2}\frac{(K_P K_C + 1) / K_P K_C}{\left(\tau_D + \frac{\tau_P}{K_P + K_C}\right)}\sqrt{\tau_I}$$



$$G_{CLTF}(s) = \frac{G_d}{1 + G_c G_m G_d G_f}$$

$$G_{OLTF}(s) = \frac{K_d \tau_I s}{(\tau^2 s^2 + 2\xi\tau s + 1)}$$

→ τ and ξ remains the same.

For unit step change in set point,

U.V.R = 1

V.V.R. = 0

For regulator,

Offset with servo = 0

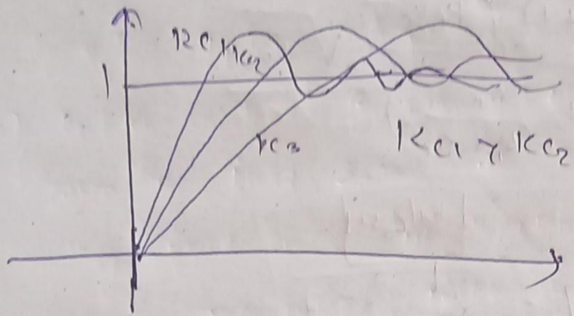
Offset with regulator = 1.

⇒ Nature of response depends upon ξ and K_C.

→ The value of ξ decreases with increase in K_C.

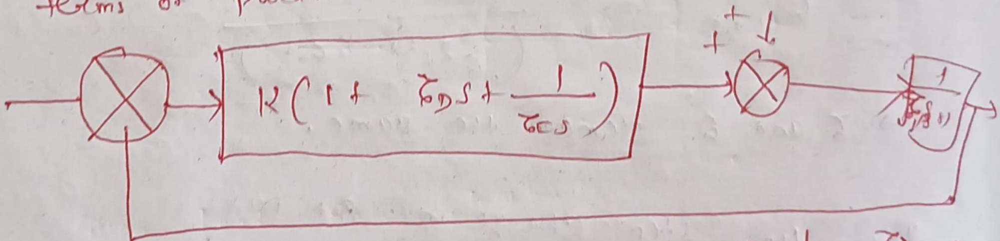
→ Higher value of K_C can be used to make the response faster. Higher value of K_C causes oscillatory response which is suppressed by the derivative control action and brings ~~stabilising~~ stabilising effect to the system.

→ By using higher value of K_C we get faster response but overshoot remains almost same. The settling time decreases.



Q.1. The control system shown in figure contains a three mode controller for the closed loop,

a. Develop formulas for the natural period of oscillation (τ) and damping factor (ξ) in terms of parameters K, τ_D, τ_I and τ_1.



2. For the following parts, $\tau_D = \tau_I = 1$, $\tau_1 = 2$.
Calculate damping factor when $K = 0.5$ and $K = 2$.

3. Do ξ and τ approach limiting values as K increases and if so, what are these values?

4. Determine the offset for a unit step change in the load when $K = 2$.

5. Sketch the response curve.

Ans. $Y_{set}(s)$ $y(s)$ $d(s)$

$$\frac{y(s)}{Y_{set}(s)} = \frac{K\left(1+\tau_D s+\frac{1}{\tau_I s}\right)\left(\frac{1}{\tau_1 s+1}\right)}{1+K\left(1+\tau_D s+\frac{1}{\tau_I s}\right)\left(\frac{1}{\tau_1 s+1}\right)}$$

$$\Rightarrow \frac{y(s)}{Y_{set}(s)} = \frac{K\left(\frac{\tau_I s+1+\tau_D s+1}{\tau_I s}\right)\left(\frac{1}{\tau_1 s+1}\right)}{\frac{\tau_I s(\tau_1 s+1)+K(\tau_I s+\tau_D s+1)(1)}{\tau_I s(\tau_1 s+1)}}$$

$$\Rightarrow \frac{y(s)}{Y_{set}(s)} = \frac{K(\tau_I s+1+\tau_D s+1)}{\tau_I\tau_1 s^2+\tau_I s+K\tau_I s+K\tau_D s+K}$$

$$\Rightarrow \frac{y(s)}{Y_{set}(s)} = \frac{K\{s(\tau_I+\tau_D)+2\}}{\tau_I\tau_1 s^2+s(\tau_I+K\tau_I+K\tau_D)+K}$$

$$\Rightarrow \frac{y(s)}{Y_{set}(s)} = \frac{2+s(\tau_I+\tau_D)}{\left(\frac{\tau_I\tau_1}{K}\right)s^2+\left(\frac{\tau_I}{K}+\tau_I+\tau_D\right)s+1}$$

$$\tau = \sqrt{\frac{\tau_I\tau_1}{K}} \qquad 2\cdot\xi\cdot\sqrt{\frac{\tau_I\tau_1}{K}} = \frac{\tau_I}{K}+\tau_I+\tau_D$$

$$\Rightarrow \xi = \frac{\frac{\tau_I}{K}+\tau_I+\tau_D}{2\sqrt{\frac{\tau_I\tau_1}{K}}}$$

For $K\to\infty$, $\tau = 1$ $\xi = 0.5$

For

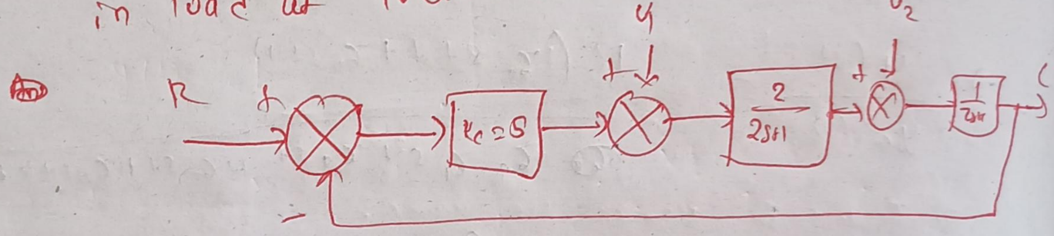
Q The location of a load change will affect the system response in the block diagram shown. A unit step change in the load either enters at location 1 or location 2.

(i) What is the frequency of the transient response when the load enters at location 1 and when the load enters at location 2?

(ii) What is the offset when the load enters at location 1 and location 2?

(iii) Sketch the transient response to a step change in load at location 1 and location 2.

Ans



Ans. When load enters location 1.

$$G(s) = \frac{\left(\frac{2}{2s+1}\right)\left(\frac{1}{2s+1}\right)}{1 + 5\left(\frac{2}{2s+1}\right)\left(\frac{1}{2s+1}\right)}$$

$$\Rightarrow G(s) = \frac{2}{(2s+1)(2s+1) + 10}$$

$$\Rightarrow G(s) = \frac{2}{(4s^2 + 4s + 1) + 10} = \frac{2}{4s^2 + 4s + 11}$$

$$\Rightarrow G(s) = \frac{2/11}{\frac{4}{11}s^2 + \frac{4}{11}s + 1}$$

$\tau^2 = \frac{4}{11} \Rightarrow \tau = \frac{2}{\sqrt{11}}$ $\quad 2 \times \zeta \times \frac{2}{\sqrt{11}} = \frac{4}{11}$

$\Rightarrow \xi = \frac{1}{\sqrt{11}}$

$f = 2\pi\omega \qquad \omega = \frac{\sqrt{1-\xi^2}}{\tau} = \frac{\sqrt{1-\frac{1}{11}}}{\frac{2}{\sqrt{11}}}$

$= \sqrt{\frac{10}{11}} \times \frac{\sqrt{11}}{2} \qquad = \cancel{\sqrt{\frac{2}{3}}} = \cancel{\sqrt{\frac{2}{3}} \times \frac{\sqrt{3}}{2}} = \cancel{\frac{1}{\sqrt{2}}}$

$= \frac{\sqrt{10}}{2}$

$f = 2\pi \times \frac{1}{\sqrt{2}} = \pi\sqrt{10}$ Hz

When load enters location 2 -

$$G(s) = \frac{\left(\frac{1}{2s+1}\right)}{1+5\left(\frac{2}{2s+1}\right)\left(\frac{1}{2s+1}\right)}$$

$$\Rightarrow G(s) = \frac{1}{(2s+1)^2 + 10}$$

$$\Rightarrow G(s) = \frac{1}{4s^2+4s+11}$$

$$\Rightarrow G(s) = \frac{1/11}{\frac{4}{11}s^2 + \frac{4}{11}s + 1}$$

$\tau^2 = \frac{4}{11} \Rightarrow \tau = \frac{2}{\sqrt{11}} \qquad 2 \times \xi \times \frac{2}{\sqrt{11}} = \frac{4}{11}$

$\Rightarrow \xi = \frac{1}{\sqrt{11}}$

$f = \frac{\sqrt{10}}{2}$ Hz

When load enters location 1,

$$\text{UVR} = \lim_{s \to 0} \frac{s \times \frac{2}{11}}{s\left(\frac{4}{11}s^2 + \frac{4}{11}s + 1\right)} = \frac{2}{11}$$

$1 - \frac{2}{11} = \frac{9}{11}$

when load enters location 2,

$$U.V.R. = \lim_{s\to 0} \frac{s \times \frac{1}{11}}{s\left(\frac{2}{11}s^2 + \frac{4}{11}s + 1\right)} = \frac{1}{11}$$

$$\text{Offset} = 1 - \frac{1}{11} = \frac{10}{11}$$

For step change in load,

$$\cancel{y(t) = 1 - \frac{1}{\sqrt{1-\frac{1}{3}}} e^{-\frac{\frac{1}{\sqrt{3}} \times 1}{\frac{2}{\sqrt{3}}}t} \sin\left(\frac{1}{\sqrt{2}}t + \tan^{-1}\left(\frac{\sqrt{1-\frac{1}{3}}}{\frac{1}{\sqrt{3}}}\right)\right)}$$

$$\cancel{\Rightarrow y(t) = 1 - \sqrt{\frac{3}{2}}\, e^{-\frac{t}{2}}\left(\sin\left(\frac{t}{\sqrt{2}} + \tan^{-1}(\sqrt{2})\right)\right)}$$

Q. Examine the effect of various values of gain of K_m of measuring device will have on the closed loop response of a process, its transfer function $\frac{1}{2s^2+3s+1}$. Assume that $G_m = K_m = G_f = 1$ and $K_c = 1$. The controller is proportional with

Q. Consider the system with the following transfer function. $\frac{K_1 e^{-t_d s}}{\tau_1 s+1} - \frac{K_2 e^{-t_d s}}{\tau_2 s+1}$. Draw the block diagram of the system that must be satisfy K_1, K_2, τ_1, τ_2 so that system exhibits inverse response. Plot the response for unit step change in input.

Stability Characteristics

Criterion of Stability.

→ Routh Hurwitz

Eg Q: $s^3 + 2s^2 + (2+K_c)s + \frac{K_c}{\tau_I} = 0$

CE eqn ↑

$$
\begin{array}{lll}
s^3 & 1 & (2+K_c) \\
s^2 & 2 & \frac{K_c}{\tau_I} \\
s^1 & \frac{2(2+K_c) - \frac{K_c}{\tau_I}}{2} & 0 \\
s^0 & \frac{\frac{K_c}{\tau_I}\left\{\frac{2(2+K_c) - \frac{K_c}{\tau_I}}{2}\right\}}{\frac{2(2+K_c) - \frac{K_c}{\tau_I}}{2}} = \frac{K_c}{\tau_I} &
\end{array}
$$

$$\left[1,\ 2,\ \frac{2(2+K_c) - \frac{K_c}{\tau_I}}{2},\ \frac{K_c}{\tau_I}\right]$$

For stability, $\boxed{\frac{K_c}{\tau_I} > 0}$

$$2(2+K_c) - \frac{K_c}{\tau_I} > 0$$

$$\Rightarrow \boxed{4 + 2K_c > \frac{K_c}{\tau_I}}$$

If $\tau_I = 0.1$, $\frac{2(2+K_c) - 10K_c}{2}$

$$= \frac{4 + 2K_c - 10K_c}{2} = \frac{4 - 8K_c}{2}$$

$4 - 8K_c > 0$

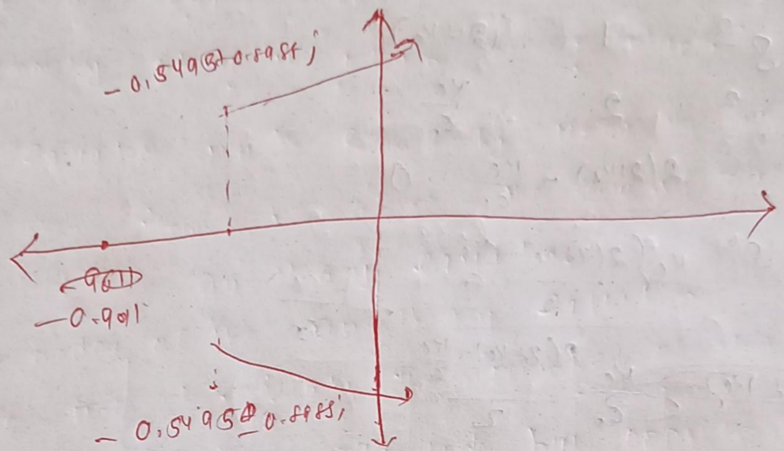
$\Rightarrow \boxed{K_c \leq 0.5}$

$\Rightarrow K_c < .1$

$K_c = 0.1, \quad \tau_I = 0.1$

$s^3 + 2s^2 + 2.1s + 1 = 0$

Roots :- $-0.9011, \quad -0.5495 + 0.8988j, \quad -0.5495 - 0.8988j$



Q. The characteristic equation is

~~$(s-2)(s+1)(s-3) = 0$.~~

$(s-2)(s+1)(s-3) = 0$. Carry the Routh's test and state the stability.

Unstable

Ans $(s^2 - 2s + s - 2)(s-3)$

$= (s^2 - s - 2)(s-3)$

$= s^3 - 3s^2 - s^2 + 3s - 2s + 6$

$= s^3 - 4s^2 + s + 6$

s^3	1	1
s^2	-4	6
s	2.5	0
s^0	6	

$\frac{-4-6}{-4} = \frac{-10}{-4} = 2.5$

Special Case

* Depending upon the equations to be tested, following difficulties may come.

(1) The first element in any one row of the routh table is zero but the other elements are not.

(2) The elements in one row of the routh table are all zero. $s^3 - 3s + 2 = 0$

s^3	1	-3
s^2	0	2
s^1	—	—
s^0		

Introducing $\varepsilon \to$ Small Positive quantity.

Since the coefficient of s^2 is zero, we know from necessary condition if one root of the eqn is in the right half of s plane, to determine how many roots are in the right half of s plane, we carry out the routh's tabulation.

To overcome this situation, we may replace the "0" element in the table by an arbitrary small positive number ε. and then proceed with the routh's test.

" $\frac{-3\varepsilon-2}{\varepsilon}$ " approaches to " $\frac{-2}{\varepsilon}$ "

s^3	1	-3
s^2	ε	2
s^1	$\frac{-3\varepsilon-2}{\varepsilon}$	0
s^0	2	0

Two sign changes (1st column)

$\rightarrow$ System is unstable

When all the elements of any row is zero.
This indicates one or more of the following conditions may exist –

1. Pair of real roots with opposite sign.
2. Pair of imaginary roots.
3. Pair of complex conjugate roots, forming symmetry about the origin in s-plane.

→ The equation that is formed by using the coefficients of the rows just above the row of zeroes is called the auxilliary equation.
The order of the auxilliary equation is always even and it indicates the number of root pairs that are equal in magnitude but opposite in sign.

To over come this situation –

(a) Take the derivative of the auxilliary equation.

(b) Replace the row of zeroes with the coefficients of the resulting equation obtained by taking the derivative of the auxilliary equation.

(c) Carry out Routh's test in the usual manner with the newly determined table.

Q.1:- CE eqn:- $s^4 + s^3 - 3s^2 - s + 2 = 0$.

Ans

s^4	1	-3	2
s^3	1	-1	0
s^2	-2	2	0
s^1	0	0	0
s^0	-	-	-

Auxi. eqn :- $-2s^2 + 2 = 0$ (A)

$\Rightarrow \frac{d}{ds}(AE)$

$= -4s$

s^4	1	-3	2
s^3	1	-1	0
s^2	-2	2	0
s^1	-4	0	0
s^0	2	0	0

The row with s^1 contains all zero, the routh's test terminates prematurely. Now the row of the zeroes, the row of zeroes in the routh table is replaced by the coefficients of the resulting eqn.

(The system is unstable with 2 roots lying in RHP)

Root Locus

CE eqn :- $1 + G_{OL} = 0$

$$\Rightarrow 1 + \frac{5 \times 1.6}{(s+1)(2s+1)} = 0$$

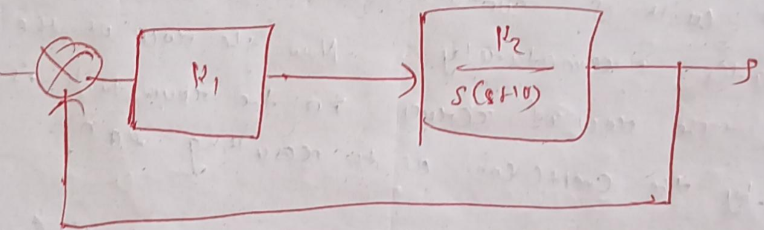
$$\Rightarrow 2s^2 + 3s + 1 + 8 = 0$$

$$\Rightarrow \boxed{2s^2 + 3s + 9 = 0}$$

s^2	2	9
s	3	0
s^0	$\frac{27}{3}$	0

Stable system

The root locus method is a graphical procedure for finding the stability of a system. This method uses open loop transfer function and determine the stability of closed loop system. It represents the diagram of the loci of the characteristic eqn roots when a certain system parameters varies (gain of the controller). If the root loci always lies to the left of the imaginary axis, the system is stable. If it presents to the right of imaginary axis, the system is unstable.



Ce eqn :- $1 + \frac{K_1 K_2}{s(s+10)} = 0$

$$\Rightarrow \boxed{s^2 + 10s + K_1 K_2 = 0}$$

Taking $K_1 K_2 = K$, for the values of K 0, 5, 25 and 50,

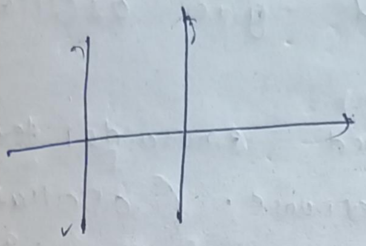
$s^2 + 10s + 0 = 0$ Roots $= 0, -10$

$s^2 + 10s + 5 = 0$ Roots $= -5 + 2\sqrt{5}, -5 - 2\sqrt{5}$

$s^2 + 10s + 25 = 0$ Roots $= -5$

$s^2 + 10s + 50 = 0$ Roots $= -5 + 5j, -5 - 5j$

$s^2 + 10s + 100 = 0$ Roots $= -5 + 5\sqrt{3}j, -5 - 5\sqrt{3}j$



(Stable System)

PROCEDURE FOR DRAWING ROOT LOCUS

1. Write the OLTF of the system in the form

$$G(s) = K \cdot \frac{N}{D}.$$

$$\Rightarrow \boxed{G(s) = K \frac{\prod_{1}^{m} (s - z_i)}{\prod_{1}^{n} (s - P_i)}}$$

2. Generally $n > m$.

CE eqn :- $1 + \frac{KN}{D} = 0$

3. The trace of the locus is found entirely from the angle criterion

$$\boxed{\sum_{i}^{m} \angle (s - z_i) - \sum_{i}^{n} \angle (s - P_i) = (2i+1)\pi}$$

4. The gain K at any point on it (locus) is obtained from the magnitude criterion.

$$\boxed{\frac{K \left(\prod_{i}^{m} |s - z_i| \right)}{\left(\prod_{i}^{n} |s - P_i| \right)} = 1}$$

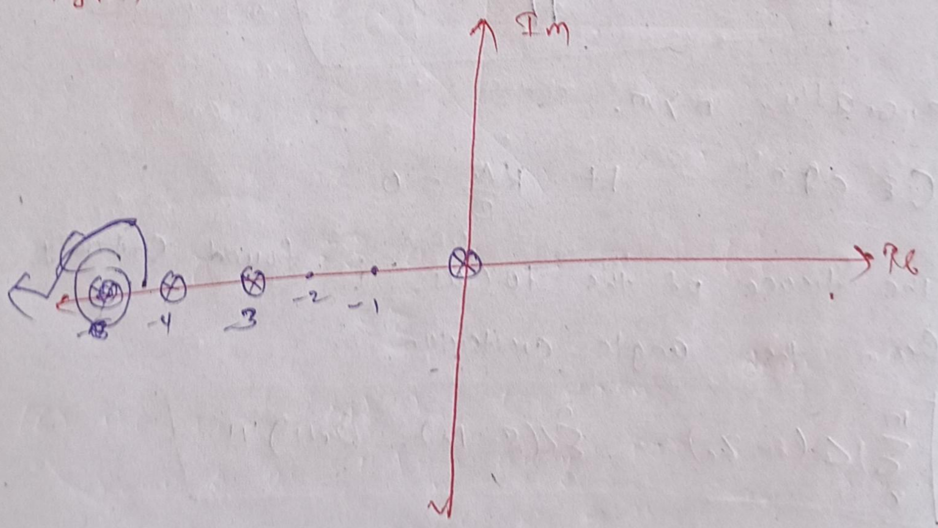
5. If the roots are on the negative real axis, the response is non-oscillatory. For complex roots with negative real part, the response

Part is under damped gives oscillatory response.

Response include damped sinusoidal term, that gives oscillatory response. (oscillation ceases gradually)

For complex roots with positive real part, the response is growing sinusoidal for purely imaginary, the response oscillates at constant amplitude

Trial and Error Procedure is adapted for drawing the root locus plot. However the same type, the following rules are applied -



(1) The root locus is symmetrical about the real axis.

(2) The number of loci branches is equal to the number of open loop poles. (order of D(s))

(3) The root loci emerge from open loop poles and terminate at open loop zeroes and termination of (n-m) loci occur at zeroes at infinity along the asymptotes.

(4) A point on the real axis lies on the root locus if the number of open loop poles plus zeroes on the real axis to the right of this point is odd.

$$G_{OLTF} = \frac{K(s+1)(s+2)}{s(s+3)(s+4)}$$

$P_1 = 0$, $P_2 = -3$, $P_3 = -4$, } 3 poles ∴ 3 Root loci branches.

$Z_1 = -1$, $Z_2 = -2$

(5) the $(n-m)$ branches of the root loci which tends to infinity ~~to sho~~ do show along the straight line asymptotes whose angles are given by $\phi_A = \frac{(2q+1)\pi}{n-m}$, $q = 0, 1, 2, \dots, (n-m-1)$.

The asymptotes crosses the real axis at a point known as the centroid determined by the relationship

$$\sigma_A = \frac{\Sigma \text{ real part of Poles} - \Sigma \text{ real parts of Zeroes}}{n-m}$$

$$\Rightarrow \sigma_A = \frac{\sum_i^n P_i - \sum_i^m Z_i}{n-m}$$

Here, $\phi_A = \pi$ $\quad \sigma_A = 0 - 4$

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

$P_1 = 0,\ P_2 = -1,\ P_3 = -2$

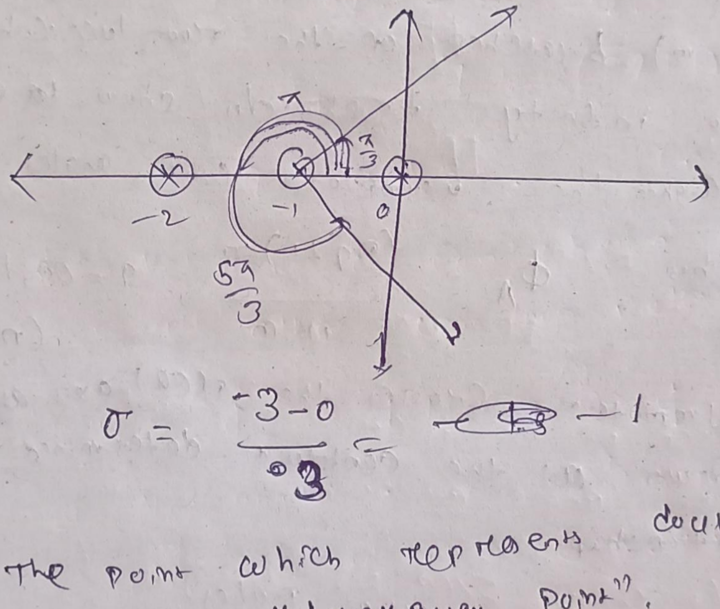
3 poles $\quad n = 3$

0 zeroes $\quad m = 0$

$n - m - 1 = 2$

$q = 0, 1, 2$

$\phi_{A_1} = \frac{\pi}{3}, \quad \phi_{A_2} = \frac{3\pi}{3} = \pi, \quad \phi_{A_3} = \frac{5\pi}{3}$



$$\sigma = \frac{-3-0}{3} = -1$$

(6) The point which represents double root is known as "breakaway point".

The breakaway points of the root locus are the solutions of $\frac{dK}{ds} = 0$

$$1 + \frac{K}{s(s+1)(s+2)} = 0$$

$\Rightarrow s(s^2+3s+2) + K = 0$

$\Rightarrow s^3 + 3s^2 + 2s + K = 0$

$\Rightarrow \frac{dK}{ds} = -3s^2 - 6s - 2$

$$\frac{dK}{ds} = 0$$

$$\Rightarrow 3s^2 + 6s + 2 = 0$$

$$\Rightarrow s = \frac{-6 \pm \sqrt{36-24}}{2(3)}$$

$$= \frac{-6 \pm \sqrt{12}}{2(3)} = \frac{-6 \pm 2\sqrt{3}}{6}$$

$$= -1 \pm \frac{\sqrt{3}}{3}$$

$$= -1 \pm \frac{1}{\sqrt{3}}$$

$-1-\frac{1}{\sqrt{3}} \rightarrow$ Unrealistic BP

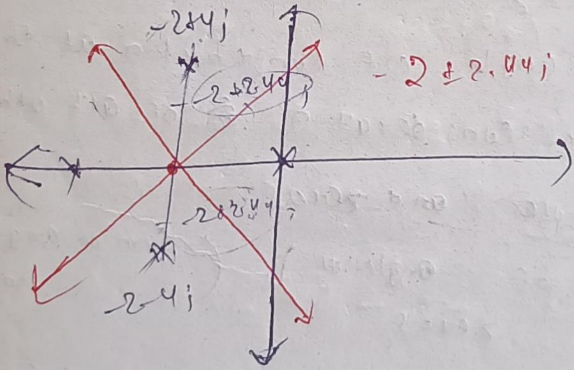
$-1+\frac{1}{\sqrt{3}} \rightarrow$ Realistic BP

Example 1:- $\dfrac{K}{s(s+4)(s^2+4s+20)}$

$$\frac{-4 \pm \sqrt{16-80}}{2} = \frac{-4 \pm \sqrt{-64}}{2} = -2 \pm 4j$$

Ans. Roots $s=0$ $s=-4$

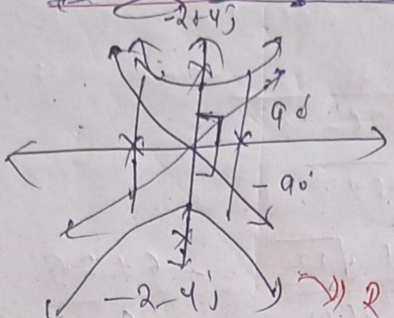
$s = -2 \pm 4j$



Complex breakaway points appears as conjugates. The root locus branches must approach or leave the breakaway point on the real axis at an angle of $\pm \frac{180^\circ}{r}$, where

r = No. of root locus branches approaching or leaving the point.

2 Root Locus Branches



The root locus branches must approach or leave the point on the real axis.

Root locus plot

(7) The angle of departure from an open loop pole is given by ϕ_E

$$\phi_E = \pm 180^\circ(2q+1) + \phi$$

$$q = 0, 1, 2, \ldots, (n-m-1).$$

(8) ϕ = Net angle of contribution at this pole of all other open loop poles and zeros.

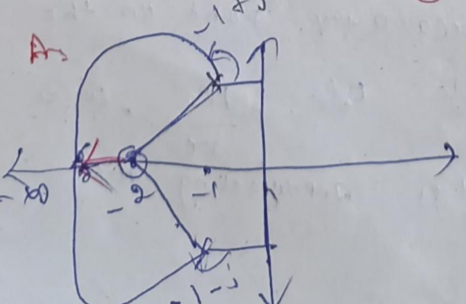
Similarly, $\phi_z = \pm 180^\circ(2q+1) - \phi$

ϕ = Net angle of contribution at this zero under consideration of all other open loop poles and zeros.

ϕ = (Sum of angle of zeros) - (Sum of angle of poles)

Ex- $G(s) = \dfrac{K(s+2)}{s^2+2s+2}$

Ans



$n=2 \quad m=1$

$$s^2 + (2+K)s + 2 + K = 0$$

$$2s + (2+K) + \frac{dK}{ds} = 0$$

(8) The intersection of the root locus branches with the imaginary axis can be determined by use of Routh criterion.

Q. Discuss the stability of the closed loop system with PID controller by plotting the root locus diagram. The transfer function of the process is given by $\left(\frac{1}{1+\tau_1 s}\right)(1+\tau_2 s)$.

Given $\tau_1 = 1$ min, $\tau_2 = 0.5$ min, $\tau_I = 5$ s, $\tau_D = 10$ s.

Ans

$$G(s) = \frac{\left(\frac{1+0.5s}{1+s}\right) K\left(1+\frac{1}{300s} + 600s\right)}{1 + K\left(\frac{1+0.5s}{1+s}\right)\left(1+\frac{1}{300s} + 600s\right)}$$

Q. The OLTF is given by $\frac{K}{s(s+3)(s^2+2s+2)}$. Draw the root locus plot for $0 < K < \infty$.

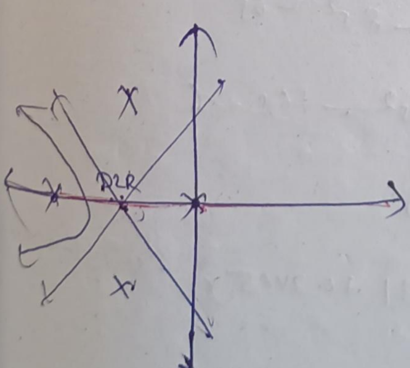
Ans.

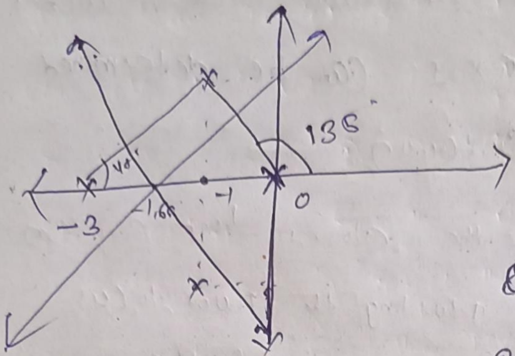
$s = 0 \qquad s = -3 \qquad s^2+2s+2 = 0$

$$\Rightarrow s = \frac{-2 \pm \sqrt{4-8}}{2}$$

$$= \frac{-2 \pm \sqrt{-4}}{2}$$

$$= \frac{-2 \pm 2j}{2} = -1 \pm j$$





$P = 4 \quad Z = 0,$

$(P - Z) = 4$

$K = 0, 1, 2, 3.$

$\theta_K = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}.$

Centroid $= \sigma = \frac{0 - 3 - 2 - 0}{4} = \frac{-5}{4} = -1.25$

$$1 + \frac{K}{s(s+3)(s^2+2s+2)} = 0$$

$\Rightarrow s(s+3)(s^2+2s+2) + K = 0$

$\Rightarrow (s^2+3s)(s^2+2s+2) + K = 0$

$\Rightarrow s^4 + 2s^3 + 2s^2 + 3s^3 + 6s^2 + 6s + K = 0$

$\cancel{\Rightarrow K = -s^4 - 2s^3}$

$\Rightarrow K = -s^4 - 5s^3 - 8s^2 - 6s$

$\Rightarrow \frac{dK}{ds} = -4s^3 - 15s^2 - 16s - 6$

$4s^3 + 15s^2 + 16s + 6 = 0$

$\Rightarrow s = -2.288, -0.731 + 0.3485j, -0.731 - 0.3485j$

$$\begin{array}{llll}
s^4 & 1 & 8 & K \\
s^3 & 5 & 6 & 0 \\
s^2 & \frac{34}{5} & K & \\
s & \dfrac{\frac{204}{5} - 5K}{\frac{34}{5}} & 0 & \\
s^0 & K & &
\end{array}$$

$$\frac{204}{5} - 5K = 0$$

$$\Rightarrow K = 8.16$$

A.E eq :- $\frac{34}{5} s^2 + 8.16$

$= 6.8 s^2 + 8.16$

$$\frac{d(AE)}{ds} = 13.8 s$$

Q. $\cancel{\dfrac{K}{(s+2)(s+4)}}$ $\dfrac{K}{s(s+2)(s+4)}$

Q. $\dfrac{K}{s(s^2+2s+2)}$

Define an appropriate performance criteria. Compute the value of performance criteria using P, PI and PID controller. With the best settings for the adjustable parameters, set τ_I and τ_D. Select the controller which gives the best value for the performance criteria.

The procedure is mathematically rigorous but has serious several drawbacks. It is very tedious. It relies on models. For the process relies on models (transfer function). The process or the sensor and the final control element which ~~result is~~ may not be known exactly. It incorporates certain ambiguities as to which is the most appropriate criteria and what input changes to consider.

Considering we have qualitative nature of different feedback controllers which we can use as basics.

(1) P controller :- It accelerates the response of the ~~process offset~~ produces offset for all, except process.

for pure integratory, that is, having $1/s$ ~~having~~ in their transfer function.

Ex- Level control for gas pressure in a vessel.

(2) PI controller :- Eliminate any offset. This eliminating usually comes at the expense of higher ~~max~~ deviation and produces sluggish, log oscillatory responses. If we increase K_c, it produce faster response, the system becomes more oscillatory and may lead to instability.

(3) D Controller :- Anticipates future errors and introduces appropriate action. Introduces a stabilising effect for the closed loop response.

(4) PID Controller is the best but introduce complex changing problem, because of having 3 adjustable parameters.

The general recommendation is -

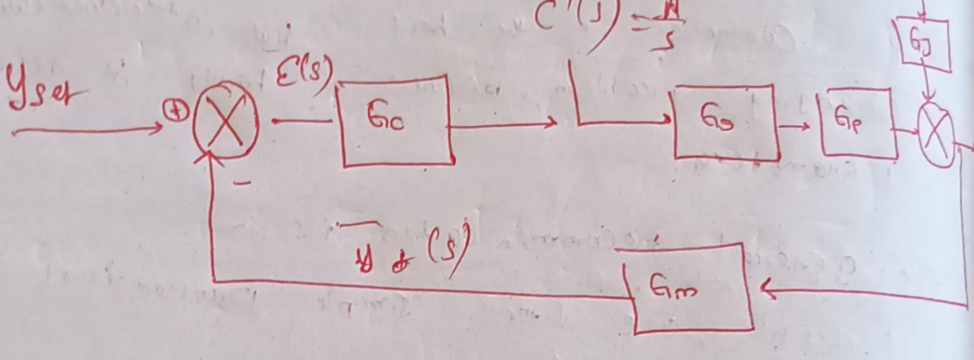
(1) If possible, use simple simple proportional controller.

(2) If simple P Controller is unacceptable, use PI Controller. It is not normally used for gas pressure or liquid level control, but for the flow controller. As the response for flow system is very fast.

(3) Use a PID Controller to increase the speed of closed loop response and ~~controller~~ maintain robust test. D. Derivative action with its stabilising effect allows the use of higher gains which produces faster response without excess oscillation. Thus, the derivative action is recommended for temperature and D. Composition action where we have sluggish multi-capacity processes.

Ex: Liquid level Control → P Controller
Gas Pressure Control → P Controller
Flow Control → PI Controller

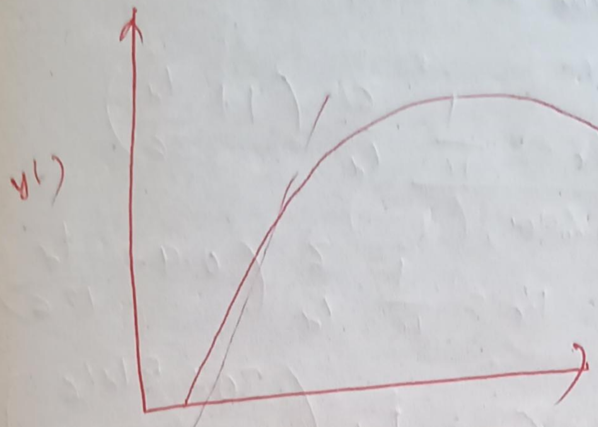
Temperature Control → PID Control
Composition Control → PID Control
Vapour Pressure Control → PID Control



(1) Use of 1/4 D.R. Criteria for a minimum settling time and minimum largest error
(2) Use of integral performance criteria, ISE/IAE or ITAE, develop the mathematical model.
(3) Experimental application is time consuming. The use of semi-empirical rules proven in practice

Process Reaction Curve method

Developed by Cohen and Coon
Step change of magnitude = A. C = controller output which actuates the final control element.
Record the output w.r.t. time.
The curve of y_m v/s t is known as Process reaction curve.



$$G(s) = \frac{Ke^{-t_d s}}{\tau s+1}$$

Cohen and Coon have observed that for both of the processes, the curve is S-shaped in nature, which can be approximated to a 1st order system with dead time. Then, $G(s) = \frac{Ae^{-t_d s}}{\tau s+1}$

- It has 3 parameters K, t_D and τ.

$K = \frac{\text{output (ss)}}{\text{Input (ss)}}$ For output

$\tau_p = \frac{B}{S}$ where S = slope at the point of inflection.

t_d = Time elapsed until the system responds.

Results of Cohen and Coon :-

1. For P Controller,

$$K_c = \frac{1}{K}\frac{\tau}{t_d}\left(1+\frac{t_d}{3\tau}\right)$$

2. For PI Controller,

$$K_c = \frac{1}{K}\frac{\tau}{t_d}\left(0.9+\frac{t_d}{12\tau}\right)$$

$$\tau_I = t_d\left(\frac{30+3t_d/\tau}{9+20t_d/\tau}\right)$$

3. For PID Controller,

$$K_c = \frac{1}{K}\frac{\tau}{t_d}\left(\frac{4}{3}+\frac{t_d}{4\tau}\right)$$

$$\tau_I = t_d\left(\frac{32+6t_d/\tau}{13+8t_d/\tau}\right)$$

$$\tau_D = t_d\left(\frac{4}{11+2t_d/\tau}\right)$$

have the features of output sinusoidal wave change with the frequency of input sinusoidal wave. The frequency w is a complex no.

The ~~real part of w is a~~,

$Re(w) = a$ $\quad Im(w) = b$

Modulus or absolute value or magnitude of

$$|w| = \sqrt{Re(w)^2 + Im(w)^2}$$

It is called Amplitude ratio.

phase angle or argument $(\omega) = \tan^{-1}\left(\frac{Im(\omega)}{Re(\omega)}\right)$.

For a 1st order system, $G_P(s) = \frac{K_P}{\tau_P s+1}$

$(s = j\omega)$

For P controller, $G_C = K_C$.

$AR = K_C$ & $\phi = 0$

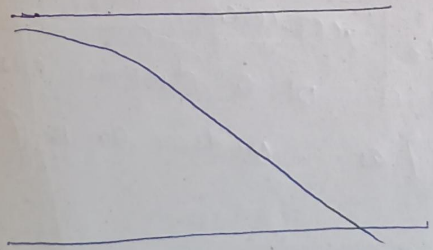
~~controller~~

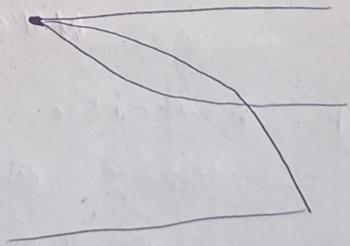
$$G(s) = \frac{K_C e^{-0.5s}}{(2s+1)}$$

ω	AR_1	AR_2	AR_3	ϕ_1	ϕ_2	ϕ_3	AR_0	ϕ_0
0.01	Kc	1	0.998	0	-0.98	-1.14		
0.05	"	"	0.995	"	-1.43	-891		
0.1	"	"	0.98	"	-2.86	-11.3		
0.5	"	"	0.707	"	-14.32	-48.		
1	"	"	0.447	"	-28.64	-63.4		
10	"	"	0.049	"	-28.648	-87.1		

$AR_0 = AR_1 \times AR_2 \times AR_3$

$\phi_0 = \phi_1 + \phi_2 + \phi_3$





For $\phi < 180$, $AR < 1 \Rightarrow$ Stable system.

$$G(s) = \frac{10(s+1)}{(2s+1)(8s+1)(4s+1)}$$

ω from 0.1 to 10 s^{-1}

using time lag = 1 min

determine cross over frequency

Determine max. A.R. for case (a) and case (b) from the plot to be allowed for stable operation?

$$G(s) = \frac{10(s+1)}{(2s+1)(5s+1)(4s+1)}$$

$s = j\omega$

$$G(s) = \frac{10(j\omega+1)}{(2j\omega+1)(5j\omega+1)(4j\omega+1)}$$

$\omega = 0.1, 0.5, 1, 5, 10$

$\phi = \tan^{-1}(-\omega 5)$

$$A.R. = \frac{1}{\sqrt{5^2\omega^2+1}}$$

ω	AR_1	AR_2	AR_3	ϕ_1	ϕ_2	ϕ_3
0.1	0.98	0.894	0.928	0.197	0.464	
0.5	0.707	0.371	0.447	0.7853	1.19	
1	0.447	0.196	0.242	1.107	1.373	
5	0.099	0.039	0.049	1.471	1.531	
10	0.05	0.019	0.025	1.521	1.551	

NYQUIST PLOTS

1. Alternative way to represent the frequency response characteristics of a dynamic system
2. Uses the Im [G(jω)] as ordinate and Re [G(jω)] as abscissa.
3. A specific value of the frequency ω defines a point on the plot

Q $G(s) = \dfrac{6e^{-0.5s}}{s+3}$

$= \dfrac{2e^{-0.5s}}{0.33s+1}$

$2 \rightarrow A.R. = 2$, $\phi = 0$

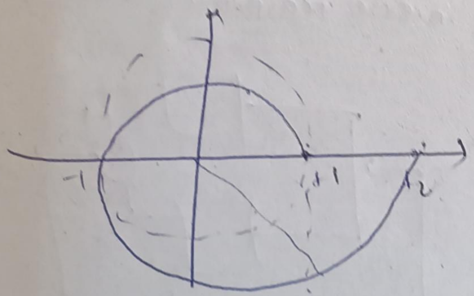
$e^{-0.5s} \rightarrow A.R. = 1$, $\phi = -0.5\omega$

$\dfrac{1}{0.33s+1}$

$A.R. = \dfrac{1}{\sqrt{0.33^2\omega^2+1}}$

$\phi = \tan^{-1}(-0.33\omega)$

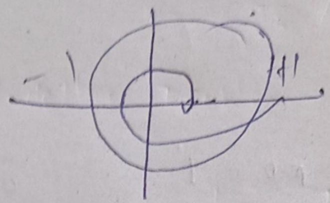
	AR	ϕ	
$\omega = 0.1$	1.9988	$-4.774°$	
$\omega = 10$	0.5765	-359.78	
$\omega = 1$	1.8974	~~$-0.\ldots$~~	$-47.08°$
$\omega = 2$	1.66		-90.715
$\omega = 5$	1.029	~~$-\ldots$~~	$-202.25°$
$\omega = 15$	0.3922	~~$-\ldots$~~	-508.585
$\omega = 20$	0.2967	~~$-\ldots$~~	-654.482



Q $G(s) = \dfrac{e^{-0.1s}}{(s+1)(2s+1)}$ is controlled by PI controller with integral time 0.5 min. (a) State the stability of the system is the system is stable (b) find PM, GM and.

$K_C = 1$ $\phi = 0$ Overall

0.1 0.5 8 16



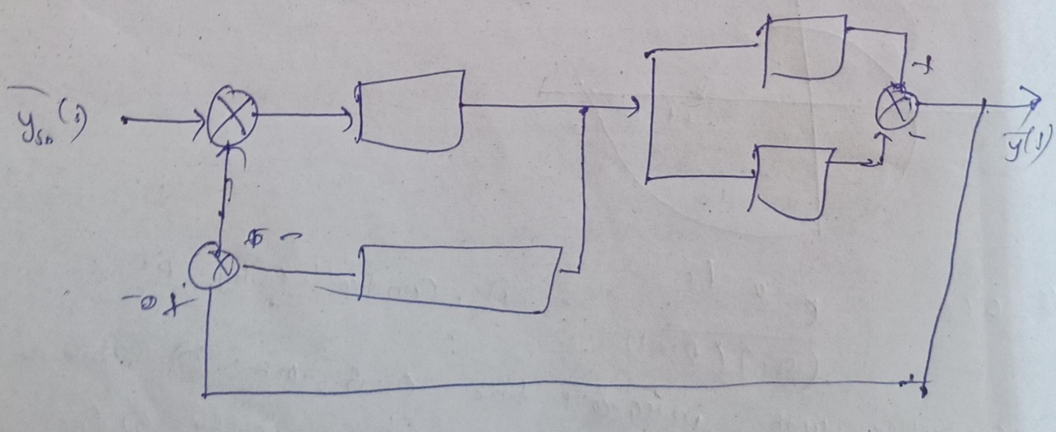
ZIEGLER NICHOLS TUNING TECHNIQUE

	K_C	τ_I	τ_D
P	$K_u/2$	–	–
PI	$K_u/2.2$	$P_u/1.2$	–
PID	$K_u/1.7$	$P_u/2$	$P_u/8$

When $t_d = 0.1$, $w_{co} = 17$ Taclum $K_c = 8.56$

~~The derivative~~

The first used as PID feed back controller with Ziegler-Nichols tuning and the second used as inverse response compensator



SAMPLER

HOLD ELEMENT

ANALOG TO DIGITAL CONVERTER (ADC)

DIGITAL TO ANALOG CONVERTER (DAC)

SINGLE LOOP CONTROL PG 563

SAMPLING OF CONTINUOUS SIGNALS

Z-TRANSFORM

Used in discrete time interval,

$$Z\{y(t)\} = \hat{y}(z) = \sum_{n=0}^{\infty} z^{-n} y(nT)$$

$$y(t) = 1 \qquad \hat{y}(z) = \frac{1}{1 - z^{-1}}$$

$$Z\{at\} = \frac{aTz}{(z-1)^2}$$

$$Z\{\sin \omega t\} = \frac{z}{s}$$

Class Test 2 - Monday - Stability Analysis, Design of feedback Control loop

Class Test 3 - Wednesday - Analysis of closed loop system with various types of Controllers, Multiple loop Control system, Digital Control system (Ch 28, 29, z transform, Response of simple system

Digital Control work Last 3 years PYQ

18 to 15 Marks → Midsem Portion.

32 to 38 Marks → Post mid sem Portion

→ Modelling Problems :- CSTR, Mixing, STHE, Jacketed CSTR

Digital C.L. (i) Continuous Time to Discrete Time,

Z-transform, Response of Simple D.L system