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Tutorial - 2

1)
```
void fun(int n){
    int j=1, i=0;
    while(i<n){
        i = i+j;
    }
    j++;
}
```

→ Values after execution

1st time → i = 1

2nd time → i = 1 + 2

3rd time → i = 1 + 2 + 3

4th time → i = 1 + 2 + 3 + 4

for i^{th} time → $i = (1+2+3+\ldots+i) < n$

$\Rightarrow \frac{i(i+1)}{2} < n \quad \Rightarrow i^2 < n$

$\Rightarrow i = \sqrt{n}$

Time complexity = $O(\sqrt{n})$

2)
```
int fib(int n){
    if(n<=1)
        return n;
    return fib(n-1) + fib(n-2);
}
```

* Recurrence Relation :-

$F(n) = F(n-1) + F(n-2)$

Let $T(n)$ denote the time complexity of $F(n)$. In $F(n-1)$ and $F(n-2)$ time will be $T(n-1)$ and $T(n-2)$. We have one more addition to sum our

results

for $n \geq 1$

$T(n) = T(n-1) + T(n-2) + 1$ ——— ①

For $n=0$ & $n=1$, no addition occurs $\therefore T(0) = T(1) = 0$

Let $T(n-1) \approx T(n-2)$ ——— ②

Adding ② in ①

$T(n) = T(n-1) + T(n-1) + 1$

$\Rightarrow 2 \times T(n-1) + 1$

Using backward substitution

$\because T(n-1) = 2 \times T(n-2) + 1$

$T(n) = 2 \times [2 \times T(n-2) + 1] + 1$

$\rightarrow 4T(n-2) + 3$

We can substitute

$T(n-2) = 2 \times T(n-3) + 1$

$T(n) = 8 \times T(n-3) + 7$

General equation –

$T(n) = 2^k \times T(n-k) + (2^k - 1)$ ——— ③

For $T(0)$

$n-k = 0 \Rightarrow k = n$

Substitute value in ③

$T(n) = 2^n \times T(0) + 2^n - 1$

$\Rightarrow 2^n + 2^n - 1$

$\boxed{T(n) = O(2^n)}$

Space complexity → $O(n)$

Reason → The function calls are executed sequencially. Sequencially, execution guarantees that the stack size will never exceed the depth of cells for first $F(n-1)$ it will create N stack.

3(i) O(nlogn) -

```
#include <iostream>
Using namespace std;
int partition (int arr[], int s, int e) {
    int pivot = arr[s];
    int count = 0;
    for (int i = s; i <= e; i++) {
        if (arr[i] <= pivot)
            count++;
    }
    int pivot_ind = s + count;
    swap (arr[pivot_ind], arr[s]);
    int i = s, j = e;
    while (i < pivot_ind && j > pivot_ind) {
        while (arr[i] <= pivot)
            i++;
        while (arr[j] > pivot)
            j--;
        if (i < pivot_ind && j > pivot_ind) {
            swap (arr[i++], arr[j--]);
        }
    }
    return pivot_ind;
}
void quick (int arr[], int s, int e) {
    if (s == e)
        return;
    int p = partition (arr, s, e);
    quicksort (arr, s, p-1);
    quicksort (arr, p+1, e);
}
```

```
int main() {
    int arr[] = {6, 8, 5, 2, 1};
    int n = 5;
    quicksort(arr, 0, n-1);
    return 0;
}
```

(ii) $O(N^3)$

```
int main() {
    int n=10;
    for(int i=0; i<n; i++) {
        for(int j=0; j<n; j++) {
            for(int k=0; k<n; k++) {
                printf("*");
            }
        }
    }
    return 0;
}
```

(iii) O(loglogn)-

```
int countPrimes(int n) {
    if (n<2)
        return 0;
    bool * non-prime = new bool[n];
    non-prime[i] = true;
    int numnonPrime = 1;
    for(int i=2; i<n; i++) {
        if (nonPrime[i])
            continue;
```

```
        int j = i * 2;
        while (j < n) {
            if (!nonprime[j]) {
                nonprime[j] = true;
                numnonPrime++;
            }
            j += i;
        }
    }
    return (n-1) - numnonPrime;
}
```

4) $T(n) = T(n/4) + T(n/2) + Cn^2$

Using master's Theorem –

Assume $T(n/2) >= T(n/4)$

Equations can be rewritten as

$T(n) <= 2T(n/2) + Cn^2$

$\Rightarrow T(n) <= O(n^2)$

$\Rightarrow T(n) = O(n^2)$

Also $T(n) >= cn^2 \Rightarrow T(n) >= O(n^2)$

$\Rightarrow T(n) = \Omega(n^2)$

$\because T(n) = O(n^2)$ & $T(n) = \Omega(n^2)$

$T(n) = \Omega(n^2)$

$\therefore T(n) = O(n^2)$ & $T(n) = \Omega(n^2)$

$T(n) = \Theta(n^2)$

5)

```
int fun(int n) {
    for(int i=1; i<=n; i++) {
        for(int j=1; j<n; j++) {
            // some O(1) task
```

}
}
}

for i=1, inner loop is executed n times
for i=2, inner loop is executed n/2 times
for i=3, inner loop is executed n/3 times

It is forming a series -

$\Rightarrow n + n/2 + n/3 + \cdots + n/n$

$$n\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right]$$

$$\Rightarrow n \times \sum_{k=1}^{n} \frac{1}{k}$$

$\Rightarrow n \times \log n$

Time complexity $= O(n \log n)$

6) for (int i = 2; i < n; i = pow(i, k)) {
    // some O(1) expression
}

With iterations -

for 1st iteration $\rightarrow 2$

for 2nd iteration $\rightarrow 2^k$

for 3rd iteration $\rightarrow (2^k)^k$

for n iteration $\rightarrow 2^{k^{\log_k(\log(n))}}$

$\because$ last term must be less than or equal to n.

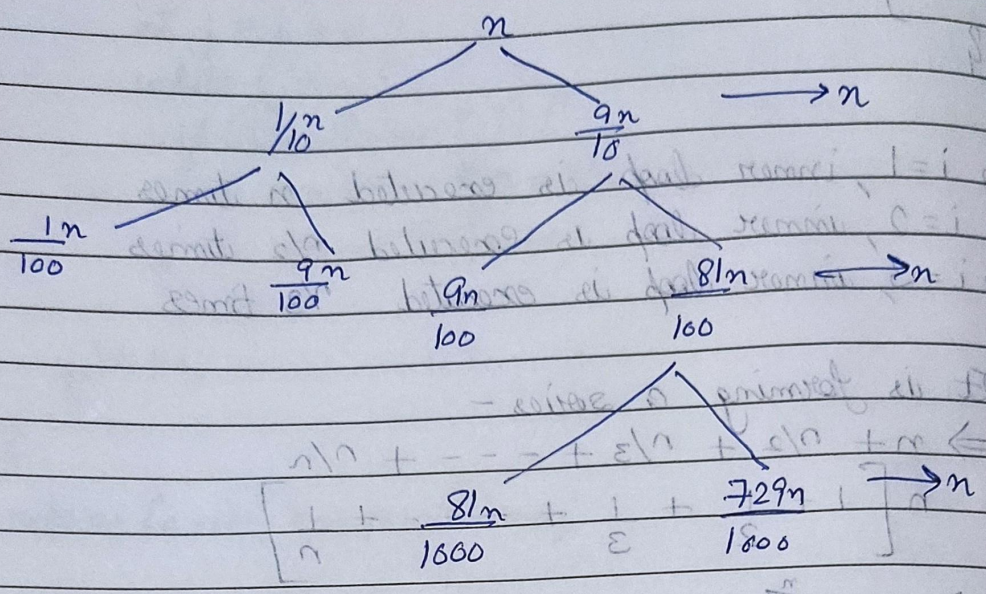
$2^{k^{\log_k(\log(n))}} = 2^{\log n} = n$

Each iteration takes constant times.

$\therefore$ Total iteration $= \log_k(\log(n))$

Time complexity $= O(\log(\log(n)))$

7)



If we split in this manner

Recurrence Relation

$$T(n) = T\left(\frac{9n}{10}\right) + T\left(\frac{n}{10}\right) + O(n)$$

When first branch is of size 9n/10 & second one is n/10

showing the above using recursion tree approach

Calculating values.

At 1st level, value = n

At 2nd level, value $\frac{9n}{10} + \frac{n}{10} = n$

Value remains same at all levels i.e. n

Time complexity = summation of values

$\Rightarrow O(n \times \log(n))$ [upper bound]

$\Rightarrow \Omega(n \times \log(n))$ [lower bound]

$\Rightarrow \Theta(n \log(n))$

8)

(a) $100 < \log(\log n) < \log n < \sqrt{n} < n < n\log n < \log^2(n) < \log(n!) < n^2 < 2^n < n^n < 4^n < 2^{2^n}$

(b) $1 < \log(\log n) < \sqrt{\log n} < \log n < 2\log n < \log(2n) < n < n\log n < \log\sqrt{n} < 2n < 4n < n^2 < n^n < 2(2^n)$

(c) $96 < \log_8 n < n\log_6 n < \log_2 n < n\log_2 n < \log(n!) < 5n < 8n^2 < 7n^3 < n^n < (8)^{2n}$