In [17]:

```
"""Question 3"""
```

Out[17]:

```
'Question 3'
```

In [1]:

```
import pandas as pd
import numpy as np
from sklearn.mixture import GaussianMixture
from sklearn import cluster
import collections
import operator
import matplotlib.pylab as plt
import os
import sys
import warnings
```

In [55]:

```
#a: Kmeans is a special case of EM Algorithm
```

In [56]:

```
%%latex
K-mean is a special case of EM algorithm.
It sets the probability of hidden clusters given input $y_i$ and model parameter $\theta$,
```

K-mean is a special case of EM algorithm. It sets the probability of hidden clusters given input y_i and model parameter θ, i.e., responsibilities γ_i^z as binary 0 and 1.

In [54]:

```
%%latex
1. E-step: Compute responsibilities $\gamma_{i}^{z}$ to determnine whether data $i$ belong
$$
\gamma_{i}^{z} =
\begin{cases}
    1, & \quad \text{when } z=\underset{z}{argmin} \parallel y_i-\mu_z \parallel \\
    0, & \quad \text{other}
\end{cases}
$$
```

1. E-step: Compute responsibilities γ_i^z to determnine whether data i belong to cluster z or not.

$$
\gamma_{i}^{z} =
\begin{cases}
    1, & \quad \text{when } z=\underset{z}{argmin} \parallel y_i-\mu_z \parallel \\
    0, & \quad \text{other}
\end{cases}
$$

In [53]:

```
%%latex
2. M-step: Update new cluster centres based on responsibilities.
$$
\mu^{z}=\frac{\sum_{i=1}^{N}{\gamma_{i}^{z}y_i}}{\sum_{i=1}^{N}{\gamma_{i}^{z}}}
$$

3. Iterate E-step and M-step until $\gamma_{i}^{z}$ does not change or reaches maximum iter
```

2. M-step: Update new cluster centres based on responsibilities.

$$
\mu^{z}=\frac{\sum_{i=1}^{N}{\gamma_{i}^{z}y_i}}{\sum_{i=1}^{N}{\gamma_{i}^{z}}}
$$

3. Iterate E-step and M-step until γ_{i}^{z} does not change or reaches maximum iteration.

In [3]:

```
#b: Import data
data = pd.read_csv(r'F:/Annie/CornellMS/Semester 4/Machine Learning/Homework/HW3/EMM/faithf
data = data.drop(['id'], axis =1)
print (data.shape, "\n", data.head())
```

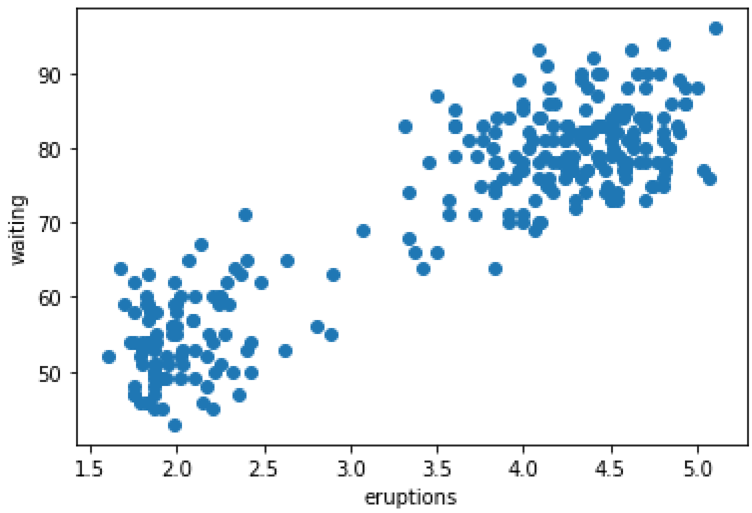
```
(272, 2)
    eruptions  waiting
0       3.600       79
1       1.800       54
2       3.333       74
3       2.283       62
4       4.533       85
```

In [4]:

```
# Parse and plot all the data in 2D plane
plt.scatter(data['eruptions'], data['waiting'])
plt.xlabel('eruptions')
plt.ylabel('waiting')
```

Out[4]:

```
Text(0, 0.5, 'waiting')
```



In [5]:

```
#c: Fitting Gaussian-bimodal distribution
def Gaussian(data):
    clf = GaussianMixture(n_components=2, covariance_type='spherical', init_params='random'
    clf.fit(data)
    print ("No of steps needed for convergence is ", clf.n_iter_)
    return clf.n_iter_
```

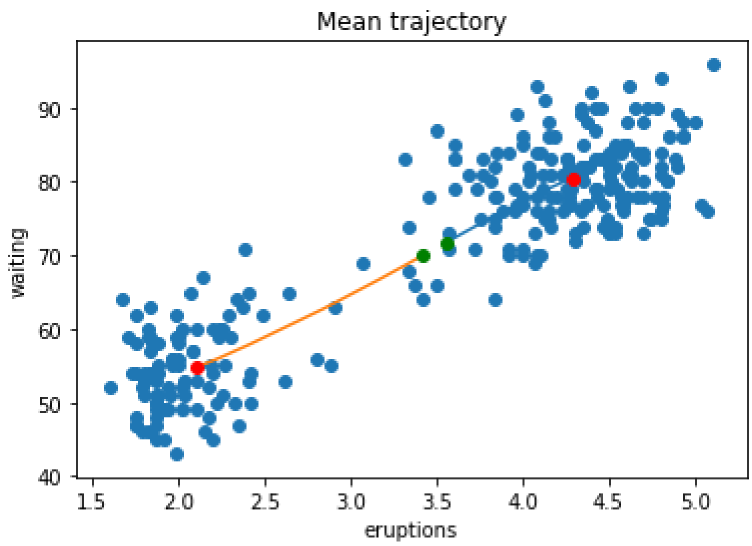
In [13]:

```
# Plot trajectories of two mean vectors in 2-dimensions
if not sys.warnoptions:
    warnings.simplefilter("ignore")

clf = GaussianMixture(n_components=2, covariance_type='spherical', init_params='random', wa
clf.fit(data)

means1 = clf.means_[0]
means2 = clf.means_[1]

while (not clf.converged_):
    means1 = np.append(means1, clf.means_[0])
    means2 = np.append(means2, clf.means_[1])
    clf.fit(data)
length = len(means2)
print (length)
print (means1, "\n", means2)
```

```
22
[ 3.553681   71.75474281  3.553681   71.75474281  3.63371201 72.78955978
  3.80122152 74.93280261  4.07148465 78.23883498  4.27797922 80.42992638
  4.3215886  80.75738068  4.31255202 80.53724865  4.30357977 80.38438983
  4.29825768 80.31355451  4.29569169 80.2841132 ]
 [ 3.42051106 70.02149023  3.42051106 70.02149023  3.33868125 68.96340945
  3.16427617 66.73167602  2.8390941  62.73787319  2.41323538 57.93379497
  2.22549049 55.9695833   2.15427524 55.31054945  2.11836474 54.97136858
  2.1048929  54.82995479  2.10031448 54.77612238]
```

In [14]:

```
# Plot the means
means1 = means1.reshape((-1,2))
means2 = means2.reshape((-1,2))
means1 = pd.DataFrame({'x': means1[:,0], 'y': means1[:,1]})
means2 = pd.DataFrame({'x': means2[:,0], 'y': means2[:,1]})

plt.plot(means1['x'], means1['y'], label = 'mean1')
plt.plot(means2['x'], means2['y'], label = 'mean2')

plt.plot(means1['x'][:1], means1['y'][:1], 'o-',c='green', label="start")
plt.plot(means2['x'][:1], means2['y'][:1], 'o-',c='green')

plt.plot(means1['x'][-1:], means1['y'][-1:], 'o-',c='red', label="end")
plt.plot(means2['x'][-1:], means2['y'][-1:], 'o-',c='red')

plt.scatter(data['eruptions'], data['waiting'])
plt.xlabel('eruptions')
plt.ylabel('waiting')

plt.title('Mean trajectory')
plt.show()
```



In [15]:

```
ermination Criteria
The termination criteria was to continue iterating until the function converges (using conve
```

Out[15]:

```
'The termination criteria was to continue iterating until the function conve
rges (using converge_ function defined in scikit learn). I believed this wou
ld produce the greatest accuracy with a reasonable number of iterations. '
```

In [8]:

```
# Run GMM model for 50 times and plot the distribution
iterations = []
for i in range(50):
    step = Gaussian(data)
    iterations.append(step)
counts = collections.Counter(iterations)
max_count = sorted(counts.items(), key=operator.itemgetter(1))[-1][1]
max_step = sorted(counts.items(), key=operator.itemgetter(0))[-1][0]
```

```
No of steps needed for convergence is  2
No of steps needed for convergence is  10
No of steps needed for convergence is  11
No of steps needed for convergence is  11
No of steps needed for convergence is  10
No of steps needed for convergence is  2
No of steps needed for convergence is  2
No of steps needed for convergence is  2
No of steps needed for convergence is  2
No of steps needed for convergence is  11
No of steps needed for convergence is  12
No of steps needed for convergence is  2
No of steps needed for convergence is  10
No of steps needed for convergence is  9
No of steps needed for convergence is  11
No of steps needed for convergence is  2
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No of steps needed for convergence is  2
No of steps needed for convergence is  11
No of steps needed for convergence is  2
No of steps needed for convergence is  10
```
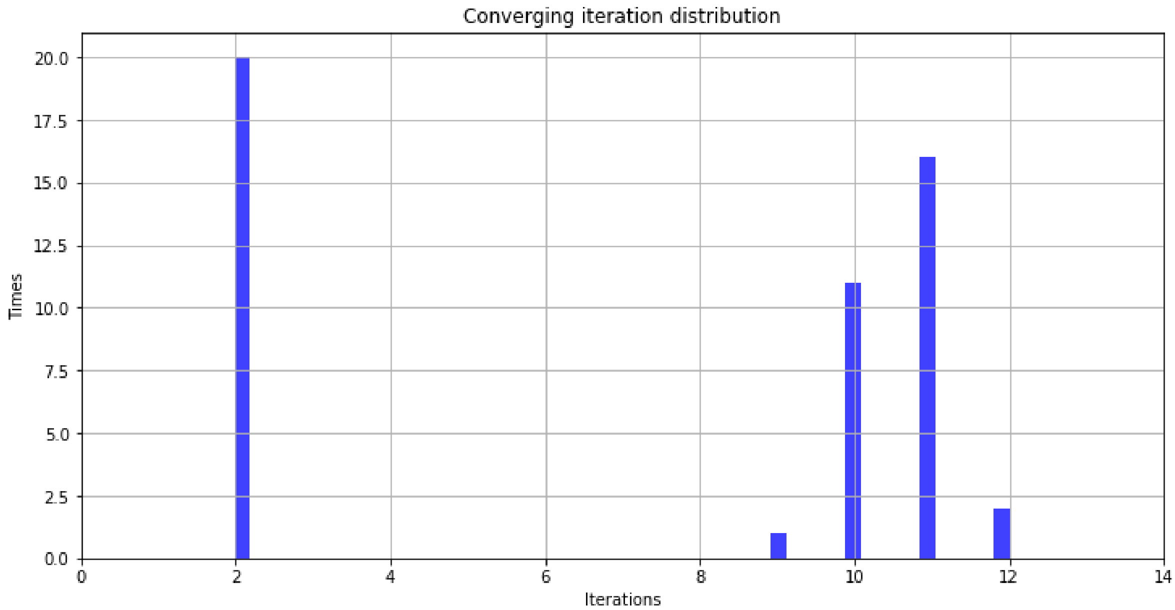

```
No of steps needed for convergence is  11
No of steps needed for convergence is  10
```

In [9]:

```
# Plot bar graph for 50 run
fig, ax = plt.subplots()
fig.set_size_inches(12, 6)
n, bins, patches = plt.hist(iterations, len(iterations)+2, facecolor='b', alpha=0.75)
plt.xlabel('Iterations')
plt.ylabel('Times')
plt.title('Converging iteration distribution')
plt.axis([0, max_step+2, 0, max_count+1])
plt.grid(True)
plt.show()
```



In [10]:

```
# d: K-mean neighbor distribution
def Gaussian_kmean(data, data_mean, data_covar, data_label):
    clf = GaussianMixture(n_components=2, covariance_type='spherical', precisions_init=1/da
                          init_params='kmeans', weights_init=pd.Series(data_label).value_cc
    clf.fit(data)
    print ("No of steps needed for convergence is ", clf.n_iter_)
    return clf.n_iter_
```

In [11]:

```
# Estimate the first guess of the mean and covariance matrices using maximum likelihood ove
clf = cluster.KMeans(n_clusters=2, init = 'random')
clf.fit(data)

clusters = clf.labels_
data1 = data[clusters == 0]
data2 = data[clusters == 1]
data1_mean = np.array(data1.mean(axis = 0))
data2_mean = np.array(data2.mean(axis = 0))
data_means = np.concatenate((data1_mean, data2_mean)).reshape(2,2)

data1_var = data1.var().mean()
data2_var = data2.var().mean()

data_covar = np.array([data1_var, data2_var])

gmm_kmeans_init = GaussianMixture(n_components=2, covariance_type = 'spherical', warm_start
                                  weights_init=pd.Series(clf.labels_).value_counts(normaliz
                                  precisions_init=1/data_covar).fit(data)
```

In [12]:

```
# Run GMM model for 50 times and plot the distribution
iterations = []
for i in range(50):
    step = Gaussian_kmean(data, data_means, data_covar, clf.labels_)
    iterations.append(step)
counts = collections.Counter(iterations)
max_count = sorted(counts.items(), key=operator.itemgetter(1))[-1][1]
max_step = sorted(counts.items(), key=operator.itemgetter(0))[-1][0]

fig, ax = plt.subplots()
fig.set_size_inches(12, 6)
n, bins, patches = plt.hist(iterations, len(iterations)+2, facecolor='b', alpha=0.75)
plt.xlabel('Iterations')
plt.ylabel('Times')
plt.title('Converging iteration distribution')
plt.axis([0, max_step+2, 0, max_count+1])
plt.grid(True)
plt.show()
```

```
No of steps needed for convergence is  6
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```

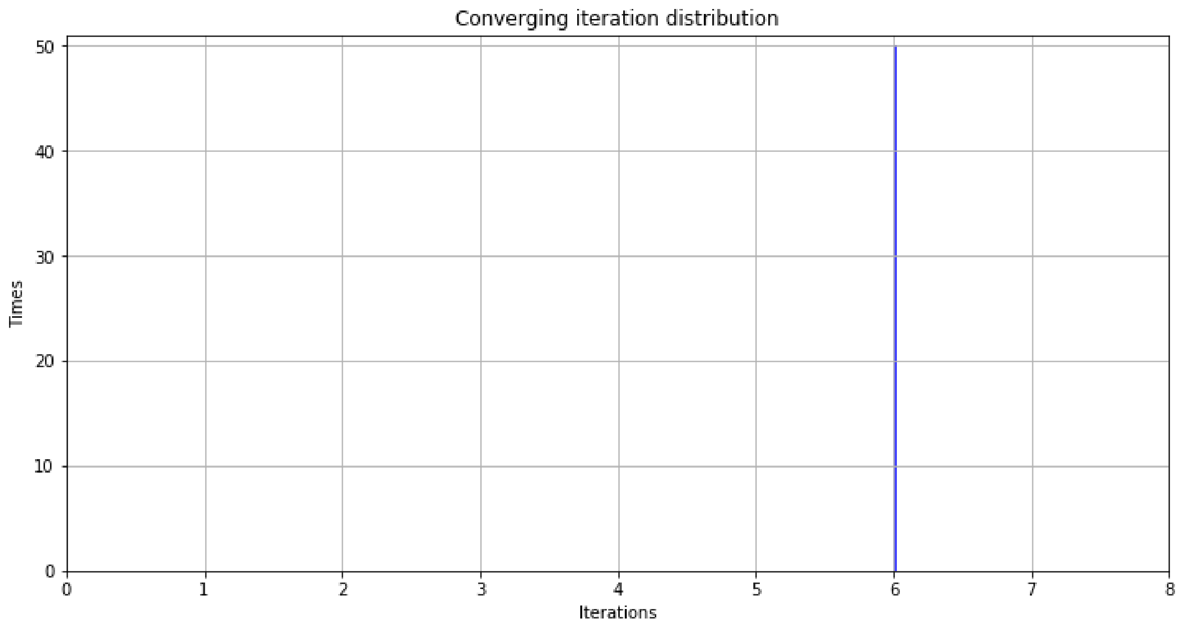

In [16]:

```
(a) and (b)
ogram above, when k means is utilized to cluster, convergence is reached quicker. This is be
```

Out[16]:

```
'As shown by the histogram above, when k means is utilized to cluster, conve
rgence is reached quicker. This is because we are using kmeans to allocate c
luster centre as opposed to randomly assigning, which makes it better as the
dataset is seperable to begin with and that is identified by the algorithm.'
```

In [ ]: