In [30]:

```
print ("Question 1")
```

Question 1

In [1]:

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.image as mpimg
from scipy.spatial.distance import pdist
from sklearn.metrics.pairwise import euclidean_distances
```

In [2]:

```
# a: Import the train/test files from Digit Recognizer
train = pd.read_csv('F:/Annie/CornellMS/Semester 4/Machine Learning/HW1/Digit/train.cs
v', delimiter=',')
test = pd.read_csv('F:/Annie/CornellMS/Semester 4/Machine Learning/HW1/Digit/test.csv',
delimiter=',')
train.shape
```

Out[2]:

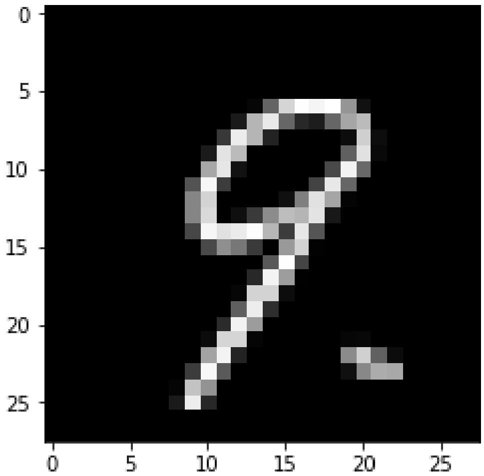
(42000, 785)

In [3]:

```
# b: Display one digit
display_image = test.iloc[2]
display_image = np.array(display_image, dtype='uint8')
pixels = display_image.reshape((28, 28))
plt.imshow(pixels, cmap='gray')
```

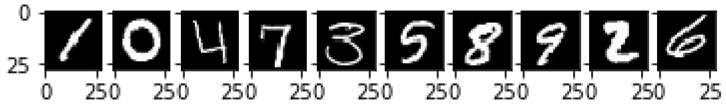
Out[3]:

<matplotlib.image.AxesImage at 0x250f4f3e860>



In [4]:

```
# b: Display one of each digit
def display_digit(file):
    unique = file['label'].unique()
    digits = {}
    for i in unique:
        digits[i] = np.where(train.label==i)[0][:1]
    fig, ax = plt.subplots(1, 10, sharex='col', sharey='row')
    for i in unique:
        display_image = train.iloc[list(digits.values())[i],1:785]
        display_image = np.array(display_image, dtype='uint8')
        pixels = display_image.reshape((28, 28))
        ax[i].imshow(pixels, cmap='gray')
display_digit(train)
```



In [5]:

```
# c: Prior probabilities of each digit
prior_prob = train['label'].value_counts(normalize=True)
print (prior_prob)
# Yes, it is almost uniform across digit because probability of each digit is .1 approx
imately.
```

```
1    0.111524
7    0.104786
3    0.103595
9    0.099714
2    0.099452
6    0.098500
0    0.098381
4    0.096952
8    0.096738
5    0.090357
Name: label, dtype: float64
```

In [25]:

```
print ("Answer", "\n", "The prior probabilities of each class is approximately 0.1, whi
ch is our expectation for nearly evenly distributed class of 10 categories of number.")
```

```
Answer
 The prior probabilities of each class is approximately 0.1, which is our
expectation for nearly evenly distributed class of 10 categories of numbe
r.
```

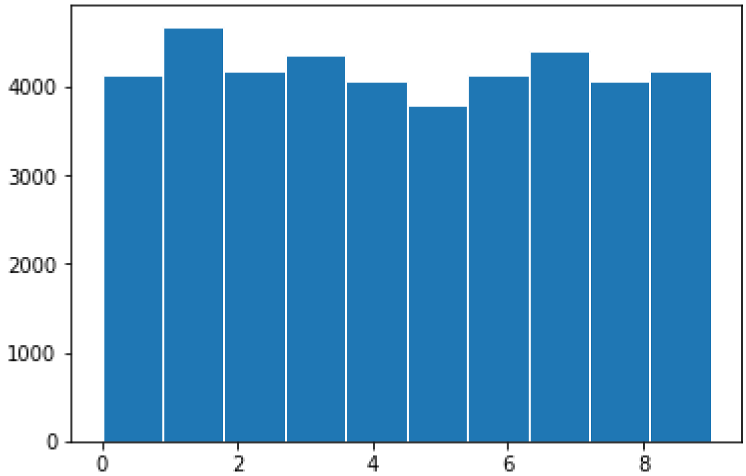
In [6]:

```
# c: Normalized histogram for each digit
plt.hist(train['label'], edgecolor='white',linewidth=1)

# Yes, the following plot reflects that it is nearly even.
```

Out[6]:

```
(array([4132., 4684., 4177., 4351., 4072., 3795., 4137., 4401., 4063.,
        4188.]),
 array([0. , 0.9, 1.8, 2.7, 3.6, 4.5, 5.4, 6.3, 7.2, 8.1, 9. ]),
 <a list of 10 Patch objects>)
```



In [27]:

```
print ("Answer", "\n", "The following plot can be mostly called even. There are 42000 d
igits in the training set, hence we would expect each digit to have a representation of
4200 which we can see approximately from the histogram. We notice that the number of 1s
are the maximum and the number of 5s is the least.")
```

```
Answer
 The following plot can be mostly called even. There are 42000 digits in t
he training set, hence we would expect each digit to have a representation
of 4200 which we can see approximately from the histogram. We notice that
the number of 1s are the maximum and the number of 5s is the least.
```

In [7]:

```
# d: Select examples of each digit
labels = np.asarray(train['label'])
def example_sample(labels):
    result = []
    for i in range(10):
        indices = np.where(labels == i)
        index = indices[0][0]
        result.append(index)
    return result
examples = example_sample(labels)
print(examples)
```

```
[1, 0, 16, 7, 3, 8, 21, 6, 10, 11]
```

In [8]:

```
# Calculate L2 distance
pixels = np.asarray(train[train.columns[1:]])
def nearest_points(example, data):
    nearest = []
    for e in example:
        target = data[e]
        dis = float('inf')
        for d in range(len(data)):
            if d == e:
                continue
            else:
                distance = np.linalg.norm(target - data[d])
                if distance < dis:
                    dis = distance
                    point = d
        nearest.append([dis, point])
    return nearest
nearest = nearest_points(examples, pixels)
print (nearest)
```

```
[[1046.5954328201515, 12950], [489.67948701165744, 29704], [1380.877257398
354, 9536], [1832.6649993929605, 8981], [1356.8809822530493, 14787], [106
6.3676664265472, 30073], [1446.5113203843239, 16240], [863.5010133172977,
15275], [1593.7775879965184, 32586], [910.5767403135224, 35742]]
```

In [9]:

```
# print error examples
error_examples = []
for i in range(len(nearest)):
    if labels[nearest[i][1]] == labels[examples[i]]:
        error_examples += [[examples[i], labels[examples[i]]]]
    else:
        error_examples += [[examples[i], labels[nearest[i][1]], '*']]
print(error_examples)
```

```
[[1, 0], [0, 1], [16, 2], [7, 5, '*'], [3, 4], [8, 5], [21, 6], [6, 7], [1
0, 8], [11, 9]]
```

In [10]:

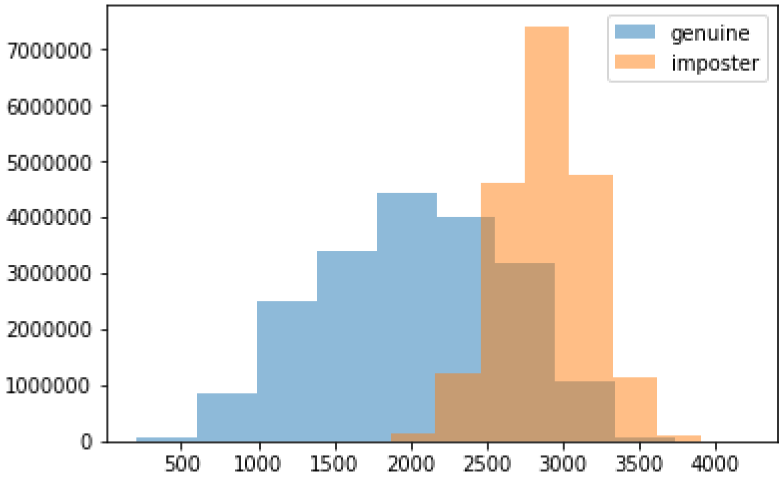
```
# e: Genuine and imposters
digit_0 = np.array(train[train.columns[1:]][train.label == 0])
digit_1 = np.array(train[train.columns[1:]][train.label == 1])
genuine_0 = np.append([], pdist(digit_0))
genuine_1 = np.append([], pdist(digit_1))
imposter = euclidean_distances(digit_0, digit_1)
```

In [11]:

```
# Plot genuine and imposters
plt.hist(np.append(genuine_0, genuine_1), alpha = 0.5, label = 'genuine')
plt.hist(imposter.flatten(), alpha = 0.5, label = 'imposter')
plt.legend(loc='upper right')
```

Out[11]:

```
<matplotlib.legend.Legend at 0x2509afc27f0>
```



In [12]:

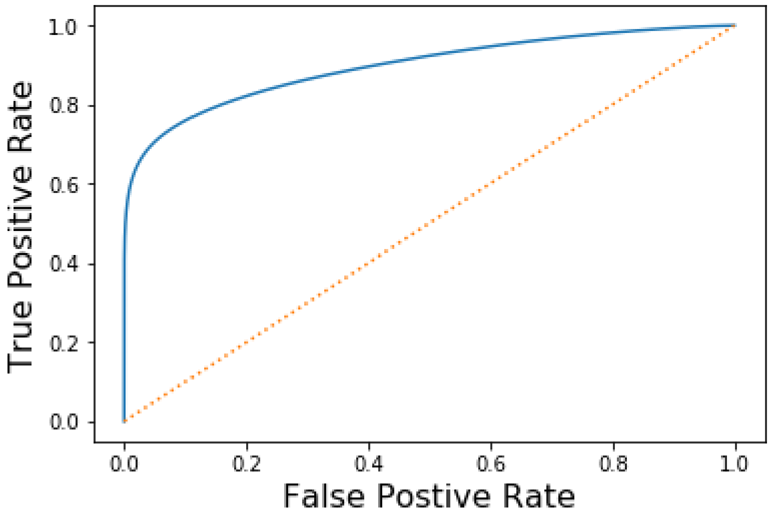
```
# f: ROC curve generation
def roc(genuine, imposter):
    theta = np.linspace(min(genuine), max(imposter) + 1, 1000)
    tpr = np.array([])
    fpr = np.array([])
    for t in theta:
        tp = np.sum(genuine < t)
        fp = np.sum(imposter < t)
        fn = len(genuine) - np.sum(genuine < t)
        tn = len(imposter) - np.sum(imposter < t)
        tpr = np.append(tpr, tp/(tp + fn))
        fpr = np.append(fpr, fp/(fp + tn))
    return tpr, fpr
```

In [13]:

```
tpr, fpr = roc(np.append(genuine_0, genuine_1), imposter.flatten())
# Plot ROC curve
fig = plt.figure()
plt.plot(fpr, tpr)
plt.plot([0, 1], [0, 1], ':')
plt.xlabel('False Postive Rate', fontsize=16)
plt.ylabel('True Positive Rate', fontsize=16)
```

Out[13]:

```
Text(0,0.5,'True Positive Rate')
```

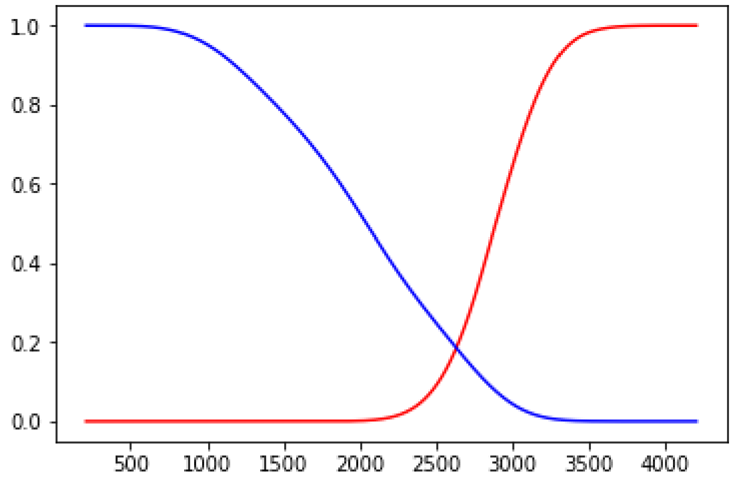


In [14]:

```
fnr = 1 - tpr
theta = np.linspace(min(np.append(genuine_0, genuine_1)), max(imposter.flatten()) + 1,
1000)
fig = plt.figure()
plt.plot(theta, fpr, 'r')
plt.plot(theta, fnr, 'b')
```

Out[14]:

```
[<matplotlib.lines.Line2D at 0x2509b081748>]
```



In [15]:

```
# Equal error generation
def equalError(fpr, fnr):
    diff = fpr - fnr
    for i in range(len(diff)):
        if diff[i] > 0:
            break
    return (fpr[i - 1] + fpr[i])/2
equalError(fpr, fnr)
```

Out[15]:

```
0.1865334958330681
```

In [29]:

```
# When a classifier guesses randomly, error rate will be high if it is a multiclass pro
blem but if it is a 2-class problem
# Also, probability of occurence of each class affects the error rate
print ("Answer", "\n", "When we have a 2-class problem, and a majority occuring class i
n it, random guess might work decently well, especially when we output the major class.
But when data-classes increases, error rate of random guess will start increasing. Rand
om guess should not be used because we are not trying to find the features and weighing
them to output class, hence usually there is possibility of incurring a high error rat
e.")
```

```
Answer
 When we have a 2-class problem, and a majority occuring class in it, rand
om guess might work decently well, especially when we output the major cla
ss. But when data-classes increases, error rate of random guess will start
increasing. Random guess should not be used because we are not trying to f
ind the features and weighing them to output class, hence usually there is
possibility of incurring a high error rate.
```

In [16]:

```
# g: knn implementation
#def split_data(data):
#    data = np.array(data)
#    np.random.shuffle(data)
#    return [data[i::n] for i in range(n)]

def knn(tr, te, k):
    tr_label = tr.T[0].T
    te_label = te.T[0].T
    tr_data = tr.T[1:].T
    te_data = te.T[1:].T
    y_hat = np.zeros(len(te_label))
    for i in range(len(te_data)):
        distance = euclidean_distances([te_data[i]], tr_data)
        ind = np.argpartition(distance[0], k)[:k]
        nearest = tr_label[ind]
        counts = np.bincount(nearest)
        y_hat[i] = np.argmax(counts)
        if i % 1000 == 0: print(i)
    accuracy = sum(y_hat==te_label)/len(te_label)
    print("Accuracy: ", accuracy)
    return y_hat, accuracy
```

In [17]:

```
# h: 3-fold classification
#data = split_data(train, 3)
three = [i for i in np.linspace(0, len(train),4)]
print (train.shape[0], three)
data = np.array(train)
np.random.shuffle(data)
sol = []
for i in range(0, 3):
    t1, t2, t3, t4, t5, t6 = three[i], three[i+1], three[(i+1)%3], three[(i+1)%3 +1], t
hree[(i+2)%3], three[(i+2)%3 +1]
    print (t1, t2, t3, t4, t5, t6 )
    test_val = data[int(t1):int(t2),:]
    train_val = np.concatenate((data[int(t3):int(t4),:], data[int(t5):int(t6),:]), axis
=0)
    print (train_val.shape, test_val.shape)
    y, acc = knn(train_val, test_val, 5)
    sol += [y, acc]
```

```
42000 [0.0, 14000.0, 28000.0, 42000.0]
0.0 14000.0 14000.0 28000.0 28000.0 42000.0
(28000, 785) (14000, 785)
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
12000
13000
Accuracy:  0.9637857142857142
14000.0 28000.0 28000.0 42000.0 0.0 14000.0
(28000, 785) (14000, 785)
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
12000
13000
Accuracy:  0.9656428571428571
28000.0 42000.0 0.0 14000.0 14000.0 28000.0
(28000, 785) (14000, 785)
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
12000
13000
Accuracy:  0.9641428571428572
```

In [18]:

```
# Average accuracy
print (np.mean(acc))
```

```
0.9641428571428572
```

In [19]:

```
# i: Confusion matrix
test_label = test_val.T[0].T
def confusion(y_hat, y):
    table = np.zeros(shape=(10,10), dtype = np.uint16)
    np.set_printoptions(precision=0)
    for i in range(0, len(y_hat)):
        table[int(y_hat[i])][y[i]] += 1
    return table
confusion(sol[4:6][0], test_label)
```

Out[19]:

```
array([[1427,    0,    8,    1,    1,    5,    8,    0,    6,    6],
       [   0, 1572,   20,    4,   11,    3,    5,   20,   18,    6],
       [   2,    5, 1349,    6,    0,    1,    0,    3,    3,    4],
       [   0,    1,    5, 1387,    0,   25,    0,    1,   25,    9],
       [   0,    0,    1,    0, 1316,    2,    2,    6,    8,   14],
       [   1,    0,    1,   13,    0, 1179,    6,    0,   19,    1],
       [   6,    1,    3,    2,    8,   15, 1341,    0,    4,    0],
       [   1,    5,   30,    7,    2,    0,    0, 1373,    5,   28],
       [   0,    0,    2,    6,    1,    3,    1,    0, 1266,    2],
       [   1,    1,    1,    7,   30,    8,    0,   18,   18, 1288]],
      dtype=uint16)
```

In [20]:

```
#j : Train-test classification using the entire data
def knn_classifier(tr, te, k):
    tr_label = tr.T[0].T
    tr_data = tr.T[1:].T
    y_hat = np.zeros(len(te))
    for i in range(len(te)):
        distance = euclidean_distances([te[i]], tr_data)
        ind = np.argpartition(distance[0], k)[:k]
        nearest = tr_label[ind]
        counts = np.bincount(nearest)
        y_hat[i] = np.argmax(counts)
        if i % 1000 == 0: print(i)
    return y_hat
```

In [21]:

```
train_data = np.array(train)
test_data = np.array(test)
test_labels = knn_classifier(train_data, test_data, 5)
```

```
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
11000
12000
13000
14000
15000
16000
17000
18000
19000
20000
21000
22000
23000
24000
25000
26000
27000
```

In [24]:

```
final_out = pd.DataFrame(test_labels, columns=['Label'])
final_out['ImageId'] = range(1, len(final_out) + 1)
final_out = final_out.set_index('ImageId')
final_out.to_csv('F:/Annie/CornellMS/Semester 4/Machine Learning/HW1/Digit/Digit_Submis
sion.csv')
```