

概率论与数理统计练习题 (6)

数学期望、方差

姓名_____学号_____班级_____

1. 填空题

(1) 设随机变量 X 的期望 EX 存在, 且 $EX = a, E(X^2) = b, c$ 为一常数, 则

$$D(cX) = \underline{\hspace{2cm}}.$$

(2) 设随机变量 X 服从参数为 λ 的泊松分布, 且 $P\{X=1\} = P\{X=2\}$, 则

$$EX = \underline{\hspace{2cm}}, \quad DX = \underline{\hspace{2cm}}.$$

(3) 设随机变量 X 的概率密度为 $f(x) = \begin{cases} ax+b, & 0 \leq x \leq 1, \\ 0, & \text{其他.} \end{cases}$, 且 $a > 0, DX = \frac{1}{18}$, 则

$$a = \underline{\hspace{2cm}}, \quad b = \underline{\hspace{2cm}}, \quad EX = \underline{\hspace{2cm}}.$$

(4) 设 X 和 Y 独立, 且 $EX = EY = 0, DX = DY = 1$, 则 $E[(X+2Y)^2] = \underline{\hspace{2cm}}.$

(5) 设 $E(X) = \mu, D(X) = \sigma^2$, 则由切比雪夫不等式有 $P\{|X - \mu| < 2\sigma\} \geq \underline{\hspace{2cm}}.$

(6) 设连续型随机变量 X 的概率密度为 $f(x) = \begin{cases} a \sin x + b, & 0 \leq x \leq \frac{\pi}{2}, \\ 0, & \text{其他.} \end{cases}$ 且

$$EX = \frac{\pi+4}{8}, \text{ 则 } a = \underline{\hspace{2cm}}, \quad b = \underline{\hspace{2cm}}.$$

2. 选择题

(1) 若随机变量 $X \sim N(\mu, \sigma^2)$, $EX = 3, DX = 1$, 则 $P\{-1 \leq X \leq 1\} = (\quad)$.

(A) $2\Phi(1)-1$; (B) $\Phi(4)-\Phi(2)$; (C) $\Phi(-4)-\Phi(-2)$; (D) $\Phi(2)-\Phi(4)$.

(2) 设随机变量 $X \sim b(n, p)$, 且 $EX = 2.4, DX = 1.44$, 则 n, p 的值为 (\quad) .

(A) $n=4, p=0.6$; (B) $n=6, p=0.4$; (C) $n=8, p=0.3$; (D) $n=24, p=0.1$.

(3) 设随机变量 X 服从指数分布, 且 $DX = 0.25$, 则 X 的概率密度为 $f(x) = (\quad)$.

(A) $\begin{cases} 2e^{-2x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$; (B) $\begin{cases} \frac{1}{2}e^{-\frac{1}{2}x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$; (C) $\begin{cases} 4e^{-4x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$; (D) $\begin{cases} \frac{1}{4}e^{-\frac{1}{4}x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$.

(4) 设随机变量 X 的分布函数为 $F(x) = \begin{cases} 0, & x < 0 \\ x^3, & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$, 则 $EX =$ ().

(A) $\int_0^{+\infty} x^4 dx$; (B) $\int_0^{+\infty} 3x^3 dx$; (C) $\int_0^1 x^4 dx + \int_1^{+\infty} x dx$; (D) $\int_0^1 3x^3 dx$.

3. 对某目标进行射击, 直到击中为止, 如果每次命中率为 p , 求射击次数 X 的数学期望和方差.

4. 一工厂生产的某种设备的寿命 X (以年计) 服从指数分布, 概率密度为

$$f(x) = \begin{cases} \frac{1}{4} e^{-\frac{1}{4}x}, & x > 0, \\ 0, & x \leq 0. \end{cases}$$

工厂规定, 出售的设备若在售出一年之内损坏可予以调换, 若工厂售出一台设备盈利 100 元, 调换一台设备厂方需花费 300 元. 试求厂方出售一台设备净盈利的数学期望.

概率论与数理统计练习题 (6) 详细解答

1. 填空题

(1)

$$D(CX) = C^2 DX = C^2 [E(X^2) - (EX)^2] = C^2(b - a^2)$$

(2) 由 $P\{X=1\} = P\{X=2\}$, 即 $\frac{\lambda^1 e^{-\lambda}}{1!} = \frac{\lambda^2 e^{-\lambda}}{2!}$, 得 $\lambda = 2$, 故 $E(X) = D(X) = 2$.

(3)

$$\begin{aligned} \int_0^1 (ax+b) dx &= 1 \Rightarrow \frac{1}{2}a + b = 1 \quad \dots\dots\dots ① \\ EX &= \int_0^1 x(ax+b) dx = \frac{1}{3}a + \frac{1}{2}b \\ E(X^2) &= \int_0^1 x^2(ax+b) dx = \frac{1}{4}a + \frac{1}{3}b \\ DX &= E(X^2) - (EX)^2 = \frac{1}{4}a + \frac{1}{3}b - \left(\frac{1}{3}a + \frac{1}{2}b\right)^2 = \frac{1}{8} \quad ② \end{aligned}$$

由①②联立可解得: $a = \pm 2$, $b = 0$ 或 2 , $EX = \frac{2}{3}$ 或 $\frac{1}{3}$
又 $\because a > 0$. $\therefore a = 2$, $b = 0$, $EX = \frac{2}{3}$

(4)

$$\begin{aligned} E(X^2) &= DX + (EX)^2 = 1 & E(Y^2) &= DY + (EY)^2 = 1 \\ E[(X+2Y)^2] &= E(X^2) + 4EX \cdot EY + 4E(Y^2) = 5 \end{aligned}$$

(5)

$$P\{|X - EX| < \varepsilon\} \geq 1 - \frac{DX}{\varepsilon^2} \Rightarrow P\{|X - \mu| < 2\sigma\} \geq 1 - \frac{\sigma^2}{(2\sigma)^2} = \frac{3}{4}$$

(6)

$$\begin{aligned} \int_0^{\frac{\pi}{2}} (a \sin x + b) dx &= 1 \Rightarrow a + \frac{\pi}{2}b = 1 \\ EX &= \int_0^{\frac{\pi}{2}} x(a \sin x + b) dx = \frac{\pi+4}{8} \Rightarrow a + \frac{\pi^2}{8}b = \frac{\pi+4}{8} \end{aligned} \quad \left. \vphantom{\int_0^{\frac{\pi}{2}} (a \sin x + b) dx = 1} \right\} \Rightarrow a = \frac{1}{2}, b = \frac{1}{\pi}$$

2. 选择题

(1)

$$P\{-1 \leq X \leq 1\} = P\left\{\frac{-1-3}{1} \leq \frac{X-3}{1} \leq \frac{1-3}{1}\right\} = \Phi(-2) - \Phi(-4) = [1 - \Phi(2)] - [1 - \Phi(4)] \\ = \Phi(4) - \Phi(2) \quad \text{故选 B}$$

(2)

$$np = 2.4, \quad npq = 1.44 \Rightarrow q = \frac{1.44}{2.4} = 0.6 \Rightarrow p = 0.4 \quad \left. \begin{array}{l} np = 2.4 \\ npq = 1.44 \end{array} \right\} \Rightarrow n = 6 \quad \text{故选 B}$$

(3)

$$f(x) = \lambda e^{-\lambda x}, \quad x > 0 \quad DX = \frac{1}{\lambda^2} = 0.25 \Rightarrow \lambda = 2 \quad \text{故选 A}$$

(4)

$$f(x) = F'(x) = \begin{cases} 3x^2, & 0 \leq x \leq 1 \\ 0, & \text{其他} \end{cases} \quad EX = \int_0^1 3x^3 dx \quad \text{故选 D}$$

3. 解: 显然 $X \sim G(p)$, 故 $E(X) = \frac{1}{p}$, $D(X) = \frac{1-p}{p^2}$.

4. 解: 设一台设备的净盈利为 Y 元, 由于 $Y = \begin{cases} 100, & X > 1 \\ -200, & X \leq 1 \end{cases}$, 所以

$$E(Y) = 100 \int_1^{\infty} \frac{1}{4} e^{-\frac{x}{4}} dx - 200 \int_0^1 \frac{1}{4} e^{-\frac{x}{4}} dx \\ = 100e^{-\frac{1}{4}} + 200e^{-\frac{1}{4}} - 200 = 300e^{-\frac{1}{4}} - 200 = 33.64 \quad (\text{元}).$$