Simulated Test of Fit for Two Normal Populations

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Contents

Theory

Goodness-of-Fit Tests

In this section the test of hypothesis concerns the CDF for the random variable X rather than its parameters; (location and scale). The test can be stated as

$$H_0 : F_X(x) = F_{X_0}(x)$$

when $F_{X_0}(x)$ is completely specified.[1]

SAS provides the following goodness of fit tests:

- Shapiro-Wilk test
- Kolmogorov-Smirnov test
- Anderson-Darling test
- Cramer-von Mises test

[1] When you specify the NORMAL option in the PROC UNIVARIATE statement or you request a fitted parametric distribution in the HISTOGRAM statement, the procedure computes goodness-of-fit tests for the null hypothesis that the values of the analysis variable are a random sample from the specified theoretical distribution.

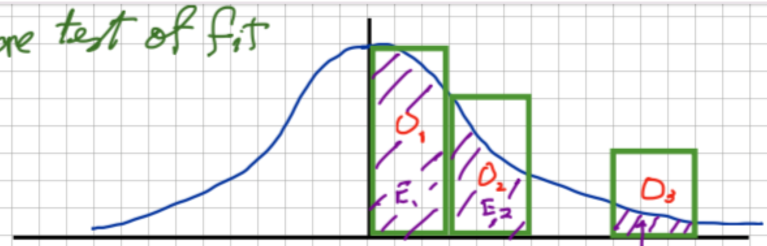
The Kolmogorov-Smirnov test D statistic, the Anderson-Darling statistic, and the Cramer-von Mises statistic are based on the empirical distribution function (EDF).

> If you want to test the normality assumptions for analysis of variance methods, beware of using a statistical test for normality alone. A test's ability to reject the null hypothesis (known as the power of the test) increases with the sample size. As the sample size becomes larger, increasingly smaller departures from normality can be detected. Because small deviations from normality do not severely affect the validity of analysis of variance tests, it is important to examine other statistics and plots to make a final assessment of normality. The skewness and kurtosis measures and the plots that are provided by the PLOTS option, the HISTOGRAM statement, the PROBPLOT statement, and the QQPLOT statement can be very helpful. For small sample sizes, power is low for detecting larger departures from normality that may be important. To increase the test's ability to detect such deviations, you may want to declare significance at higher levels, such as 0.15 or 0.20, rather than the often-used 0.05 level. Again, consulting plots and additional statistics can help you assess the severity of the deviations from normality.

χ^2 Procedure

Perhaps you are beginning to wonder why I haven't mentioned the Ch1 Square Goodness of Fit procedure? Good question! Answer it is not very good when you have continuous data. It is much better (not good) when doing contingency tables. Consider the following figure

Chi-square test of fit



O_i = # of observed point in cell i

$$\chi^2 \text{ statistic} = \sum_{\text{all cells}} (O_i - E_i)^2 / E_i$$

observed	expected	$(O-E)^2$	$(O-E)^2/E$
5	10	$(5-10)^2$	$25/10 = 2.5$
15	10	$(15-10)^2$	$25/10 = 2.5$
95	100	$(95-100)^2$	$25/100 = .25$
105	100	$(105-100)^2$	$25/100 = .25$
25	10	$(25-10)^2$	$225/10 = 22.5$ very large
115	100	$(115-100)^2$	$225/100 = 2.25$

This method is very sensitive to difference when the expected count is small and not sensitive when the expected count is large

QQ and PP Plots

Probability plots can be used to assess normality of data, especially normality of the residuals in linear regression. Let $z_{(1)} \le z_{(2)} \le \ldots \le z_{(n)}$ represent the ordered values of n independent and identically distributed $N(0,1)$ random variables. It can be shown that the expected value of $z_{(i)}$ is

$$E(z_{(i)}) \approx \gamma_i = \Phi^{-1}[(i-3/8)/(n+1/4)]$$

where Φ is the cdf for the standard normal given by

$$\Phi(x) = (2\pi)^{-1/2} \int_{-\infty}^{x} e^{-1/2t^2} dt.$$

The QQ plot consists of plotting of the ordered data (standardized residuals), $z_{(i)}$ versus γ_i, i.e. $(z_{(i)}, \gamma_i)$. If the data are normal then the resulting scatterplot should fall on the diagonal degree line ($z_{(i)} = \gamma_i$). The PP plot is obtained when plotting of the ordered pairs $(\Phi(z_{(i)}), [i/n])$.

Shapiro-Wilk Statistic

The Shapiro-Wilk statistic, W (also denoted as W_n to emphasize its dependence on the sample size n) is the ratio of the best estimator of the variance (based on the square of a linear combination of the order statistics) to the usual corrected sum of squares estimator of the variance (Shapiro and Wilk, 1965). The statistic W is always greater than zero and less than or equal to one $(0 < W \leq 1)$. When the data are normally distributed, one would expect the ratio of these two estimators for the variance to be close to one, in which case, small values of W lead to the rejection of the null hypothesis of normality. The distribution of W is highly skewed. Seemingly large values of W (such as 0.90) may be considered small and lead you to reject the null hypothesis. The method for computing the p-value (the probability of obtaining a W statistic less than or equal to the observed value) depends on n. For $n = 3$, the probability distribution of W is known and is used to determine the p-value. For $n > 4$, a normalizing transformation is computed:

$$Z_n = \begin{cases} (-\log(\gamma - \log(1 - W_n)) - \mu)/\sigma & \text{if } 4 \leq n \leq 11 \\ (\log(1 - W_n) - \mu)/\sigma & \text{if } n \geq 12 \end{cases}$$

The values of σ, γ, and μ are functions of n obtained from simulation results. Large values of Z_n indicate departure from normality, and because the statistic Z_n has an approximately standard normal distribution, this distribution is used to determine the p-values for $n > 4$.[2]

EDF Goodness-of-Fit Tests

The EDF tests offer advantages over traditional chi-square goodness-of-fit test, including improved power and invariance with respect to the histogram midpoints. For a thorough discussion, refer to D'Agostino and Stephens (1986).

The **Empirical Distribution Function (EDF)** is defined for a set of n independent observations $X_1, \ldots, X_n$ with a common distribution function $F(x)$. Denote the observations ordered from smallest to largest as $X_{(1)}, \ldots, X_{(n)}$. The EDF, $F_n(x)$, is,

$$\begin{array}{lll} F_n(x) = 0, & x < X_{(1)} & \\ F_n(x) = \frac{i}{n}, & X_{(i)} \leq x < X_{(i+1)} & i = 1, \ldots, n-1 \ . \\ F_n(x) = 1, & X_{(n)} \leq x & \end{array}$$

Note: $F_n(x)$ is a step function that takes a step of height $\frac{1}{n}$ at each observation. This function estimates the distribution function $F(x)$. At any value x, $F_n(x)$ is the proportion of observations less than or equal to x, while $F(x)$ is the probability of an observation less than or equal to x. EDF statistics measure the discrepancy between $F_n(x)$ and $F(x)$.[3]

[2] The Shapiro-Wilks procedure is based upon "moment-type" estimators and would not be as powerful as the procedures that are based upon the estimated CDF for X. It should probably not be used without considering the other procedures.

[3] The computational formulas for the EDF statistics are based upon the probability integral transformation $U = F(X)$. That is, if $X \sim F(X)$ then $U \sim U(0, 1)$. In the test of fit problems, $F(X)$ is the null (or specified) distribution function.

Kolmogorov D Statistic

The Kolmogorov-Smirnov statistic (D) is defined as

$$D = \sup_x |F_n(x) - F(x)|$$

The Kolmogorov-Smirnov statistic is computed as the maximum of D^+ and D^-, where D^+ is the largest vertical distance between the EDF and the distribution function when the EDF is greater than the distribution function, and D^- is the largest vertical distance when the EDF is less than the distribution function[4].

$$\begin{aligned} D^+ &= \max_i \left(\tfrac{i}{n} - U_{(i)}\right) \\ D^- &= \max_i \left(U_{(i)} - \tfrac{i-1}{n}\right) \\ D &= \max\left(D^+, D^-\right) \end{aligned}$$

Quadratic EDF Statistics

The Anderson-Darling statistic and the Cramer-von Mises statistic are special cases of the general quadratic class of EDF statistics. This class of statistics is based on the squared difference $(F_n(x) - F(x))^2$. The general form of the quadratic class of EDF statistics is

$$Q = n \int_{-\infty}^{+\infty} (F_n(x) - F(x))^2 \psi(x) dF(x) \tag{1}$$

where $\psi(x)$ is a weight function defined on the squared difference $(F_n(x) - F(x))^2$.

Anderson-Darling Statistic

The Anderson-Darling statistic considers $\psi(x) = [F(x)(1 - F(x))]^{-1}$ in which case equation (1) is

$$A^2 = -n - \frac{1}{n} \sum_{i=1}^{n} \left[(2i-1)\log U_{(i)} + (2n+1-2i)\log(1 - U_{(i)})\right].$$

where $U_{(i)} = F_X(X_{(i)})$.

Cramer-von Mises Statistic

The Cramer-von Mises statistic considers $\psi(x) = 1$ in which case equation (1) is

$$W^2 = \sum_{i=1}^{n} \left(U_{(i)} - \frac{2i-1}{2n}\right)^2 + \frac{1}{12n}.$$

[4] PROC UNIVARIATE uses a modified Kolmogorov D statistic to test the data against a normal distribution with mean and variance equal to the sample mean and variance.

Tests Based on the Empirical Distribution Function (EDF)

I have previously indicated that the EDF is the best estimate for the population CDF for a random variable X. Therefore, it follows that tests based upon the EDF should be superior to those that are not. Consider the following Figure

This section describes three nonparametric tests that are based on the empirical distribution function[5]. The procedures are; the Kolmogorov-Smirnov and Cramer-von Mises tests, and also the Kuiper test for two-sample data.[6] The null hypothesis is $H_0 : F_X(\cdot) = F_Y(\cdot) = F(\cdot)$. The (EDF) of a sample $\{x_j\}$, $j = 1, 2, \ldots, n$ is defined as

$$\hat{F}(x) = \frac{1}{n}(\text{number of } x_j \leq x) = \frac{1}{n}\sum_{j=1}^{n} I(x_j \leq x)$$

where $I(\cdot)$ is an indicator function. Let $\hat{F}_i$ denote the sample EDF for the i^{th} group. The EDF for the overall sample, pooled over groups, can also be expressed as

$$\hat{F}(x) = \frac{1}{n}\sum_{i}\left(n_i \hat{F}_i(x)\right)$$

where n_i is the number of observations in the i^{th} group, and n is the total number of observations.

[5][PROC NPAR1WAY - EDF option].

[6]For further information about the formulas and the interpretation of EDF statistics, see Hollander and Wolfe (1999) and Gibbons and Chakraborti (2010). For details about the k-sample analogs of the Kolmogorov-Smirnov and Cramer-von Mises statistics, see Kiefer (1959).

Kolmogorov-Smirnov Test

The Kolmogorov-Smirnov statistic measures the maximum deviation of the EDF within the groups from the pooled EDF. The Kolmogorov-Smirnov statistic is computed as,

$$KS = \max_j \sqrt{\frac{1}{n} \sum_i n_i \left(\hat{F}_i(x_j) - \hat{F}(x_j)\right)^2} \quad \text{for} \quad j = 1, 2, \ldots, n$$

The asymptotic Kolmogorov-Smirnov statistic is computed as, $KS_a = KS \times \sqrt{n}$. If there are only two class levels, the two-sample Kolmogorov-Smirnov test statistic D is

$$D = \max_j \left|\hat{F}_1(x_j) - \hat{F}_2(x_j)\right| \quad \text{for} \quad j = 1, 2, \ldots, n$$

The p-value for this test is the probability that D is greater than the observed value d under the null hypothesis of no difference between class levels (samples). The asymptotic p-value for D is approximated as,

$$\Pr(D > d) = 2 \sum_{i=1}^{\infty} (-1)^{(i-1)} \, e^{(-2i^2 z^2)}$$

where

$$z = d\sqrt{n_1 n_2 / n}$$

See Hodges (1957) for information about this approximation.

Cramer-von Mises Test

The Cramer-von Mises statistic is

$$CM = \frac{1}{n^2} \sum_i \left(n_i \sum_{j=1}^{p} t_j \left(\hat{F}_i(x_j) - \hat{F}(x_j)\right)^2 \right)$$

where t_j is the number of ties at the j^{th} distinct value and p is the number of distinct values. The asymptotic value is computed as

$$CM_a = CM \times n.$$

Kuiper Test

For data with two class levels, the Kuiper statistic is

$$K = \max_j \left(\hat{F}_1(x_j) - \hat{F}_2(x_j)\right) - \min_j \left(\hat{F}_1(x_j) - \hat{F}_2(x_j)\right) \quad \text{where} \quad j = 1, 2, \ldots, n$$

The asymptotic value is

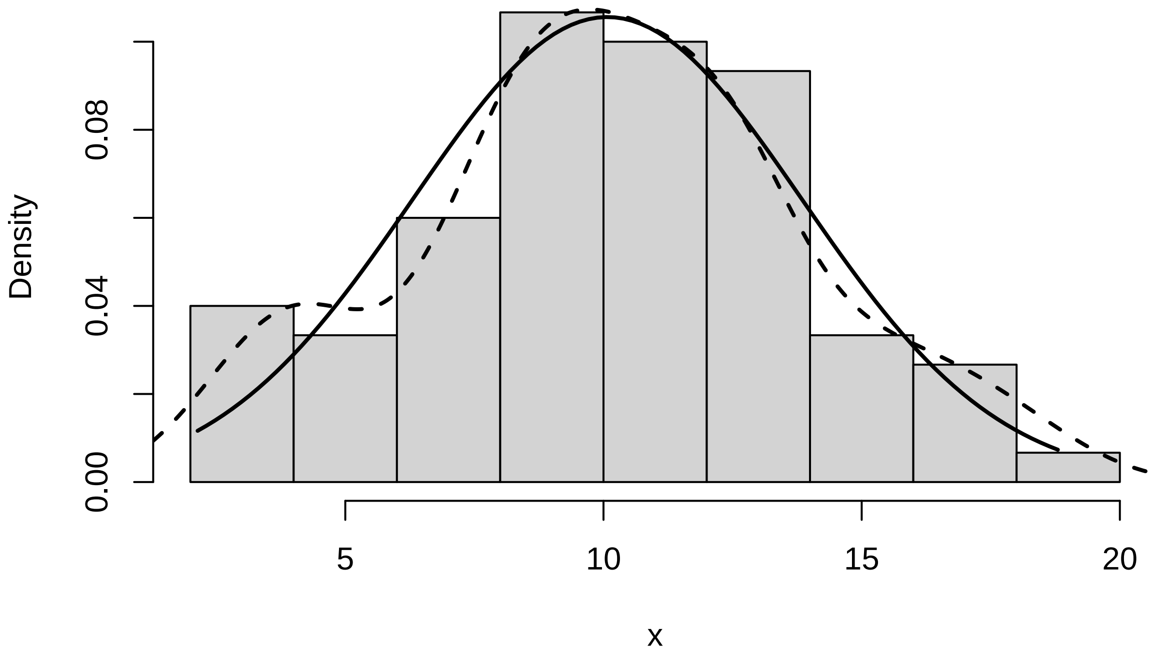
$$K_a = K\sqrt{n_1 n_2 / n}$$

The p-value for the Kuiper test is the probability of observing a larger value of K_a under the null hypothesis of no difference between the two classes Owen (1962, p 441).

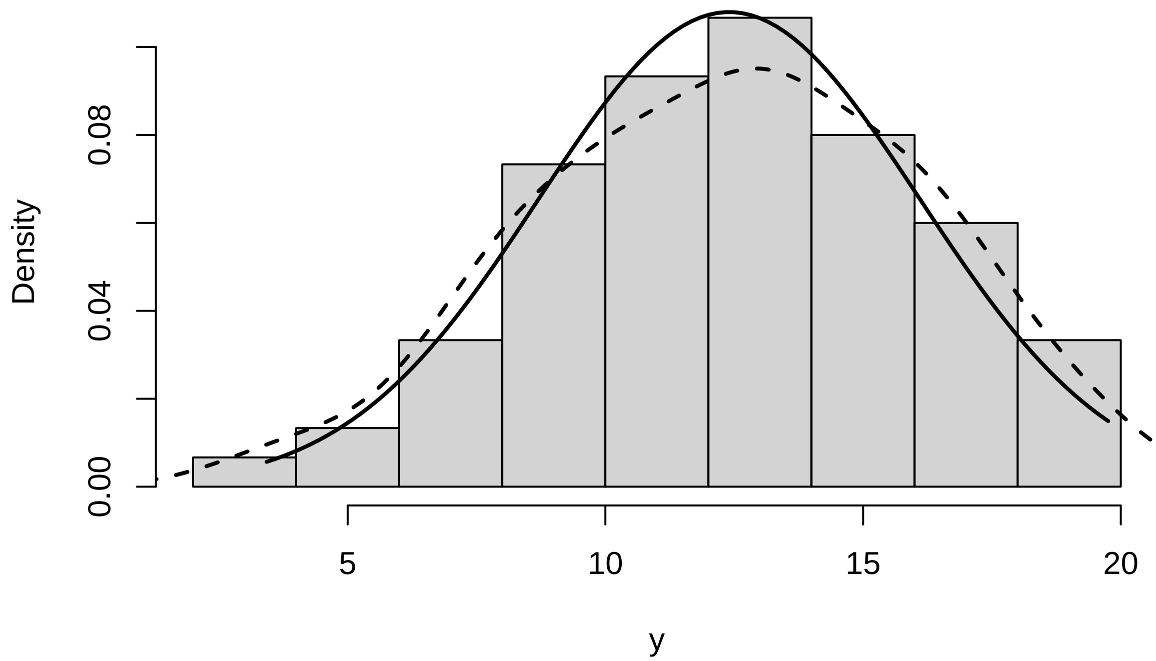
R

Simulated normal data

```
x = rnorm(75, mean=10, sd = 4)    #x ~ N(10,4)
y = rnorm(75, mean=12, sd = 4)    #y ~ N(14,4)
cert = data.frame(x,y)
#hist(x)
#hist(y)

with(cert, hist(x, main="", freq=FALSE))
with(cert, lines(density(x), main="X", lty=2, lwd=2))
xvals = with(cert, seq(from=min(x), to=max(x), length=100))
with(cert, lines(xvals, dnorm(xvals, mean(x), sd(x)), lwd=2))
```




```
with(cert, hist(y, main="", freq=FALSE))
with(cert, lines(density(y), main="", lty=2, lwd=2))
xvals = with(cert, seq(from=min(y), to=max(y), length=100))
with(cert, lines(xvals, dnorm(xvals, mean(y), sd(y)), lwd=2))
```



Descriptive statistics

```
if (!require("vcd")) install.packages("vcd", dep=TRUE)
```

```
## Loading required package: vcd
```

```
## Loading required package: grid
```

```
library("vcd")
#favstats(x)
mean(x, trim=.05)
```

```
## [1] 10.04904
```

```
quantile(x, seq(from=.025, to= .975, by=.1))
```

```
##      2.5%     12.5%     22.5%     32.5%     42.5%     52.5%     62.5%     72.5%
##  3.091728  4.865886  7.821566  8.595344  9.309669 10.373080 11.350422 12.250111
##     82.5%     92.5%
## 13.090933 15.725539
```

```
t.test(x, mu=12, conf.level=.9)    #test for mu=12 and 90 percent ci
```

```
##
##  One Sample t-test
##
## data:  x
## t = -4.4143, df = 74, p-value = 3.395e-05
## alternative hypothesis: true mean is not equal to 12
```

```
## 90 percent confidence interval:
##    9.347699 10.800978
## sample estimates:
## mean of x
##  10.07434
```

```
#favstats(y)
mean(y, trim=.05)
```

```
## [1] 12.45752
```

```
quantile(y, seq(from=.025, to= .975, by=.1))
```

```
##      2.5%     12.5%     22.5%     32.5%     42.5%     52.5%     62.5%     72.5%
##  5.029569  8.070631  9.650323 10.552400 11.745636 12.940976 13.577305 14.965538
##     82.5%     92.5%
## 16.073867 17.267639
```

```
t.test(y, mu=12, conf.level=.9)    #test for mu=12 and 90 percent ci
```

```
##
##  One Sample t-test
##
## data:  y
## t = 0.94589, df = 74, p-value = 0.3473
## alternative hypothesis: true mean is not equal to 12
## 90 percent confidence interval:
##  11.69285 13.11439
## sample estimates:
## mean of x
##  12.40362
```

```
if (!require("coin")) install.packages("coin", dep=TRUE)
```

```
## Loading required package: coin
```

```
## Loading required package: survival
```

```
library("coin")
wilcox.test(x,y)
```

```
##
##  Wilcoxon rank sum test with continuity correction
##
## data:  x and y
## W = 1846, p-value = 0.0002824
## alternative hypothesis: true location shift is not equal to 0
```

```
t.test(x,y)
```

```
##
##  Welch Two Sample t-test
```

```
##
## data:  x and y
## t = -3.817, df = 147.93, p-value = 0.000198
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -3.535179 -1.123386
## sample estimates:
## mean of x mean of y
##  10.07434  12.40362
```

```r
ks.test(x,y)
```

```
##
##  Two-sample Kolmogorov-Smirnov test
##
## data:  x and y
## D = 0.29333, p-value = 0.002994
## alternative hypothesis: two-sided
```

Univariate Test of Fit for Normality

```r
if (!require("nortest")) install.packages("nortest", dep=TRUE)
```

```
## Loading required package: nortest
```

```r
library("nortest")
ad.test(x)
```

```
##
##  Anderson-Darling normality test
##
## data:  x
## A = 0.28646, p-value = 0.6142
```

```r
cvm.test(x)
```

```
##
##  Cramer-von Mises normality test
##
## data:  x
## W = 0.036008, p-value = 0.7499
```

```r
lillie.test(x)
```

```
##
##  Lilliefors (Kolmogorov-Smirnov) normality test
##
## data:  x
## D = 0.06015, p-value = 0.7211
```

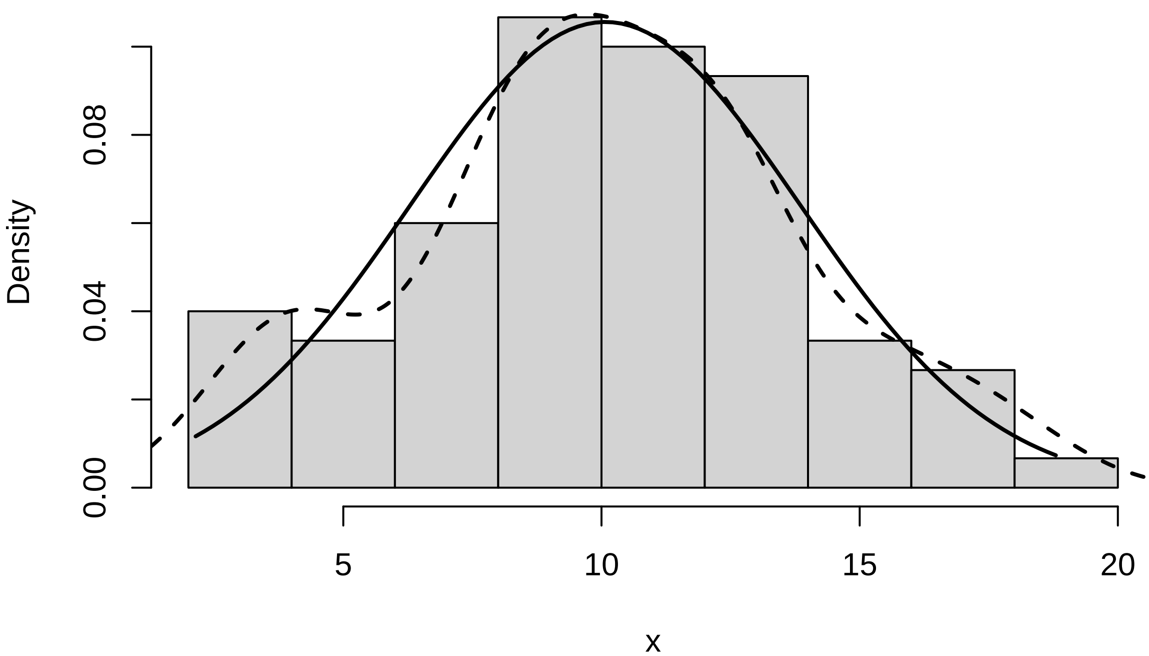
```r
pearson.test(x)
```

```
##
##  Pearson chi-square normality test
##
## data:  x
## P = 10.28, p-value = 0.3283
```

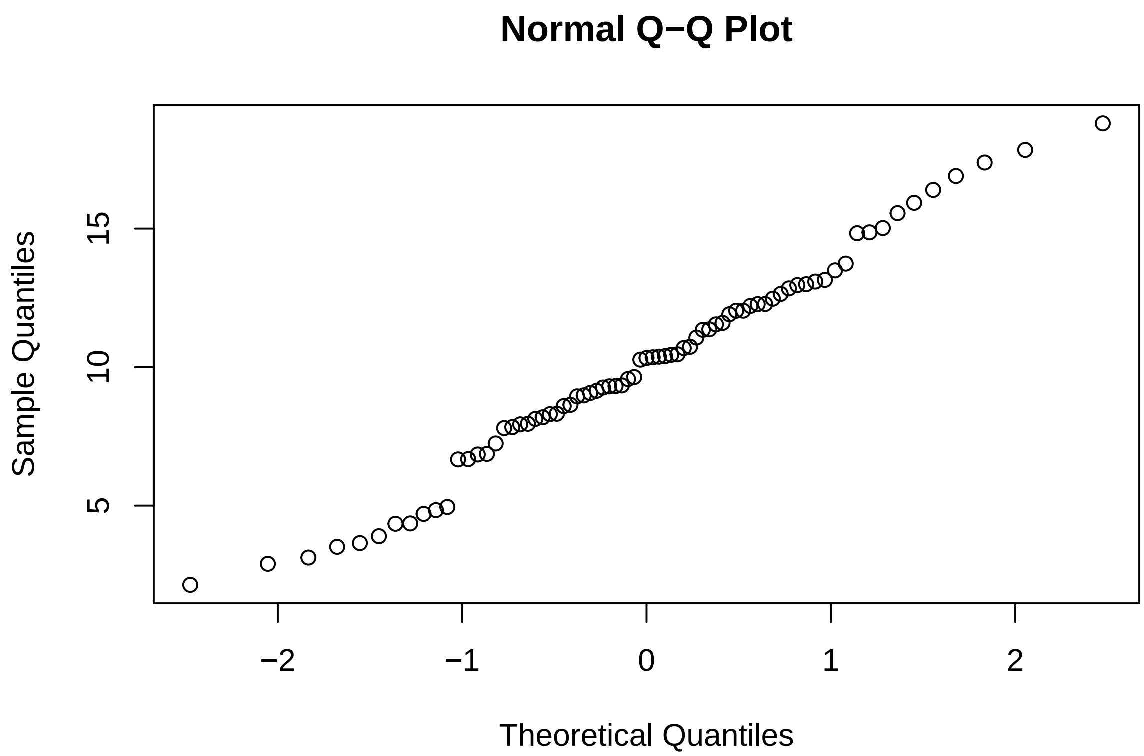
```r
sf.test(x)
```

```
##
##  Shapiro-Francia normality test
##
## data:  x
## W = 0.98948, p-value = 0.7056
```

```r
with(cert, hist(x, main="", freq=FALSE))
with(cert, lines(density(x), main="X", lty=2, lwd=2))
xvals = with(cert, seq(from=min(x), to=max(x), length=100))
with(cert, lines(xvals, dnorm(xvals, mean(x), sd(x)), lwd=2))
```




```r
qqnorm(x)
```



Repeating above with lognormal simulated data

```
x = 2*rlnorm(75, mean=0, sd = 1) + 10
cert = data.frame(x,y)     #redefined data.frame with new x

ad.test(x)
```

```
##
##  Anderson-Darling normality test
##
## data:  x
## A = 7.8903, p-value < 2.2e-16
```

```
cvm.test(x)
```

```
## Warning in cvm.test(x): p-value is smaller than 7.37e-10, cannot be computed
## more accurately
```

```
##
##  Cramer-von Mises normality test
##
## data:  x
## W = 1.4108, p-value = 7.37e-10
```

```r
lillie.test(x)
```

```
##
##  Lilliefors (Kolmogorov-Smirnov) normality test
##
## data:  x
## D = 0.24428, p-value = 4.709e-12
```

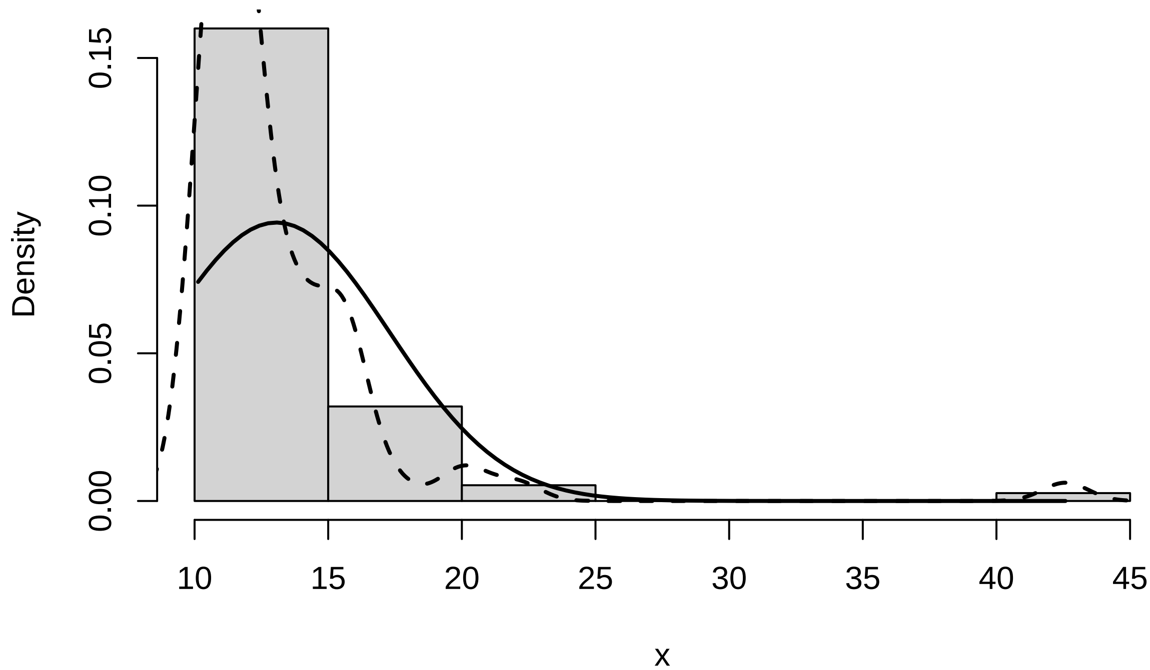
```r
pearson.test(x)
```

```
##
##  Pearson chi-square normality test
##
## data:  x
## P = 107.24, p-value < 2.2e-16
```

```r
sf.test(x)
```

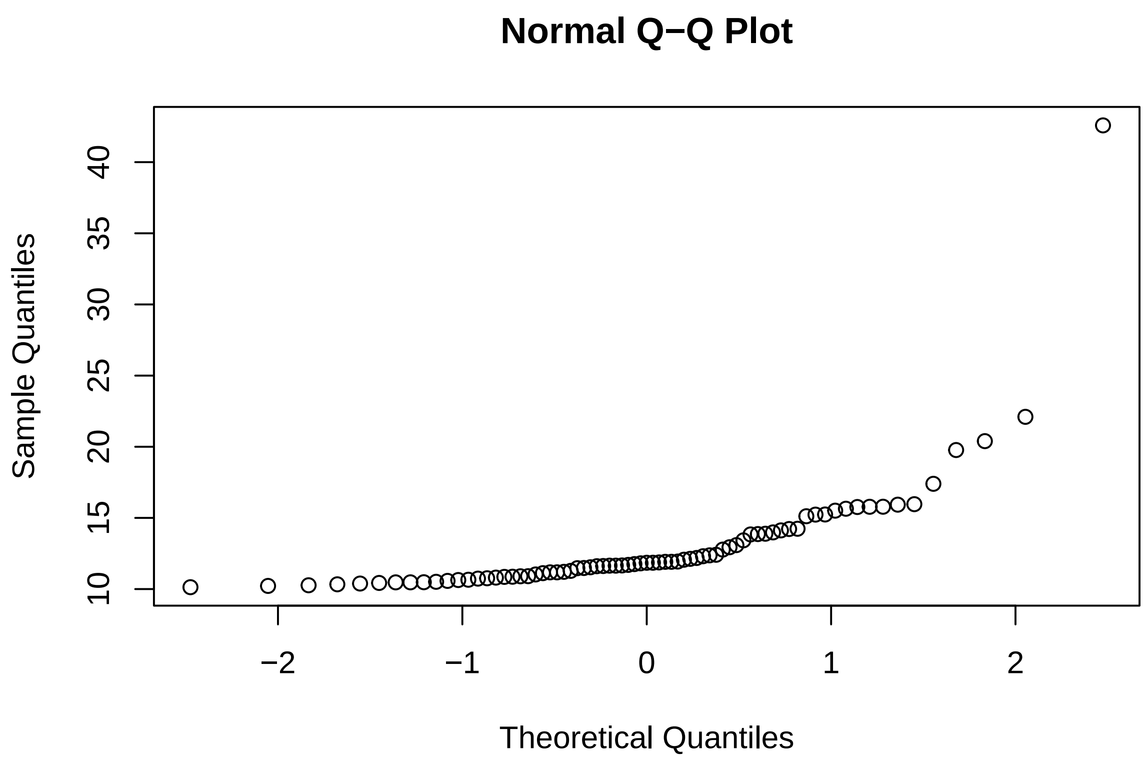
```
##
##  Shapiro-Francia normality test
##
## data:  x
## W = 0.52418, p-value = 3.574e-12
```

```r
with(cert, hist(x, main="", freq=FALSE))
with(cert, lines(density(x), main="X", lty=2, lwd=2))
xvals = with(cert, seq(from=min(x), to=max(x), length=100))
with(cert, lines(xvals, dnorm(xvals, mean(x), sd(x)), lwd=2))
```



Density
0.00
0.05
0.10
0.15
10
15
20
25
30
35
40
45
x


```
qqnorm(x)
```

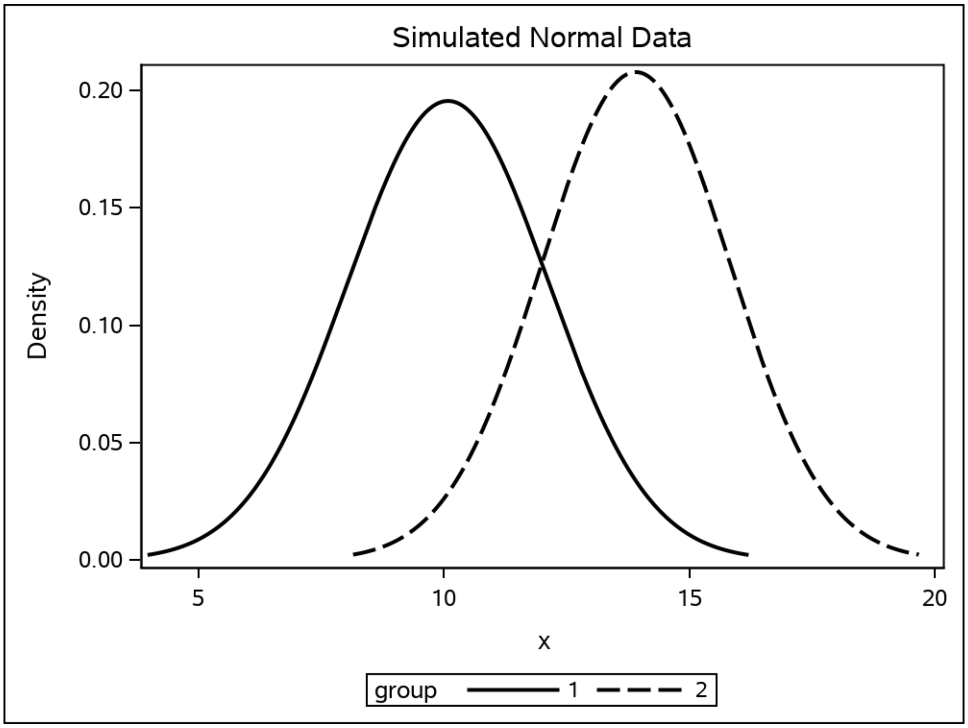


Normal Q−Q Plot
Sample Quantiles
10
15
20
25
30
35
40
−2
−1
0
1
2
Theoretical Quantiles

SAS

Code

```
title 'Simulated Normal Data';
%let n=50;

data cert;
  do group = 1 to 2;
     do i = 1 to &n;
        x = rand('normal', 0, 1);
      output;
      end;
  end;
run;
data cert; set cert;
   if group = 1 then x = 2*x + 10;
   else x = 2*x + 14;
run;

proc sgplot data=cert;
density x /group=group;
run;

title2 'Descriptive Statistics';
proc univariate data=cert normal trim=.05 winsor=.05 mu0=12;
var x;
run;

title2 'Nonparametric Test of Hypothesis';
proc npar1way data=cert wilcoxon edf ;
class group;
var x;
run;

title2 'T Test';
proc ttest data=cert ;
class group;
var x;
run;



title2 'Fit for 2 populations';
proc univariate data=cert normal;
class group;
var x;
run;
```


Simulated Normal Data
Density
0.20
0.15
0.10
0.05
0.00
5
10
15
20
x
group
1
2

Simulated Normal Data

Descriptive Statistics

The UNIVARIATE Procedure

Variable: x

Moments			
N	100	**Sum Weights**	100
Mean	11.9984529	**Sum Observations**	1199.84529
Std Deviation	2.75347943	**Variance**	7.58164898
Skewness	−0.0855179	**Kurtosis**	−0.7284714
Uncorrected SS	15146.8705	**Corrected SS**	750.583249
Coeff Variation	22.9486205	**Std Error Mean**	0.27534794

Basic Statistical Measures			
Location		**Variability**	
Mean	11.99845	**Std Deviation**	2.75348
Median	12.15610	**Variance**	7.58165
Mode	.	**Range**	11.55031
		Interquartile Range	4.13781

Tests for Location: Mu0=12				
Test	**Statistic**		**p Value**	
Student's t	**t**	−0.00562	**Pr > |t|**	0.9955
Sign	**M**	2	**Pr >= |M|**	0.7644
Signed Rank	**S**	18	**Pr >= |S|**	0.9510

Tests for Normality				
Test	**Statistic**		**p Value**	
Shapiro-Wilk	**W**	0.984332	**Pr < W**	0.2840
Kolmogorov-Smirnov	**D**	0.050788	**Pr > D**	>0.1500
Cramer-von Mises	**W-Sq**	0.05193	**Pr > W-Sq**	>0.2500
Anderson-Darling	**A-Sq**	0.347208	**Pr > A-Sq**	>0.2500

Trimmed Means								
% Trimmed	# Trimmed	Trimmed Mean	SE Trimmed	95% CI		DF	t for H0	Pr > |t|
5.00	5	12.00690	0.294375	11.42198	12.59181	89	0.023428	0.9814

Winsorized Means								
% Winsor	# Winsor	Winsor Mean	SE Winsor	95% CI		DF	t for H0	Pr > |t|
5.00	5	11.98844	0.294540	11.40320	12.57369	89	−0.03923	0.9688

Quantiles (Definition 5)	
Level	Quantile
100% Max	17.90617
99%	17.47536
95%	16.59441
90%	15.53162
75% Q3	13.94750
50% Median	12.15610
25% Q1	9.80969
10%	8.06803
5%	7.21831
1%	6.39296
0% Min	6.35587

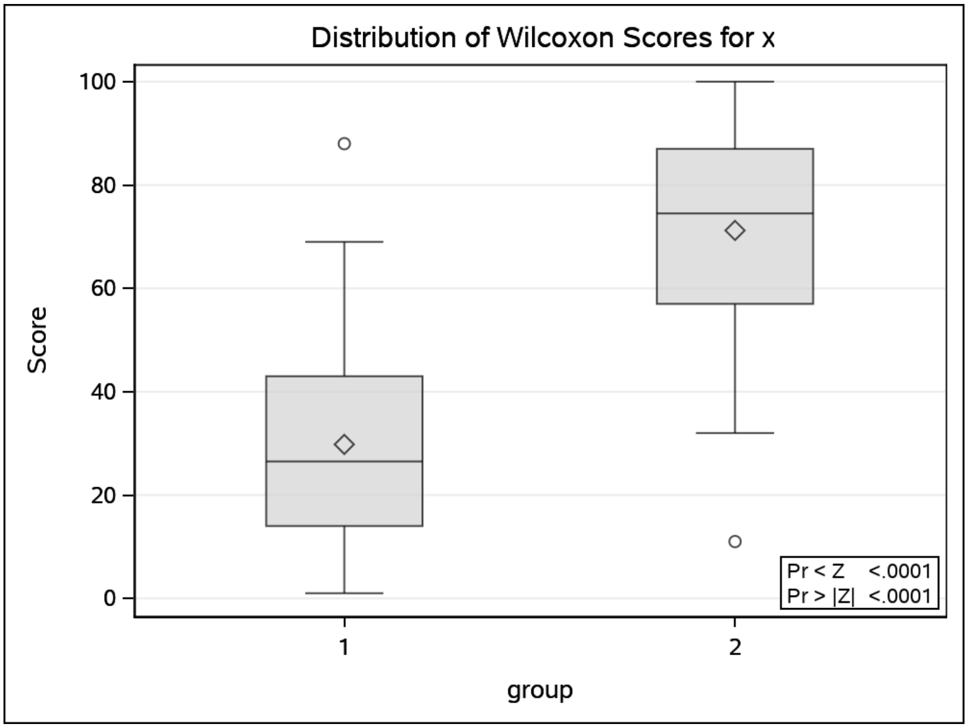
Extreme Observations			
Lowest		Highest	
Value	Obs	Value	Obs
6.35587	24	16.7632	86
6.43005	43	16.7768	79
6.69599	5	16.9643	71
7.07019	20	17.0445	51
7.21752	41	17.9062	73

Simulated Normal Data
Nonparametric Test of Hypothesis
The NPAR1WAY Procedure

Wilcoxon Scores (Rank Sums) for Variable x				Classified by Variable group	
group	**N**	**Sum of Scores**	**Expected Under H0**	**Std Dev Under H0**	**Mean Score**
1	50	1490.0	2525.0	145.057460	29.80
2	50	3560.0	2525.0	145.057460	71.20

Wilcoxon Two-Sample Test					
Statistic	**Z**	**Pr < Z**	**Pr > |Z|**	**t Approximation**	
				Pr < Z	**Pr > |Z|**
1490.000	−7.1317	<.0001	<.0001	<.0001	<.0001
Z includes a continuity correction of 0.5.					

Kruskal-Wallis Test		
Chi-Square	**DF**	**Pr > ChiSq**
50.9097	1	<.0001


Distribution of Wilcoxon Scores for x
Score
100
80
60
40
20
0
1
2
group
Pr < Z <.0001
Pr > |Z| <.0001

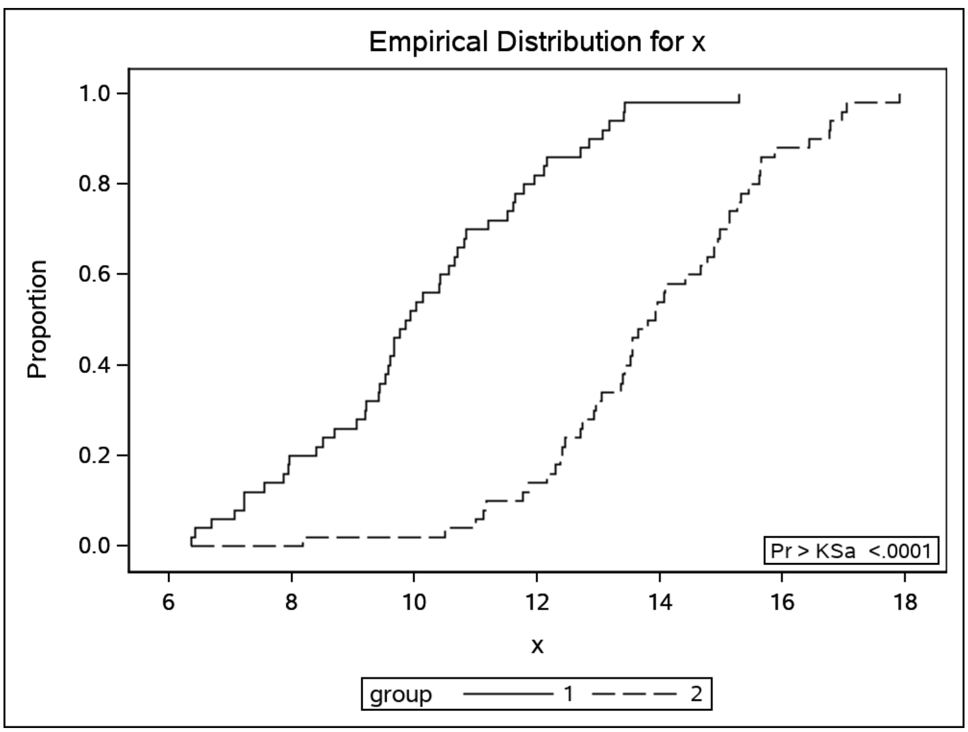
Simulated Normal Data

Nonparametric Test of Hypothesis

The NPAR1WAY Procedure

Kolmogorov-Smirnov Test for Variable x Classified by Variable group			
group	**N**	**EDF at Maximum**	**Deviation from Mean at Maximum**
1	50	0.840	2.474874
2	50	0.140	−2.474874
Total	100	0.490	
Maximum Deviation Occurred at Observation 31			
Value of x at Maximum = 12.113502			

Kolmogorov-Smirnov Two-Sample Test (Asymptotic)			
KS	0.350000	**D**	0.700000
KSa	3.500000	**Pr > KSa**	<.0001



Cramer-von Mises Test for Variable x Classified by Variable group		
group	**N**	**Summed Deviation from Mean**
1	50	2.69790
2	50	2.69790

Cramer-von Mises Statistics (Asymptotic)			
CM	0.053958	**CMa**	5.395800

Kuiper Test for Variable x Classified by Variable group		
group	**N**	**Deviation from Mean**
1	50	0.70
2	50	0.00

Kuiper Two-Sample Test (Asymptotic)					
K	0.700000	**Ka**	3.500000	**Pr > Ka**	<.0001

Simulated Normal Data
T Test
The TTEST Procedure

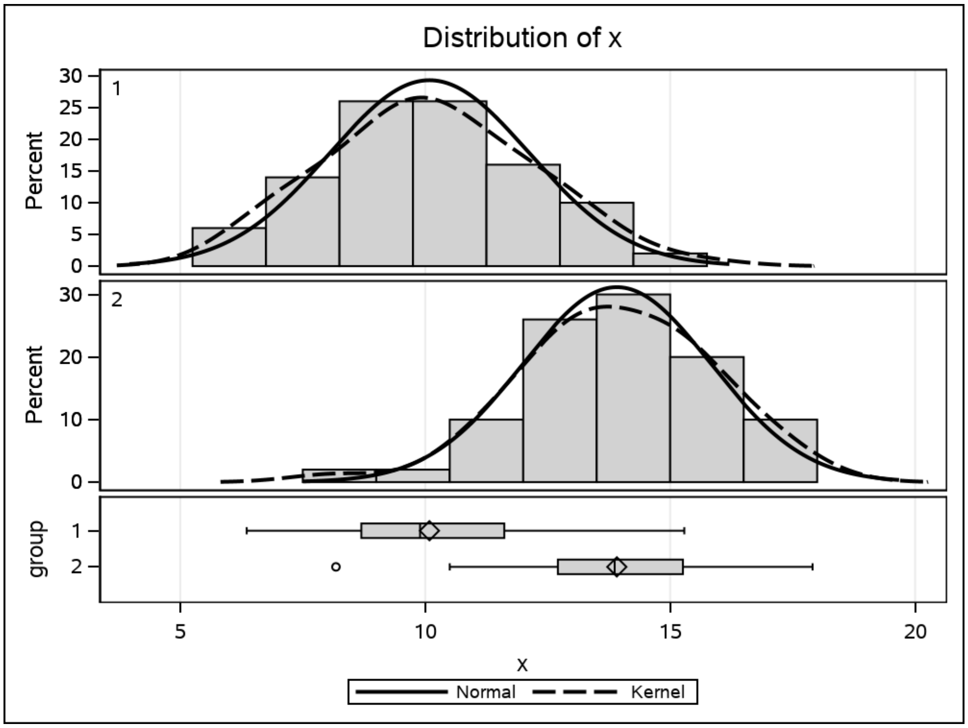
Variable: x

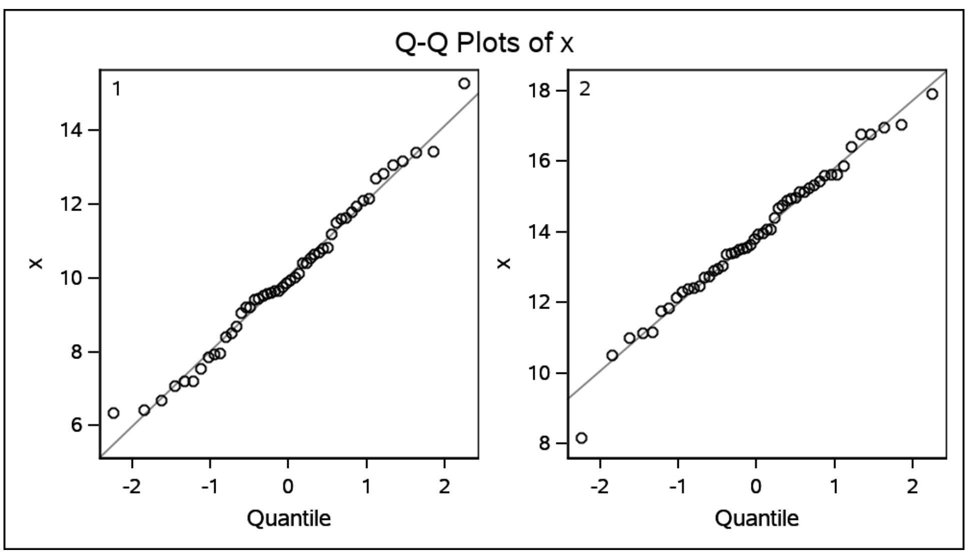
group	Method	N	Mean	Std Dev	Std Err	Minimum	Maximum
1		50	10.0858	2.0409	0.2886	6.3559	15.2884
2		50	13.9111	1.9201	0.2715	8.1772	17.9062
Diff (1-2)	Pooled		−3.8254	1.9814	0.3963		
Diff (1-2)	Satterthwaite		−3.8254		0.3963		

group	Method	Mean	95% CL Mean		Std Dev	95% CL Std Dev	
1		10.0858	9.5058	10.6658	2.0409	1.7048	2.5432
2		13.9111	13.3654	14.4568	1.9201	1.6039	2.3927
Diff (1-2)	Pooled	−3.8254	−4.6118	−3.0390	1.9814	1.7386	2.3037
Diff (1-2)	Satterthwaite	−3.8254	−4.6118	−3.0389			

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	98	−9.65	<.0001
Satterthwaite	Unequal	97.638	−9.65	<.0001

Equality of Variances				
Method	Num DF	Den DF	F Value	Pr > F
Folded F	49	49	1.13	0.6711


Distribution of x
1
2
Percent
group
x
Normal
Kernel


Q-Q Plots of x
1
2
x
Quantile

Simulated Normal Data

Fit for 2 populations

The UNIVARIATE Procedure

Variable: x

group = 1

Moments			
N	50	Sum Weights	50
Mean	10.0857719	Sum Observations	504.288594
Std Deviation	2.04087326	Variance	4.16516367
Skewness	0.18932897	Kurtosis	−0.2944267
Uncorrected SS	5290.23274	Corrected SS	204.09302
Coeff Variation	20.2351717	Std Error Mean	0.28862306

Basic Statistical Measures			
Location		Variability	
Mean	10.08577	Std Deviation	2.04087
Median	9.89483	Variance	4.16516
Mode	.	Range	8.93255
		Interquartile Range	2.91747

Tests for Location: Mu0=0				
Test	Statistic		p Value	
Student's t	t	34.94444	Pr > |t|	<.0001
Sign	M	25	Pr >= |M|	<.0001
Signed Rank	S	637.5	Pr >= |S|	<.0001

Tests for Normality				
Test	Statistic		p Value	
Shapiro-Wilk	W	0.98372	Pr < W	0.7155
Kolmogorov-Smirnov	D	0.056801	Pr > D	>0.1500
Cramer-von Mises	W-Sq	0.03085	Pr > W-Sq	>0.2500
Anderson-Darling	A-Sq	0.212723	Pr > A-Sq	>0.2500

Quantiles (Definition 5)	
Level	**Quantile**
100% Max	15.28842
99%	15.28842
95%	13.41470
90%	12.95479
75% Q3	11.61271
50% Median	9.89483
25% Q1	8.69524
10%	7.21831
5%	6.69599
1%	6.35587
0% Min	6.35587

Extreme Observations			
Lowest		Highest	
Value	Obs	Value	Obs
6.35587	24	13.0686	50
6.43005	43	13.1773	30
6.69599	5	13.4147	44
7.07019	20	13.4331	9
7.21752	41	15.2884	8

Simulated Normal Data

Fit for 2 populations

The UNIVARIATE Procedure

Variable: x

group = 2

Moments			
N	50	**Sum Weights**	50
Mean	13.911134	**Sum Observations**	695.556699
Std Deviation	1.92011559	**Variance**	3.68684387
Skewness	−0.3436185	**Kurtosis**	0.46626543
Uncorrected SS	9856.63778	**Corrected SS**	180.65535
Coeff Variation	13.8027251	**Std Error Mean**	0.27154535

Basic Statistical Measures			
Location		**Variability**	
Mean	13.91113	**Std Deviation**	1.92012
Median	13.86706	**Variance**	3.68684
Mode	.	**Range**	9.72897
		Interquartile Range	2.55258

Tests for Location: Mu0=0				
Test	**Statistic**		**p Value**	
Student's t	**t**	51.22951	**Pr > |t|**	<.0001
Sign	**M**	25	**Pr >= |M|**	<.0001
Signed Rank	**S**	637.5	**Pr >= |S|**	<.0001

Tests for Normality				
Test	**Statistic**		**p Value**	
Shapiro-Wilk	**W**	0.985385	**Pr < W**	0.7880
Kolmogorov-Smirnov	**D**	0.054115	**Pr > D**	>0.1500
Cramer-von Mises	**W-Sq**	0.022948	**Pr > W-Sq**	>0.2500
Anderson-Darling	**A-Sq**	0.176737	**Pr > A-Sq**	>0.2500

Quantiles (Definition 5)	
Level	**Quantile**
100% Max	17.9062
99%	17.9062
95%	16.9643
90%	16.5944
75% Q3	15.2590
50% Median	13.8671
25% Q1	12.7064
10%	11.4682
5%	10.9932
1%	8.1772
0% Min	8.1772

Extreme Observations			
Lowest		**Highest**	
Value	**Obs**	**Value**	**Obs**
8.1772	70	16.7632	86
10.5000	85	16.7768	79
10.9932	97	16.9643	71
11.1300	96	17.0445	51
11.1654	82	17.9062	73