Outlines

- Motivation
- Problem Formulation
- Controller Design
- Stability Analysis
- Experiments
- Conclusion

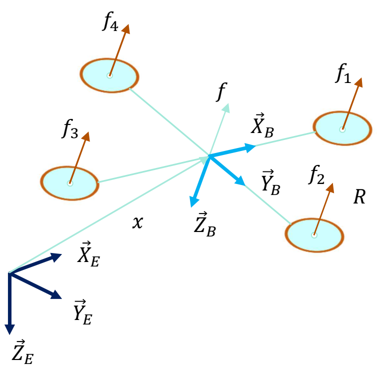
Motivation

- Knowledge of the geometric and inertia parameters is essential to achieving good control performance.
- The payload or sensors attaching to multirotors may change the geometric and inertia parameters.
- Some geometric and inertia parameters like moment of inertia can not be measured through instrument.
- Existing adaptive control method can only guarantee the stability of multirotors system, can not ensure the parameters converge.

Motivation
Problem Formulation
Controller Design
Stability Analysis
Experiments
Conclusion

Problem Formulation - Definition of Symbols

Symbol	Description
x	Position of the multirotor
v	Velocity of the multirotor
R	Rotation matrix from the body-fixed frame to the inertial frame
Ω	Angular velocity in the body-fixed frame
f	Net thrust control input
M	Moment control input
m	Mass of the multirotor
J	Moment of inertia of the multirotor



Body-fixed frame : $\{\vec{X}_B,\ \vec{Y}_B,\ \vec{Z}_B\}$

Inertial frame: $\{\vec{X}_E,\ \vec{Y}_E,\ \vec{Z}_E\}$

Motivation
Problem Formulation
Controller Design
Stability Analysis
Experiments
Conclusion

Problem Formulation - Dynamics of the Multirotor

- The multirotor is described by both translational and rotational dynamics.
- The translational dynamics considers forces such as the effects of gravity, thrusts, and the external force.
- The rotational dynamics takes the moment control input, rotational speed, and moment of inertia into account.

$$\left.\begin{aligned}\dot{x} &= v\\ m\dot{v} &= mge_3 - fRe_3\end{aligned}\right\}\ \text{Translational dynamics}$$

$$\left.\begin{aligned}\dot{R} &= R\hat{\Omega}\\ J\dot{\Omega} + \Omega \times J\Omega &= M\end{aligned}\right\}\ \text{Rotational dynamics}$$

$$J = \begin{bmatrix} J_{xx} & 0 & 0\\ 0 & J_{yy} & 0\\ 0 & 0 & J_{zz}\end{bmatrix}$$

Motivation
Problem Formulation
Controller Design
Stability Analysis
Experiments
Conclusion

Problem Formulation – Tracking Errors and Estimate Errors

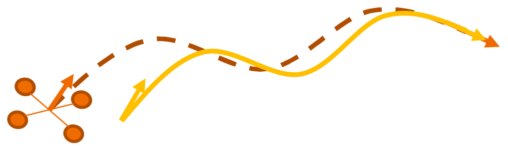
- Position and velocity tracking errors
  $e_x \triangleq x - x_d$
  $e_v \triangleq v - v_d$
- Attitude error function on SO(3) based on Geometric Tracking Control
  $\Psi(R, R_d) \triangleq \frac{1}{2} tr[I - R_d^T R]$
- Attitude tracking error and the angular velocity tracking error
  $e_R \triangleq \frac{1}{2}(R_d^T R - R^T R_d)^{\vee}$
  $e_\Omega \triangleq \Omega - R^T R_d \Omega_d$
- Estimate error of mass
  $\tilde{\theta}_m \triangleq \theta_m - \hat{\theta}_m, \theta_m = m$ (mass of the multirotor)
- Estimate error of moment of inertia
  $\tilde{\theta}_J \triangleq \theta_J - \hat{\theta}_J, \theta_J = [J_{xx} \quad J_{yy} \quad J_{zz}]^T$ (moment of inertia of the multirotor)

Motivation
Problem Formulation
Controller Design
Stability Analysis
Experiments
Conclusion

Problem Formulation – Control Objectives

- Track a desired 3D trajectory
- Track a desired yaw angle
- Estimate the mass of the multirotor
- Estimate the moment of inertia of the multirotor

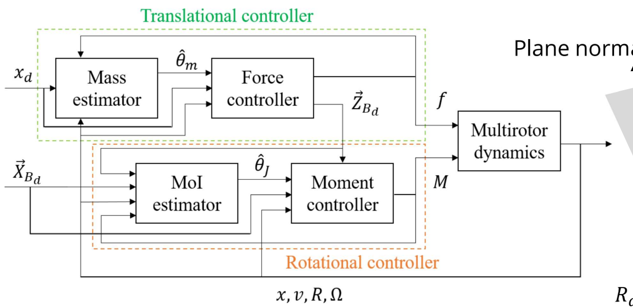
$$\begin{cases} e_x \to 0 \\ e_v \to 0 \\ e_R \to 0 \\ e_\Omega \to 0 \\ \tilde{\theta}_m \to 0 \\ \tilde{\theta}_J \to 0 \end{cases} \quad as\ t \to \infty$$

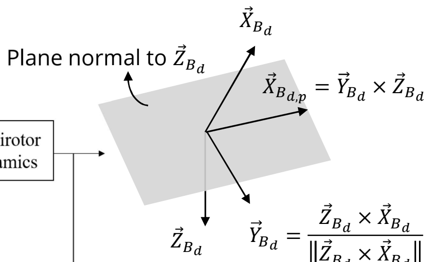


3D position
3D trajectory
yaw angle
desired yaw angle

Motivation
Problem Formulation
Controller Design
Stability Analysis
Experiments
Conclusion

Controller Design – Control Architecture





$\vec{X}_{B_{d,p}} = \vec{Y}_{B_d} \times \vec{Z}_{B_d}$

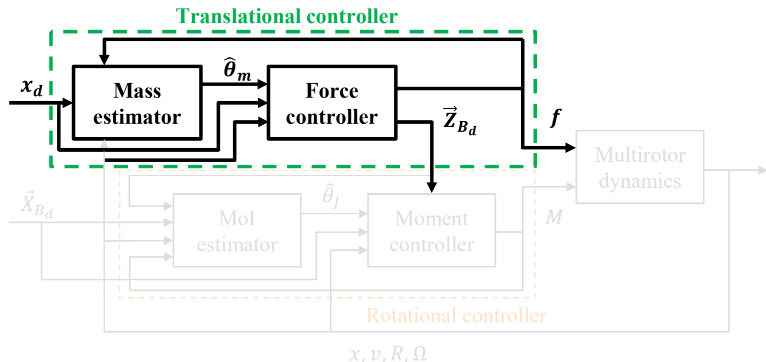
$\vec{Y}_{B_d} = \frac{\vec{Z}_{B_d} \times \vec{X}_{B_d}}{\|\vec{Z}_{B_d} \times \vec{X}_{B_d}\|}$

$R_d = [\vec{X}_{B_{d,p}}; \quad \vec{Y}_{B_d}; \quad \vec{Z}_{B_d}] \in R^{3\times3}$

Motivation
Problem Formulation
Controller Design
Stability Analysis
Experiments
Conclusion

Controller Design – Translational Controller

- Translational controller



Motivation
Problem Formulation
Controller Design
Stability Analysis
Experiments
Conclusion

Controller Design – Translational Controller

- Translational controller

$$f = (k_x e_x + k_v e_v + Y_m \hat{\theta}_m) \cdot R e_3 \quad , Y_m = \begin{bmatrix} -\ddot{x}_{d_1} \\ -\ddot{x}_{d_2} \\ g - \ddot{x}_{d_3} \end{bmatrix} \text{ is a regression matrix}$$

$$= \textit{feedback term} + \textit{adaptive term}$$

- Integral CL-based adaptive control update law $\dot{\hat{\theta}}_m$

$$\dot{\hat{\theta}}_m = \Gamma_m Y_m^T (e_v + C_1 e_x) + k_m^{cl} \Gamma_m \sum_{j=1}^{N_m} \left(y_m^{cl}(t_j) \right)^T \left(F(t_j) - y_m^{cl}(t_j)\, \hat{\theta}_m \right)$$

$$= \textit{adaptive term} + \textit{ICL} - \textit{based te}$$

$$y_m^{cl}(t_j) \triangleq \begin{cases} 0_{n\times 1} & t \in [0, \Delta t] \\ \int_{t-\Delta t}^{t} Y_m^{cl}(\tau) d\tau & t > \Delta t \end{cases} , \quad F(t_j) \triangleq \begin{cases} 0_{n\times 1} & t \in [0, \Delta t] \\ \int_{t-\Delta t}^{t} f R e_3(\tau) d\tau & t > \Delta t \end{cases}$$

Controller Design – Translational Controller

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Motivation
Problem Formulation
Controller Design
Stability Analysis
Experiments
Conclusion

- Y_m^{cl} defined as follows contains acceleration which is not implementable

$$f R e_3 = m g e_3 - m \dot{v} = Y_m^{cl} \theta_m , \quad Y_m^{cl} = \begin{bmatrix} -\ddot{x}_1 \\ -\ddot{x}_2 \\ g - \ddot{x}_3 \end{bmatrix}$$

- By integrating Y_m^{cl} to be y_m^{cl} as defined in last page, y_m^{cl} becomes implementable
- Integrating both sides of the translational dynamics $f R e_3 = Y_m^{cl} \theta_m$ yields

$$\int_{t-\Delta t}^{t} f R e_3(\tau) d\tau = \int_{t-\Delta t}^{t} Y_m^{cl}(\tau) \theta_m d\tau \Rightarrow \int \boldsymbol{fRe_3(\tau)} \Big|_{\tau=t} - \int \boldsymbol{fR\ {}_3(\tau)} \Big|_{\tau=t-\Delta t} = y_m^{cl} \theta_m$$

$$\dot{\hat{\theta}}_m = \Gamma_m Y_m^T (e_v + C_1 e_x) + k_m^{cl} \Gamma_m \sum_{j=1}^{N_m} \left(y_m^{cl}(t_j) \right)^T \left(\boldsymbol{F(t_j)} - y_m^{cl}(t_j)\, \hat{\theta}_m \right)$$

$$= \Gamma_m Y_m^T (e_v + C_1 e_x) + k_m^{cl} \Gamma_m \sum_{j=1}^{N_m} \left(y_m^{cl}(t_j) \right)^T y_m^{cl}(t_j) \tilde{\theta}_m$$

Controller Design – Rotational Controller

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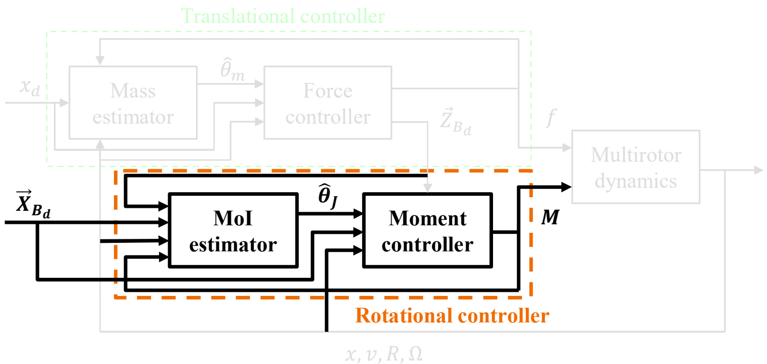
Motivation
Problem Formulation
Controller Design
Stability Analysis
Experiments
Conclusion

- Rotational controller



Controller Design – Rotational Controller

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Motivation
Problem Formulation
Controller Design
Stability Analysis
Experiments
Conclusion

- Rotational controller

$$M = -k_R e_R - k_\Omega e_\Omega - Y_J \hat{\theta}_J \quad , Y_J = \begin{bmatrix} \bar{\Omega}_1 & \Omega_2 \cdot \Omega_3 & -\Omega_2 \cdot \Omega_3 \\ -\Omega_1 \cdot \Omega_3 & \bar{\Omega}_2 & \Omega_1 \cdot \Omega_3 \\ \Omega_1 \cdot \Omega_2 & -\Omega_1 \cdot \Omega_2 & \bar{\Omega}_3 \end{bmatrix}$$

$$= \textit{feedback term} + \textit{adaptive term}$$

- Integral CL-based adaptive control update law $\dot{\hat{\theta}}_J$

$$\dot{\hat{\theta}}_J = \Gamma_J Y_J^T (e_\Omega + C_2 e_R) + k_J^{cl} \Gamma_J \sum_{j=1}^{N_J} \left(y_J^{cl}(t_j) \right)^T \left(\bar{M}(t_j) - y_J^{cl}(t_j)\, \hat{\theta}_J \right)$$

$$= \textit{adaptive term} + \textit{ICL} - \textit{based term}$$

$$y_J^{cl}(t_j) \triangleq \begin{cases} 0_{n\times m} & t \in [0, \Delta t] \\ \int_{t-\Delta t}^{t} Y_J^{cl}(\tau) d\tau & t > \Delta t \end{cases} , \quad \bar{M}(t_j) \triangleq \begin{cases} 0_{n\times m} & t \in [0, \Delta t] \\ \int_{t-\Delta t}^{t} M(\tau) d\tau & t > \Delta t \end{cases}$$

Controller Design – Rotational Controller

- Y_J^{cl} defined as follows contains angular acceleration which is not implementable

$$M = J\dot{\Omega} + \Omega \times J\Omega = Y_J^{cl}\theta_J, \qquad Y_J^{cl} = \begin{bmatrix} \dot{\Omega}_1 & -\Omega_2 \cdot \Omega_3 & \Omega_2 \cdot \Omega_3 \\ \Omega_1 \cdot \Omega_3 & \dot{\Omega}_2 & -\Omega_1 \cdot \Omega_3 \\ -\Omega_1 \cdot \Omega_2 & \Omega_1 \cdot \Omega_2 & \dot{\Omega}_3 \end{bmatrix}$$

- By integrating Y_J^{cl} to be y_J^{cl} as defined in last page, y_J^{cl} becomes implementable
- Integrating both sides of the translational dynamics $M = Y_J^{cl}\theta_J$ yields

$$\int_{t-\Delta t}^{t} M(\tau)d\tau = \int_{t-\Delta t}^{t} Y_J^{cl}(\tau)\theta_J d\tau \Rightarrow \int \boldsymbol{M(\tau)}\Big|_{\tau=t} - \int \boldsymbol{M(\tau)}\Big|_{\tau=t-\Delta t} = y_J^{cl}\theta_J$$

$$\dot{\hat{\theta}}_J = \Gamma_J Y_J^T(e_\Omega + C_2 e_R) + k_J^{cl}\Gamma_J \sum_{j=1}^{N_J} \left(y_J^{cl}(t_j)\right)^T \left(\boldsymbol{\bar{M}(t_j)} - y_J^{cl}(t_j)\,\hat{\theta}_J\right)$$

$$= \Gamma_J Y_J^T(e_\Omega + C_2 e_R) + k_J^{cl}\Gamma_J \sum_{j=1}^{N_J} \left(y_J^{cl}(t_j)\right)^T y_J^{cl}(t_j)\tilde{\theta}_J$$

Stability Analysis – Closed-Loop Error Systems

- Taking the time derivative of error dynamics e_x, e_v defined in Problem Formulation

$$\dot{e}_x = \dot{e}_v$$

$$m\dot{e}_v = mge_3 - fRe_3 - m\ddot{x}_d$$

$$= Y_m\theta_m - \boldsymbol{f}Re_3$$

$$= -k_x e_x - k_v e_v + Y_m\tilde{\theta}_m - X \quad , X = \frac{f}{e_3^T R_d^T R e_3}\left((e_3^T R_d^T R e_3)Re_3 - R_d e_3\right)$$

- Taking the time derivative of error dynamics e_R, e_Ω defined in Problem Formulation

$$\dot{e}_R = \frac{1}{2}\left(R_d^T R\hat{e}_\Omega + \hat{e}_\Omega R^T R_d\right)^\vee$$

$$= \frac{1}{2}\left(tr[R^T R_d]I - R^T R_d\right) \equiv C(R_d^T R)e_\Omega$$

$$J\dot{e}_\Omega = J\dot{\Omega} + J(\hat{\Omega}R^T R_d\Omega_d - R^T R_d\dot{\Omega}_d)$$

$$= J\dot{\Omega} + J\bar{\Omega} = \boldsymbol{M} + Y_J\theta_J = -k_R e_R - k_\Omega e_\Omega + Y_J\tilde{\theta}_J$$

Stability Analysis – Translational Dynamics

- Let Lyapunov function V_1 defined as

$$V_1 = \frac{1}{2}k_x e_x^T e_x + \frac{1}{2}m e_v^T e_v + C_1 m e_x \cdot e_v + \frac{1}{2}\tilde{\theta}_m^T \Gamma_m^{-1}\tilde{\theta}_m$$

- V_1 is P.D. and it can be lower and upper bounded by

$$z_1^T M_{11} z_1 + \frac{1}{2}\tilde{\theta}_m^T \Gamma_m^{-1}\tilde{\theta}_m \le V_1 \le z_1^T M_{12} z_1 + \frac{1}{2}\tilde{\theta}_m^T \Gamma_m^{-1}\tilde{\theta}_m$$

$$z_1 \triangleq [\|e_x\|, \ \|e_v\|]^T$$

$$M_{11} = \frac{1}{2}\begin{bmatrix} k_x & -C_1 m \\ -C_1 m & m \end{bmatrix}$$

$$M_{12} = \frac{1}{2}\begin{bmatrix} k_x & C_1 m \\ C_1 m & m \end{bmatrix}$$

Stability Analysis – Translational Dynamics

- Taking the time derivative of V_1 yields

$$\dot{V}_1 = k_x e_x \cdot \boldsymbol{\dot{e}_x} + e_v \cdot \boldsymbol{m\dot{e}_v} + C_1 m\boldsymbol{\dot{e}_x} \cdot e_v + C_1 e_x \cdot \boldsymbol{m\dot{e}_v} - \tilde{\theta}_m^T \Gamma_m^{-1}\boldsymbol{\dot{\hat{\theta}}_m}$$

- Substitute $\dot{e}_x$ and $m\dot{e}_v$ defined in the previous page into $\dot{V}_1$

$$\dot{V}_1 \le -z_1^T W_1 z_1 + z_1^T W_{12} z_2 - k_m^{cl}\tilde{\theta}_m^T \left(\sum_{j=1}^{N_m}\left(y_m^{cl}(t_j)\right)^T y_m^{cl}(t_j)\right)\tilde{\theta}_m$$

$$W_1 = \begin{bmatrix} k_x C_1(1-\alpha) & -\frac{1}{2}C_1 k_v(1+\alpha) \\ -\frac{1}{2}C_1 k_v(1+\alpha) & k_v(1-\alpha) - C_1 m \end{bmatrix}, \qquad W_{12} = \begin{bmatrix} k_x e_{v,max} + C_1 B & 0 \\ B & 0 \end{bmatrix}$$

- M_{11}, M_{12}, W_1 in V_1 and $\dot{V}_1$ are positive-definite matrices if C_1 satisfies

$$C_1 < min\left\{\sqrt{\frac{k_x}{m}}, \frac{k_v(1-\alpha)}{m}, \frac{4k_x k_v(1-\alpha)^2}{k_v^2(1+\alpha)^2 + 4mk_x(1-\alpha)}\right\}$$

Stability Analysis – Rotational Dynamics

- Let Lyapunov function V_2 defined as

$$V_2 = \frac{1}{2} e_\Omega \cdot J e_\Omega + k_R \Psi(R, R_d) + J C_2 e_R \cdot e_\Omega + \frac{1}{2} \tilde{\theta}_J^T \Gamma_J^{-1} \tilde{\theta}_J$$

- V_2 is P.D. and it can be lower and upper bounded by

$$z_2^T M_{21} z_2 + \frac{1}{2} \tilde{\theta}_J^T \Gamma_J^{-1} \tilde{\theta}_J \le V_2 \le z_2^T M_{22} z_2 + \frac{1}{2} \tilde{\theta}_J^T \Gamma_J^{-1} \tilde{\theta}_J$$

$$z_2 \triangleq [\|e_R\|, \quad \|e_\Omega\|]^T$$

$$M_{21} = \frac{1}{2} \begin{bmatrix} k_R & -C_2 \lambda_{max}(J) \\ -C_2 \lambda_{max}(J) & \lambda_{min}(J) \end{bmatrix}$$

$$M_{22} = \frac{1}{2} \begin{bmatrix} \frac{2 k_R}{2 - \psi_2} & -C_2 \lambda_{max}(J) \\ -C_2 \lambda_{max}(J) & \lambda_{min}(J) \end{bmatrix}$$

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Stability Analysis – Rotational Dynamics

- Taking the time derivative of V_2 yields

$$\dot{V}_2 = (e_\Omega + C_2 e_R) \cdot (J \dot{e}_\Omega) + k_R e_\Omega \cdot e_R + J C_2 \dot{e}_R \cdot e_\Omega - \tilde{\theta}_J^T \Gamma_J^{-1} \dot{\hat{\theta}}_J$$

- Substitute $\dot{e}_R$ and $J\dot{e}_\Omega$ defined in the previous page into $\dot{V}_2$

$$\dot{V}_2 \le -z_2^T W_2 z_2 - k_J^{cl} \tilde{\theta}_J^T \left(\sum_{j=1}^{N_J} \left(y_J^{cl}(t_j) \right)^T y_J^{cl}(t_j) \right) \tilde{\theta}_J$$

$$W_2 = \begin{bmatrix} C_2 k_R & -\frac{C_2 k_\Omega}{2} \\ -\frac{C_2 k_\Omega}{2} & k_\Omega - C_2 \lambda_{max}(J) \end{bmatrix}$$

- M_{21}, M_{22}, W_2 in V_2 and $\dot{V}_2$ are positive-definite matrices if C_2 satisfies

$$C_2 < min \left\{ \frac{k_\Omega}{\lambda_{max}(J)}, \frac{4 k_\Omega k_R}{k_\Omega^2 + 4 k_R \lambda_{max}(J)}, \sqrt{\frac{k_R \lambda_{min}(J)}{\lambda_{max}(J)^2}} \right\}$$

Motivation
Problem Formulation
Controller Design
Stability Analysis
Experiments
Conclusion

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Stability Analysis – Overall System

- Let $V = V_1 + V_2$ be a Lyapunov function for the system containing rotational and translational dynamics

$$V = V_1 + V_2$$
$$= \frac{1}{2} k_x e_x^T e_x + \frac{1}{2} m e_v^T e_v + C_1 m e_x \cdot e_v + \frac{1}{2} \tilde{\theta}_m^T \Gamma_m^{-1} \tilde{\theta}_m$$
$$+ \frac{1}{2} e_\Omega \cdot J e_\Omega + k_R \Psi(R, R_d) + J C_2 e_R \cdot e_\Omega + \frac{1}{2} \tilde{\theta}_J^T \Gamma_J^{-1} \tilde{\theta}_J \quad \text{... P.D.}$$

- Taking the time derivative of V and substituting $\dot{V}_1$ and $\dot{V}_2$ yields

$$\dot{V} = \dot{V}_1 + \dot{V}_2 \le -z_1^T W_1 z_1 + z_1^T W_{12} z_2 - k_m^{cl} \tilde{\theta}_m^T \left(\sum_{j=1}^{N_m} \left(y_m^{cl}(t_j) \right)^T y_m^{cl}(t_j) \right) \tilde{\theta}_m$$
$$-z_2^T W_2 z_2 - k_J^{cl} \tilde{\theta}_J^T \left(\sum_{j=1}^{N_J} \left(y_J^{cl}(t_j) \right)^T y_J^{cl}(t_j) \right) \tilde{\theta}_J \quad \text{... N.D.}$$

, where $\lambda_{min}(W_2) > {}^{4\|W_{12}\|^2}/_{\lambda_{min}(W_1)}$

Motivation
Problem Formulation
Controller Design
Stability Analysis
Experiments
Conclusion

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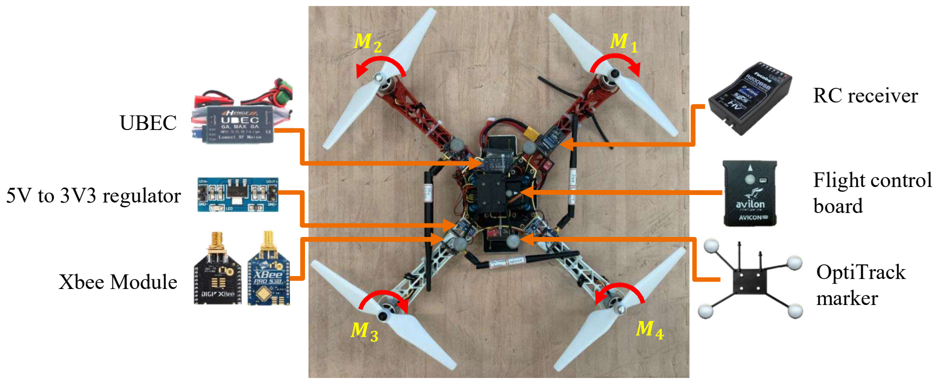
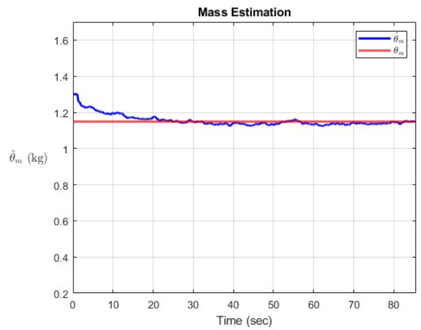
Experiments – Hardware Architecture



Motivation
Problem Formulation
Controller Design
Stability Analysis
Experiments
Conclusion

Experiments – Mass Estimation

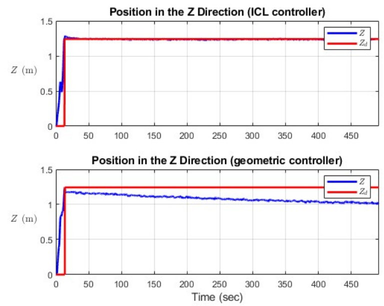
- The mass estimation of the multirotor with ICL controller converged to 1.15kg and has 1% error with ground truth



Mass estimation	Mass ground truth
1.15 (kg)	1.16 (kg)

Experiments – Mass Estimation

- The accurate mass estimation also make the multirotor have better tracking performance in the z direction



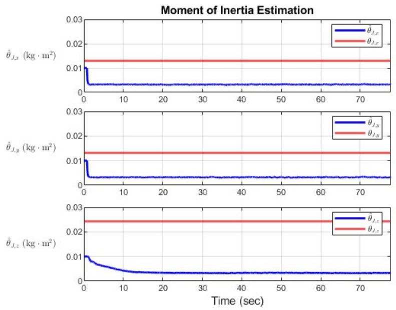
Steady-state error in the z direction	
ICL controller	Geometric controller
<0.5 (cm)	>10 (cm)

ICL controller : $f = \left(k_x e_x + k_v e_v + \boldsymbol{Y_m \hat{\theta}_m}\right) \cdot Re_3$

Geometric controller : $f = (k_x e_x + k_v e_v + \boldsymbol{mg\ _3 - m\ddot{x}_d}) \cdot Re_3$

Experiments – Moment of Inertia Estimation

- The moment of inertia estimation converge to $[0.0033, 0.0032, 0.0033]$ $(kg \cdot m^2)$ which is smaller than the ground truth



Moment of inertia estimation $(kg \cdot m^2)$	Moment of inertia ground truth $(kg \cdot m^2)$
$\begin{bmatrix} 0.0033 & 0 & 0 \\ 0 & 0.0032 & 0 \\ 0 & 0 & 0.0033 \end{bmatrix}$	$\begin{bmatrix} 0.013 & 0 & 0 \\ 0 & 0.013 & 0 \\ 0 & 0 & 0.024 \end{bmatrix}$

Experiments – Moment of Inertia Estimation

- We suppose the incorrect convergence is resulted from the measurement noise from gyroscope
- To present the experimental scene more realistically, white Gaussian noise is applied to the simulation
- Moreover, we designed an estimator to eliminate the influence of noise on the system

$$M = -k_R e_R - k_\Omega e_\Omega - Y_J \hat{\theta}_J \ , Y_J = \begin{bmatrix} \bar{\Omega}_1 & \Omega_2 \cdot \Omega_3 & -\Omega_2 \cdot \Omega_3 \\ -\Omega_1 \cdot \Omega_3 & \bar{\Omega}_2 & \Omega_1 \cdot \Omega_3 \\ \Omega_1 \cdot \Omega_2 & -\Omega_1 \cdot \Omega_2 & \bar{\Omega}_3 \end{bmatrix}$$

$$\dot{\hat{\theta}}_J = \Gamma_J Y_J^T (e_\Omega + C_2 e_R) + k_J^{cl} \Gamma_J \sum_{j=1}^{N_J} \left(y_J^{cl}(t_j)\right)^T \left(\bar{M}(t_j) - y_J^{cl}(t_j)\, \hat{\theta}_J\right) \ , Y_J^{cl} = \begin{bmatrix} \dot{\Omega}_1 & -\Omega_2 \cdot \Omega_3 & \Omega_2 \cdot \Omega_3 \\ \Omega_1 \cdot \Omega_3 & \dot{\Omega}_2 & -\Omega_1 \cdot \Omega_3 \\ -\Omega_1 \cdot \Omega_2 & \Omega_1 \cdot \Omega_2 & \dot{\Omega}_3 \end{bmatrix}$$

Experiments – Estimator of the Angular Velocity

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- The rotational dynamics of the multirotor ca be written as

$$\dot{\Omega} = J^{-1}M - J^{-1}\Omega \times J\Omega = \begin{bmatrix} \frac{M_1}{\hat{J}_{xx}} + \frac{\hat{J}_{yy}}{\hat{J}_{xx}}\Omega_2\Omega_3 - \frac{\hat{J}_{zz}}{\hat{J}_{xx}}\Omega_2\Omega_3 \\ \frac{M_2}{\hat{J}_{yy}} + \frac{\hat{J}_{xx}}{\hat{J}_{yy}}\Omega_1\Omega_3 - \frac{\hat{J}_{zz}}{\hat{J}_{yy}}\Omega_1\Omega_3 \\ \frac{M_3}{\hat{J}_{zz}} + \frac{\hat{J}_{xx}}{\hat{J}_{zz}}\Omega_1\Omega_2 - \frac{\hat{J}_{yy}}{\hat{J}_{zz}}\Omega_1\Omega_2 \end{bmatrix}$$

$$\hat{\Omega}_k^- = \hat{\Omega}_{k-1} + \dot{\Omega}_{k-1}\Delta t = \begin{bmatrix} \hat{\Omega}_{k-1,1} + \left(\frac{M_1}{\hat{J}_{xx}} + \frac{\hat{J}_{yy}}{\hat{J}_{xx}}\Omega_2\Omega_3 - \frac{\hat{J}_{zz}}{\hat{J}_{xx}}\Omega_2\Omega_3\right)\cdot\Delta t \\ \hat{\Omega}_{k-1,2} + \left(\frac{M_2}{\hat{J}_{yy}} + \frac{\hat{J}_{xx}}{\hat{J}_{yy}}\Omega_1\Omega_3 - \frac{\hat{J}_{zz}}{\hat{J}_{yy}}\Omega_1\Omega_3\right)\cdot\Delta t \\ \hat{\Omega}_{k-1,3} + \left(\frac{M_3}{\hat{J}_{zz}} + \frac{\hat{J}_{xx}}{\hat{J}_{zz}}\Omega_1\Omega_2 - \frac{\hat{J}_{yy}}{\hat{J}_{zz}}\Omega_1\Omega_2\right)\cdot\Delta t \end{bmatrix}$$

- The estimated angular velocity can be generated as

$\hat{\Omega}_k = \hat{\Omega}_k^- + K_\Omega(\Omega - \hat{\Omega}_k^-)$, K_Ω is a estimator gain

Experiments – Moment of Inertia Estimation

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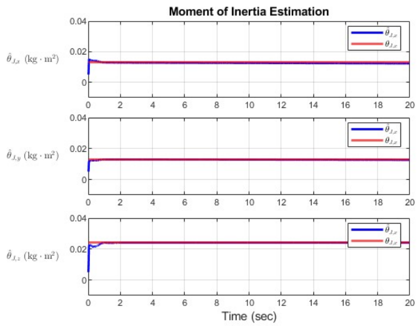
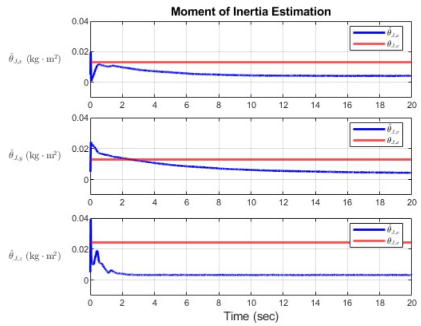


Moment of inertia estimation **with noise and estimator** in the simulation



Moment of inertia estimation **with noise, without estimator** in the simulation

Experiments – Moment of Inertia Estimation

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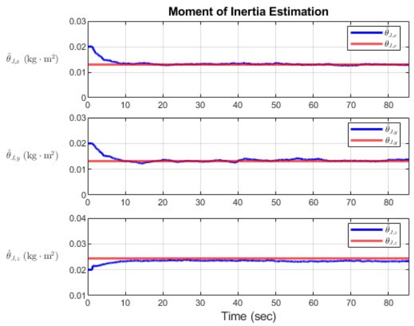
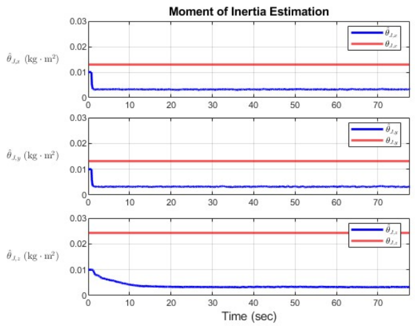


Moment of inertia estimation **with noise and estimator** in the experiments



Moment of inertia estimation **with noise, without estimator** in the experiments

Experiments – Moment of Inertia Estimation

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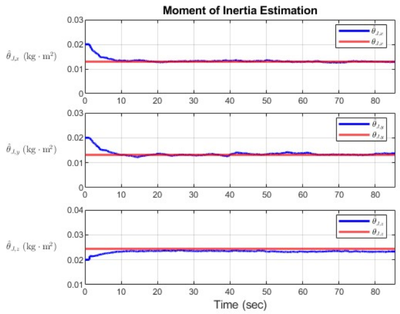
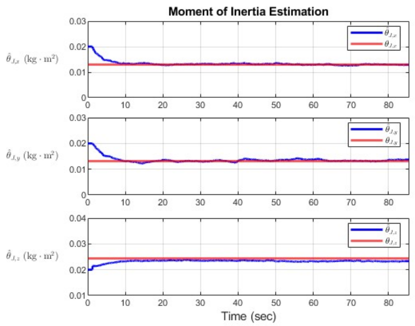
- The moment of inertia estimation converge to $[0.013, 0.014, 0.022]\ (kg \cdot m^2)$ and has 8% error with ground truth

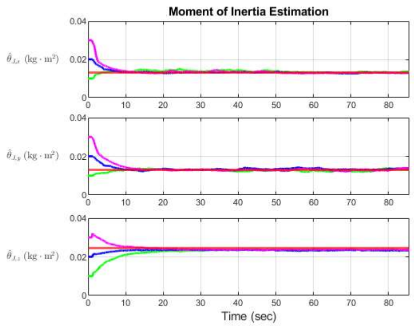


Moment of inertia estimation $(kg \cdot m^2)$	Moment of inertia ground truth $(kg \cdot m^2)$
$\begin{bmatrix} 0.013 & 0 & 0 \\ 0 & 0.014 & 0 \\ 0 & 0 & 0.022 \end{bmatrix}$	$\begin{bmatrix} 0.013 & 0 & 0 \\ 0 & 0.013 & 0 \\ 0 & 0 & 0.024 \end{bmatrix}$

Experiments – Moment of Inertia Estimation



Moment of inertia estimation **with noise and estimator** in the experiments



Moment of inertia estimation **with different initial values** in the experiments

Experiments – Trajectory Generation

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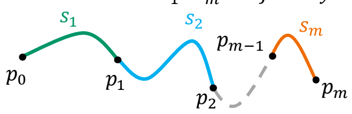
- Formulate the trajectory generation problem as a quadratic programming (QP) problem
- Write the trajectory passing through given waypoints as piecewise polynomial function of order n as

$$s_i(t) = \sum_{j=0}^{n} \sigma_{ij} t^j, t_{i-1} \le t < t_i, i \in \{1,2,\cdots,m\},$$

with cost function and constraints defined as

$$\min \int_{t_0}^{t_m} \left\| \frac{d^4 s_i}{dt^4} \right\|^2 dt,$$
$$s.t. A\sigma = b$$

$p_0 \sim p_m$: *waypoints*
$s_1 \sim s_m$: *trajectory*



Motivation
Problem Formulation
Controller Design
Stability Analysis
Experiments
Conclusion

Experiments – Trajectory Generation

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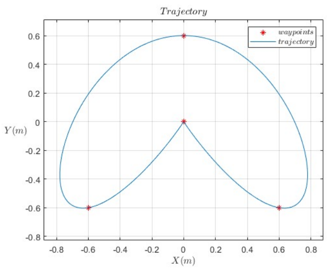
- The waypoints are given as

$$p_0 = \begin{bmatrix}0\\0\end{bmatrix} \quad p_1 = \begin{bmatrix}0.6\\-0.6\end{bmatrix} \quad p_2 = \begin{bmatrix}0\\0.6\end{bmatrix} \quad p_3 = \begin{bmatrix}-0.6\\-0.6\end{bmatrix} \quad p_4 = \begin{bmatrix}0\\0\end{bmatrix}$$

- The desired trajectory is generation as



Motivation
Problem Formulation
Controller Design
Stability Analysis
Experiments
Conclusion

Experiments – Tracking Performance

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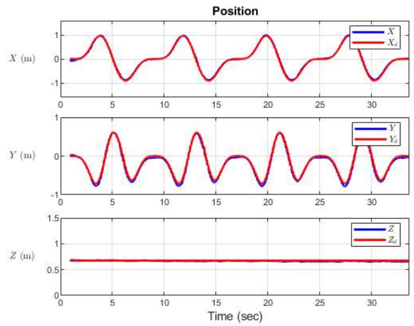
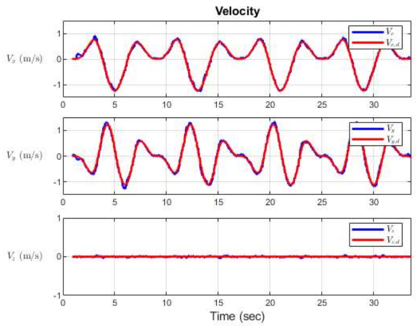


Position tracking performance of the multirotor using ICL controller



Velocity tracking performance of the multirotor using ICL controller

Motivation
Problem Formulation
Controller Design
Stability Analysis
Experiments
Conclusion

Experiments – Comparison

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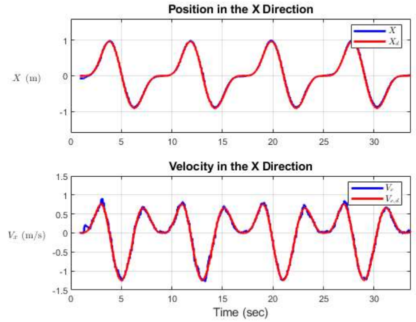
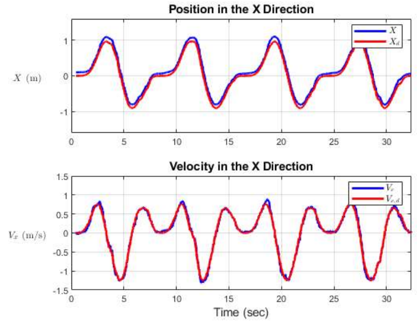
- Motivation
- Problem Formulation
- Controller Design
- Stability Analysis
- Experiments
- Conclusion



Tracking performance of the multirotor using **ICL controller**



Tracking performance of the multirotor using **geometric controller**

Experiments – Comparison

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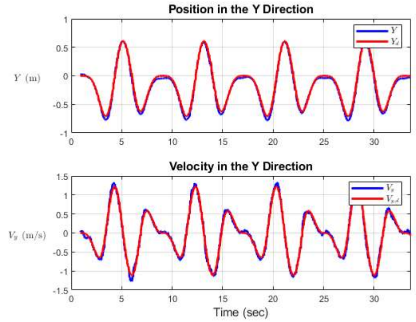
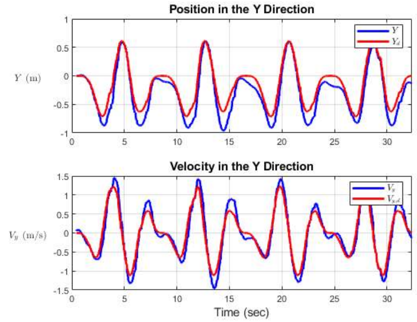
- Motivation
- Problem Formulation
- Controller Design
- Stability Analysis
- Experiments
- Conclusion



Tracking performance of the multirotor using **ICL controller**



Tracking performance of the multirotor using **geometric controller**

Experiments – Comparison

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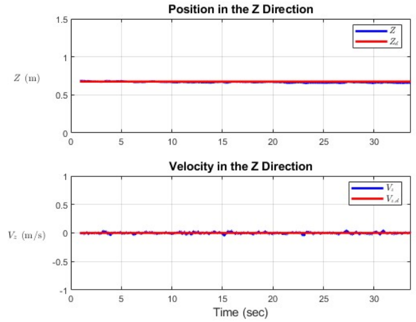
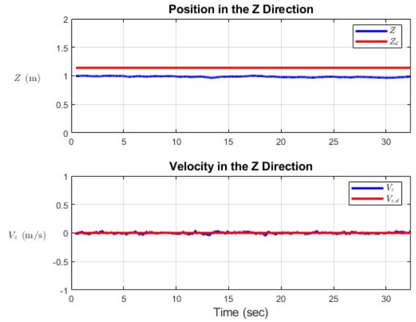
- Motivation
- Problem Formulation
- Controller Design
- Stability Analysis
- Experiments
- Conclusion



Tracking performance of the multirotor using **ICL controller**



Tracking performance of the multirotor using **geometric controller**

Experiments – Video (ICL controller)

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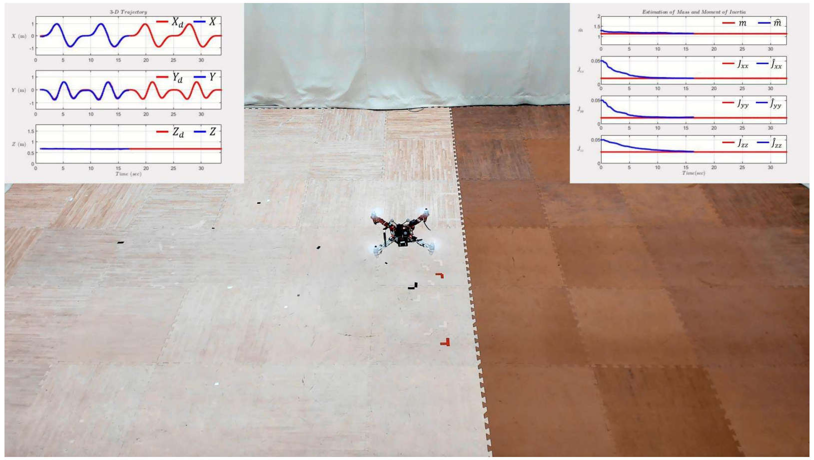
- Motivation
- Problem Formulation
- Controller Design
- Stability Analysis
- Experiments
- Conclusion



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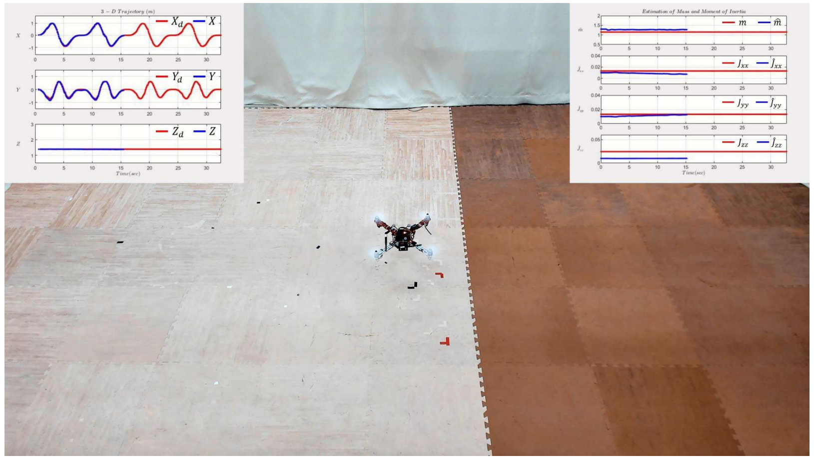
Experiments – Video (adaptive controller)



Motivation
Problem Formulation
Controller Design
Stability Analysis
Experiments
Conclusion

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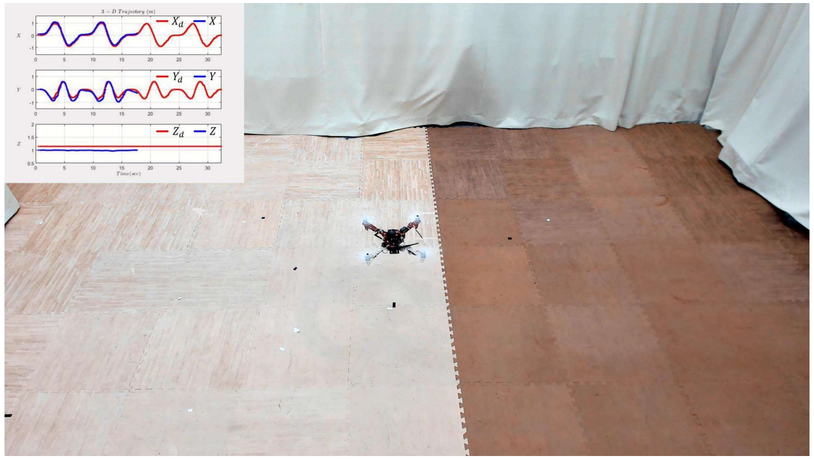
Experiments – Video (geometric controller)



Motivation
Problem Formulation
Controller Design
Stability Analysis
Experiments
Conclusion

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Conclusion

- An ICL controller has been developed for controlling a multirotor with an unknown mass and moment of inertia
- The control architecture can be applied to many types of multirotors of unknown mass
- The ICL controller ensures the steady-state errors resulted from the wrong parameters be eliminated
- The ICL controller can guarantee asymptotic convergence of the system parameters, while the adaptive controller cannot
- Future work can be estimate other parameters of the multirotor, such as off-diagonal elements in the inertia matrix and the center of mass.

Motivation
Problem Formulation
Controller Design
Stability Analysis
Experiments
Conclusion

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Thanks for listening!