CS1231 Assignment #2
AY2017/18 Semester 1
Deadline: 5pm on Thursday, 2 November 2017

Instructions:

This is a **graded** assignment worth 10% of your final grade. Please work on it by yourself (not in a group), and submit your answers by the deadline stated above. Answer all questions. A handwritten submission is fine; there is no need to use Word or Latex to typeset. But please write legibly. Also, state **your name, student number, and tutorial group** at the top of the first page. Make sure you write the correct tutorial group.

How to submit:

You may drop your submission through a slot labeled "CS1231 Assignment" at the Undergraduate Studies Office, COM1-02-19, from 27 October 2017. **Note: Late submission or softcopy submission will not be accepted.**

Question 1. (8 marks)

Answer the following parts. Show your workings.

(a) (2 marks) You are buying lunch for 20 guests. The menu offers 6 dishes: chicken rice, nasi lemak, mee rebus, ayam penyet, laksa, and bak chor mee. Two guests ask for chicken rice, three ask for nasi lemak, two ask for mee rebus and the rest have no preference. How many different selections can you make?
(A "selection" means a multiset of 20 dishes that you may order for your guests.)

(b) (2 marks) An urn B_1 contains 2 red and 3 blue balls and another urn B_2 contains 3 red and 4 blue balls. Both urns are equally likely to be chosen. Aiken selects one urn at random and draws a blue ball from it. What is the probability that the blue ball comes from urn B_1? Leave your answer as a fraction in its simplest form.

(c) (2 marks) Dueet is taking a test and does not know the answer to three multiple-choice questions (MCQs). Each MCQ has five choices for the answer.

Dueet can eliminate two answer choices as incorrect for the first question, and eliminate one answer choice as incorrect for the second question, but has no clue about the correct answer at all for the third question.

Assuming that Dueet's selection of a choice for one question does not affect his selection of a choice for another question, what is the probability that Dueet will answer at least one of the three questions correctly?

(d) (2 marks) For a biased 6-sided die, the probability of the smaller numbers (1, 2, 3) turning up is three times the probability of the larger numbers (4, 5, 6) turning up. If we roll two such dice, what is the probability of getting a sum of 6? Leave your answer as a fraction in its simplest form.

Question 2. (7 marks)

You must use the Pigeonhole Principle (PHP) to solve the following problems. Other methods will <u>not</u> be accepted.

(a) (1 mark) With reference to question 1 part (a), explain why there will be a lunch option chosen by at least 4 persons.

(b) (3 marks) Show that at any party there are two persons who have the same number of friends at the party, assuming that all friendships are mutual and nobody can be friend to himself/herself, and a party must consist of at least two persons.

(c) (3 marks) In a square house there are 121 square rooms laid out in an 11×11 grid. There is one person in every room. Two rooms are adjacent if they share a wall. At the sound of a gong which all can hear, each person selects an adjacent room and moves into it, everyone at the same time. Is it possible that every room is still occupied by someone after the gong? Explain.

Question 3. (5 marks)

(a) (2 marks) Consider the identity function $I_A : A \longrightarrow A$ defined on a set A, where $\forall x \in A,\ I_A(x) = x$. Prove that it is a bijection.

(b) (3 marks) Let $\preceq$ be a partial order on a set A. Is $\preceq^{-1}$ a partial order? If so, prove it; otherwise, give a counter-example. (Recall that for any binary relation $\mathcal{R}$, its inverse relation is denoted $\mathcal{R}^{-1}$.)

Question 4. (10 marks)

A sequence a_n is defined by the third-order recurrence relation:

$$a_n = 5a_{n-1} - 8a_{n-2} + 4a_{n-3},\ \text{ for } n \in \mathbb{Z}_{\geq 3}.$$

with initial values: $a_0 = a_1 = 1$ and $a_2 = 3$.

(a) (1 mark) Explicitly calculate a_3, a_4, a_5 and a_6.

(b) (4 marks) Using Mathematical Induction, prove that:

$$\sum_{r=0}^{n-1} r2^r \ = \ 2^n(n-2) + 2,\ \text{ for all } n \in \mathbb{Z}^+.$$

(c) (5 marks) Derive an explicit closed-form formula for a_n, for all $n \in \mathbb{N}$.

Hint: Define a new sequence b_n in terms of a_n, and get a second-order linear homogeneous recurrence relation with constant coefficients for b_n. Solve it, then solve for a_n. Do not use the formula for a 3rd-order recurrence relation.

—— **END** ——