F# Tutorial

Pipe-Forward Operator

March 1, 2018

1 Modified Project Euler Questions

`https://projecteuler.net/problem=1`
`https://projecteuler.net/problem=2`

Change the url so that you get `problem=3, problem=4`, etc.

Question 1

Original Question. *Implement a function that sums up all multiples of* 3 *or* 5 *in a list.*

Question 2

Original Question. *The Fibonacci sequence (starting with* 1 *and* 2*) looks something like:*

$$1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots$$

(For example, $1 + 2 = 3, 2 + 3 = 5, 3 + 5 = 8$*, etc.)*

Find the sum of all even-valued fibonacci numbers below 4 *million.*

(a) List down the first 40 fibonacci numbers. Show that the 40th Fibonacci number already exceed 4 million (in fact, probably the 32nd or 33rd number already exceed 4 million)

(b) Sum all even-valued fibonacci numbers below 4 million. (In particular, by part (a), the first 40 numbers are already sufficient)

Question 3

Exercise (Euler Project Question 3)

Modified Question. *Write a function that takes a list of (positive) integers, and returns the largest prime number in that list (assuming each integer is less than* `INT_MAX`*).*

Question 4

Original Question. *A palindromic number reads the same from left-to-right or right-to-left.*

The largest palindromic number made from the product of two 2-digit numbers is $9009 = 91 \times 99$.

Find the largest palindrome made from the product of two 3-digit numbers.

Question 5

Modified Question. *Given a list of integers, find the lowest common multiple (LCM) of all those numbers. (Assume no integer overflow)*

Question 6

Original Question. *Given a list of integers $x_1, x_2, \ldots, x_n$, write a function that calculates the following:*

$$\left(\sum_{i=1}^{n} x_i\right)^2 - \left(\sum_{i=1}^{n} {x_i}^2\right)$$

Question 7

Original Question. *The list of prime numbers are $2, 3, 5, 7, 11, 13, \ldots$. We can see that the 6th prime number is 13.*

What is the 10001th prime number?

(a) How many prime numbers are there between 2 and 500000? Verify that there are more than 10000 prime numbers between this range. (In fact, more than 40000 prime numbers)

(b) What is the 10001th prime number between 2 and 500000?

Question 8

Modified Question. *Given a list of digits, find four adjacent digits with the largest product. For example, in the following number:*

$$73167176531330624919225119**9674**4265747423553491949 34$$

The 4 consecutive digits that gives the largest product is $9 \times 6 \times 7 \times 4 = 9674$

(Notice that this line is the first line in the original question)

Question 9

Original Question. *Find the only Pythagorean triplet a, b, c that satisfy:*

$$a < b < c, \qquad a + b + c = 1000, \qquad a^2 + b^2 = c^2$$

Question 10

Exercise (Euler Project Question 10)

Modified Question. *Given a number $N < 200,000$, find the sum of all prime numbers between 2 and N.*

2 Original Project Euler Solutions

Question 1, 2, 4, 6, 7, 9

We did not modify Question 1, 2, 4, 6, 7, 9.

Question 5

Original Question. *Find the least common multiple (LCM) of* 1 *to* 20. *(You may encounter integer overflow)*

Question 8

Original Question. *In the webpage, a* $1000-$*digit number is provided.*

The four adjacent digits in the 1000*-digit number that have the greatest product are* $9 \times 9 \times 8 \times 9 = 5832$.

731671765313.....31998900.....2963450

Find the thirteen adjacent digits in the 1000*-digit number that have the greatest product. (You may encounter integer overflow)*

Question 10

Original Question. *The sum of the primes below* 10 *is* $2 + 3 + 5 + 7 = 17$

Find the sum of all the primes below two million $(2,000,000)$. *(You may encounter integer overflow)*

Question 3

Question. *Given an integer Z, write a function that finds the largest prime factor of Z. e.g. The prime factors of* 13195 *are* $5, 7, 13, 29$*, and so the largest for* 13195 *is* 29*.*

Problem Analysis

Remark: Given an integer Z, it is possible that the largest prime factor of Z is greater than $\sqrt{Z}$

- Example: $6 \times 11 = 66$. The largest prime factor is $11 > \sqrt{66} \approx 8.12$.

To solve this question, we need some additional mathematical consideration (which is not quite directly related to programming).

(a) Let $S_1 = \{a_1, \ldots, a_n\}$ be all the factors of Z (not necessarily prime factors) between 1 and $\sqrt{Z}$. This set will always contain at least one element: $a_1 = 1$.

(b) Let $S_2 = \left\{\frac{Z}{a_1}, \ldots, \frac{Z}{a_n}\right\}$. These are all the factors of Z between $\sqrt{Z}$ and Z. This set will always contain at least one element: $\frac{Z}{a_1} = Z$.

(c) So, $S_1 \cup S_2 = \left\{a_1, \ldots, a_n, \frac{Z}{a_1}, \ldots, \frac{Z}{a_n}\right\}$ are all the factor of Z (not necessarily prime factors).

(d) Out of our list of candidates $S_1 \cup S_2$, which number is the largest, prime number?