China Winter School on LISA MLDC – Lecture 1

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Objectives

- Generate simplified Mock data → single EMRI source in Gaussian stationary noise
- Detection and estimation of signal using Particle Swarm Optimization
- Simplified data
  - White noise with whitened waveform: single detector
  - Include Doppler modulation in signal waveform
- Characterize detection and estimation performance of PSO as a function of masses and SNR
- Extensions if time allows:
  - Network version: simple case of 3 independent 60° detectors
  - Colored noise version: identical Power Spectral Densities
- Parallel tracks depending on student preparation and background knowledge

PLAN: Dec 3

Lecture

- GWs
  - Plane tensor wave
  - Response of detector
- LIGO, LISA
  - Main LISA feature: Doppler modulation and time-dependent polarization phase
  - *Response of moving detector*

Lab

- Matlab programming and other issues (for newbies)
- Discussions of background knowledge (GW, data analysis) and school objectives (Advanced students)

Gravitation

- Newtonian gravity cannot be correct as it is not consistent with Special Relativity
  - $F = \frac{GM_1M_2}{r^2}$
  - Postulates instantaneous action at a distance which is incompatible with the finite speed of light
- Einstein's search for an alternative led to the General Theory of Relativity

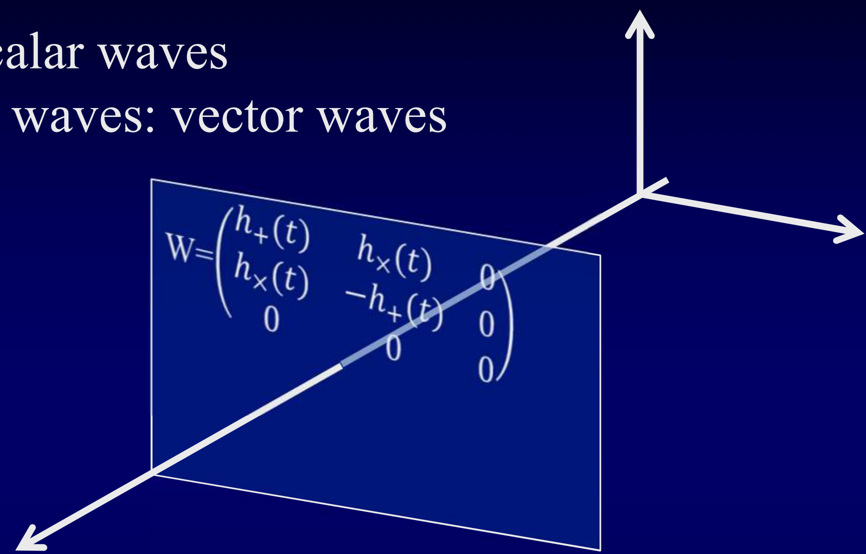
Gravitational waves

- General Theory of Relativity: Gravity is a manifestation of matter curving space-time geometry
- Changes in matter-energy distribution appear to distant observers as a time dependent change in their local space-time geometry
  - Local expansion and contraction of space
- The changes propagate away from the source as a wave travelling at the speed of light

GW plane wave

Sound waves: scalar waves

Electromagnetic waves: vector waves



Tensor field propagating on flat space-time

(Appropriate gauge and coordinate choices)

$$\Box h_{\mu\nu} = 0$$

$$\left.\begin{aligned} h_{11} &= -h_{22} = h_+(t) \\ h_{12} &= h_{21} = h_\times(t) \end{aligned}\right\} \text{two polarizations}$$

Effect of a plane monochromatic gravitational wave

$h_+(t)$

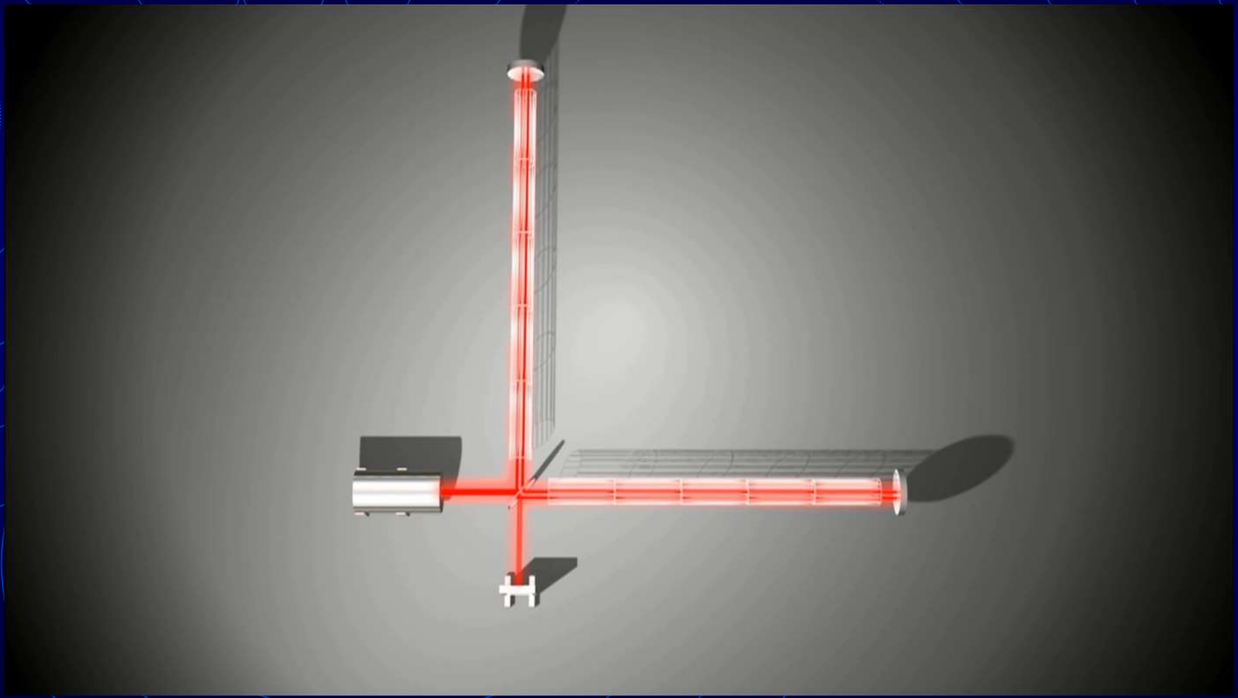
The general effect is a linear combination of the two polarizations

$h_\times(t)$

Catching Gravitational Waves

Laser Interferometry

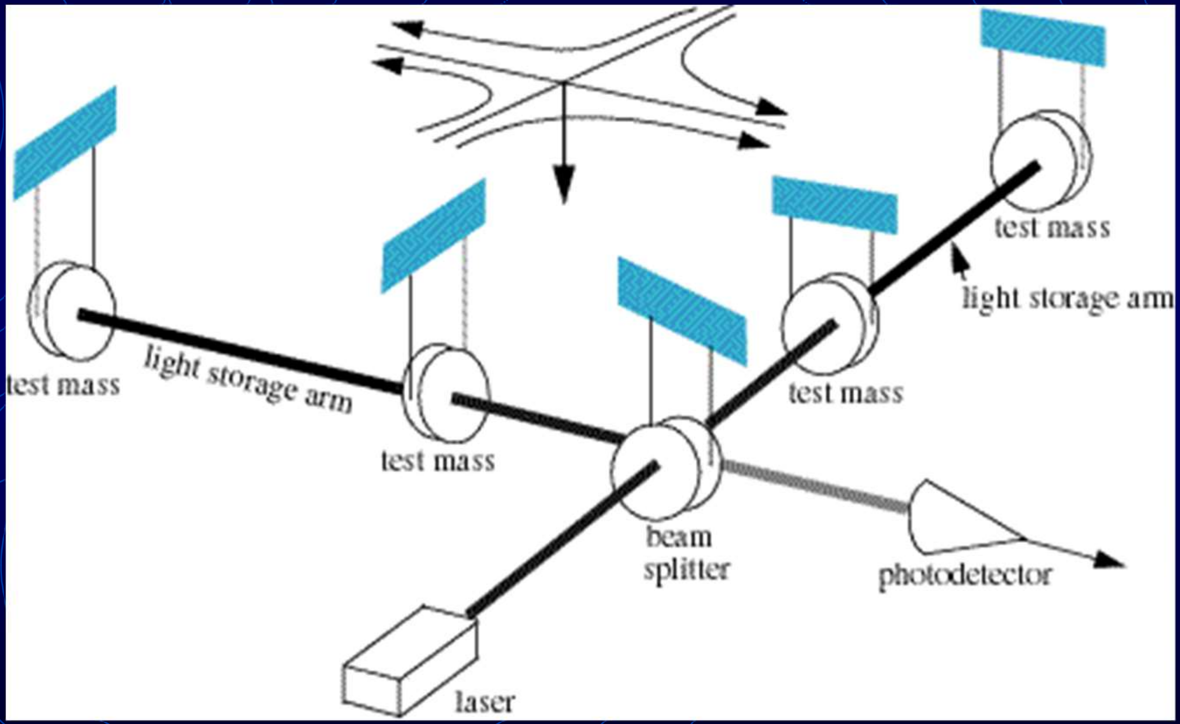
- Modern version of the classic Michelson-Morley experiment for the existence of Ether



GW detection

Measure ΔL at the level of 10^{-19} meters !

- Important Modifications:
  - Frequency stabilized high power laser (about 120 Watt)
  - Fabry Perot cavities to trap light
  - Suspended mirrors to reduce seismic noise
  - Control system to keep cavities on resonance
  - …

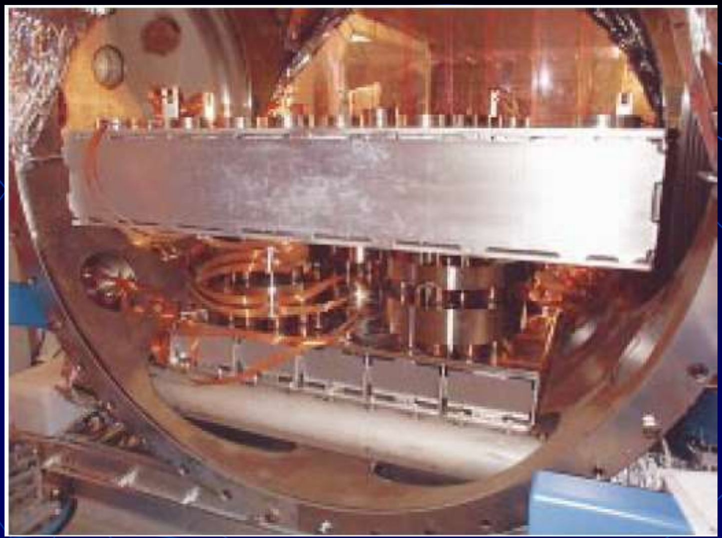
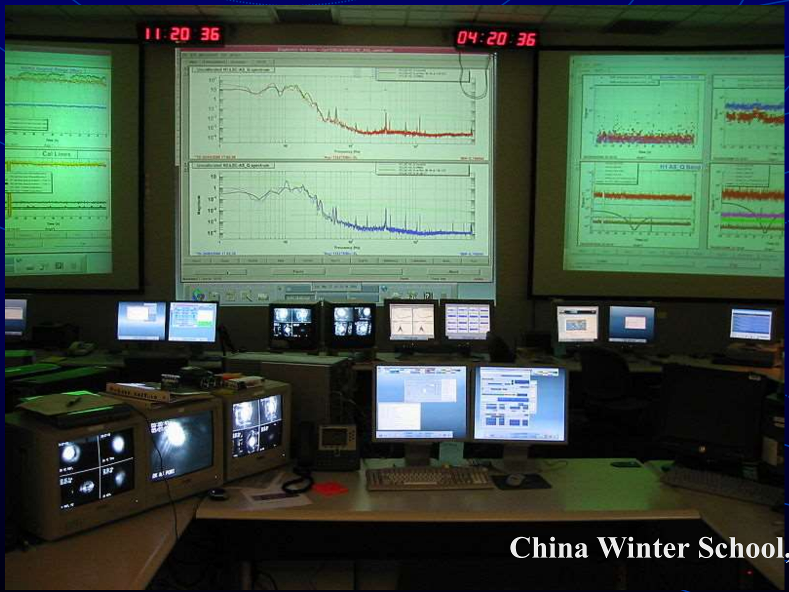


LIGO-P990006-00.pdf

LIGO: Laser Interferometric GW Observatory

Technological Challenges

- Largest resonant optical cavities
- One of the largest vaccum systems (~ 20,000m^3, 10^{-9} torr)$\Rightarrow$ special steel with very low trapped Hydrogen
- Ultra-stable laser: 10^{-7} Hz / Hz$^{1/2}$
- Ultra-high quality factor suspensions: Q~10^8, the mirrors will swing for several years!
- Ultra-high Q mirror substrates
- Test mass optics surface absorption < 2ppm
- Surface figure error < λ/1000
- Many more!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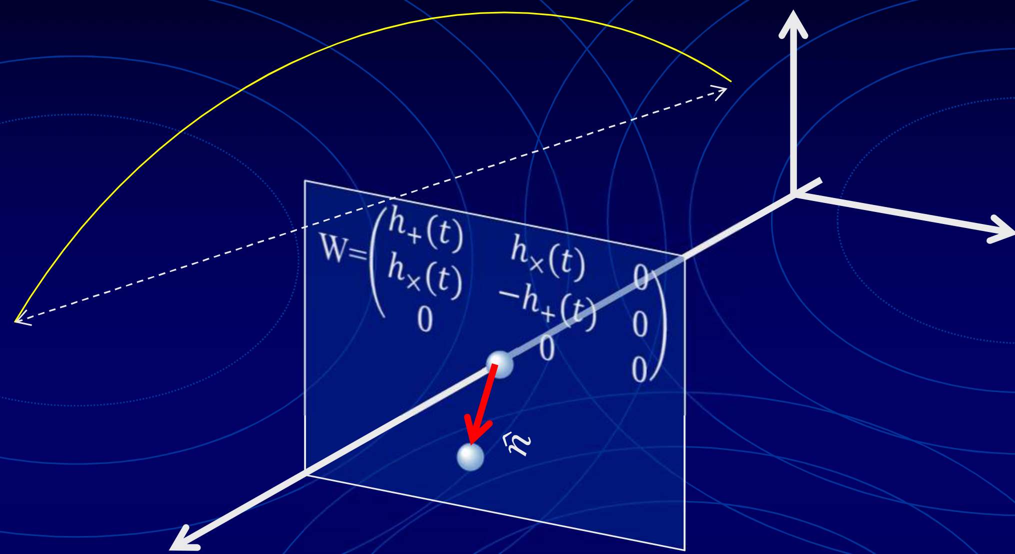
Long wavelength approximation



$$h(t)=\frac{\Delta L(t)}{L}=\hat{n}^T W(t)\hat{n} \;=W_{ij}(\hat{n}\otimes\hat{n})^{ij}$$

$$h(t)=F_+h_+(t)+F_\times h_\times(t)\,;$$

$(F_+,F_\times)$ depend on the relative orientation of the wave frame and $\hat{n}$

Detector response

A GW incident on an interferometer produces a relative change in the lengths of the light propagation distance along the two arms: $\Delta L(t)$

How do we go from the two polarizations $h_+(t)$ and $h_\times(t)$ to the scalar $\Delta L(t)$?

Detector response, $\Delta L(t)/L$, is the contraction of the wave and detector tensors, $\overleftrightarrow{W}$ and $\overleftrightarrow{D}$, both expressed in a common coordinate system:

$$s(t) = \underbrace{W_{ij}(t)}_{\text{Wave}} \overbrace{D^{ij}}^{\text{Detector}} \;;$$

In any Cartesian system: $\overleftrightarrow{D} = \hat{n}_X \otimes \hat{n}_X - \hat{n}_Y \otimes \hat{n}_Y$

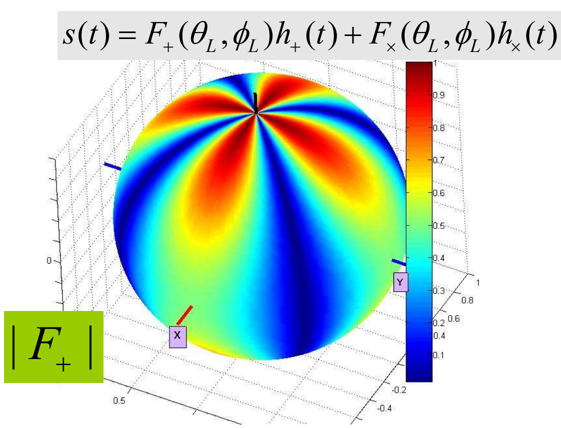
$\hat{n}_I$: unit vector along I^{th} arm

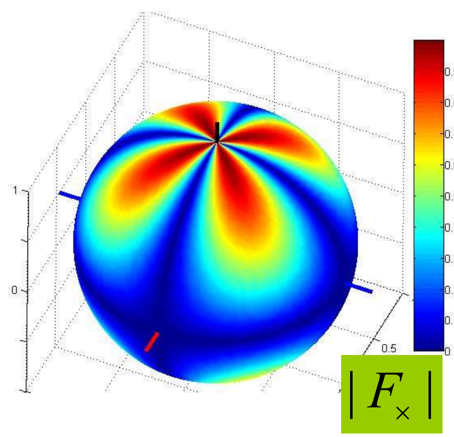
$\therefore$ Before contraction, the two tensors must be rotated into a common frame

If $\mathbf{R}$ is the rotation matrix from one Cartesian frame to another : $\overleftrightarrow{T} \to \mathbf{R}^T \overleftrightarrow{T} \mathbf{R}$

Detector response: Stationary detector

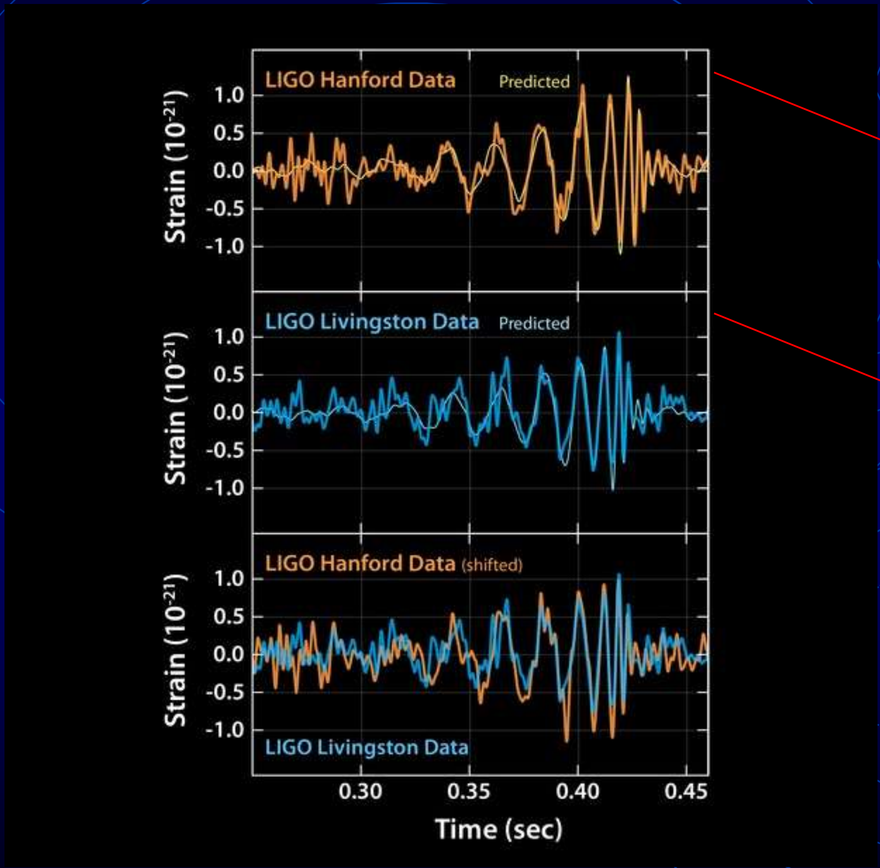
- Detector located at $\bar{x}_d$ in the plane GW wave field
- Response: $s(t) = W_{ij}D^{ij} = F_+(\hat{n})h_+(t) + F_\times(\hat{n})h_\times(t)$
- $\hat{n}$: Direction to GW source
- $h_{+,\times}(t)$: GW polarization waveform at detector location



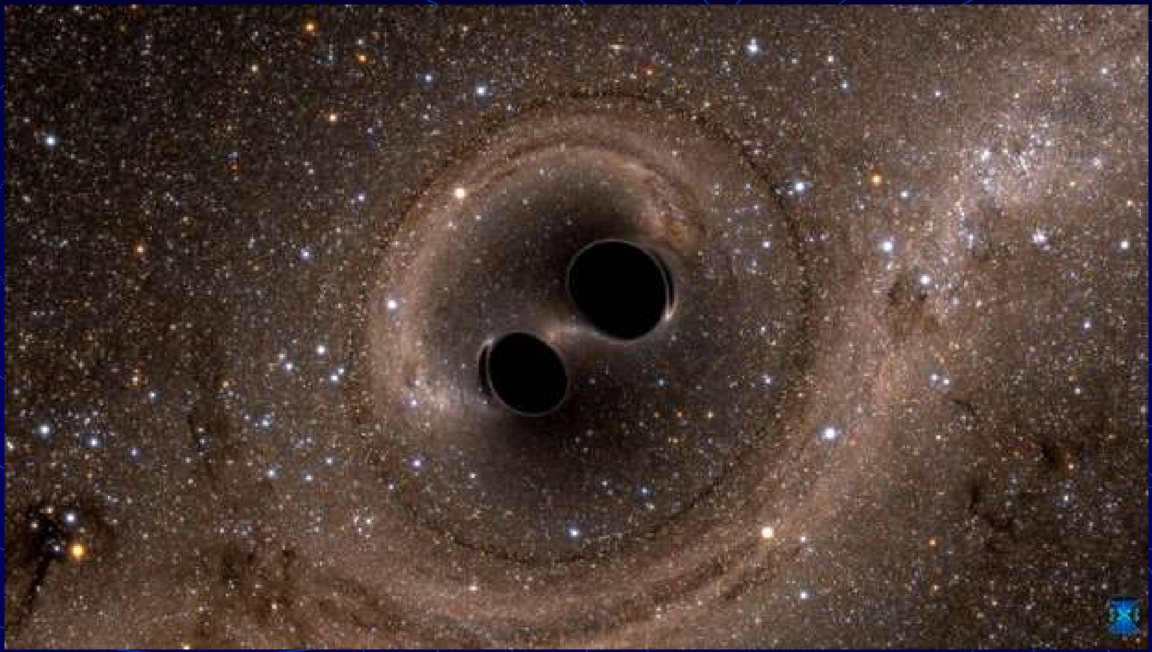


Response functions for an interferometer

GW150914: GW Astronomy launched!



GW150914: A Binary Black Hole merger

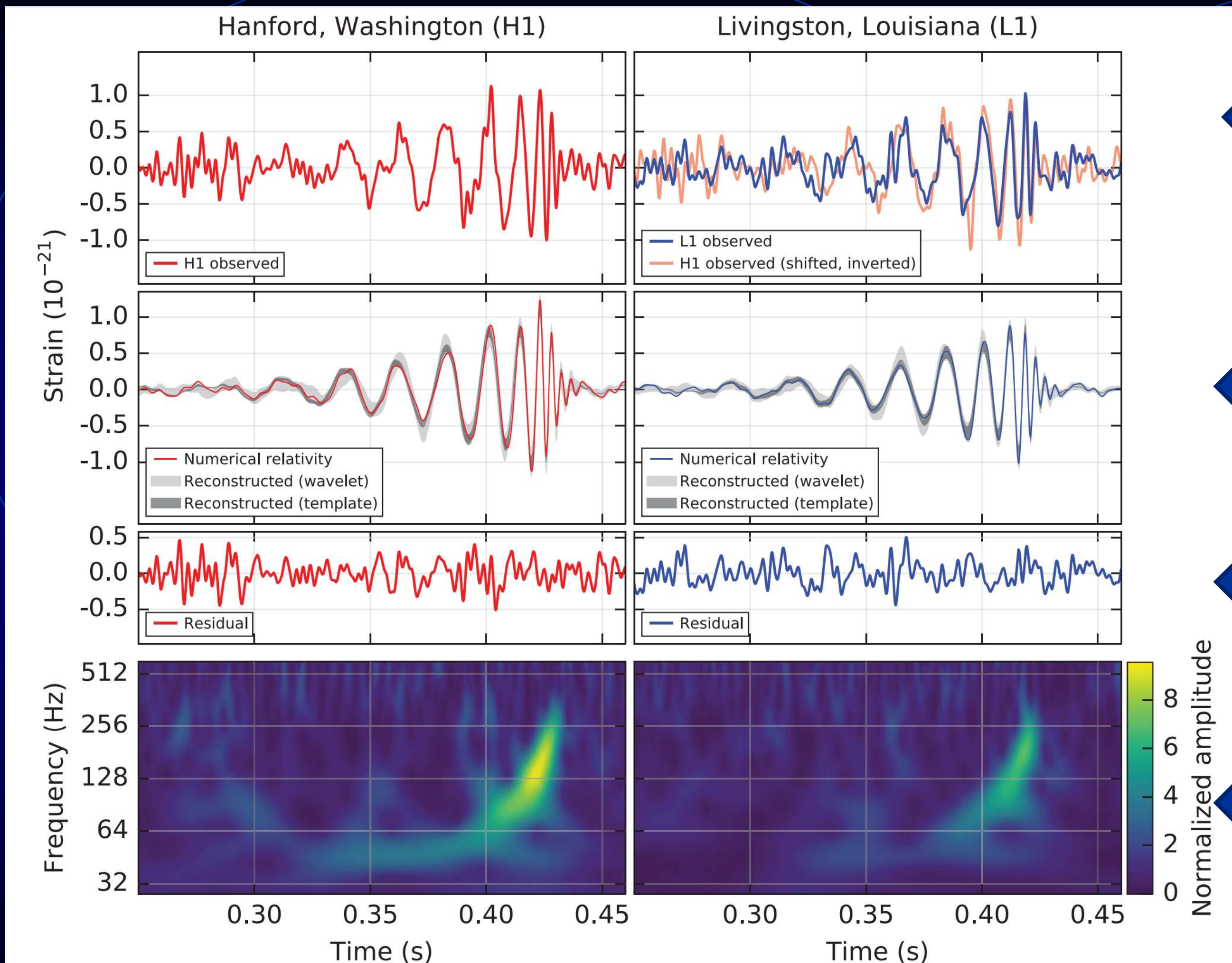


Simulated view!

PRL **116**, 061102 (2016)

"The signals came from two merging black holes, each about 30 times the mass of our sun, lying 1.3 billion light-years away."

"About 3 times the mass of the sun was converted into gravitational waves in a fraction of a second—with a peak power output about 50 times that of the whole visible universe."– ligo.org



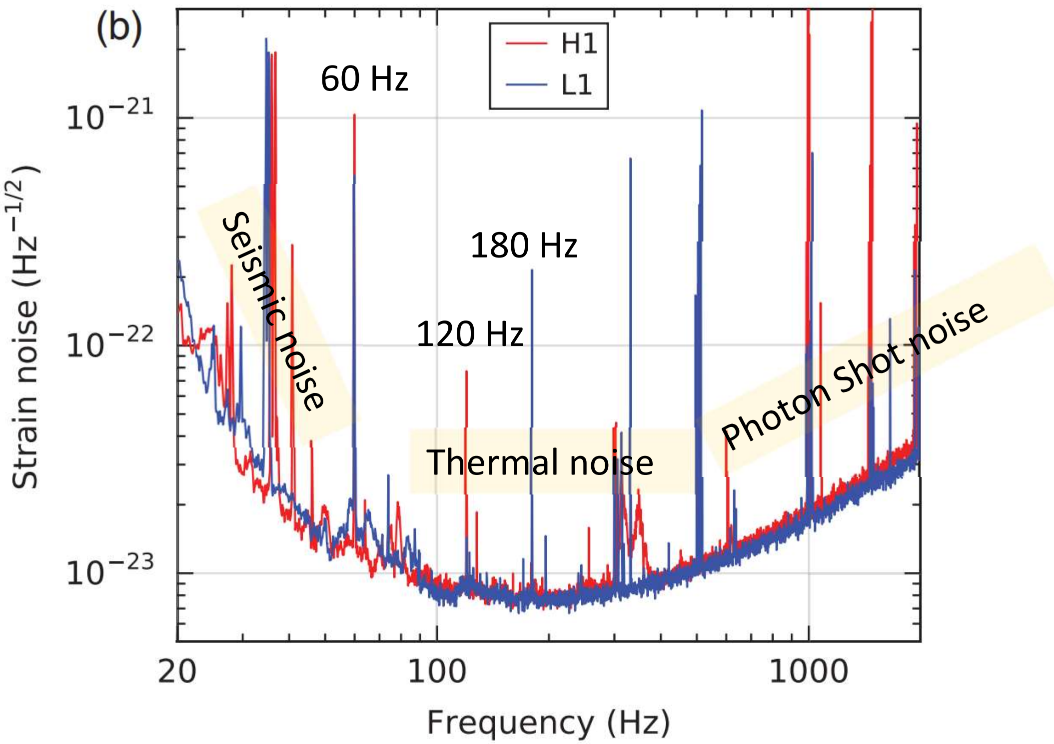
Whitened Data

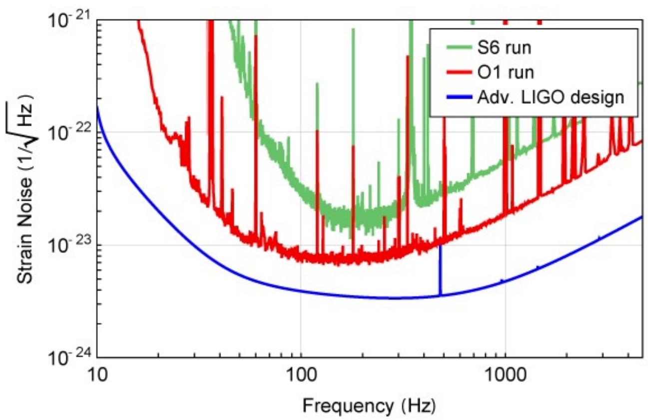
Best fit signal

Residual = Data-Fit = noise

Time-frequency plot of data

Detector Noise





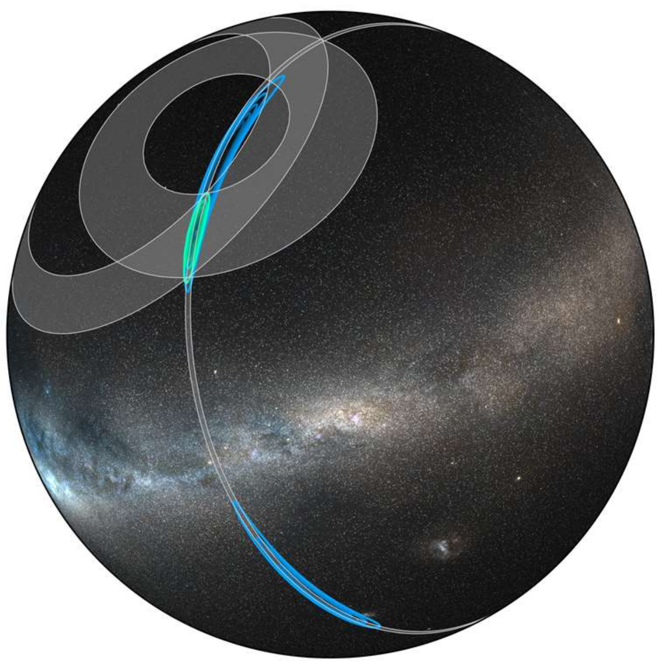
Combining data from GW detectors

- Interferometric detectors are nearly omni-directional $\Rightarrow$ they cannot be pointed at a specific location
- Sky location estimation comes from combining signal information from multiple detectors → Network analysis
- Inversion of network data for reconstructing source direction and polarization waveforms is not a trivial problem because of *ill-posedness*
  - Important issue for any approach to GW detection
  - Required *Regularized Maximum Likelihood* analysis
- GW150914: First detected (LVC, PRD, 122004, 2016) by a regularized Network analysis algorithm introduced in
  - Klimenko, Mohanty, Rakhmanov, Mitselmakher, PRD, 2005
  - LIGO analysis pipeline implementing this algorithm: Coherent WaveBurst (CWb)

Gravitational wave detector network

K. Hayama

GW170817: a 3-detector event



Credit: LIGO/Virgo/NASA/Leo Singer (Milky Way image: Axel Mellinger)

Blue: Localization by LHO and LLO
Green: Localization by LHO-LLO-Virgo

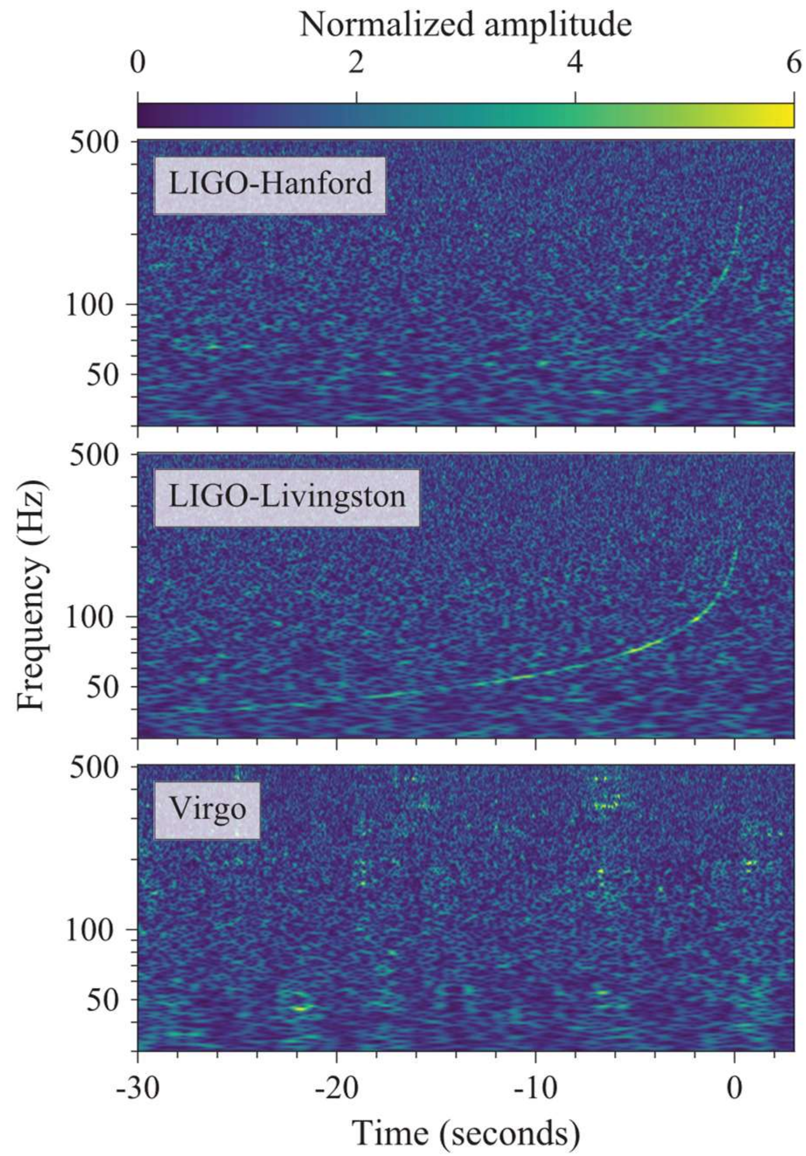
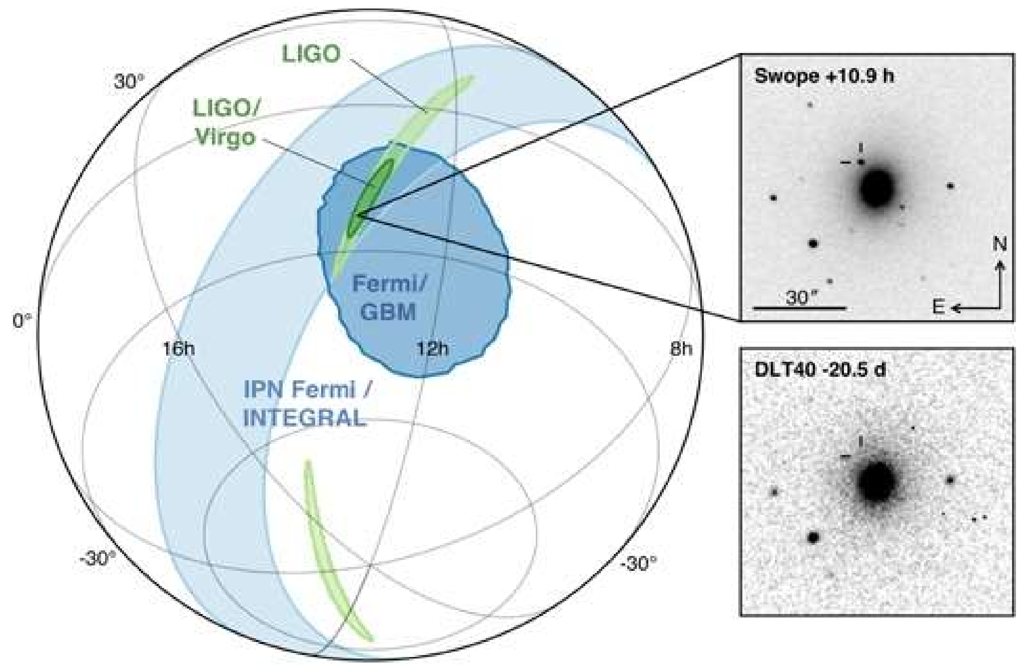


Figure 1 from Multi-messenger Observations of a Binary Neutron Star Merger
B. P. Abbott et al. 2017 ApJL 848 L12 doi:10.3847/2041-8213/aa91c9



Information:

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- Remove components (including this text box!)
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- Apply a style or theme
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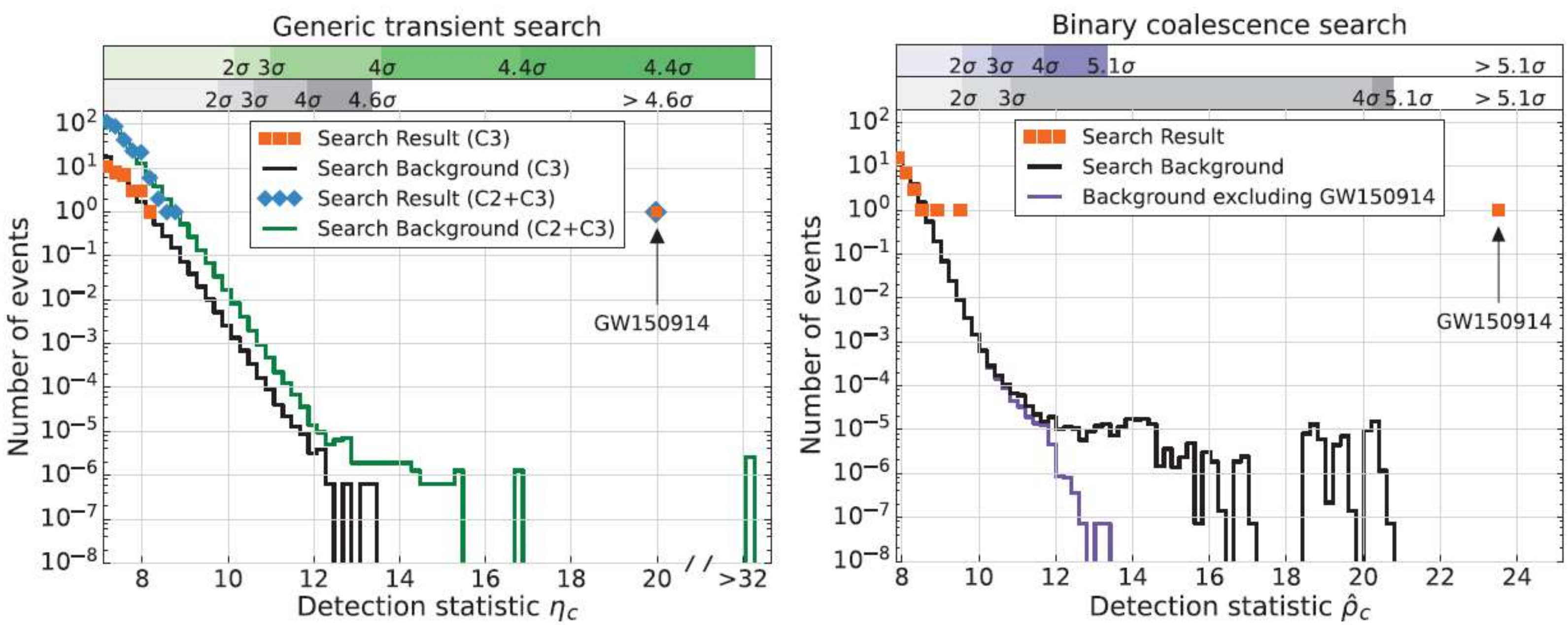
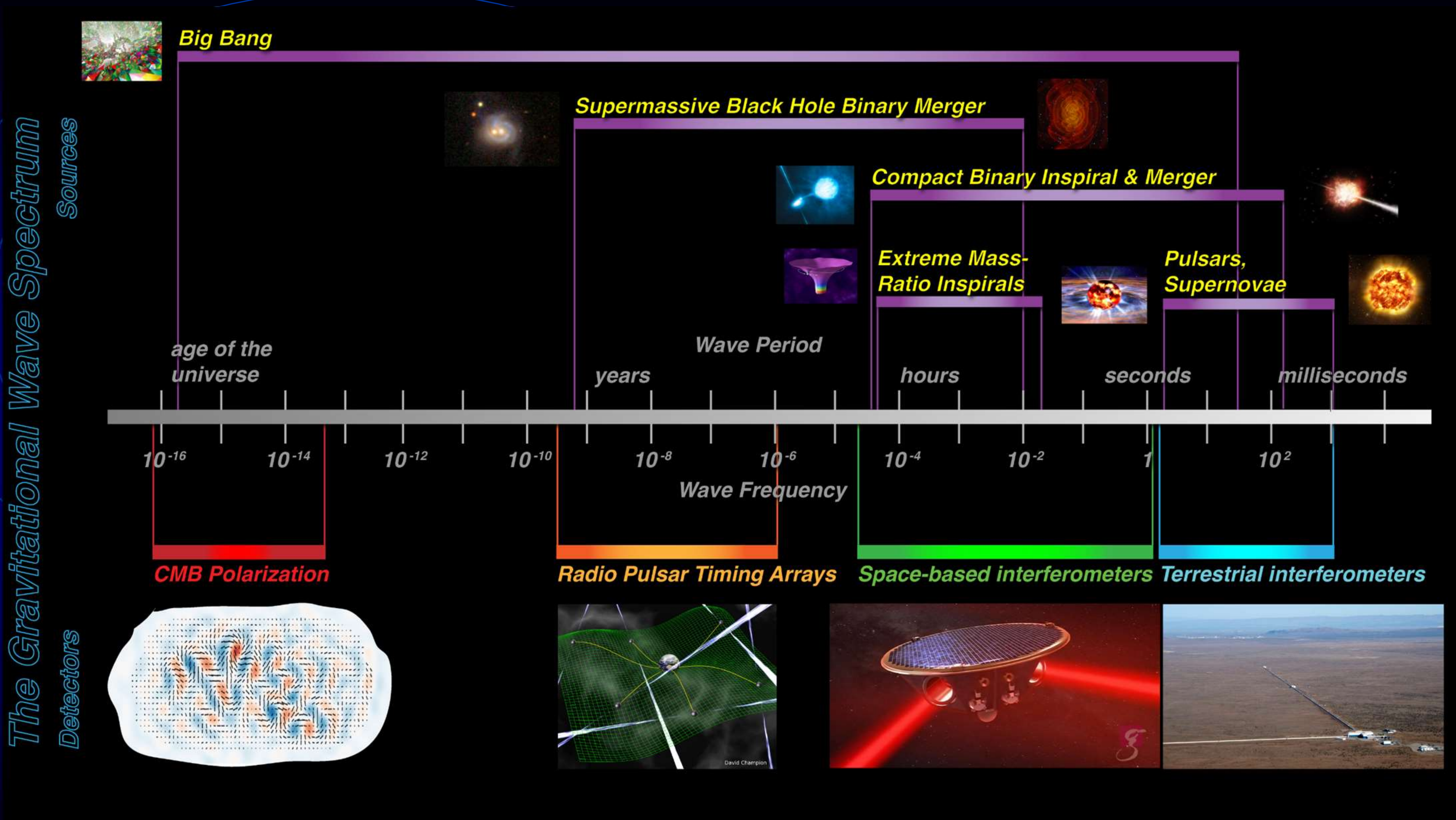
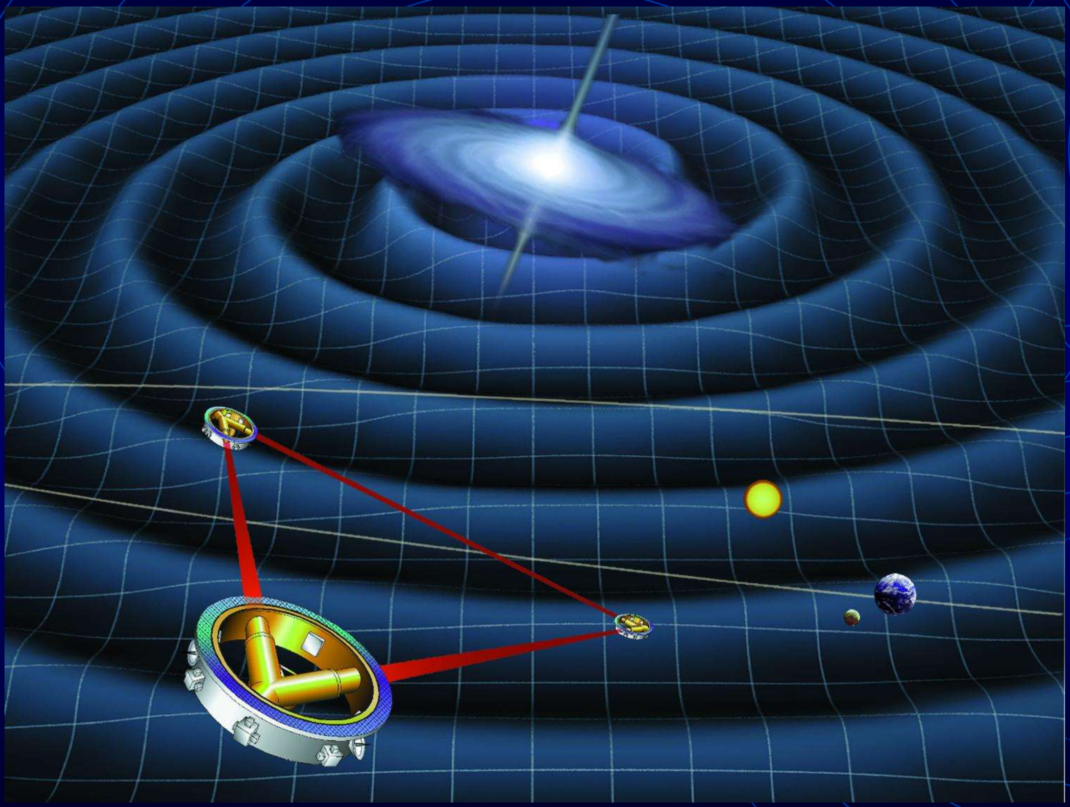
Combining data from GW detectors

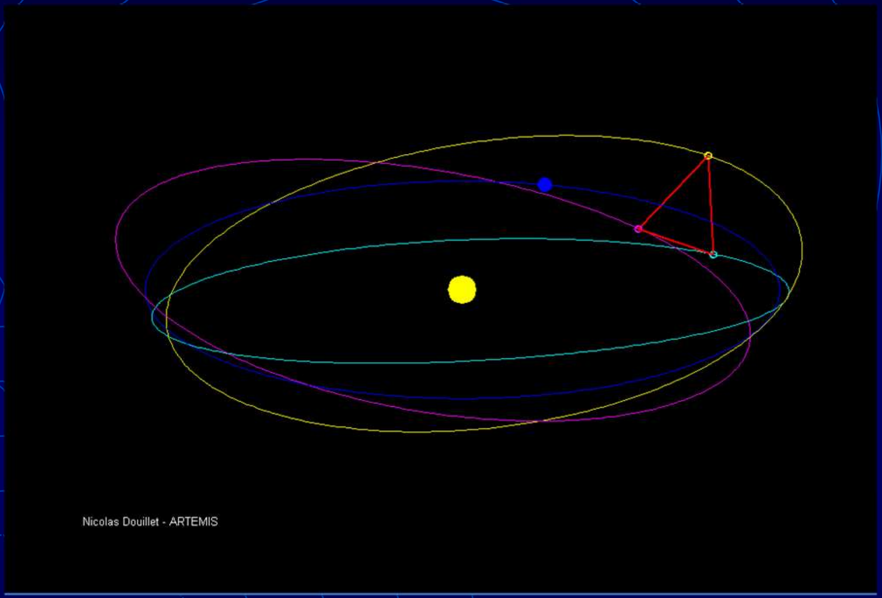
- LIGO data consists of noise + instrumental artifacts (lines, glitches, non-stationarity)
- Another important reason to have multiple detectors is to reject terrestrial signals




The Gravitational Wave Spectrum
Sources
Detectors
Big Bang
Supermassive Black Hole Binary Merger
Compact Binary Inspiral & Merger
Extreme Mass-Ratio Inspirals
Pulsars, Supernovae
Wave Period
age of the universe
years
hours
seconds
milliseconds
10^-16
10^-14
10^-12
10^-10
10^-8
10^-6
10^-4
10^-2
1
10^2
Wave Frequency
CMB Polarization
Radio Pulsar Timing Arrays
Space-based interferometers
Terrestrial interferometers

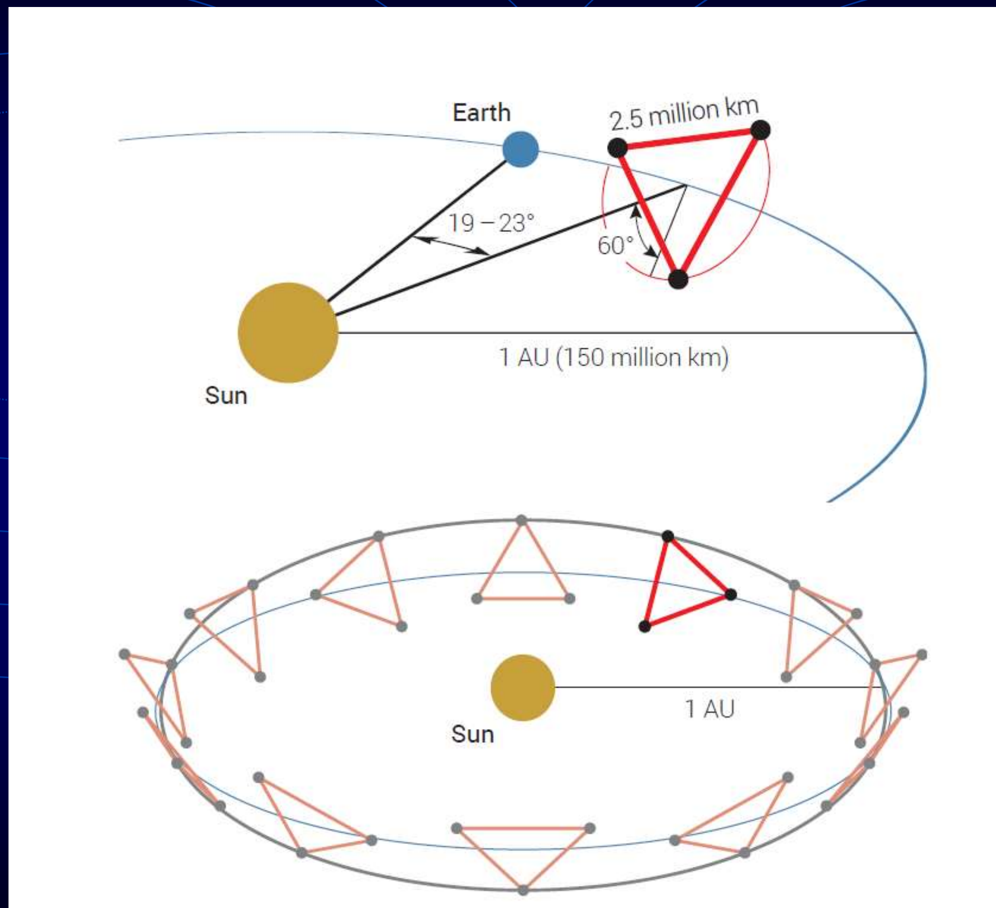
LISA



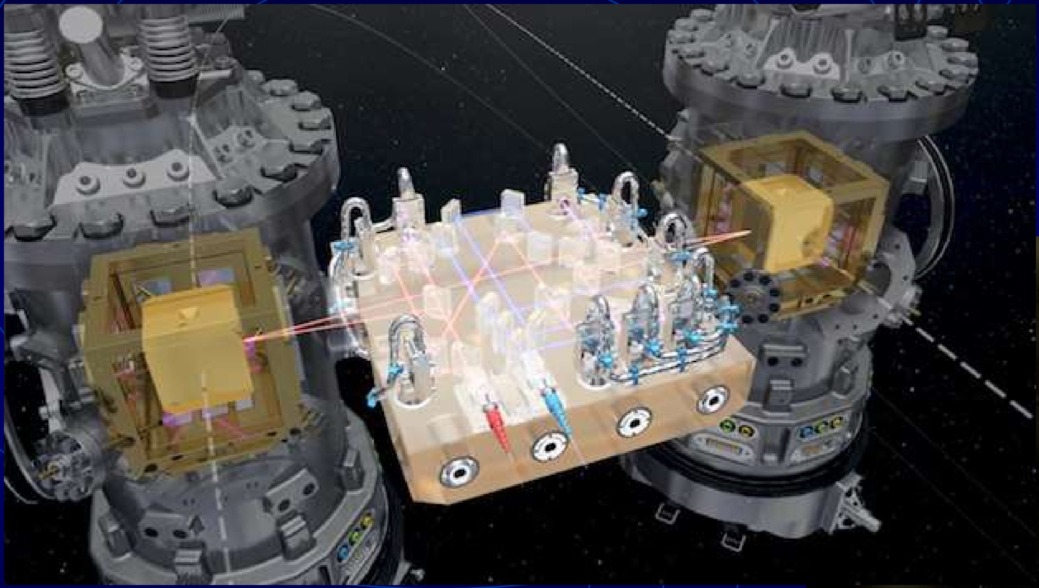


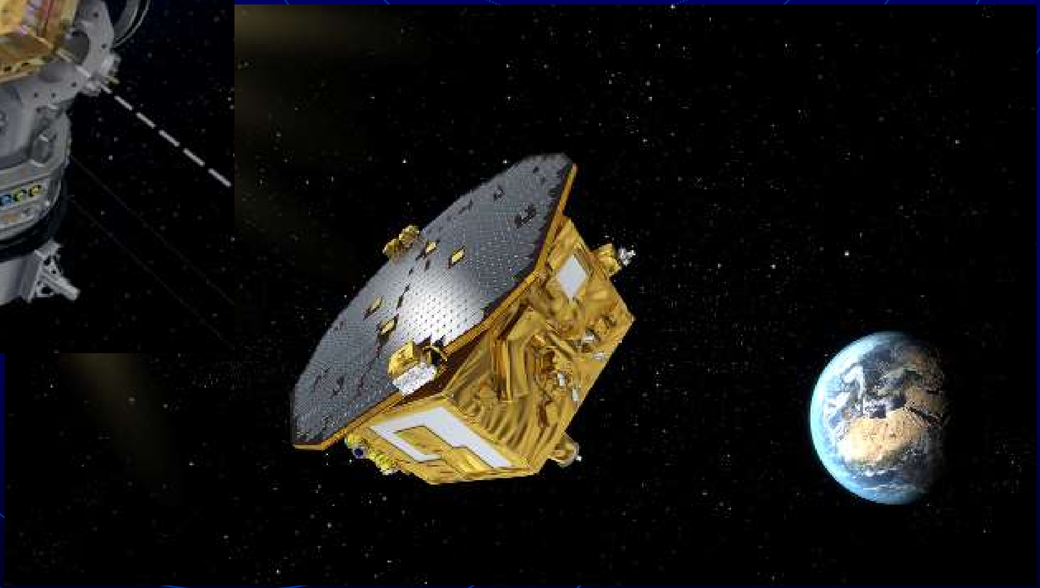
- Interferometer with ~ 1 million km arms
- Satellites exchanging laser beams while following Keplerian orbits
- Monitoring the position of freely falling proof masses

(Evolved) LISA

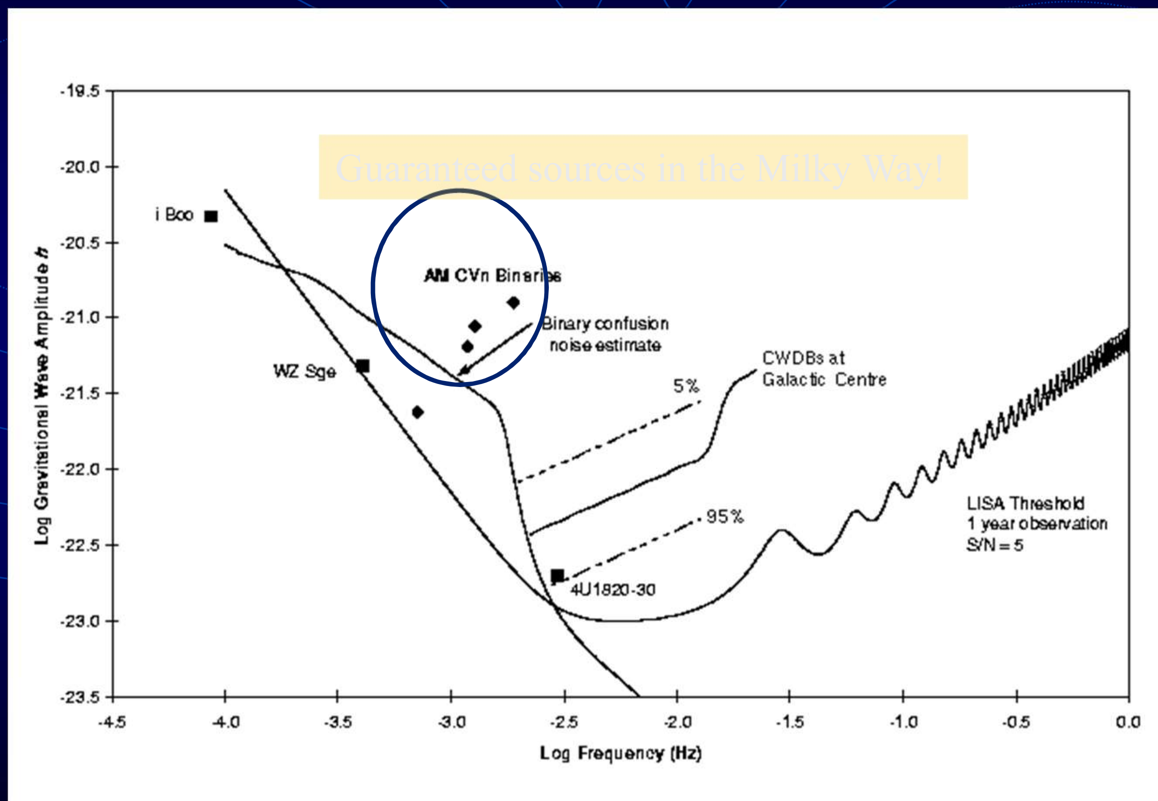


LISA Pathfinder Mission





LISA sensitivity



Detector response: Moving detector

- Detector located at $\bar{x}_d$ in the plane GW wave field
- Response: $s(t) = W_{ij}D^{ij} = F_+(\hat{n})h_+\left(\omega t\ -\ \bar{k}.\bar{x}_d\right) + F_\times(\hat{n})h_\times(\omega t\ -\ \bar{k}.\bar{x}_d)$
- $\hat{n}$: Direction to GW source
- $\bar{k} = -2\pi\hat{n}/\lambda;\qquad \omega = 2\pi f;\qquad f\lambda = c$
- For a moving detector, $\bar{x}_d$ is a function of time
- Example: constant velocity motion (non-relativistic case)
  - $\sin\left(\omega t\ -\ \bar{k}.(\bar{v}t)\right) \rightarrow \sin(2\pi f\left(1 + \frac{\bar{v}.\hat{n}}{c}\right)t)$: Doppler shift of frequency
  - Detector moving towards source will see a higher frequency (Blue shift)
- Further, changing detector orientation also makes $F_{+,\times}$ time dependent
  - Example: $\mathrm{F}_+(t)\sin\omega t + F_\times(t)\cos\omega t\ =\ \sqrt{F_+^2(t) + F_\times^2(t)}\sin(\omega t + \phi(t))$
  - $\phi(t) = \tan^{-1}\frac{F_\times(t)}{F_+(t)}$: Polarization phase

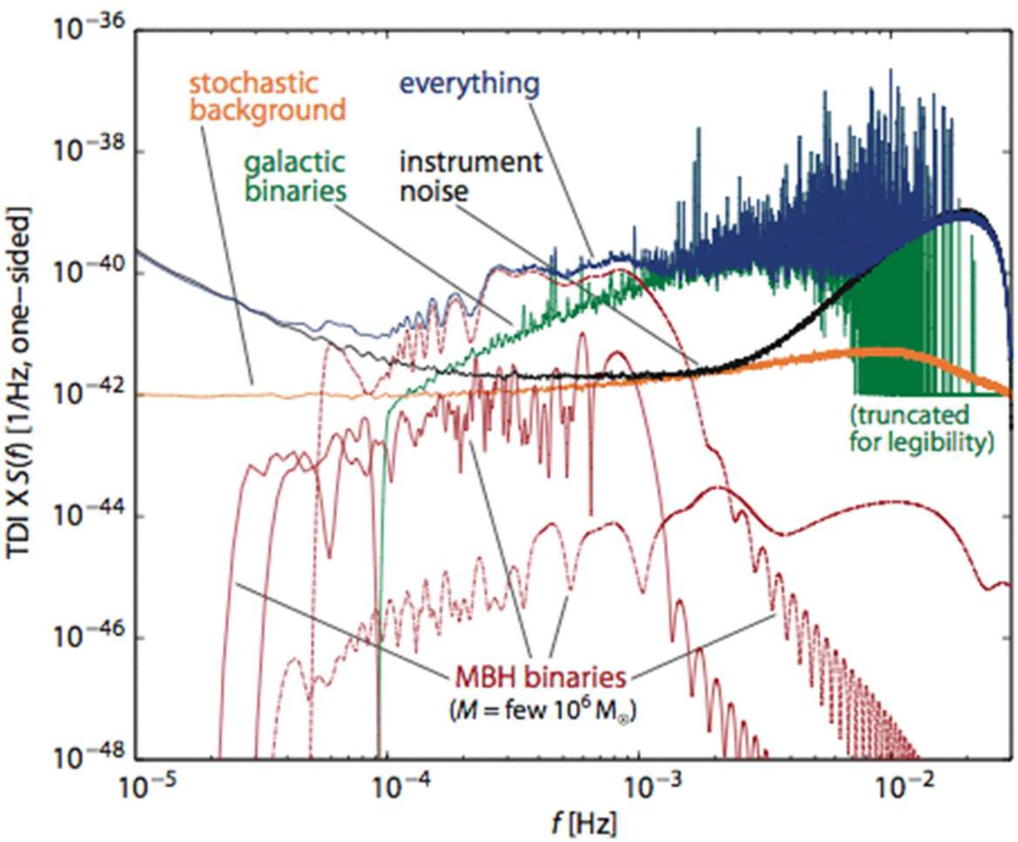
Doppler modulation and polarization phase

- Uniform velocity of detector is not important (relativity)
- Non-uniform motion encodes extra information about the source direction
- Doppler modulation allows extremely high source localization accuracy
- Polarization phase changes much more slowly due to the broad directional response of interferometric detectors
- Distinguishing feature for most LISA sources
  - Most LISA sources are long lived
  - Significant detector motion while the GW signal is active
- In LIGO, only important for Continuous Wave sources
  - Deformed Neutron stars
  - Detector motion is not important for signals lasting ≈ 1 hour

Mock LISA data challenge (MLDC)

- “Mock”: Synthetic noise and signals
- https://astrogravs.nasa.gov/docs/mldc/
- Several stages: 1 to 4 in the original MLDCs
  - From simple (few GW sources) to full LISA data (many GW sources)
  - Non-blind and blind challenges: source catalogue made available or not along with mock data
  - Non-blind challenge is for testing and developing data analysis algorithms
  - Blind challenge is for comparing the performance of different algorithms

Stage 4 MLDC



"…Round 4 training dataset (not the "official" released training dataset). [more]This particular realization includes 60+ million chirping Galactic binaries, 4 MBH binaries, 9 EMRIs, 15 cosmic-string bursts, an isotropic stochastic background, and of course instrument noise. "