Preliminary evaluation of MLDC SMBH challenge: Round 1

S. Babak, E. Porter, M. Vallisneri

I. CHALLENGE EVALUATION

Three collaborations have submitted results for the challenge 1.2.1. Two submissions contain estimation of all nine parameters and the third one contains estimation of only chirp mass and time of coalescence. We have only one entry for challenge 1.2.2.

In order to evaluate the results we have computed several quantities. The noise weighted inner products were computed using X-stream (used in MLDC) and two orthogonal streams with equal and uncorrelated noise:

$$A = (2X - Y - Z)/3; \quad E = (Z - Y)/\sqrt{3} \tag{1}$$

and approximate expression for the noise

$$S = 2(S_X - S_{XY})/3 \tag{2}$$

where for the frequency response used in synthetic LISA:

$$S_X = 16\sin^2(2\pi f L)(2(1+\cos^2(2\pi f L))S_{pm} + S_{op}) \tag{3}$$

$$S_{XY} = -4\sin(4\pi f L)\sin(2\pi f L)(S_{op} + 4S_{pm}) \tag{4}$$

$$S_{pm} = 2.5\times 10^{-48}\left(1 + \left(\frac{f}{10^{-4}Hz}\right)^{-2}\right)\left(\frac{f}{1Hz}\right)^{-2}, \quad S_{op} = 1.8\times 10^{-37}\left(\frac{f}{1Hz}\right)^{2} \tag{5}$$

We have computed the following quantities. The ξ^2 per degree of freedom

$$\chi^2 = \frac{(A_{data} - A_{rec}|A_{data} - A_{rec}) + (E_{data} - E_{rec}|E_{data} - E_{rec})}{N - D} \tag{6}$$

and another (similar) quantity

$$\xi = \frac{\sqrt{(A_{data} - A_{rec}|A_{data} - A_{rec})^2 + (E_{data} - E_{rec}|E_{data} - E_{rec})^2}}{N - D} \tag{7}$$

where N is a number of points and $D = 9$ is number of parameters.

The second value is recovered combined SNR:

$$SNR = \sqrt{SNR_A^2 + SNR_E^2}, \tag{8}$$

$$SNR_A = \frac{(A_{data}|A_{rec})}{\sqrt{(A_{rec}|A_{rec})}}, \quad SNR_E = \frac{(E_{data}|E_{rec})}{\sqrt{(E_{rec}|E_{rec})}} \tag{9}$$

Together with this we also compare noiseless injected signal with the recovered one. Here we compute several overlaps:

$$O_A = \frac{(A_{key}|A_{rec})}{\sqrt{(A_{rec}|A_{rec})(A_{key}|A_{key})}}, \quad O_E = \frac{(E_{key}|E_{rec})}{\sqrt{(E_{rec}|E_{rec})(E_{key}|E_{key})}} \tag{10}$$

and overlap of difference between X channels:

$$O_{dX} = \frac{(X_{rec} - X_{key}|X_{rec} - X_{key})}{\sqrt{(X_{rec}|X_{rec})(X_{key}|X_{key})}} \tag{11}$$

Overlaps show how well we track the phase neglecting error in the amplitude.

In order to take into account the possible error in the initial phase we have computed overlaps maximized over the phase:

TABLE I: Inner products for challenge 1.2.1

Entry	χ^2	ξ	SNR	O_A	O_E	O_{dX}	$min_{\phi_0}(O_{dX})$	$max_{\phi_0}(O_X)$
Key	0.5824	0.41182	667.734	–	–	–	–	–
Montana/AEI	0.67193	0.47606	528.023	0.79153	0.79051	0.41694	0.000128	0.99994
JPL	0.58451	0.41331	664.471	0.9944	0.9958	0.0112	0.00909	0.9955
Montana/AEI*	0.58331	0.412466	666.324	0.998	0.998	0.0112	0.000128	0.99994

TABLE II: Errors in estimation of parameters for challenge 1.2.1. Values in brackets corresponds to the opposite direction on the sky.

Entry	ΔM_c	$\Delta\mu$	ΔDl	$\Delta\theta$	$\Delta\phi$	Δinc	$\Delta\psi$	Δt_c	$\Delta\phi_c$
JPL	37.3	36.8	-63.2	566.308 (23.75)	-1493.76 (-15.08)	165.719	-271.752	-8.1	0.074
Montana/AEI	-5.0	-3.5	2.4	0.63	0.63	2.1	322.0	-0.62	0.076
Goddard	-2203	–	–	–	–	–	–	–	-1433

$$max_{\phi_0}(O_X) = \sqrt{(X_{rec}|X_{key}(\phi_0=0))^2 + (X_{rec}|X_{key}(\phi_0=\pi/2))^2} \tag{12}$$

$$min_{\phi_0}(O_{dX}) = \frac{(X_{rec}|X_{rec}) + (X_{key}|X_{key}) - 2max_{\phi_0}(X_{rec}|X_{key})}{\sqrt{(X_{rec}|X_{rec})(X_{key}|X_{key})}} \tag{13}$$

We have also computed errors in the parameter estimations in units of sigma. Sigma was taken from the diagonal elements of (inverse of) Fisher matrix. Numerical values of Fisher matrix were checked against LISA calculator.

Finally we plot noiseless data and provide visual comparison of the recovered signals against the key.

II. CHALLENGE1.2.1

We denote three entries as Montana/AEI, JPL, Goddard. Results of computing various inner products are summarized in the Table I. Montana/AEI entry have constant phase difference (which is clear from values of maximized overlaps), we have added one more entry (Montana/AEI*) with corrected initial phase.

The variance-covariance matrix provides us with the following values for sigma

$$M_c = 1.208590 \times 10^6, \quad \sigma = 23.78306 \tag{14}$$
$$\mu = 5.811961 \times 10^5, \quad \sigma = 173.3452 \tag{15}$$
$$Dl = 8.000000, \quad \sigma = 0.1394930 \tag{16}$$
$$colat = -0.492289, \quad \sigma = 0.0018149 \tag{17}$$
$$long = 0.865777, \quad \sigma = 0.00212455 \tag{18}$$
$$inc = 1.94439, \quad \sigma = 0.00801087 \tag{19}$$
$$\psi = 3.23422, \quad \sigma = 0.0097565 \tag{20}$$
$$t_c = 1.337403 \times 10^7, \quad \sigma = 5.525738 \tag{21}$$
$$\phi_c = 4.364670, \quad \sigma = 0.058297 \tag{22}$$

Ed, I have noticed that colatitude here is actually latitude!

The errors in the parameter estimation (in units of sigma) are presented in the Table II

Finally in the Figure 1 we compare beginning and the end of the recovered signals as compared to key.

JPL signal provides a very good fit to the key in the LISA's most sensitive frequency band, but not across the whole bandwidth.

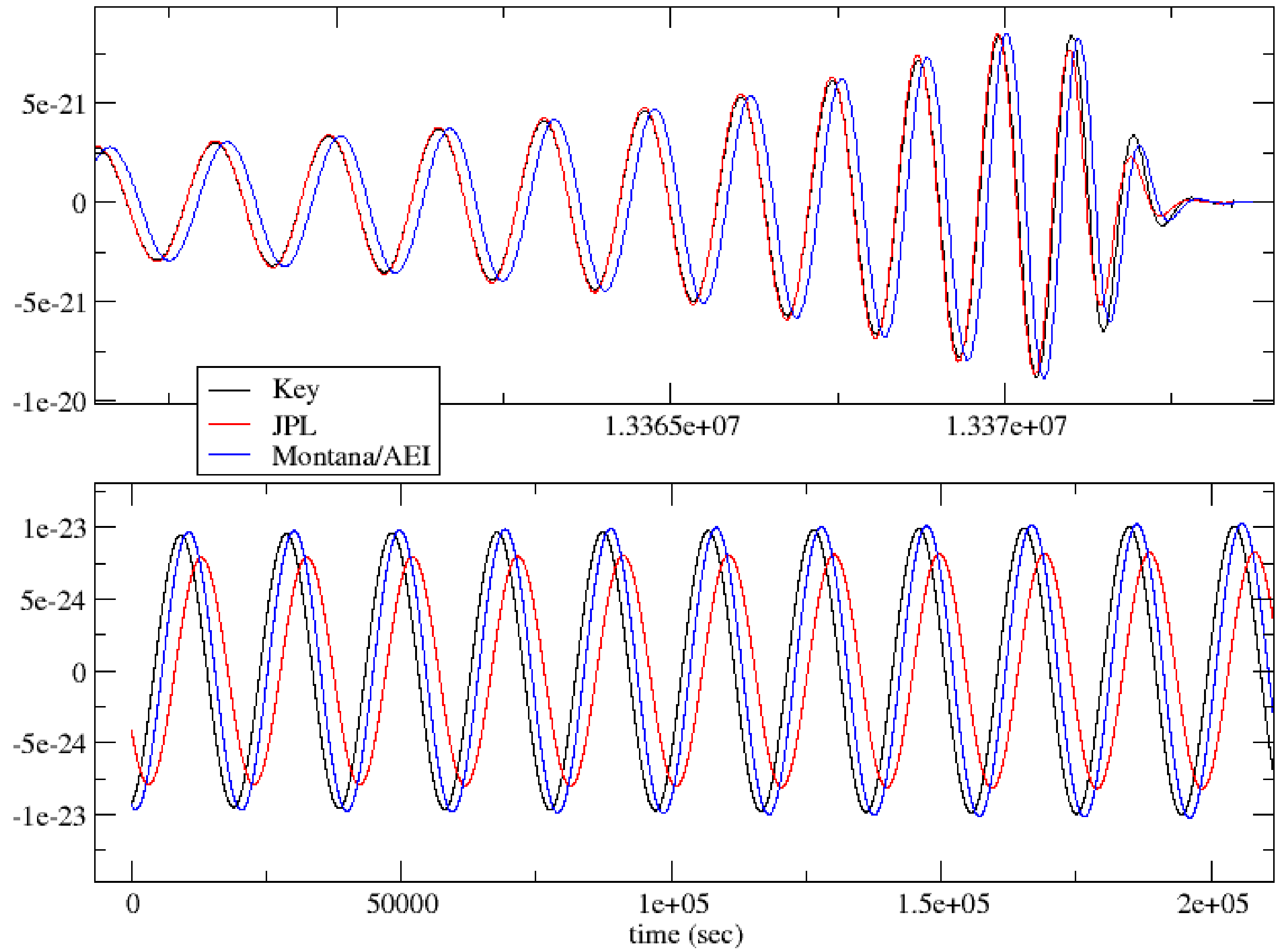

FIG. 1: Comparison of the X-response to the signal from inspiralling SMBH.

TABLE III: Inner products for challenge 1.2.2

Entry	χ^2	ξ	SNR	O_A	O_E	O_{dX}	$min_{\phi_0}(O_{dX})$	$max_{\phi_0}(O_X)$
Key	0.58063	0.410566	106.77	–	–	–	–	–
Montana/AEI	0.58064	0.41058	106.60	0.9984	0.9987	0.0033	0.0029	0.9985

III. CHALLENGE1.2.2

There is only one entry for this challenge. Here we present only inner product results, which are summarized in the Table III

And visual comparison (again beginning and the end) is presented in the figure 2.

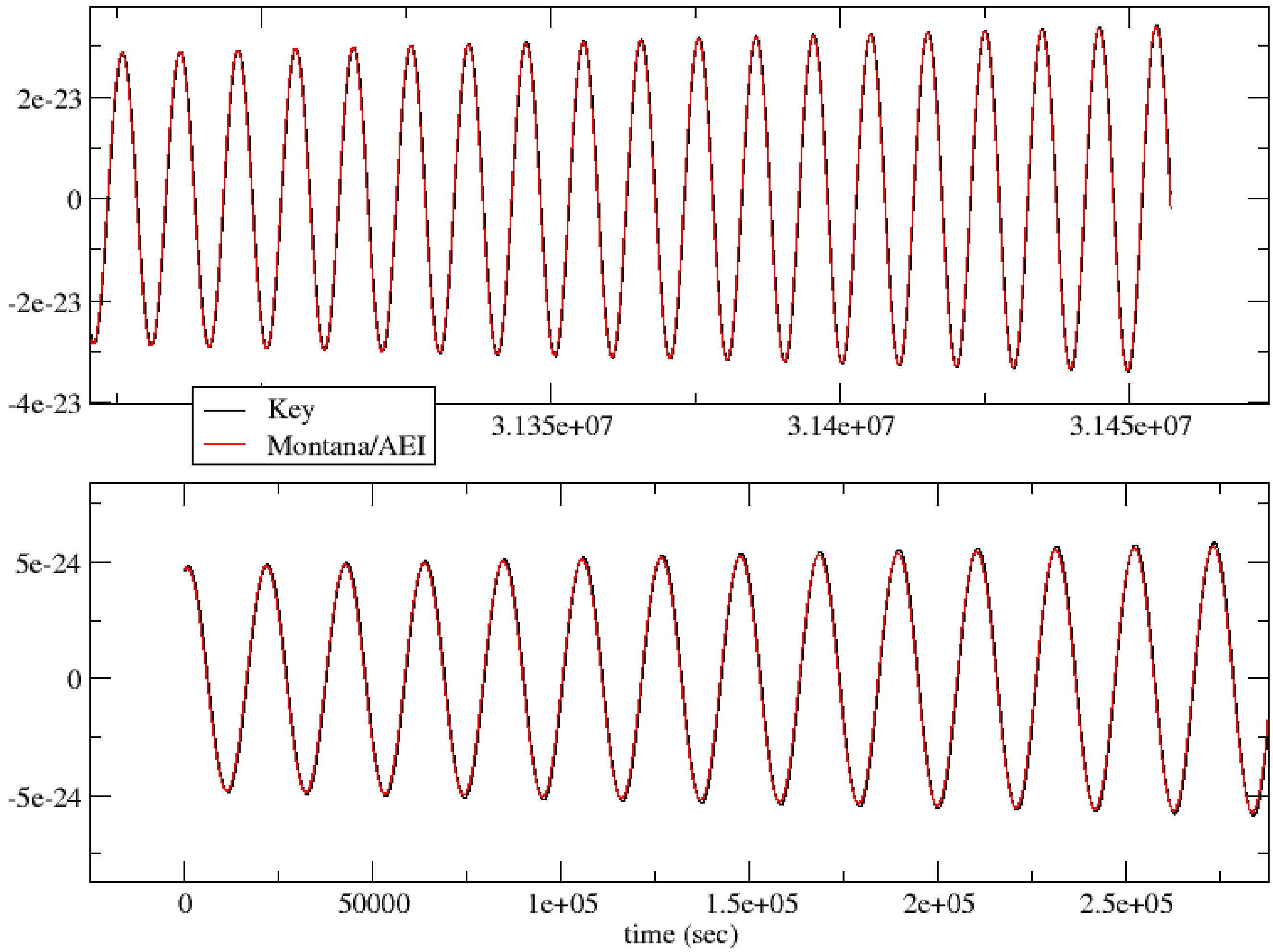

FIG. 2: Comparison of the X-response to the signal from inspiralling SMBH.