Waveform Description for Cosmic String Cusps

Cusps are regions on long strings or loops that achieve huge Lorentz boosts γ. These boosts, combined with the large mass per unit length of cosmic strings, lead to the production of a gravitational radiation. The tip of the cusp moves at the speed of light, and a small region of string around the cusp has an enormous Lorentz boost. An analytical treatment of the gravitational wave bursts produced by cusps has been provided by Damour and Vilenkin [1]. In the limit where the tip of the cusp is moving directly toward the observer the resulting metric perturbation is a linearly polarized gravitation wave with

$$h(t) = A|t - t_\star|^{1/3}\,, \tag{1}$$

where $t_\star$ is the central time of arrival of the burst and the amplitude A is given by [2]

$$A \sim \frac{G\mu L^{2/3}}{D_L}\,. \tag{2}$$

Here G is Newton's constant, μ is the mass per unit length of the string, D_L is the luminosity distance to the source, and L is the size of the feature that produces the cusp (eg. the length of a cosmic string loop). If the observer's line of sight does not coincide with the direction that the cusp is moving, the waveform becomes much more complicated, with a mixture of $+$ and $\times$ polarizations [3]. If the viewing angle α departs only slightly from zero, the waveform remains dominantly linearly polarized, and the sharp spike in the waveform (1) is rounded off. In the Fourier domain this has the effect of introducing an exponential suppression of power for frequencies above $f_{\rm max}$, where $f_{\rm max}$ is related to the viewing angle α and feature length L by

$$f_{\rm max} = \frac{2}{\alpha^3 L}\,. \tag{3}$$

Following the model used by the LSC, we define our cusp waveforms in the Fourier domain according to

$$h(f) \begin{cases} \mathcal{A}\left(1+\left(\frac{f_{\rm low}}{f}\right)^2\right)^{-4} f^{-4/3} & f \le f_{\rm max} \\ \mathcal{A}\left(1+\left(\frac{f_{\rm low}}{f}\right)^2\right)^{-4} \exp\left(1-\frac{f}{f_{\rm max}}\right) f^{-4/3} & f > f_{\rm max} \end{cases} \tag{4}$$

The amplitude $\mathcal{A}$ has dimensions $\mathrm{Hz}^{1/3}$, and $f_{\rm low}$ sets the low frequency cut-off of what is effectively a fourth order Butterworth filter. This high pass filter prevents dynamic range issues with the inverse Fourier transforms. For the MLDC data sets a reasonable value to use is $f_{\rm low} = 1 \times 10^{-5}$ Hz.

The phase of the waveform is set using

$$\begin{aligned} h_R(f) &= -h(f)\cos(2\pi f t_\star) \\ h_I(f) &= h(f)\sin(2\pi f t_\star)\,. \end{aligned} \tag{5}$$

The waveform is then inverse Fourier transformed to the time domain to produce $h(t)$. The final step needed to produce the Barycenter waveforms used by the simulators is to apply a polarization rotation by angle ψ:

$$\begin{aligned} h_+(t) &= h(t)\cos(2\psi) \\ h_\times(t) &= -h(f)\sin(2\psi)\,. \end{aligned} \tag{6}$$

The orientation of the various LISA arms imparts positional information into the measured waveforms, which adds the ecliptic latitude and longitude to the collection of parameters required to describe the signal from a cosmic string cusp.

To summarize, a cosmic string cusp is described by the six parameters: Amplitude $\mathcal{A}$, Central Time $t_\star$, Maximum Frequency $f_{\rm max}$, Polarization ψ, Ecliptic Latitude and Ecliptic Longitude.

The amplitude should be chosen to give the desired matched filtered SNR. The central time should be chosen uniformly in the observation period. The maximum frequency should be chosen between $f_{\rm min}$ and $f_{\rm Nyquist}$ (Once $f_{\rm max}$ exceeds the Nyquist frequency it can not be determined from the data, so to limit aliasing it is a good idea to cap its maximum value at the Nyquist frequency). The polarization angle is uniform in the range $[0,\pi]$, and the sky location should be uniform in longitude and the sine of the latitude.

References

[1] T. Damour and A. Vilenkin, Phys. Rev. Lett. **85**, 3761 (2000).

[2] X. Siemens, J. Creighton, I. Maor, S. Ray Majumder, K. Cannon, and J. Read, Phys. Rev. D**73** 105001 (2006).

[3] X. Siemens and K. D. Olum, Phys. Rev. D**68**, 085017 (2003).