Preliminary evaluation of MLDC SMBH challenge: Round 1

S. Babak, E. Porter, M. Vallisneri

I. CHALLENGE EVALUATION

Three collaborations have submitted results for the challenge 1.2.1. Two submissions contain estimation of all nine parameters and the third one contains estimation of only chirp mass and time of coalescence. We have only one entry for challenge 1.2.2.

In order to evaluate the results we have computed several quantities. The noise weighted inner products were computed using the X-stream (used in MLDC) and two orthogonal streams with equal and uncorrelated noise:

$$A = (2X - Y - Z)/3; \quad E = (Z - Y)/\sqrt{3} \tag{1}$$

and an approximate expression for the noise

$$S = 2(S_X - S_{XY})/3 \tag{2}$$

where for the frequency response used in synthetic LISA:

$$\begin{aligned} S_X &= 16 \sin^2(2\pi f L)(2(1 + \cos^2(2\pi f L))S_{pm} + S_{op}) & (3) \\ S_{XY} &= -4 \sin(4\pi f L) \sin(2\pi f L)(S_{op} + 4S_{pm}) & (4) \\ S_{pm} &= 2.5 \times 10^{-48} \left(1 + \left(\frac{f}{10^{-4}Hz}\right)^{-2}\right) \left(\frac{f}{1Hz}\right)^{-2}, \quad S_{op} = 1.8 \times 10^{-37} \left(\frac{f}{1Hz}\right)^{2} & (5) \end{aligned}$$

We have computed the following quantities. The χ^2 per degree of freedom

$$\chi^2 = \frac{(A_{data} - A_{rec}|A_{data} - A_{rec}) + (E_{data} - E_{rec}|E_{data} - E_{rec})}{N - D} \tag{6}$$

and another (similar) quantity

$$\xi = \frac{\sqrt{(A_{data} - A_{rec}|A_{data} - A_{rec})^2 + (E_{data} - E_{rec}|E_{data} - E_{rec})^2}}{N - D} \tag{7}$$

where N is a number of data points and $D = 9$ is number of parameters.

The next value is recovered combined SNR:

$$SNR = \sqrt{SNR_A^2 + SNR_E^2}, \tag{8}$$

$$SNR_A = \frac{(A_{data}|A_{rec})}{\sqrt{(A_{rec}|A_{rec})}}, \quad SNR_E = \frac{(E_{data}|E_{rec})}{\sqrt{(E_{rec}|E_{rec})}} \tag{9}$$

Together with this we also compare the noiseless injected signal with the recovered one. Here we compute several overlaps:

$$O_A = \frac{(A_{key}|A_{rec})}{\sqrt{(A_{rec}|A_{rec})(A_{key}|A_{key})}}, \quad O_E = \frac{(E_{key}|E_{rec})}{\sqrt{(E_{rec}|E_{rec})(E_{key}|E_{key})}} \tag{10}$$

and the overlap of the difference between the X channels:

$$O_{dX} = \frac{(X_{rec} - X_{key}|X_{rec} - X_{key})}{\sqrt{(X_{rec}|X_{rec})(X_{key}|X_{key})}} \tag{11}$$

The overlaps show how well we track the phase neglecting the error in the amplitude.

TABLE I: Inner products for challenge 1.2.1

Entry	χ^2	ξ	SNR	O_A	O_E	O_{dX}	$min_{\phi_0}(O_{dX})$	$max_{\phi_0}(O_X)$
Key	0.5824	0.41182	667.734	–	–	–	–	–
Montana/AEI	0.67193	0.47606	528.023	0.79153	0.79051	0.41694	0.000128	0.99994
JPL	0.58451	0.41331	664.471	0.9944	0.9958	0.0112	0.00909	0.9955
Montana/AEI*	0.58331	0.412466	666.324	0.998	0.998	0.0112	0.000128	0.99994

In order to take into account the possible error in the initial phase we have also computed overlaps maximized over the phase:

$$max_{\phi_0}(O_X) = \sqrt{(X_{rec}|X_{key}(\phi_0 = 0))^2 + (X_{rec}|X_{key}(\phi_0 = \pi/2))^2} \tag{12}$$

$$min_{\phi_0}(O_{dX}) = \frac{(X_{rec}|X_{rec}) + (X_{key}|X_{key}) - 2max_{\phi_0}(X_{rec}|X_{key})}{\sqrt{(X_{rec}|X_{rec})(X_{key}|X_{key})}} \tag{13}$$

We have also computed the errors in the parameter estimations in units of σ_F, where σ_F was taken from the square root of the diagonal elements of the covariance matrix (i.e. inverse of the Fisher matrix). These error estimates from the Fisher matrix were also checked against the LISA calculator. Finally we plot the noiseless data and provide a visual comparison of the recovered signals against the key.

II. CHALLENGE 1.2.1

We denote three entries as Montana/AEI, JPL, Goddard. The results of computing various inner products are summarized in the Table I. The Montana/AEI entry has a constant phase difference (which is clear from values of maximized overlaps), we have added one more entry (Montana/AEI*) with corrected initial phase.

The 9-d covariance matrix for the parameter set $\vec{x} = \{\ln(M_c), \ln(\mu), \ln(D_L), \theta_{CL}, \phi, \iota, \psi, \ln(t_c), \varphi_c\}$ evaluated with the key file is given by

$$\begin{pmatrix}
3.87e-10 & 5.49e-09 & -4.83e-08 & -6.51e-09 & 3.87e-09 & -2.8e-08 & -1.95e-08 & 6.92e-12 & 6.84e-07 \\
5.49e-09 & 8.9e-08 & -4.30e-07 & 1.22e-08 & -6.68e-08 & -7.49e-08 & 3.69e-07 & 1.16e-10 & 1.40e-05 \\
-4.83e-08 & -4.30e-07 & 3.04e-04 & 1.68e-05 & 7.63e-06 & 1.28e-04 & -1.85e-05 & -6.57e-12 & -1.71e-05 \\
-6.51e-09 & 1.22e-08 & 1.68e-05 & 3.29e-06 & -2.38e-06 & 1.12e-05 & 1.17e-05 & -1.32e-11 & 4.98e-05 \\
3.87e-09 & -6.68e-08 & 7.63e-06 & -2.38e-06 & 4.51e-06 & -2.71e-06 & -1.82e-05 & 2.63e-11 & -7.24e-05 \\
-2.8e-08 & -7.49e-08 & 1.28e-04 & 1.12e-05 & -2.71e-06 & 6.42e-05 & 2.22e-05 & 7.46e-12 & 1.14e-04 \\
-1.95e-08 & 3.69e-07 & -1.85e-05 & 1.17e-05 & -1.82e-05 & 2.22e-05 & 9.52e-05 & 5.74e-11 & 3.78e-04 \\
6.92e-12 & 1.16e-10 & -6.57e-12 & -1.32e-11 & 2.63e-11 & 7.46e-12 & 5.74e-11 & 1.71e-13 & 1.79e-08 \\
6.83e-07 & 1.40e-05 & -1.71e-05 & 4.98e-05 & -7.24e-05 & 1.14e-04 & 3.78e-04 & 1.79e-08 & 3.4e-03
\end{pmatrix} \tag{14}$$

The variance-covariance matrix provides us with the following values for a 1-σ error in the parameter estimation

$$\begin{aligned}
M_c &= 1.208590 \times 10^6, \quad \sigma = 23.78306 && (15)\\
\mu &= 5.811961 \times 10^5, \quad \sigma = 173.3452 && (16)\\
D_L &= 8.000000, \quad \sigma = 0.1394930 && (17)\\
\theta_{CL} &= 2.063085, \quad \sigma = 0.0018149 && (18)\\
\phi &= 0.865777, \quad \sigma = 0.00212455 && (19)\\
\iota &= 1.94439, \quad \sigma = 0.00801087 && (20)\\
\psi &= 3.23422, \quad \sigma = 0.0097565 && (21)\\
t_c &= 1.337403 \times 10^7, \quad \sigma = 5.525738 && (22)\\
\varphi_c &= 4.364670, \quad \sigma = 0.058297 && (23)
\end{aligned}$$

TABLE II: Errors in estimation of parameters for challenge 1.2.1. Values in brackets corresponds to the opposite direction on the sky.

Entry	ΔM_c	$\Delta\mu$	ΔD_L	$\Delta\theta$	$\Delta\phi$	$\Delta\iota$	$\Delta\psi$	Δt_c	$\Delta\varphi_c$
JPL	37.3	36.8	-63.2	566.308 (23.75)	-1493.76 (-15.08)	165.719	-271.752	-8.1	0.074
Montana/AEI	-5.0	-3.5	2.4	0.63	0.63	2.1	322.0	-0.62	0.076
Goddard	-2203	–	–	–	–	–	–	–	-1433

Note that in the above list, the latitude has been converted to co-latitude according to $\theta_{CL} = \pi/2 - \theta_L$, luminosity distance and its error are quoted in Gpc, all angles are in radians, t_c is quoted in seconds and we transform from initial orbital phase to gravitational wave phase at coalescence according to $\varphi_c = 2\,(\Phi_0 - \phi(\omega(t=0)))$, where $\omega(t=0)$ is the orbital frequency evaluated at $t=0$.

The errors in the parameter estimation (in units of sigma) are presented in the Table II

Finally in Figure 1 we compare the beginning and the end of the recovered signals as compared to the signals generated using the key file.

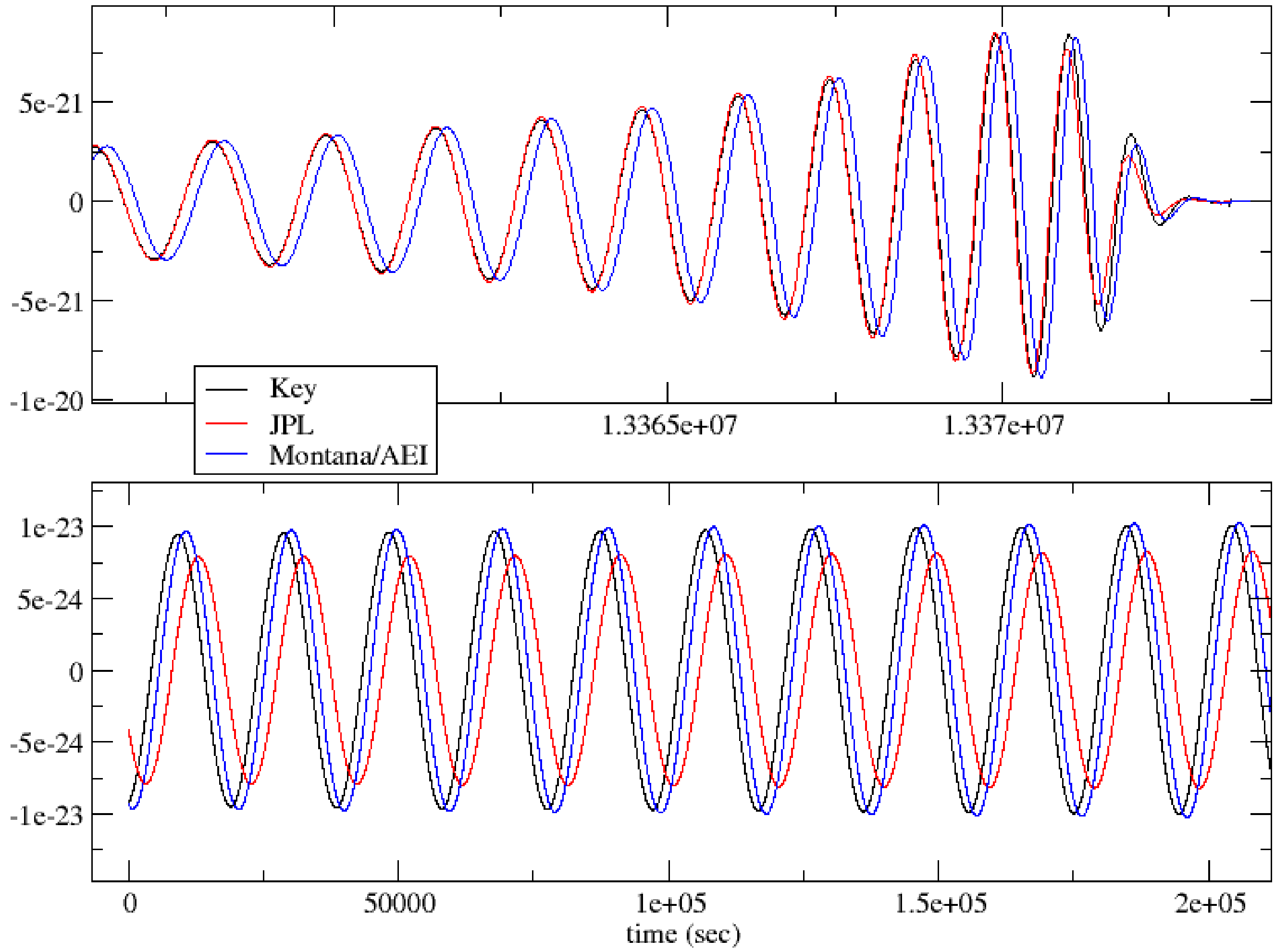

FIG. 1: Comparison of the X-response to the signal from inspiralling SMBH.

The JPL signal provides a very good fit to the key in the LISA's most sensitive frequency band, but not across the whole bandwidth. We can see however from Table II that even though there is a large error in the estimation of

TABLE III: Inner products for challenge 1.2.2

Entry	χ^2	ξ	SNR	O_A	O_E	O_{dX}	$min_{\phi_0}(O_{dX})$	$max_{\phi_0}(O_X)$
Key	0.58063	0.410566	106.77	–	–	–	–	–
Montana/AEI	0.58064	0.41058	106.60	0.9984	0.9987	0.0033	0.0029	0.9985

TABLE IV: Errors in estimation of parameters for challenge 1.2.2.

Entry	ΔM_c	$\Delta\mu$	ΔD_L	$\Delta\theta$	$\Delta\phi$	Δt_c
σ_F	97.78	5,273	8.682	0.00507	0.00527	2,827
Montana/AEI	-2.35	-2.66	0.125	3.728	1.025	-2.33

parameters for the JPL entry, the combination of these parameters results in a waveform that matches the key-file waveform extremely well at end of the waveform where most of the SNR is accumulated. The Montana/AEI waveform matches the amplitude and phase of the key-file signal extremely well throughout the entire bandwidth (except for the constant phase offset which has since been attributed to a missing sign allocation in the code). When we maximize over phase we can see that the Montana/AEI overlaps truely reflect the close parameter estimation.

III. CHALLENGE 1.2.2

There is only one entry for this challenge. Here we again present the inner product and parameter estimation results, which are summarized in Table III and Table IV. Again note that the errors for luminosity distance are given in Gpc, the error for t_c is given in seconds and the angular errors are in radians.

The visual comparison (again beginning and end of the waveform) is presented in the figure 2. We can see in this case the phase and amplitude from the recovered signal match the key-file signal almost perfectly.

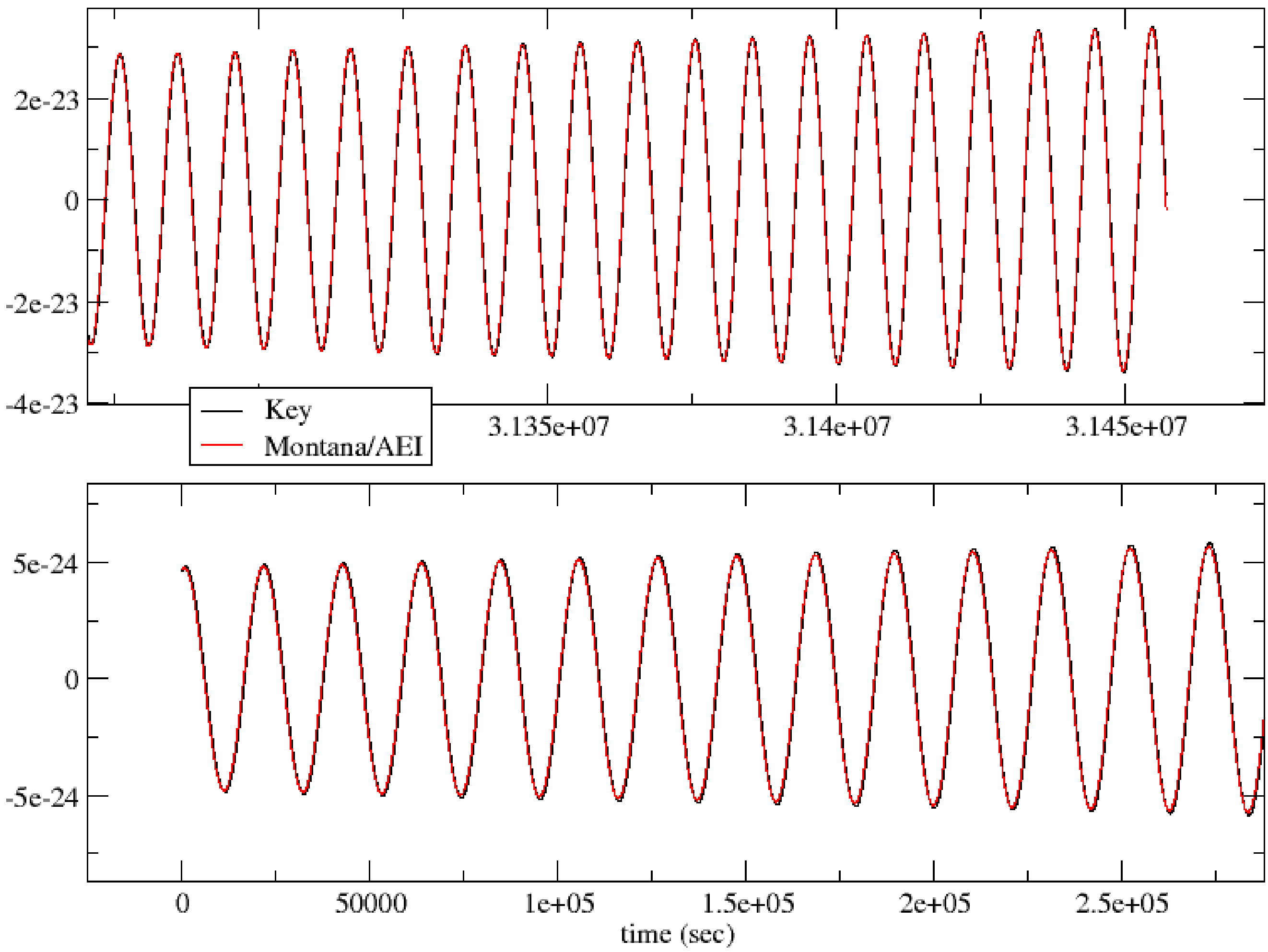

FIG. 2: Comparison of the X-response to the signal from inspiralling SMBH.