Deep Learning Example :

1. **[Detector]** The signal coming from a radar detector follows the Gaussian distribution with the mean 4 and the variance 1 when there is a plane in the air, and with the mean 0 and the variance 1 when there is no plain in the air. The Gaussian distribution is given by

$$\boldsymbol{f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2\sigma^2}(x-\mu)^2}}$$

where μ is the mean and σ^2 is the variance. The decision logic of the detector is

$$\boldsymbol{\phi(x)=\begin{cases}1, & x\geq 1\\ 0, & x<1\end{cases}}$$

A. Write a function "detect" that takes the input x and returns the detector output $\phi(x)$.

B. We generate detector signal x according to the Gaussian models. First generate 2048×1 vector x_1 (2048 samples of one dimensional data) that follows the distribution when a plane is in the air. Second, generate 2048×1 vector x_0 that follows the distribution when no plane is in the air.

C. Since we are generating the detector signal data, we can even label the data. We label the data as 1 when there is a plane in the air, and 0 when there is no plane in the air. Hence, the label y_1 is a 2048×1 vector with 1's as its elements, and label y_2 is a 2048×1 vector with 0's as its elements. Generate the label vectors.

D. Histogram is a method to estimate the probability density function of a data. Plot the histogram of x_1, x_2. (Note that the spreads of the data are about the same, but the centers of the data are at different locations. The spread is related to the variance, and the center is related to the mean.)

E. Generate a test signal x and a label y by selecting 2048 elements randomly from x_1, x_2 for the signal x, and by selecting corresponding elements from y_1, y_2. Now we have a labeled test signal. Plot the histogram of the test signal x.

F. Find the detector output by inputting the test signal x in step D to the function "detect" in step A. Denote the detector output be $\hat{y}$.

G. Compare the label y in step D and the detector output $\hat{y}$ in F, and count how many times the following happens.

	$\boldsymbol{\hat{y} = 1}$	$\boldsymbol{\hat{y} = 0}$
$\boldsymbol{y = 1}$	**(hit)**	**(miss)**
$\boldsymbol{y = 0}$	**(false alarm)**	**(correct reject)**

The above table is called the confusion matrix.

2. [Linear Regression] In a PHYSICS Lab, data is collected as follows

t (sec)	y (m)
5	12.3314
10	20.5385
15	29.7415
20	40.5997
25	51.7653
30	59.1725

where t is the time in second and y is the location of a car measured from the origin in meter. We want to build a linear model $y = at + b$. With 6 data points, the error between the model output $\hat{y}$ and the measurement y can be written as

$$\boldsymbol{e} = \boldsymbol{y} - \widehat{\boldsymbol{y}} = \begin{bmatrix} 12.3314 \\ 20.5385 \\ 29.7415 \\ 40.5997 \\ 51.7653 \\ 59.1725 \end{bmatrix} - \begin{bmatrix} \mathbf{5} & \mathbf{1} \\ \mathbf{10} & \mathbf{1} \\ \mathbf{15} & \mathbf{1} \\ \mathbf{20} & \mathbf{1} \\ \mathbf{25} & \mathbf{1} \\ \mathbf{30} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \end{bmatrix}$$

A. Collect the data t and prepare a $\mathbf{6 \times 2}$ matrix

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{5} & \mathbf{1} \\ \mathbf{10} & \mathbf{1} \\ \mathbf{15} & \mathbf{1} \\ \mathbf{20} & \mathbf{1} \\ \mathbf{25} & \mathbf{1} \\ \mathbf{30} & \mathbf{1} \end{bmatrix}$$

B. Collect the measurement y and prepare a $\mathbf{6 \times 1}$ vector

$$\boldsymbol{y} = \begin{bmatrix} 12.3314 \\ 20.5385 \\ 29.7415 \\ 40.5997 \\ 51.7653 \\ 59.1725 \end{bmatrix}$$

C. Let $x = \begin{bmatrix} a \\ b \end{bmatrix}$, then the x that minimizes the norm of the error e in step A can be found by

$$x = (A^T A)^{-1} A^T y$$

Find and print the values of x. By the way, what is the size of the matrix

$A^T A$? What is the rank of the matrix $A^T A$? Why were we able to find the inverse of $A^T A$?

D. Using the model, find the location of the car at $t = 17$ and $t = 35$.

E. Plot the data and the model in a t vs y plot.