MA500HW7

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```r
library(ggplot2)

library(car)
```

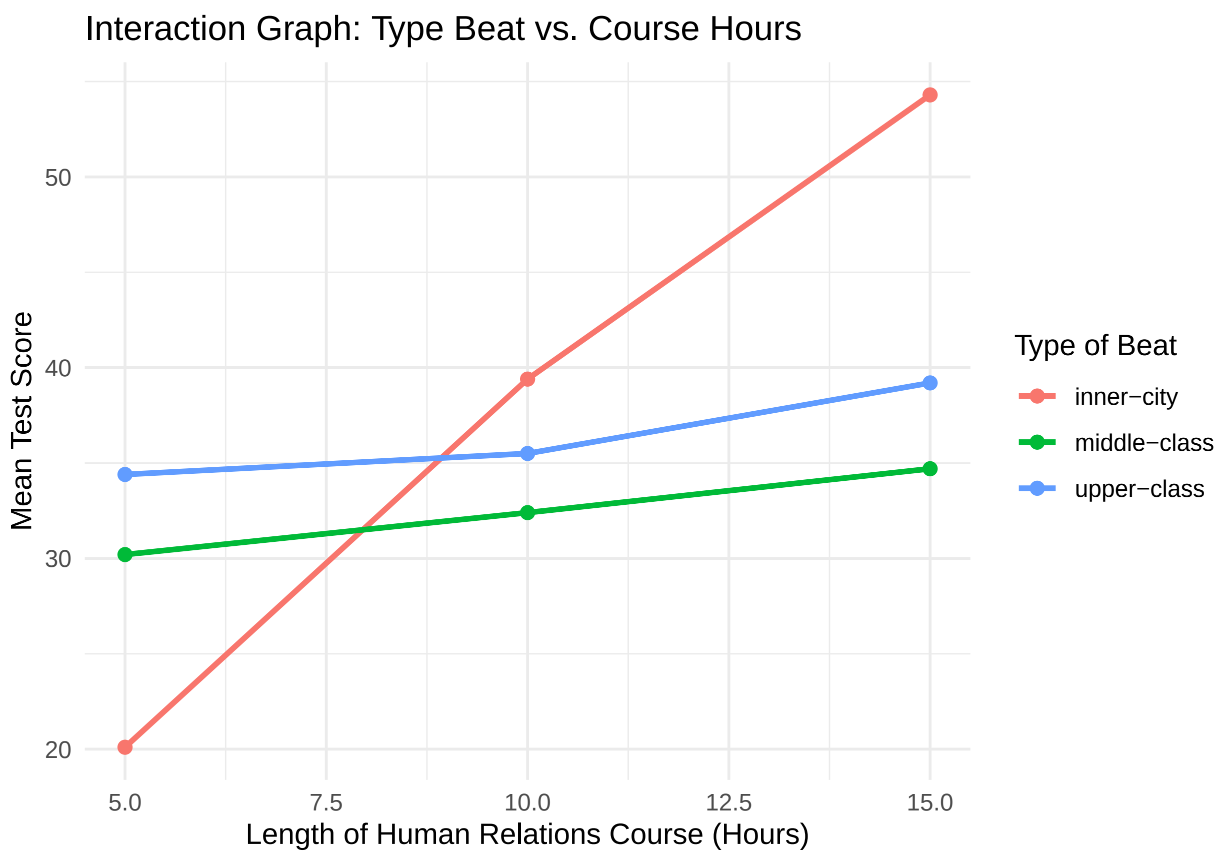
```
## Loading required package: carData
```

Question 1

```r
# Create the data
df <- data.frame(
  Type_Beat = rep(c("upper-class", "middle-class", "inner-city"), each = 3),
  Course_Hours = rep(c(5, 10, 15), times = 3),
  Score = c(34.4, 35.5, 39.2,
            30.2, 32.4, 34.7,
            20.1, 39.4, 54.3)
)

# Plot interaction graph
ggplot(df, aes(x = Course_Hours, y = Score, color = Type_Beat, group = Type_Beat)) +
  geom_line(size = 1) +
  geom_point(size = 2) +
  labs(title = "Interaction Graph: Type Beat vs. Course Hours",
       x = "Length of Human Relations Course (Hours)",
       y = "Mean Test Score",
       color = "Type of Beat") +
  theme_minimal()
```

```
## Warning: Using `size` aesthetic for lines was deprecated in ggplot2 3.4.0.
## i Please use `linewidth` instead.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was
## generated.
```



Question 2

```
# (b)
set.seed(123)

df <- expand.grid(Start_Angle = 1:3, Stop_Angle = 1:3)

# Replicate each combination twice
df <- df[rep(1:nrow(df), each = 2), ]

# Add a random order column and shuffle
df$Trial <- sample(1:nrow(df))

# Order by randomized trial number
df <- df[order(df$Trial), ]

# Show the randomized design
df
```

```
##     Start_Angle Stop_Angle Trial
## 6             3          2     1
## 3             3          1     2
## 2             2          1     3
## 5             2          2     4
```

```
## 4.1           1           2     5
## 3.1           3           1     6
## 9.1           3           3     7
## 9             3           3     8
## 8             2           3     9
## 2.1           2           1    10
## 4             1           2    11
## 7.1           1           3    12
## 5.1           2           2    13
## 1.1           1           1    14
## 1             1           1    15
## 7             1           3    16
## 6.1           3           2    17
## 8.1           2           3    18
```

```
# (e)
set.seed(1022)

# Create the 3x3 factorial design with 2 replicates per cell
design <- expand.grid(Start_Angle = 1:3, Stop_Angle = 1:3, Replicate = 1:2)

# Simulate response variable (distance) with some made-up main effects and noise
design$Distance <- with(design,
                        50 +                                         # base distance
                        5 * Start_Angle +                            # effect of start angle
                        3 * Stop_Angle +                             # effect of stop angle
                        2 * Start_Angle * Stop_Angle +               # interaction effect
                        rnorm(nrow(design), mean = 0, sd = sqrt(12))  # random error
)

head(design)
```

```
##   Start_Angle Stop_Angle Replicate Distance
## 1           1          1         1 57.81626
## 2           2          1         1 66.25575
## 3           3          1         1 70.34698
## 4           1          2         1 63.21392
## 5           2          2         1 73.86091
## 6           3          2         1 85.24894
```

```
summary(design)
```

```
##   Start_Angle   Stop_Angle   Replicate      Distance
##  Min.   :1     Min.   :1    Min.   :1.0   Min.   :57.82
##  1st Qu.:1     1st Qu.:1    1st Qu.:1.0   1st Qu.:66.76
##  Median :2     Median :2    Median :1.5   Median :70.67
##  Mean   :2     Mean   :2    Mean   :1.5   Mean   :74.29
##  3rd Qu.:3     3rd Qu.:3    3rd Qu.:2.0   3rd Qu.:84.31
##  Max.   :3     Max.   :3    Max.   :2.0   Max.   :94.01
```

```
# (f)
# Make sure Start_Angle and Stop_Angle are factors
```

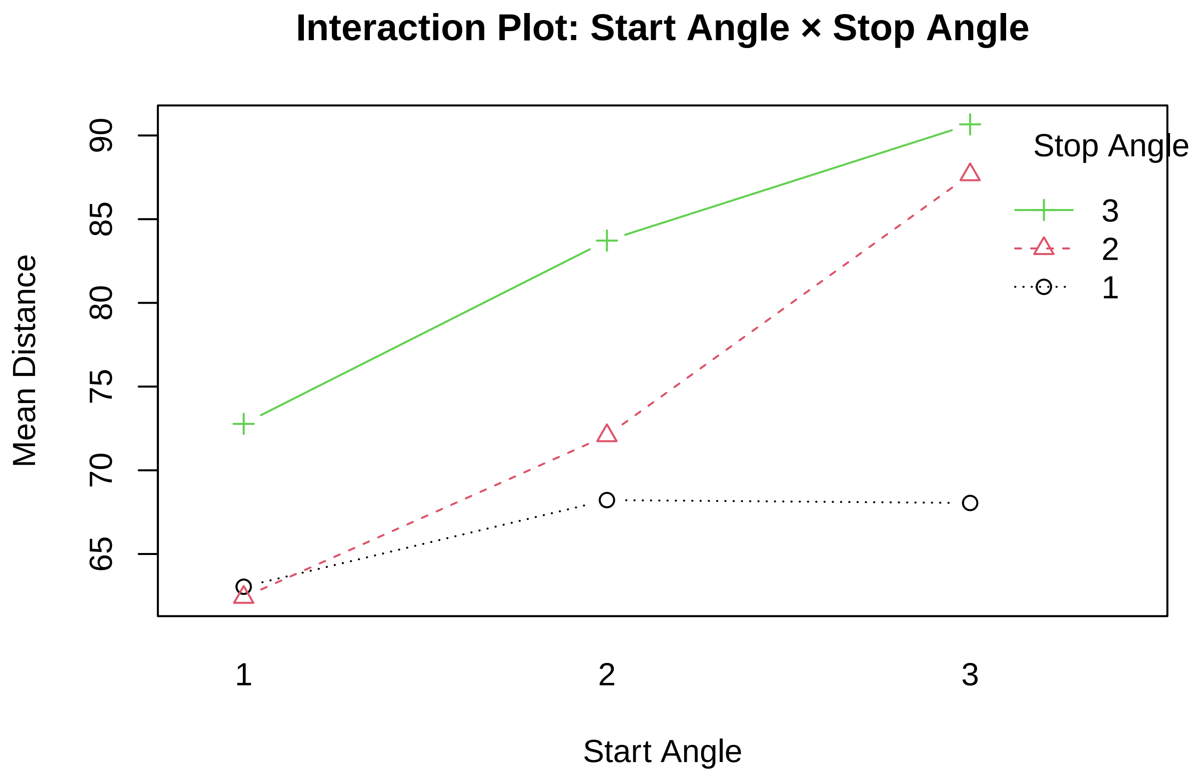
```r
design$Start_Angle <- as.factor(design$Start_Angle)
design$Stop_Angle <- as.factor(design$Stop_Angle)

# Fit two-way ANOVA model with interaction
anova_model <- aov(Distance ~ Start_Angle * Stop_Angle, data = design)

summary(anova_model)
```

```
##                         Df Sum Sq Mean Sq F value   Pr(>F)
## Start_Angle              2  773.7   386.9  28.149 0.000134 ***
## Stop_Angle               2  763.5   381.8  27.778 0.000141 ***
## Start_Angle:Stop_Angle   4  235.2    58.8   4.279 0.032700 *
## Residuals                9  123.7    13.7
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

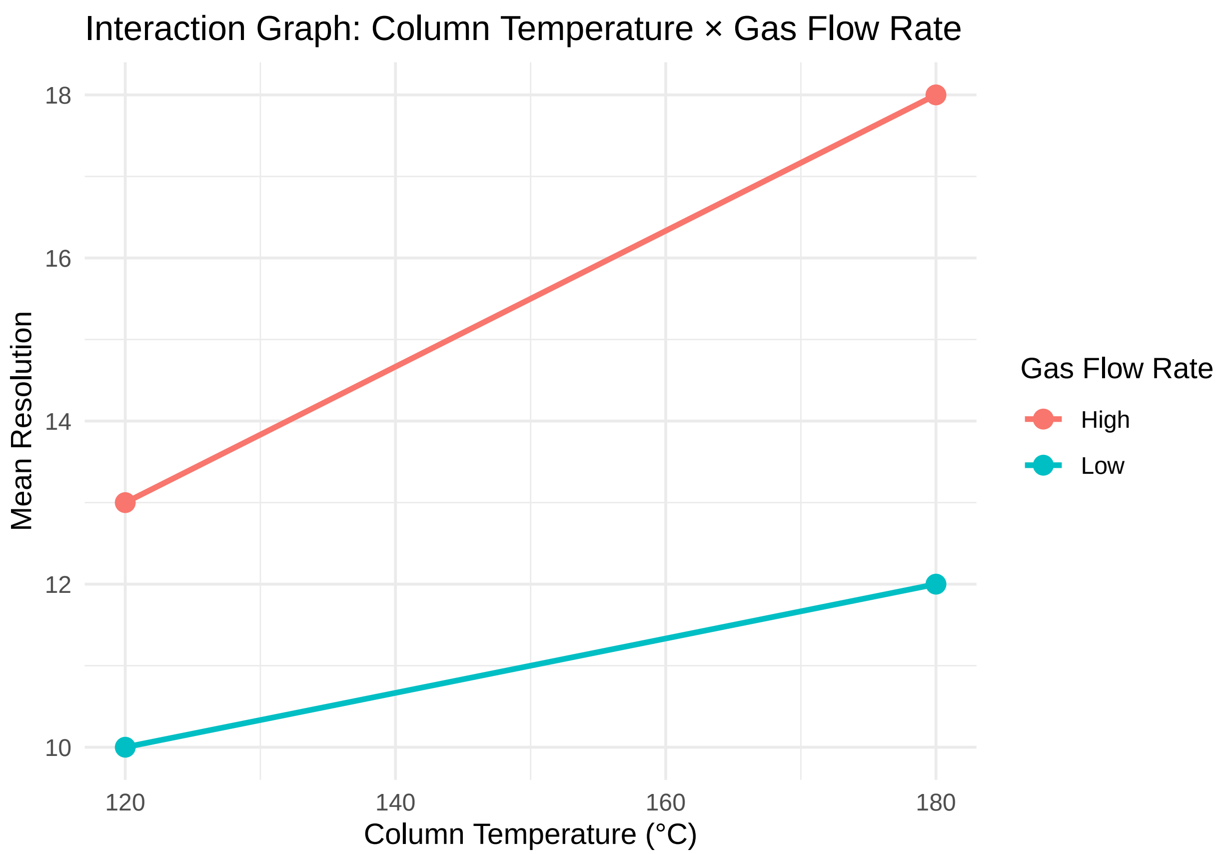
```r
# (g)
# Interaction Plot
interaction.plot(
  x.factor = design$Start_Angle,
  trace.factor = design$Stop_Angle,
  response = design$Distance,
  fun = mean,
  type = "b", col = 1:3, pch = 1:3,
  xlab = "Start Angle", ylab = "Mean Distance",
  trace.label = "Stop Angle",
  main = "Interaction Plot: Start Angle × Stop Angle"
)
```



Question 3

```r
# Dataframe
df <- data.frame(
  Temperature = rep(c(120, 180), 2),
  FlowRate = factor(rep(c("Low", "High"), each = 2)),
  Resolution = c(10, 12, 13, 18)
)

ggplot(df, aes(x = Temperature, y = Resolution, color = FlowRate, group = FlowRate)) +
  geom_line(size = 1) +
  geom_point(size = 3) +
  labs(title = "Interaction Graph: Column Temperature × Gas Flow Rate",
       x = "Column Temperature (°C)",
       y = "Mean Resolution",
       color = "Gas Flow Rate") +
  theme_minimal()
```



Question 4

```r
# (c)
set.seed(1022)

design_1 <- expand.grid(
  Brand = c("A", "B"),
  Power = c("Low", "Medium", "High"),
  Time = c(2, 2.5, 3, 3.5),
  Rep = 1:8
)

# Randomize order
design_1$Trial <- sample(1:nrow(design_1))
design_1 <- design_1[order(design_1$Trial), ]
head(design_1)
```

```
##     Brand  Power Time Rep Trial
## 129     A Medium  2.5   6     1
## 13      A    Low  3.0   1     2
## 132     B   High  2.5   6     3
## 61      A    Low  3.0   3     4
## 138     B   High  3.0   6     5
## 42      B   High  3.0   2     6
```

```r
summary(design_1)
```

```
##  Brand      Power          Time             Rep            Trial       
##  A:96   Low   :64   Min.   :2.000   Min.   :1.00   Min.   :  1.00  
##  B:96   Medium:64   1st Qu.:2.375   1st Qu.:2.75   1st Qu.: 48.75  
##         High  :64   Median :2.750   Median :4.50   Median : 96.50  
##                     Mean   :2.750   Mean   :4.50   Mean   : 96.50  
##                     3rd Qu.:3.125   3rd Qu.:6.25   3rd Qu.:144.25  
##                     Max.   :3.500   Max.   :8.00   Max.   :192.00
```

```r
# (e)
# Genarate edible as response variable
set.seed(1022)

design_1$Edible <- with(design_1,
  70 +                                      # base % edible
  ifelse(Brand == "B", 2, 0) +              # Brand B slightly better
  ifelse(Power == "Low", -3,
         ifelse(Power == "Medium", 0, 3)) + # increasing trend in power
  -1.5 * (as.numeric(Time) - 2.75)^2 +      # optimal time at 2.75
  rnorm(nrow(design_1), mean = 0, sd = 2)     # random error
)

# Polynomial contrast analysis
design_1$Power <- factor(design_1$Power, levels = c("Low", "Medium", "High"), ordered = TRUE)

model <- aov(Edible ~ Brand + poly(as.numeric(Power), 2) + poly(Time, 3), data = design_1)
summary(model)
```

```
##                              Df Sum Sq Mean Sq F value   Pr(>F)    
## Brand                         1  295.0   295.0   67.61 3.42e-14 ***
## poly(as.numeric(Power), 2)    2 1260.8   630.4  144.51  < 2e-16 ***
## poly(Time, 3)                 3   33.8    11.3    2.58    0.055 .  
## Residuals                   185  807.0     4.4                     
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Question 7

```r
# (b)
set.seed(1022)  # For reproducibility of design

# Step 1: Create full factorial design
factors <- expand.grid(
  Flame   = factor(c("Low", "High")),
  PanSize = factor(c("Small", "Large")),
  Cover   = factor(c("None", "Glass")),
  Salt    = factor(c("No", "Yes"))
)
```

```r
# Step 2: Replicate each combination 6 times
design_new <- factors[rep(1:nrow(factors), each = 6), ]
design_new$Replicate <- rep(1:6, times = nrow(factors))

# Step 4: Randomize run order
design_new$Trial <- sample(1:nrow(design_new))
design_new <- design_new[order(design_new$Trial), ]

design_new
```

```
##      Flame PanSize Cover Salt Replicate Trial
## 10.2  High   Small  None  Yes         3     1
## 3.1    Low   Large  None   No         2     2
## 9.3    Low   Small  None  Yes         4     3
## 11.3   Low   Large  None  Yes         4     4
## 5.1    Low   Small Glass   No         2     5
## 7.1    Low   Large Glass   No         2     6
## 12.3  High   Large  None  Yes         4     7
## 5.5    Low   Small Glass   No         6     8
## 16.3  High   Large Glass  Yes         4     9
## 14.3  High   Small Glass  Yes         4    10
## 12.5  High   Large  None  Yes         6    11
## 6.2   High   Small Glass   No         3    12
## 5      Low   Small Glass   No         1    13
## 7.5    Low   Large Glass   No         6    14
## 13.4   Low   Small Glass  Yes         5    15
## 11     Low   Large  None  Yes         1    16
## 10.1  High   Small  None  Yes         2    17
## 15     Low   Large Glass  Yes         1    18
## 16.1  High   Large Glass  Yes         2    19
## 8.1   High   Large Glass   No         2    20
## 11.1   Low   Large  None  Yes         2    21
## 1.4    Low   Small  None   No         5    22
## 11.4   Low   Large  None  Yes         5    23
## 15.1   Low   Large Glass  Yes         2    24
## 2.4   High   Small  None   No         5    25
## 16.2  High   Large Glass  Yes         3    26
## 11.2   Low   Large  None  Yes         3    27
## 9.4    Low   Small  None  Yes         5    28
## 8     High   Large Glass   No         1    29
## 13.3   Low   Small Glass  Yes         4    30
## 4.4   High   Large  None   No         5    31
## 2.2   High   Small  None   No         3    32
## 3.4    Low   Large  None   No         5    33
## 15.3   Low   Large Glass  Yes         4    34
## 9.1    Low   Small  None  Yes         2    35
## 1      Low   Small  None   No         1    36
## 6.3   High   Small Glass   No         4    37
## 13     Low   Small Glass  Yes         1    38
## 2.1   High   Small  None   No         2    39
## 8.4   High   Large Glass   No         5    40
## 6.1   High   Small Glass   No         2    41
## 1.5    Low   Small  None   No         6    42
```

```
## 5.4    Low   Small Glass   No          5    43
## 2     High   Small  None   No          1    44
## 10    High   Small  None  Yes          1    45
## 5.2    Low   Small Glass   No          3    46
## 16.5  High   Large Glass  Yes          6    47
## 3.2    Low   Large  None   No          3    48
## 10.4  High   Small  None  Yes          5    49
## 5.3    Low   Small Glass   No          4    50
## 12.4  High   Large  None  Yes          5    51
## 3.3    Low   Large  None   No          4    52
## 4.3   High   Large  None   No          4    53
## 14.5  High   Small Glass  Yes          6    54
## 1.1    Low   Small  None   No          2    55
## 14.1  High   Small Glass  Yes          2    56
## 4.2   High   Large  None   No          3    57
## 1.2    Low   Small  None   No          3    58
## 3.5    Low   Large  None   No          6    59
## 6.5   High   Small Glass   No          6    60
## 16.4  High   Large Glass  Yes          5    61
## 8.5   High   Large Glass   No          6    62
## 13.2   Low   Small Glass  Yes          3    63
## 7.3    Low   Large Glass   No          4    64
## 8.2   High   Large Glass   No          3    65
## 14.2  High   Small Glass  Yes          3    66
## 14    High   Small Glass  Yes          1    67
## 9      Low   Small  None  Yes          1    68
## 8.3   High   Large Glass   No          4    69
## 14.4  High   Small Glass  Yes          5    70
## 9.2    Low   Small  None  Yes          3    71
## 2.3   High   Small  None   No          4    72
## 13.1   Low   Small Glass  Yes          2    73
## 12.2  High   Large  None  Yes          3    74
## 4.5   High   Large  None   No          6    75
## 15.5   Low   Large Glass  Yes          6    76
## 11.5   Low   Large  None  Yes          6    77
## 2.5   High   Small  None   No          6    78
## 6.4   High   Small Glass   No          5    79
## 7      Low   Large Glass   No          1    80
## 16    High   Large Glass  Yes          1    81
## 13.5   Low   Small Glass  Yes          6    82
## 15.4   Low   Large Glass  Yes          5    83
## 3      Low   Large  None   No          1    84
## 1.3    Low   Small  None   No          4    85
## 6     High   Small Glass   No          1    86
## 4     High   Large  None   No          1    87
## 12    High   Large  None  Yes          1    88
## 7.2    Low   Large Glass   No          3    89
## 4.1   High   Large  None   No          2    90
## 7.4    Low   Large Glass   No          5    91
## 10.3  High   Small  None  Yes          4    92
## 10.5  High   Small  None  Yes          6    93
## 9.5    Low   Small  None  Yes          6    94
## 15.2   Low   Large Glass  Yes          3    95
## 12.1  High   Large  None  Yes          2    96
```

```r
# (d)
boiling_data <- read.csv("Boiling_Time_Data.csv")

boiling_model <- aov(Time ~ Flame * PanSize * Cover * Salt, data = boiling_data)
boiling_model
```

```
## Call:
##    aov(formula = Time ~ Flame * PanSize * Cover * Salt, data = boiling_data)
##
## Terms:
##                    Flame  PanSize    Cover     Salt Flame:PanSize Flame:Cover
## Sum of Squares  60.07670  2.09098 22.33514  1.27674       0.01465     0.00937
## Deg. of Freedom        1        1        1        1             1           1
##                 PanSize:Cover Flame:Salt PanSize:Salt Cover:Salt
## Sum of Squares        0.05451    0.00029      0.06094    0.09131
## Deg. of Freedom             1          1            1          1
##                 Flame:PanSize:Cover Flame:PanSize:Salt Flame:Cover:Salt
## Sum of Squares              0.00455            0.02250          0.03672
## Deg. of Freedom                   1                  1                1
##                 PanSize:Cover:Salt Flame:PanSize:Cover:Salt Residuals
## Sum of Squares             0.01903                  0.00590   2.72407
## Deg. of Freedom                  1                        1        80
##
## Residual standard error: 0.1845287
## Estimated effects may be unbalanced
```

```r
summary(boiling_model)
```

```
##                          Df Sum Sq Mean Sq  F value   Pr(>F)
## Flame                     1  60.08   60.08 1764.324  < 2e-16 ***
## PanSize                   1   2.09    2.09   61.408 1.69e-11 ***
## Cover                     1  22.34   22.34  655.935  < 2e-16 ***
## Salt                      1   1.28    1.28   37.495 3.23e-08 ***
## Flame:PanSize             1   0.01    0.01    0.430    0.514
## Flame:Cover               1   0.01    0.01    0.275    0.601
## PanSize:Cover             1   0.05    0.05    1.601    0.209
## Flame:Salt                1   0.00    0.00    0.008    0.927
## PanSize:Salt              1   0.06    0.06    1.790    0.185
## Cover:Salt                1   0.09    0.09    2.682    0.105
## Flame:PanSize:Cover       1   0.00    0.00    0.134    0.716
## Flame:PanSize:Salt        1   0.02    0.02    0.661    0.419
## Flame:Cover:Salt          1   0.04    0.04    1.078    0.302
## PanSize:Cover:Salt        1   0.02    0.02    0.559    0.457
## Flame:PanSize:Cover:Salt  1   0.01    0.01    0.173    0.678
## Residuals                80   2.72    0.03
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```r
# Since R gave the warning "Estimated effects may be unbalanced", we should consider using Type III Sum
Anova(boiling_model, type = 3)
```

```
## Anova Table (Type III tests)
```

```
##
## Response: Time
##                         Sum Sq Df   F value    Pr(>F)    
## (Intercept)            200.627  1 5891.9861 < 2.2e-16 ***
## Flame                    6.673  1  195.9609 < 2.2e-16 ***
## PanSize                  0.481  1   14.1261 0.0003236 ***
## Cover                    2.531  1   74.3366 4.864e-13 ***
## Salt                     0.106  1    3.1188 0.0812095 .  
## Flame:PanSize            0.008  1    0.2203 0.6400544    
## Flame:Cover              0.047  1    1.3745 0.2445174    
## PanSize:Cover            0.014  1    0.4222 0.5177273    
## Flame:Salt               0.040  1    1.1833 0.2799490    
## PanSize:Salt             0.028  1    0.8275 0.3657264    
## Cover:Salt               0.007  1    0.2164 0.6430538    
## Flame:PanSize:Cover      0.010  1    0.3056 0.5819344    
## Flame:PanSize:Salt       0.026  1    0.7553 0.3873916    
## Flame:Cover:Salt         0.036  1    1.0580 0.3067606    
## PanSize:Cover:Salt       0.002  1    0.0549 0.8153513    
## Flame:PanSize:Cover:Salt   0.006  1    0.1732 0.6783956    
## Residuals                2.724 80                        
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```