114-1 (Fall 2025) Semester

Reinforcement Learning

Assignment #2

Model-Free Prediction & Control

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Outline

- Environment
- Tasks
- Code structure
- Grading
- Submission
- Policy
- Contact

Environment

Grid World

State space

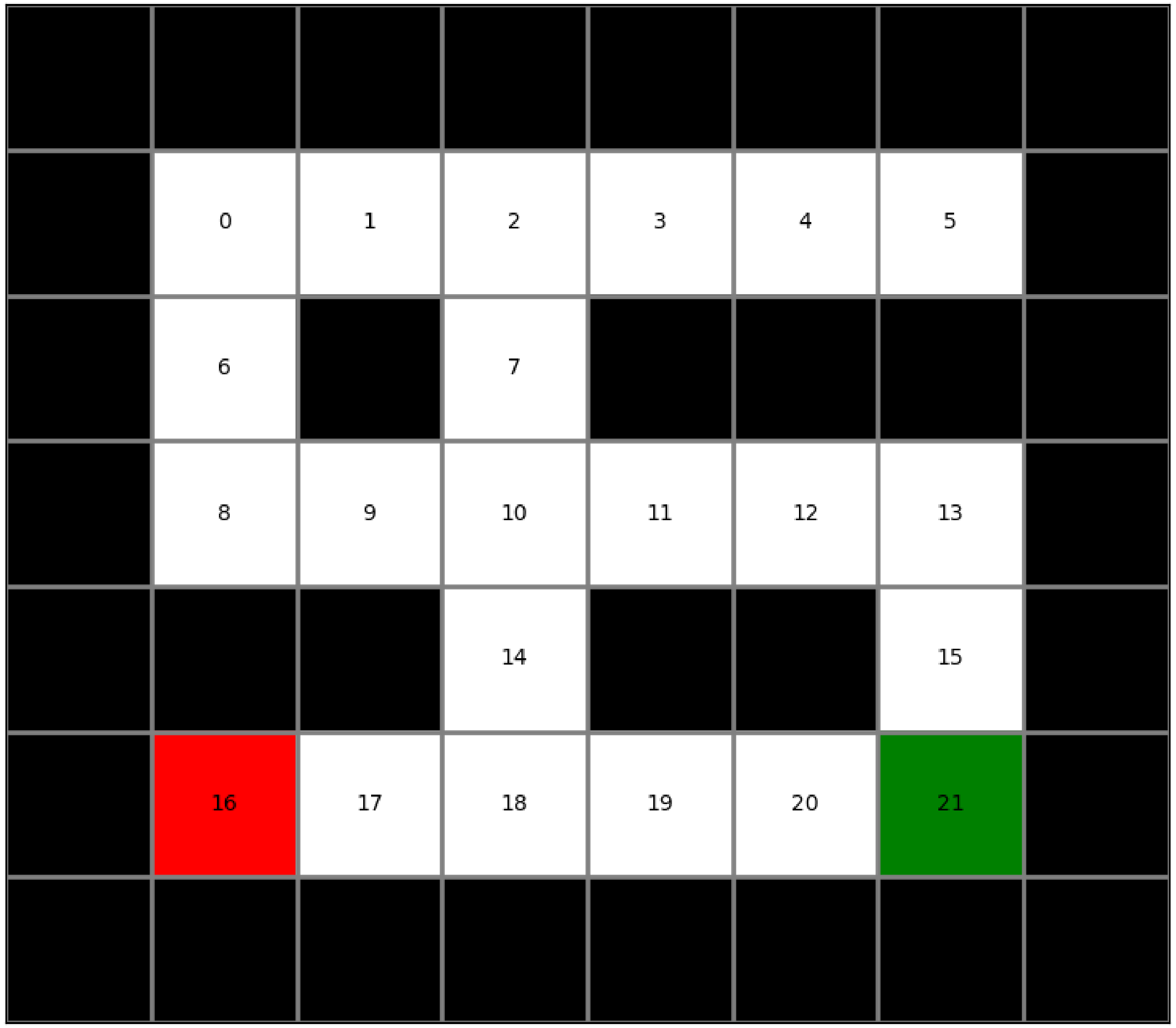
- Nonterminal states: Empty
- Terminal states: **Goal** (Green), **Trap** (Red)
- 0-indexed

Action space

- Up, down, left, right
- Hitting the wall will remain at the same state

Reward

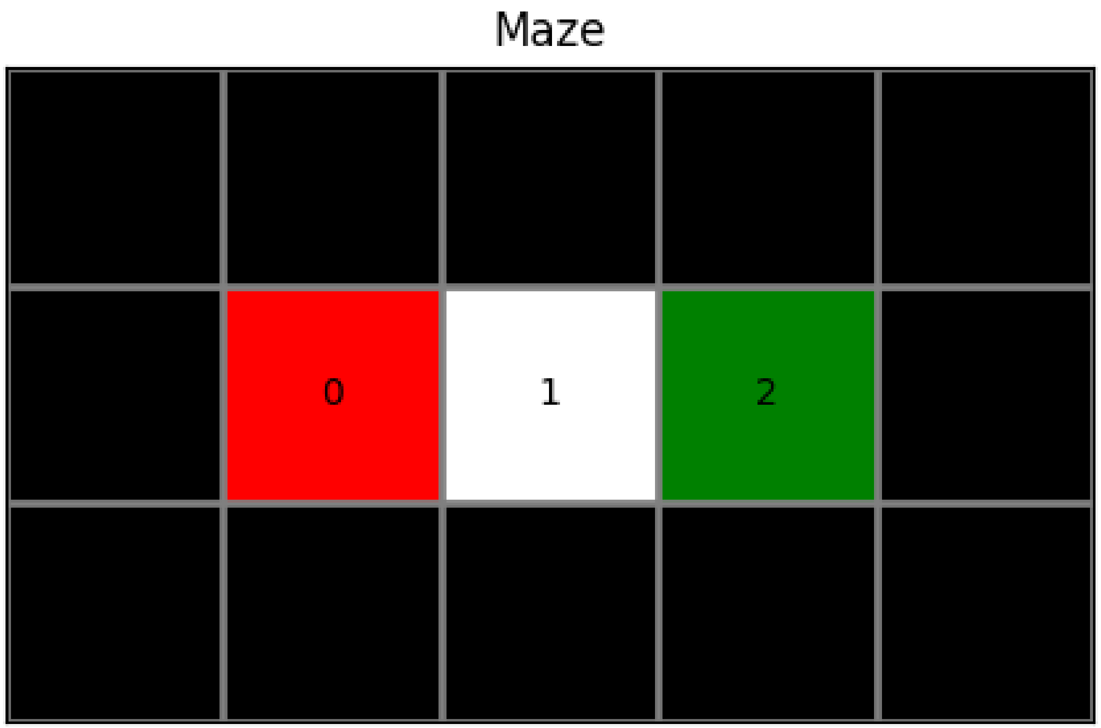
- Step reward given at every transition
- Goal reward given after leaving goal state
- Trap reward given after leaving trap state

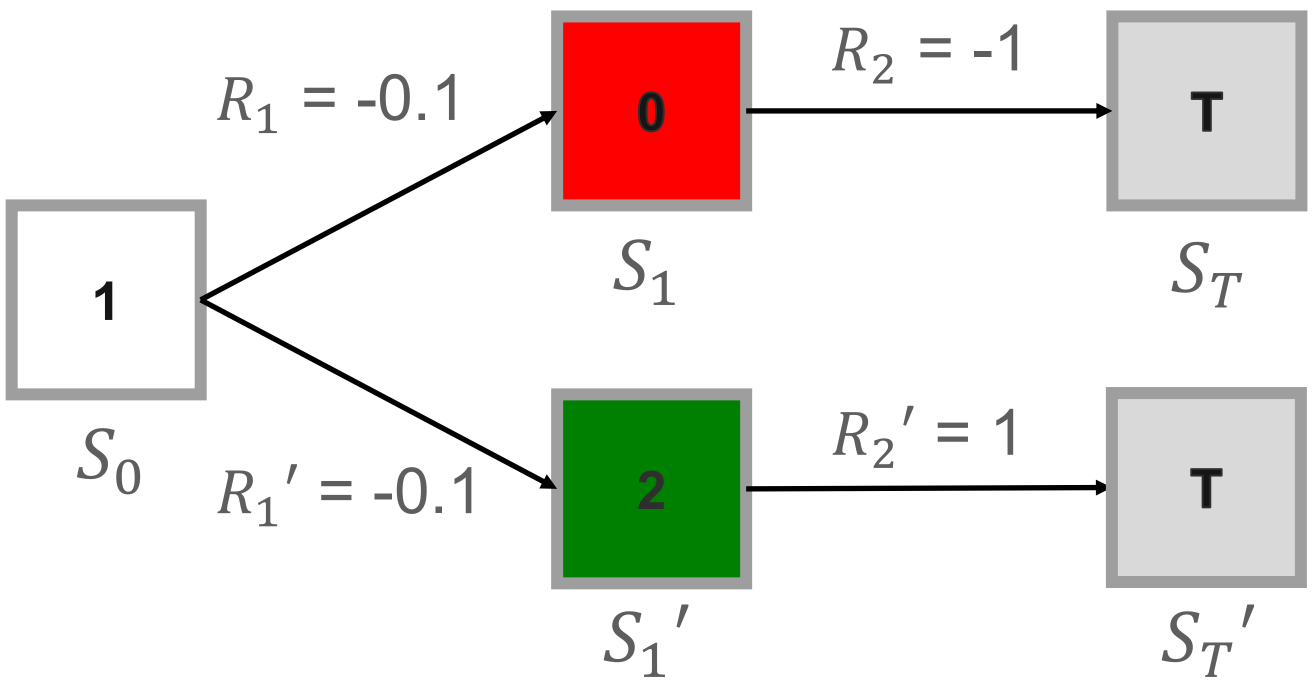


Interaction with Environments

- Learn to interact with a OpenAI gym-like environment
- Grid World in this assignment is a MDP (defined by maze.txt, do not modified)
- Grid World functions:
  - step(action): Interact with the environment, taking one parameter (action)
  - reset(): Reset the environment to the initial state (only used once at the first step)
  - Update the **value/q-value** function with **states, rewards and done flags**

Terminal State





- The final transition at the end of the i^{th} episode will be:

  $$(S_T^i, A_T^i, R_{T+1}^i, S_0^{i+1})$$

  where the first state of the $(i+1)^{th}$ episode is S_0^{i+1}

Tasks

Model-Free Prediction & Control

Prediction 1 - First-Visit Monte-Carlo Prediction

- Evaluate a policy by predicting the value function for each state
- Update the value function with First-visit Monte-Carlo method using **state, reward, and done from the collect_data() function.**
- Update the value function per episode

First-visit MC prediction, for estimating $V \approx v_\pi$

Input: a policy π to be evaluated

Initialize:

$V(s) \in \mathbb{R}$, arbitrarily, for all $s \in \mathcal{S}$

$Returns(s) \leftarrow$ an empty list, for all $s \in \mathcal{S}$

Be careful with the index of S and R !
(S_{T-1} is goal or trap in our case)

Loop forever (for each episode):

Generate an episode following π: $S_0, A_0, R_1, S_1, A_1, R_2, \ldots, S_{T-1}, A_{T-1}, R_T$

$G \leftarrow 0$

Loop for each step of episode, $t = T-1, T-2, \ldots, 0$:

$G \leftarrow \gamma G + R_{t+1}$

Unless S_t appears in $S_0, S_1, \ldots, S_{t-1}$:

Append G to $Returns(S_t)$

$V(S_t) \leftarrow \text{average}(Returns(S_t))$

Prediction 2 - TD(0)

- Evaluate a policy by predicting the value function for each state
- Update the value function with TD(0) method using **the collect_data() function**
- Update the value function per step

Tabular TD(0) for estimating v_π

Input: the policy π to be evaluated
Algorithm parameter: step size $\alpha \in (0, 1]$
Initialize $V(s)$, for all $s \in \mathcal{S}^+$, arbitrarily except that $V(terminal) = 0$

Loop for each episode:
  Initialize S
  Loop for each step of episode:
    $A \leftarrow$ action given by π for S
    Take action A, observe R, S'
    $V(S) \leftarrow V(S) + \alpha\big[R + \gamma V(S') - V(S)\big]$
    $S \leftarrow S'$
  until S is terminal

Be careful with the done flag

Prediction 3 - N-step TD

- Evaluate a policy by predicting the value function for each state
- Update the value function with n-step TD method using **the collect_data() function**
- Update the value function per step expect steps that out of range of the n-step TD

n-step TD for estimating $V \approx v_\pi$

Input: a policy π
Algorithm parameters: step size $\alpha \in (0,1]$, a positive integer n
Initialize $V(s)$ arbitrarily, for all $s \in \mathcal{S}$
All store and access operations (for S_t and R_t) can take their index mod $n+1$

Loop for each episode:
  Initialize and store $S_0 \neq$ terminal
  $T \leftarrow \infty$
  Loop for $t = 0, 1, 2, \ldots$:
  | If $t < T$, then:
  | Take an action according to $\pi(\cdot|S_t)$
  | Observe and store the next reward as R_{t+1} and the next state as S_{t+1}
  | If S_{t+1} is terminal, then $T \leftarrow t+1$
  | $\tau \leftarrow t - n + 1$ (τ is the time whose state's estimate is being updated)
  | If $\tau \geq 0$: Skip n-1 step
  | $G \leftarrow \sum_{i=\tau+1}^{\min(\tau+n,T)} \gamma^{i-\tau-1} R_i$ Be care of the index R !
  | If $\tau + n < T$, then: $G \leftarrow G + \gamma^n V(S_{\tau+n})$ Skip n-1 step $(G_{\tau:\tau+n})$
  | $V(S_\tau) \leftarrow V(S_\tau) + \alpha [G - V(S_\tau)]$
  Until $\tau = T - 1$

[Text book p.144](#)

Control 1 – Every-Visit Monte-Carlo Prediction with ϵ-Greedy Improvement

- Problem
  - Evaluate a policy by predicting the Q value function for **each** (state, action) pair
  - Update the Q value function with the **Every-Visit** Monte-Carlo method using **constant $\boldsymbol{\alpha}$**
  - Using collected **trajectories** to update the value function **per episode**

MC Policy Evaluation + ϵ-Greedy Improvement

- Sample kth episode using π: $\{S_1, A_1, R_2, \ldots, S_T\} \sim \pi$
- For each state S_t and action A_t in the episode,

$$G_t = R_{t+1} + \gamma R_{t+2} + \cdots + \gamma^{T-1} R_T$$

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \boxed{(G_t - Q(S_t, A_t))}$$

Estimation Loss

- Improve policy based on new action-value function

$$\epsilon \leftarrow \text{constant}$$

$$\pi \leftarrow \epsilon\text{-greedy}(Q)$$

Control 1 – Every-Visit Monte-Carlo Prediction with ϵ-Greedy Improvement

- Problem
  - Evaluate a policy by predicting the Q value function for **each** (state, action) pair
  - Update the Q value function with the **Every-Visit** Monte-Carlo method using **constant α**
  - Using collected **trajectories** to update the value function **per episode**

ϵ-Greedy Improvement

- Simplest idea for ensuring continual exploration
- All m actions are tried with non-zero probability
- With probability $1-\epsilon$ choose the greedy action
- With probability ϵ choose an action at random

$$\pi(a|s) = \begin{cases} \epsilon/m + 1 - \epsilon & \text{if } a^* = \underset{a \in \mathcal{A}}{\operatorname{argmax}}\, Q(s,a) \\ \epsilon/m & \text{otherwise} \end{cases}$$

Control 2 - SARSA: Temporal-difference Prediction TD(0) with ϵ-Greedy Improvement

- Problem
  - Evaluate a policy by predicting the Q value function for each (state, action)
  - Update the Q value function with TD(0) method using the collected transition per step

TD(0) Policy Evaluation + ϵ-Greedy Improvement

Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
  Initialize S
  Choose A from S using policy derived from Q (e.g., ε-greedy)
  Repeat (for each step of episode):
    Take action A, observe R, S'
    Choose A' from S' using policy derived from Q (e.g., ε-greedy)
    $Q(S,A) \leftarrow Q(S,A) + \alpha \boxed{\left[R + \gamma Q(S',A') - Q(S,A)\right]}$
    $S \leftarrow S'$; $A \leftarrow A'$;
  until S is terminal

Estimation Loss

Control 3 - Q-Learning with ϵ-Greedy Improvement

- Problem
  - Store the collected transition $(S, A, R, S', done)$ in the replay buffer at each time step
  - Uniformly random sample transitions from the replay buffer and store them in the batch
  - Update the Q value function method using the sampled transitions in the batch

Q-Learning + ϵ-Greedy Improvement

Given update frequency m and sample batch size n
Initialize transition count $i = 0$, value function $Q(S, A)$, replay buffer ψ
Repeat (for each episode)
    Initialize S
    Repeat (for each step of the episode):
        Choose A from S using policy derived from Q (e.g., ϵ-greedy)
        Take action A, observe $R, S', done$
        Store the transition $(S, A, R, S', done)$ to ψ
        $i = i + 1$
        Initialize sampled batch transition $B = [\,]$
        If i mod $m == 0$:
            Uniformly random sample n transitions from ψ and store them to B
        For each $(S, A, R, S', done)$ in B:
            $Q(S, A) \leftarrow Q(S, A) + \alpha \underbrace{\boxed{[R + \gamma \max_{A'} Q(S', A') - Q(S, A)]}}_{\text{Estimation Loss}}$
        $S \leftarrow S'$
    Until S is terminal

Code Structure

algorithms.py

class **ModelFreePrediction()**

- Parent class for prediction algorithms
- Collect_data(): use the policy to interact with the env

class **MonteCarloPrediction()**

- TODO: run()

class **TDZeroPrediction()**

- TODO: run()

class **TDNstepPrediction()**

- TODO: run()

class **ModelFreeControl()**

- Parent class for control algorithms

class **MonteCarloPolicyIteration()**

- TODO: run(),
- Auxiliary functions:
  policy_evaluation(), policy_improvement()

class **SARSA()**

- TODO: run()
- Auxiliary functions:
  policy_eval_improve()

class **Q_learning()**

- TODO: run()
- Auxiliary functions:
  add_buffer(), sample_batch(), policy_eval_improve()

Feel free to add any function if needed

The implementation will be judged by run()

Grading

Grading (100%)

- Monte-Carlo prediction (10%)
  - Test cases (2% x 5 cases)
- TD(0) prediction (10%)
  - Test cases (2% x 5 cases)
- N-step TD prediction (10%)
  - Test cases (2% x 5 cases)
- Monte-Carlo Control (15%)
  - Test cases (3% x 5 cases)
- SARSA (15%)
  - Test cases (3% x 5 cases)
- Q-Learning (15%)
  - Test cases (3% x 5 cases)

- Report (25%) (Report Template: https://www.overleaf.com/read/gqfnzvjvwcps#939ed3)
  - Run Monte-Carlo prediction and TD(0) prediction for 50 seeds. Compare the resulting values with the GT values. Discuss the variance and bias. (16%)
    #prediction_GT.npy is provided (calculated using iterative policy evaluation)
  - Discuss and plot learning (Return) curves under ϵ values of (0.1, 0.2, 0.3, 0.4) on MC, SARSA, and Q-Learning (4%)
  - Discuss and plot loss curves under ϵ values of (0.1, 0.2, 0.3, 0.4) on MC, SARSA, and Q-Learning (4%)
  - Using Weights & Bias (https://wandb.ai/site) to plot all figures in the report (1%)

Prediction Bias & Variance

- Let V_{GT} be the ground truth value for a state.
- Let $\hat{V}_i$ be the estimated value from the i-th run.
- Let $\hat{V}_{avg}$ be the average predicted value over all runs for that state:

$$\hat{V}_{avg} = \frac{1}{n}\sum_{i=1}^{n} \hat{V}_i$$

- The $Bias$ measures how far the average estimate is from the true value. For a single state, the $Bias$ is:

$$Bias = \hat{V}_{avg} - V_{GT}$$

- The $Variance$ measures the variability of the predicted values across different runs. For a single state, the $Variance$ is:

$$Variance = \frac{1}{n}\sum_{i=1}^{n}(\hat{V}_i - \hat{V}_{avg})^2$$

Learning Curves & Loss Curves

1. Learning curves: #episode (X-axis) vs. Average non-discounted Episodic Reward $\mathcal{R}$ of last 10 episodes (Y-axis)

   Example:

   Episode 0: step1: r_{01}, step2: r_{02}, …, step T: r_{0a}
   Episode 1: step1: r_{11}, step2: r_{12}, …, step T: r_{1b}
   ⋮
   Episode 9: step1: r_{91}, step2: r_{12}, …, step T: r_{9j}

$$\mathcal{R} = \frac{\frac{\sum_{k=1}^{a} r_{0k}}{a} + \frac{\sum_{k=1}^{b} r_{1k}}{b} + \ldots + \frac{\sum_{k=1}^{j} r_{9k}}{j}}{10}$$

2. Loss Curves: #episode (X-axis) vs. Average Absolute Estimation Loss $\mathcal{L}$ over each transition of the last 10 episodes (Y-axis)

   Example:

   Episode 0: $\mathrm{abs}(EL_{01})$, $\mathrm{abs}(EL_{02})$, …, $\mathrm{abs}(EL_{0a})$
   Episode 1: $\mathrm{abs}(EL_{11})$, $\mathrm{abs}(EL_{12})$, …, $\mathrm{abs}(EL_{1b})$
   ⋮
   Episode 9: $\mathrm{abs}(EL_{91})$, $\mathrm{abs}(EL_{92})$, …, $\mathrm{abs}(EL_{9j})$

$$\mathcal{L} = \frac{\frac{\sum_{k=1}^{a} \mathrm{abs}(EL_{0k})}{a} + \frac{\sum_{k=1}^{b} \mathrm{abs}(EL_{1k})}{b} + \ldots + \frac{\sum_{k=1}^{j} \mathrm{abs}(EL_{9k})}{j}}{10}$$

Criteria

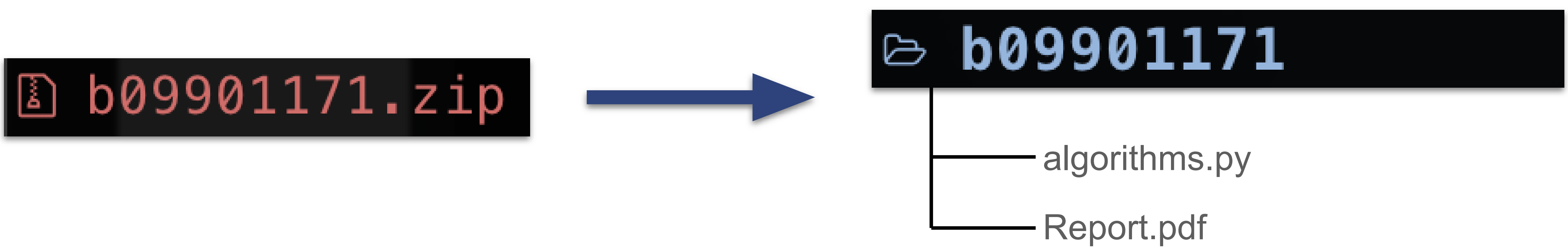
- Test cases:
  - Call run() and check the final output
  - Prediction Tasks: Check the **state values** after evaluation (tolerance: 0.005)
  - Control 1, 2: Check the resulting ***max_state_values*** on every state (tolerance: 0.1)
  - Control 3: Check the resulting ***max_state_values*** on every state (tolerance: 0.001)
  - Run time limit **3 minute** for Prediction cases to avoid infinite loops
  - Run time limit **5 minute** for Control cases to avoid infinite loops

- Sample solutions are provided for reference
  - Optimal policy may not be unique

Submission

Submission

- Submit on NTU COOL with following zip file structure
  - algorithms.py: containing your implementation for HW2
  - Get rid of pycache, DS_Store, etc.
  - Student ID with lower case
  - 10% deduction for wrong format



- **Deadline: 2025/10/15 Wed 11:59pm**
- **No late submission is allowed**

Policy

Policy

Package

- You can use any Python standard library (e.g., heap, queue…)
- Don't print anything out
- System level packages are prohibited (e.g., sys, os, multiprocess, subprocess, shutil, pathlib, …) for security concern, import any one of them will result in 0 score (even if you did not call it)

Collaboration

- Discussions are encouraged
- Write your own codes

Plagiarism & cheating

- All assignment submissions will be subject to duplication checking (e.g., MOSS)
- Plagiarism will receive an **F** grade for this course
- LLM for code writing is not prohibited. However, it's best to just write the code yourself, without the help of any LLM. (We will not accept using LLM as an excuse for plagiarism.)

Grade appeal

- Assignment grades are considered finalized two weeks after release

Contact

Questions?

- General questions
  - Use channel **[#assignment](#)** in slack as first option
  - Reply in thread to avoid spamming other people
- Personal questions
  - DM me on Slack: **TA 楊可 d14948001**