Model- II: (M/M/c): (∞ /FCFS)

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P_0	$\dfrac{1}{\left[\sum_{n=0}^{c-1}\left(\frac{\lambda}{\mu}\right)^n \frac{1}{n!} + \frac{\left(\frac{\lambda}{\mu}\right)^c}{c!} \times \frac{c\mu}{c\mu-\lambda}\right]}$
L_s	$\dfrac{\lambda\mu\left(\frac{\lambda}{\mu}\right)^c}{(c-1)!\,(c\mu-\lambda)^2} p_0 + \dfrac{\lambda}{\mu}$
L_q	$\dfrac{1}{(c-1)!}\left(\dfrac{\lambda}{u}\right)^c \dfrac{\lambda\mu}{(c\mu-\lambda)^2} p_0$
L_w	$\dfrac{\lambda}{\mu c-\lambda}$
W_s	$\dfrac{1}{\mu c!\,c}\left(\dfrac{\lambda}{\mu}\right)^c \dfrac{1}{\left(1-\frac{\lambda}{\mu c}\right)^2} p_0 + \dfrac{\lambda}{\mu}$
W_q	$\dfrac{\mu\left(\frac{\lambda}{u}\right)^c}{(c-1)!\,(c\mu-\lambda)^2} P_0$
P(N≥c)	$\dfrac{\mu\left(\frac{\lambda}{u}\right)^c}{(c-1)!\,(c\mu-\lambda)} P_0$
ρ	$\dfrac{\lambda}{\mu c}$

Example 1

- A supermarket has two servers servicing at counters. The customer arrive in a Poisson fashion at the rate of 10 per hour. The service time for each customer is expected with mean 4 minutes. Find the probability that a customer has to wait for the service, average queue length, and the average time spent by a customer in the queue.

Solution

- **Step 1: <u>Model Identification</u>**
  - Since there are 2 servers, the number of service channels is two. Also, since any number of customers can enter the system, the capacity of the system is infinity. Hence this problem comes under the model (M/M/c): (∞ /FCFS).
- **Step 2: <u>Given Data</u>**

No. of servers $c = 2$

Arrival rate , $\lambda = 10$ per hour

Service rate, $\mu = 4$ minutes per customer

$= 15$ person per hour

Solution

- **Step 3: To find the following**
  - The probability of a customer has to wait for the service i.e., $P(W_s > 0)$.
  - Average queue length L_q.
  - The average time spent by a customer in the queue W_q
- **Step 4: Required Computations**
  - First we have to find P_0
  - We know that
  - $$P_0 = \frac{1}{\left[\sum_{n=0}^{c-1}\frac{\left(\frac{\lambda}{\mu}\right)^n}{n!}+\frac{\left(\frac{\lambda}{\mu}\right)^c}{c!}\times\frac{c\mu}{c\mu-\lambda}\right]}$$
  - $$= \frac{1}{\sum_{n=0}^{1}\frac{\left(\frac{\lambda}{\mu}\right)^n}{n!}+\frac{\left(\frac{\lambda}{n}\right)^2}{2!}\times\frac{2\mu}{2\mu-\lambda}}$$

Solution

$= \dfrac{1}{\left[\sum_{n=0}^{1} \frac{\left(\frac{2}{3}\right)^n}{n!} + \frac{\left(\frac{2}{3}\right)^2}{2!} \times \frac{2(15)}{2(15)-10}\right]}$

- $= \dfrac{1}{1+\frac{2}{3}+\frac{1}{3}} = 0.5$

I. Probability of a customer has to wait for the service.

- $\therefore P(Ws > 0) = \dfrac{\left(\frac{\lambda}{\mu}\right)^c}{C!\left(1-\frac{\lambda}{\mu C}\right)} \times P_0$
- $= \dfrac{\left(\frac{2}{3}\right)^2}{2! \times \frac{1}{3}} \times 0 \cdot 5 = \dfrac{1}{3}$
- $= 0.33$

Solution

II. Average queue length

- $L_q = \frac{1}{(c-1)!}\left(\frac{\lambda}{\mu}\right)^c \frac{\lambda\mu}{(c\mu-\lambda)} P_0$
- $= \left(\frac{10}{15}\right)^2 \frac{10\times15}{(2\times15-10)^2} \times 0.5 = \frac{1}{6} \times 0.5$
- **=0.083 Customer**

III. Average waiting time in the queue

- $W_q = \frac{L_q}{\lambda} = \frac{0.083}{10}$
- **= 0.0083 Hours**

Example 4:

- A tax-consulting firm has 3 counters in its office to receive people who have problems concerning their income, wealth and sales tax. On the average 48 persons arrive in an 8-hour day. Each Tax advisor spends 15 minutes on the average on an arrival. If the arrivals are Poisson distributes and service times are according to exponential distribution , Find
  - Average number of customers in the system.
  - Average number of customers waiting to be serviced.
  - Average time a customer spends in the system.
  - Average waiting time for a customer in the queue.
  - The number of hours each week a tax advisor spends performing his job.
  - The expected number of idle tax advisor at any specified time.
  - The probability that a customer has to wait before he gets service.

Solution:

- **Step 1: Model Identification**
  - Since there are 3 counters, the number of service channels is three. Also, since number of the customers can enter the system, the capacity of the system is infinity. Hence this problem comes under the model model (M/M/c): (∞ /FCFS).
- **Step 2: Given Data**
  - Arrival rate , λ= 48/8 per hour

    = **6 per hour**
  - Service rate, $\mu = \frac{1}{15} \times 60$

    = 4 per hour
  - No. of service channels = c = 3

Solution

- **Step 3: <u>To find the following</u>**
  - Average number of customer spends in the system i.e., L_s.
  - The average number of customers waiting to be serviced L_q.
  - Average time a customer spends in the system W_s.
  - Average waiting time for a customer in the queue W_q.
  - The number of hours each week a tax advisor spends performing his job (to apply a different method).
  - The expected number of idle tax advisers at any specified time i.e., Probability that a customer has to wait (to apply a different method).
  - The probability that a customer has to wait before he gets service (to apply a different method).

Solution

- First we will find P_0 (i.e.) the probability of no customer in the system. I other words, we will find the probability of the system being free.
  - We know that

$$P_0 = \frac{1}{\left[\sum_{n=0}^{c-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{\left(\frac{\lambda}{\mu}\right)^c}{c!} \times \frac{c\mu}{c\mu-\lambda}\right]}$$

$$= \frac{1}{\sum_{n=0}^{2} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!} + \frac{\left(\frac{\lambda}{n}\right)^3}{3!} \times \frac{3\mu}{3\mu-\lambda}}$$

$$= \frac{1}{\left[\sum_{n=0}^{2} \frac{\left(\frac{3}{2}\right)^n}{n!} + \frac{\left(\frac{3}{2}\right)^3}{3!} \times \frac{3(4)}{12-6}\right]}$$

Solution

- $= \dfrac{1}{1+\frac{5}{2}+\frac{9}{8}+\frac{27}{48}\cdot\frac{12}{6}}$
- $= \dfrac{1}{1+\frac{3}{2}+\frac{9}{8}+\frac{9}{8}}$
- $\therefore \mathrm{P}_0 = \dfrac{1}{\frac{38}{8}}$
- $= \dfrac{\mathbf{8}}{\mathbf{38}} = \mathbf{0.21}$