## Some basic equivalences

| Name | Equivalences | Equivalences |
|---|---|---|
| Idempotence | (1) $A \wedge A \equiv A$ | (2) $A \vee A \equiv A$ |
| Associativity | (3) $A \wedge (B \wedge C) \equiv (A \wedge B) \wedge C$ | (4) $A \vee (B \vee C) \equiv (A \vee B) \vee C$ |
| Commutativity | (5) $A \wedge B \equiv B \wedge A$ | (6) $A \vee B \equiv B \vee A$ |
| Distributivism | (7) $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$ | (8) $A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$ |
| Identity | (9) $A \wedge \text{True} \equiv A$ | (10) $A \vee \text{False} \equiv A$ |
| Domination | (11) $A \wedge \text{False} \equiv \text{False}$ | (12) $A \vee \text{True} \equiv \text{True}$ |
| Double negation | (13) $\neg(\neg A) \equiv A$ | |
| Complement | (14) $A \wedge \neg A \equiv \text{False}$<br>(16) $\neg \text{True} \equiv \text{False}$ | (15) $A \vee \neg A \equiv \text{True}$<br>(17) $\neg \text{False} \equiv \text{True}$ |
| De Morgan's | (17) $\neg(A \vee B) \equiv \neg A \wedge \neg B$ | (18) $\neg(A \wedge B) \equiv \neg A \vee \neg B$ |
| Absorption | (19) $A \vee (A \wedge B) \equiv A$ | (20) $A \wedge (A \vee B) \equiv A$ |
| Conditional Identity | (21) $A \rightarrow B \equiv \neg A \vee B$ | (22) $A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$ |

## Rules of Inferences - PROPOSITIONS

| Rule | Name |
|---|---|
| P<br>P→Q<br>∴ **Q** | Modus Ponens |
| ¬Q<br>P→Q<br>∴¬ **P** | Modus Tollens |
| P<br>∴ P ∨ Q | Addition |
| P ∧ Q<br>∴ **P** | Simplification |

| Rule | Name |
|---|---|
| P<br>Q<br>∴ **P ∧ Q** | Conjunction |
| P→Q<br>Q→R<br>∴ **P→R** | Hypothetical Syllogism |
| P ∨ Q<br>¬ P<br>∴ **Q** | Disjunctive Syllogism |
| P ∨ Q<br>¬P ∨ R<br>∴ **Q ∨ R** | Resolution |

## Rules of Inferences with QUANTIFIERS

| Rule | Name | Example |
|---|---|---|
| **c** is *arbitrary* or *particular*<br>∀x P (x)<br>∴ P (c) | Universal Instantiation / Elimination | Sam is a student in the class.<br>Every student in the class completed the assignment.<br>Therefore, Sam completed his assignment. |
| **c** is *arbitrary*<br>P(c)<br>∴ ∀x P (x) | Universal Generalization / Introduction | Let c be an arbitrary integer<br>c ≤ c2<br>Therefore, every integer is ≤ to its square (careful!) |
| ∃x P(x)<br>∴ **c** is *particular* P (c) | Existential Instantiation / Elimination | There is an integer that is equal to its square.<br>Therefore, c2 = c, for some integer c. |
| **c** is *arbitrary* or *particular*<br>P (c)<br>∴ ∃x P(x) | Existential Generalization / Introduction | Sam is a particular student in the class.<br>Sam completed the assignment.<br>Therefore, there is a student in the class who completed the assignment. |