极客大学算法训练营
第十二课
动态规划

覃超

Sophon Tech 创始人，前 Facebook 工程师

分治 + 回溯 + 递归 + 动态规划

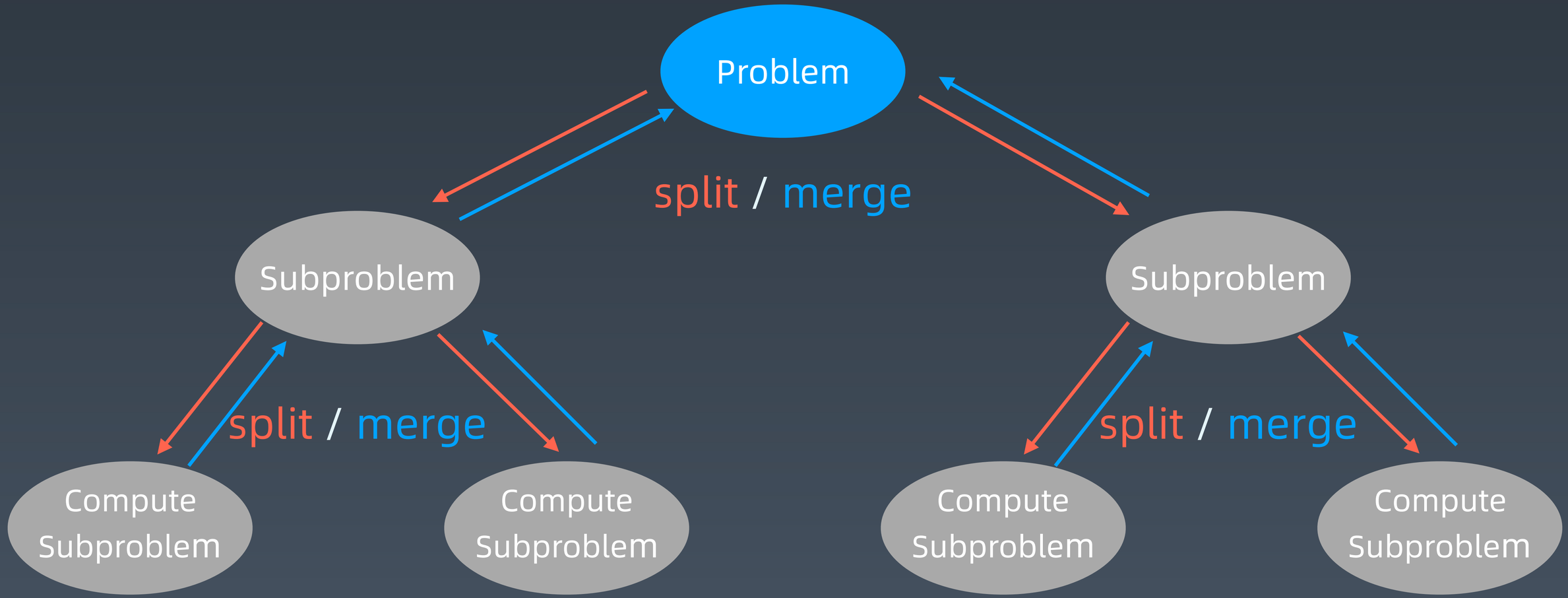
递归代码模版

```
public void recur(int level, int param) {

    // terminator
    if (level > MAX_LEVEL) {
      // process result
      return;
    }

    // process current logic
    process(level, param);

    // drill down
    recur( level: level + 1, newParam);

    // restore current status

}
```

分治

Divide & Conquer

递归状态树


Problem
split / merge
Subproblem
Subproblem
split / merge
split / merge
Compute Subproblem
Compute Subproblem
Compute Subproblem
Compute Subproblem

分治代码模板

```
def divide_conquer(problem, param1, param2, ...):
  # recursion terminator
  if problem is None:
    print_result
    return

  # prepare data
  data = prepare_data(problem)
  subproblems = split_problem(problem, data)

  # conquer subproblems
  subresult1 = self.divide_conquer(subproblems[0], p1, ...)
  subresult2 = self.divide_conquer(subproblems[1], p1, ...)
  subresult3 = self.divide_conquer(subproblems[2], p1, ...)
  …

  # process and generate the final result
  result = process_result(subresult1, subresult2, subresult3, …)

  # revert the current level states
```

感触

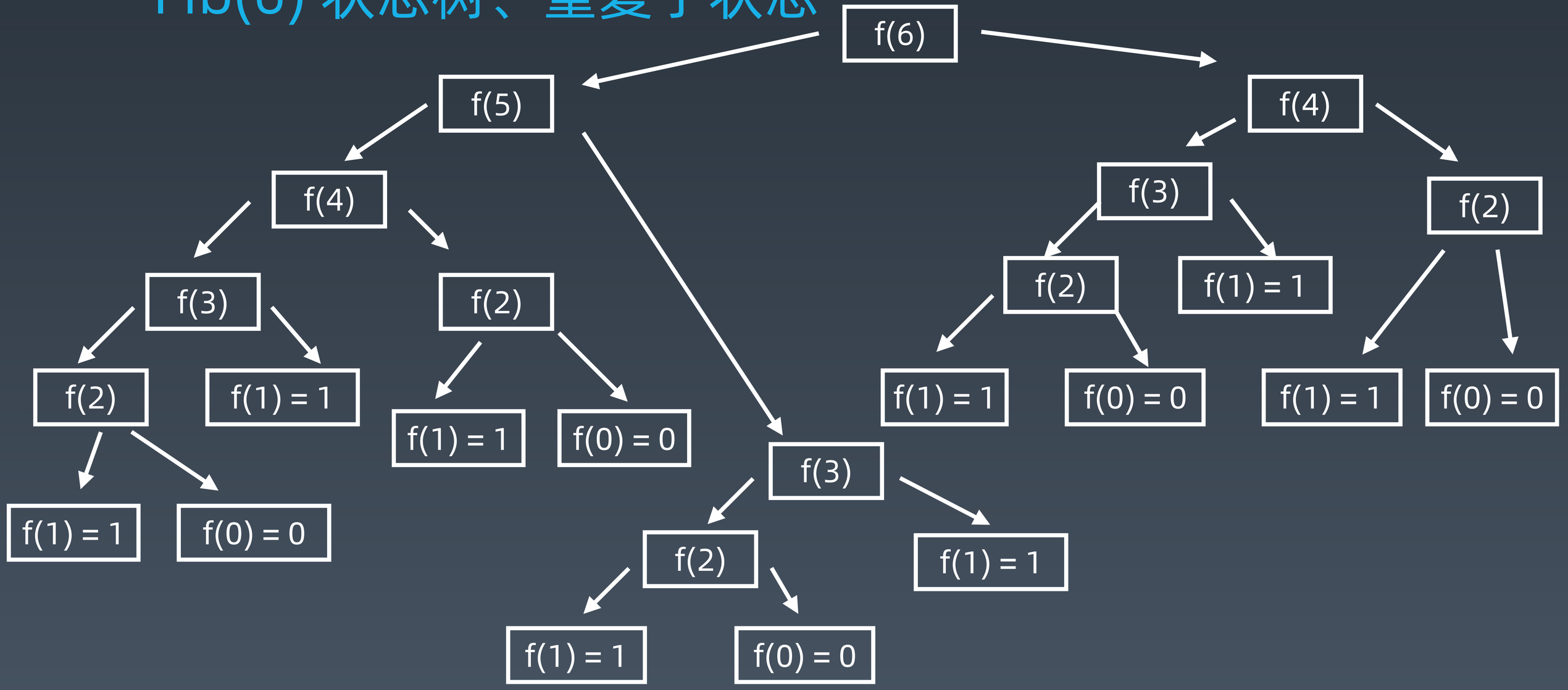
1. 人肉递归低效、很累

2. 找到最近最简方法，将其拆解成可重复解决的问题

3. 数学归纳法思维（抵制人肉递归的诱惑）

本质：寻找重复性 —> 计算机指令集

Fib(6) 状态树、重复子状态



动态规划 Dynamic Programming

1. Wiki 定义：
   https://en.wikipedia.org/wiki/Dynamic_programming

2. “Simplifying a complicated problem by breaking it down into simpler sub-problems”
   (in a recursive manner)

3. Divide & Conquer + Optimal substructure
   分治 + 最优子结构

关键点

动态规划 和 递归或者分治 没有根本上的区别（关键看有无最优的子结构）

共性：找到重复子问题

差异性：最优子结构、中途可以**淘汰**次优解

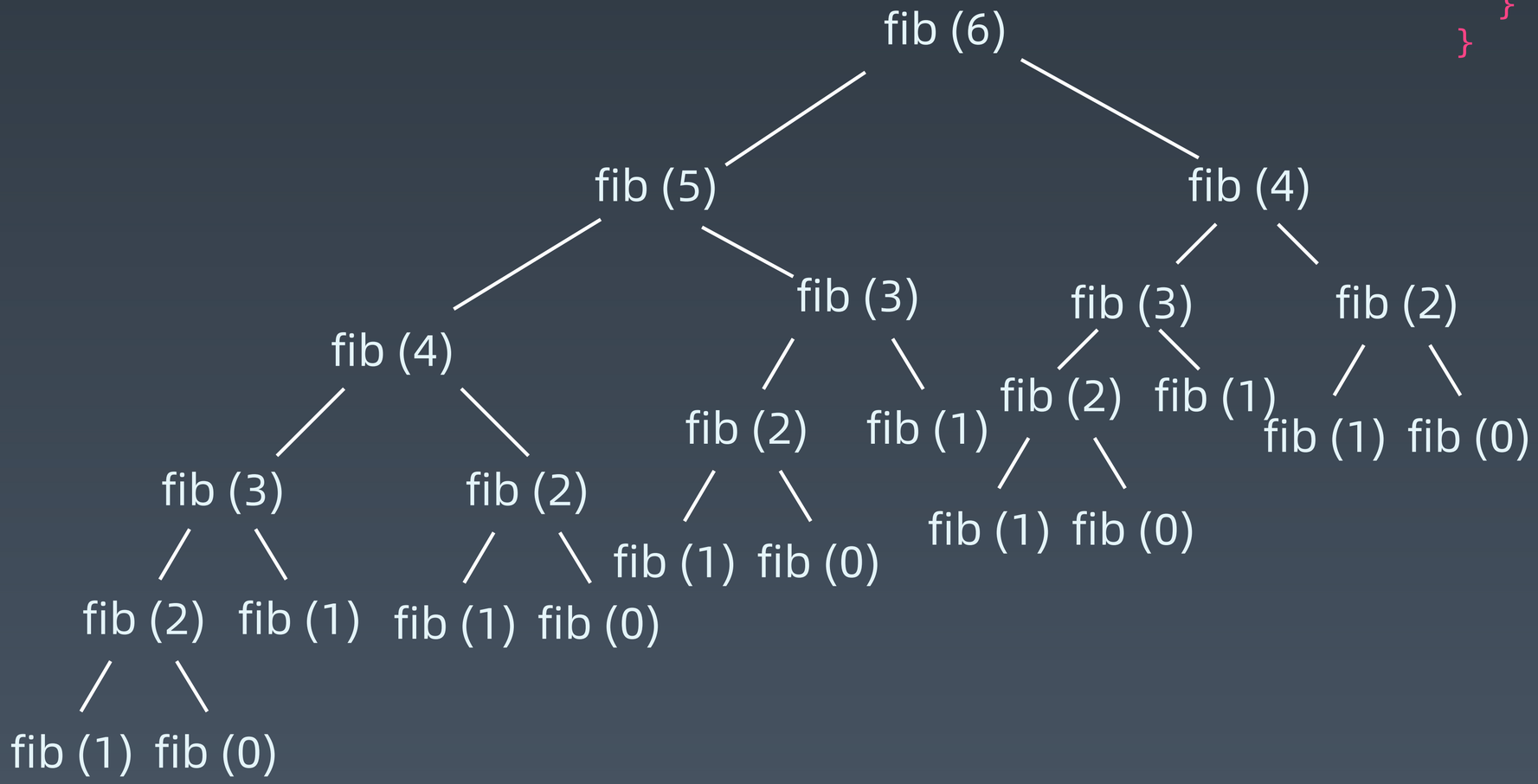
实战例题一
斐波拉契数列

fib (n) = fib (n-1) + fib (n-2)

fib (0) = 0

fib (1) = 1

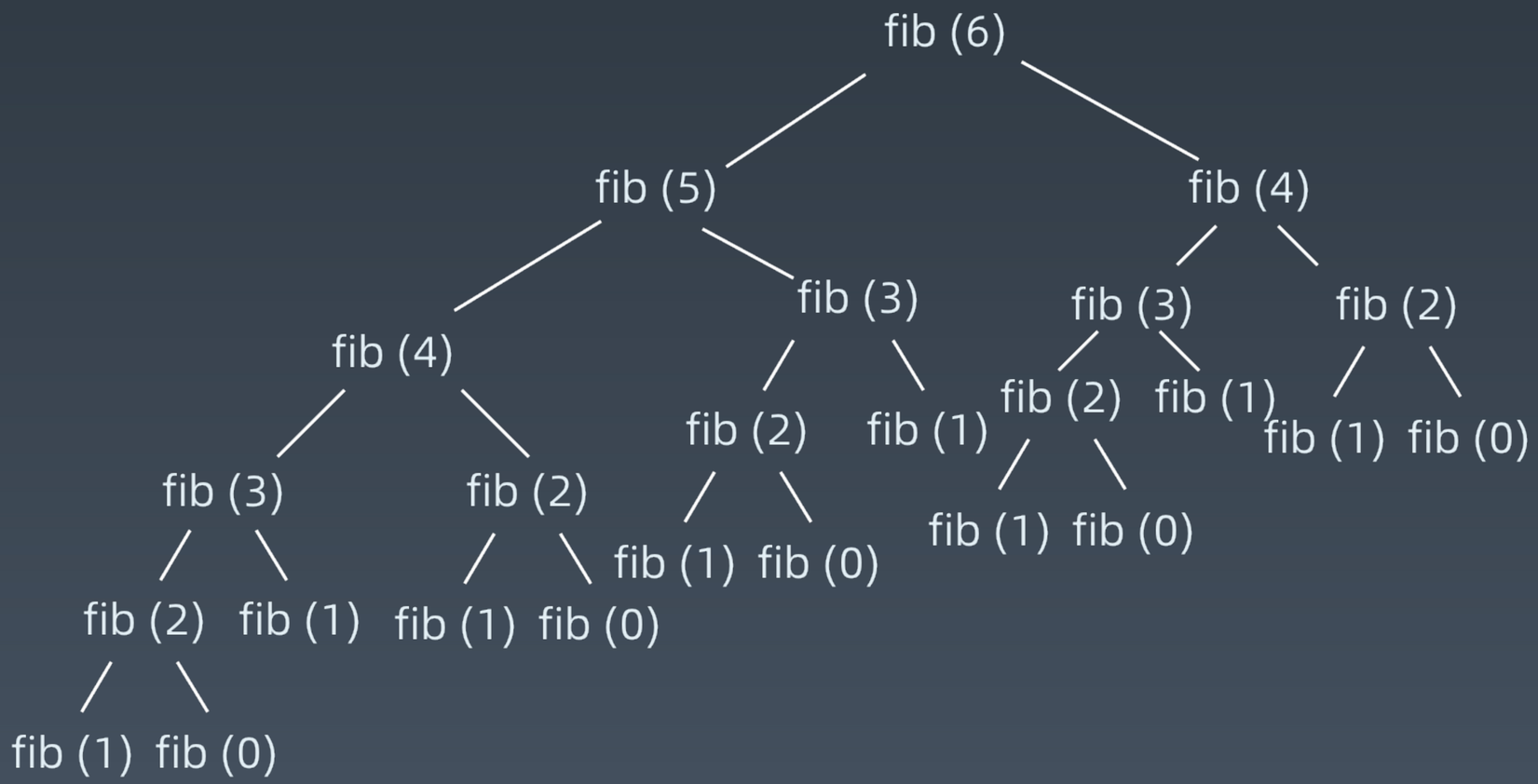
```
int fib (int n) {
  if (n <= 0) {
    return 0;
  } else if (n == 1) {
    return 1;
  } else {
    return fib (n - 1) + fib (n - 2);
  }
}
```



fib (n) = fib (n-1) + fib (n-2)

fib (0) = 0

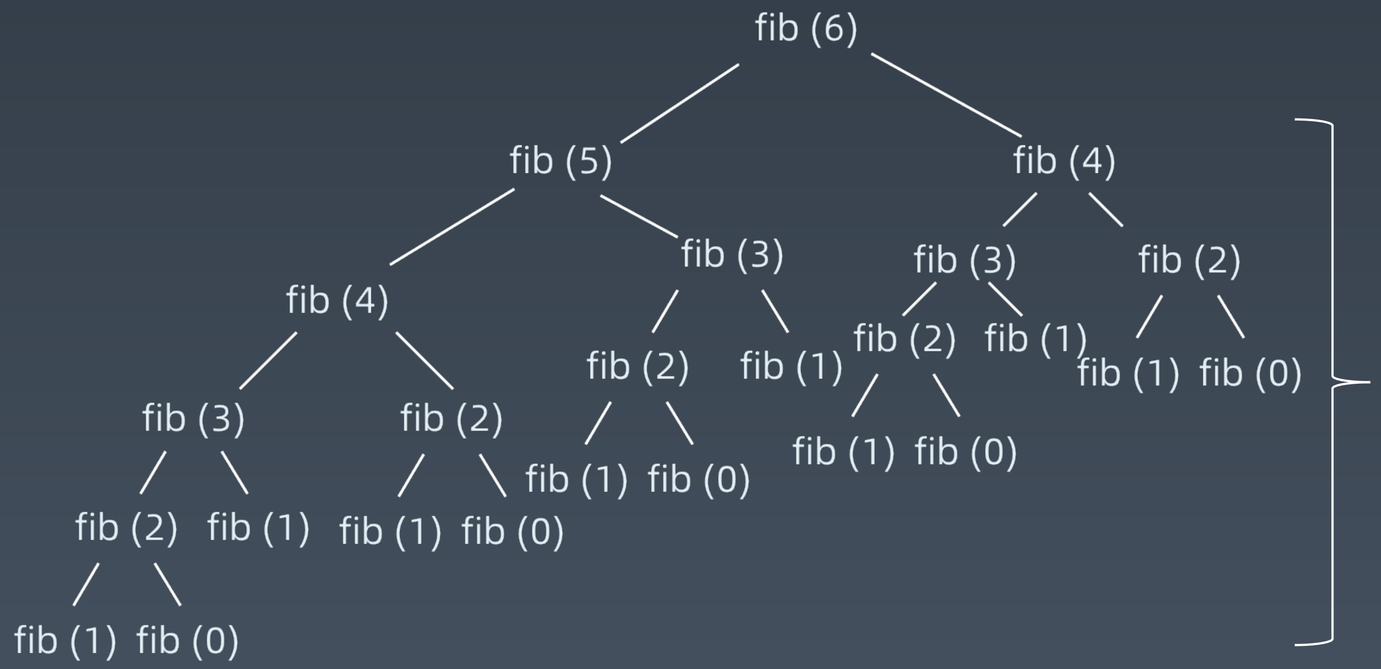
fib (1) = 1




```c
int fib (int n) {
  if (n <= 0) {
    return 0;
  } else if (n == 1) {
    return 1;
  } else {
    return fib (n - 1) + fib (n - 2);
  }
}
```

fib (n) = fib (n-1) + fib (n-2)
fib (0) = 0
fib (1) = 1

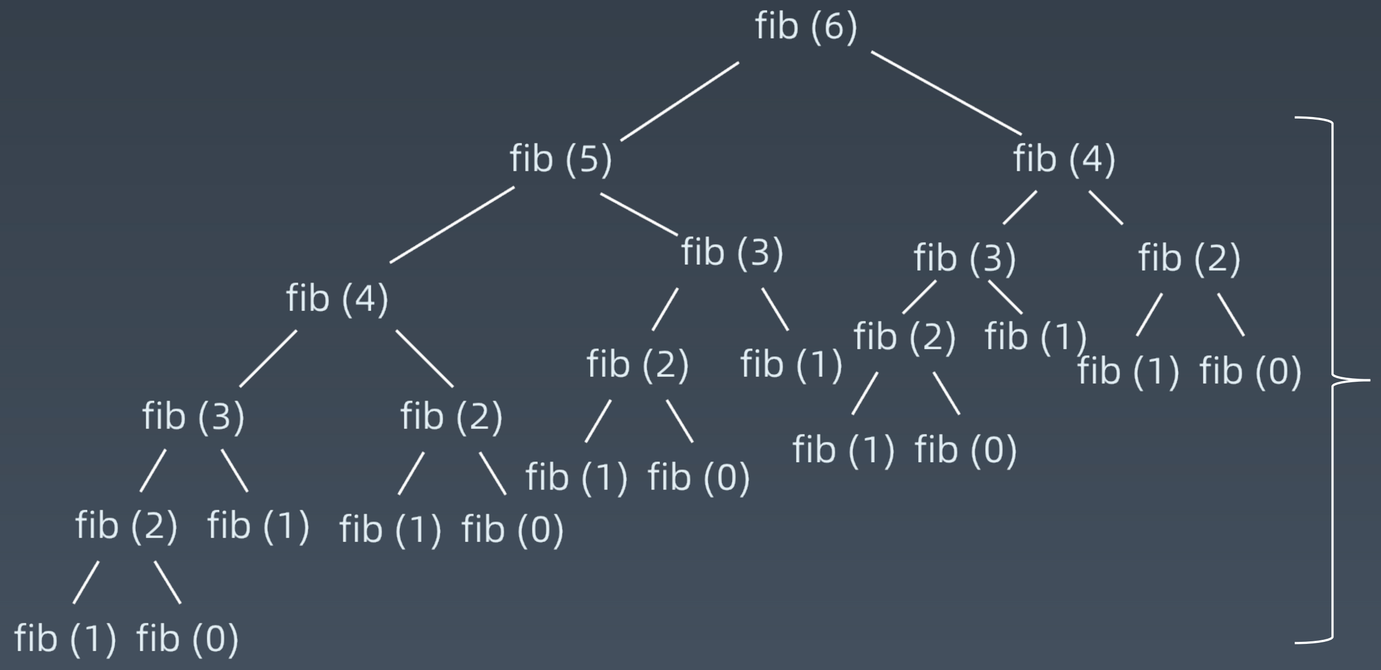
```c
int fib (int n) {
  if (n <= 0) {
    return 0;
  } else if (n == 1) {
    return 1;
  } else {
    return fib (n - 1) + fib (n - 2);
  }
}
```



Call tree has n levels

fib (n) = fib (n-1) + fib (n-2)
fib (0) = 0
fib (1) = 1

```
int fib (int n) {
  if (n <= 0) {
    return 0;
  } else if (n == 1) {
    return 1;
  } else {
    return fib (n - 1) + fib (n - 2);
  }
}
```



Call tree has n levels

Level 1: 1 node
Level 2: 2
Level 3: 4
Level 4: 8

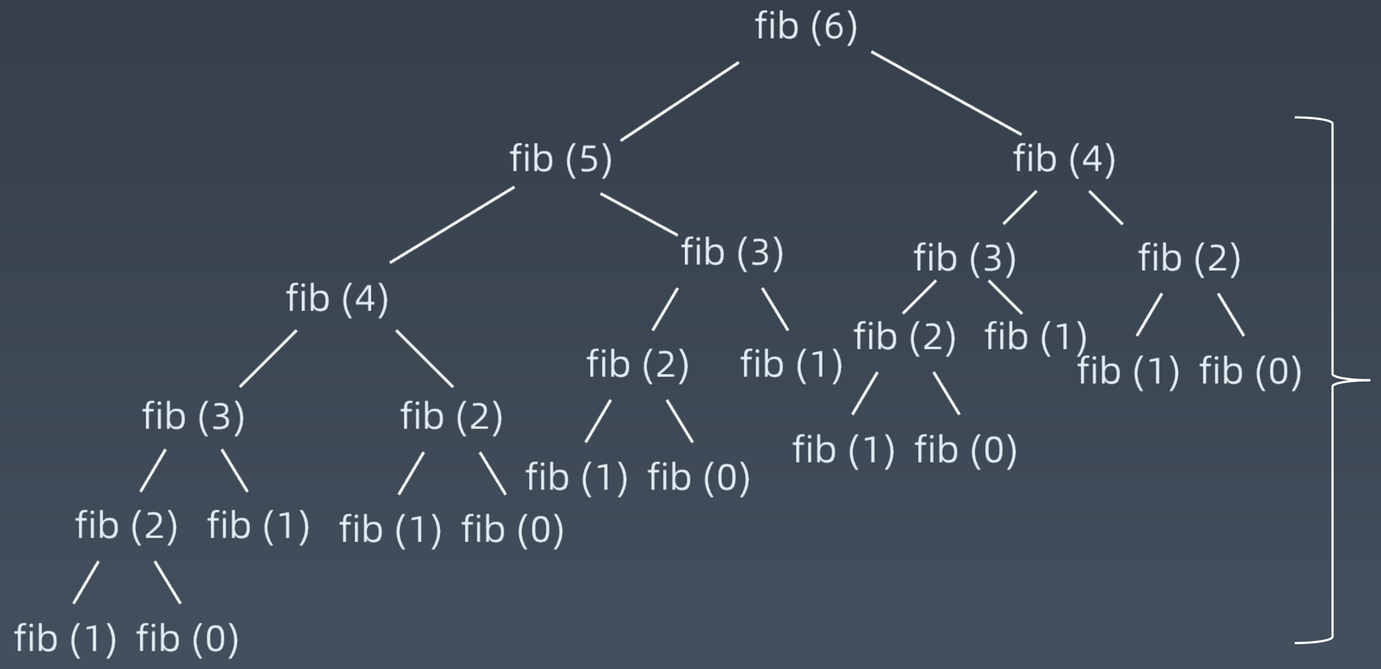
1 x 2 x 2 x ... x 2
= O(2^n)

Java

```
int fib (int n) {
  return n <= 1 ? n : fib (n - 1) + fib (n - 2);
}
```

fib (n) = fib (n-1) + fib (n-2)
fib (0) = 0
fib (1) = 1

```
int fib (int n) {
  if (n <= 0) {
    return 0;
  } else if (n == 1) {
    return 1;
  } else {
    return fib (n - 1) + fib (n - 2);
  }
}
```



Call tree has n levels

Level 1: 1 node
Level 2 : 2
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Level 4: 8

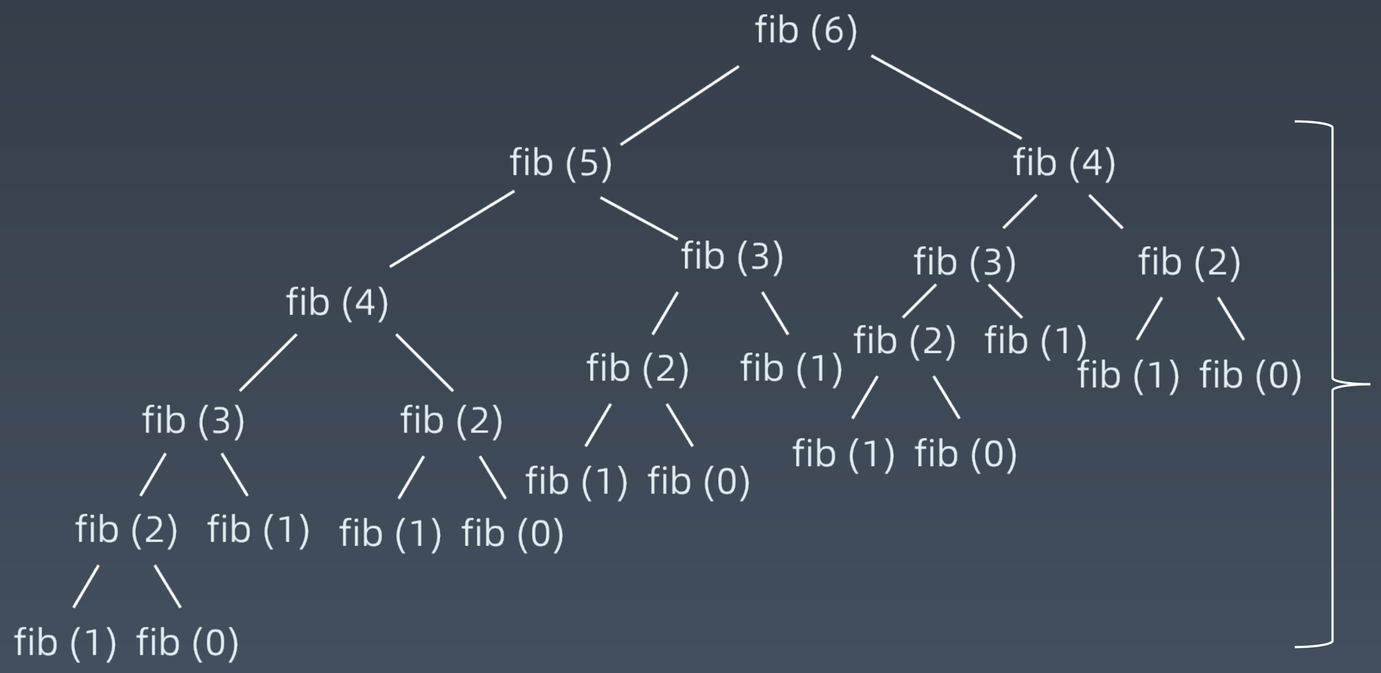
1 x 2 x 2 x ... x 2
= O(2^n)

fib (n) = fib (n-1) + fib (n-2)
fib (0) = 0
fib (1) = 1

```
int fib (int n) {
  if (n <= 0) {
    return 0;
  } else if (n == 1) {
    return 1;
  } else {
    return fib (n - 1) + fib (n - 2);
  }
}
```

```
int fib (int n, int[] memo) {
  if (n <= 1) {
    return n;
  }

  if (memo[n] == 0) {
    memo[n] = fib (n - 1) + fib (n - 2);
  }
  return memo[n];
}
```

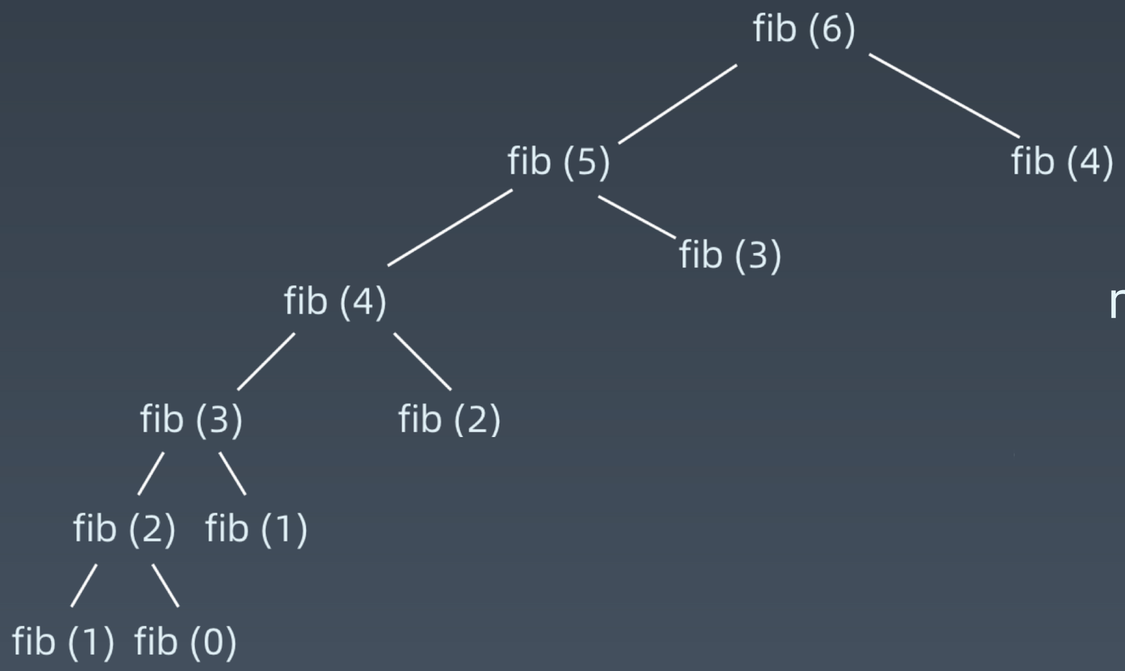


Call tree has n levels

Level 1: 1 node
Level 2 : 2
Level 3: 4
Level 4: 8

1 x 2 x 2 x ... x 2
= O(2^n)

fib (n) = fib (n-1) + fib (n-2)
fib (0) = 0
fib (1) = 1



memo[2] = 1
[3] = 2
[4] = 3
[5] = 5

```
int fib (int n, int[] memo) {
  if (n <= 1) {
    return n;
  }

  if (memo[n] == 0) {
    memo[n] = fib (n - 1) + fib (n - 2);
  }
  return memo[n];
}
```

Memoization

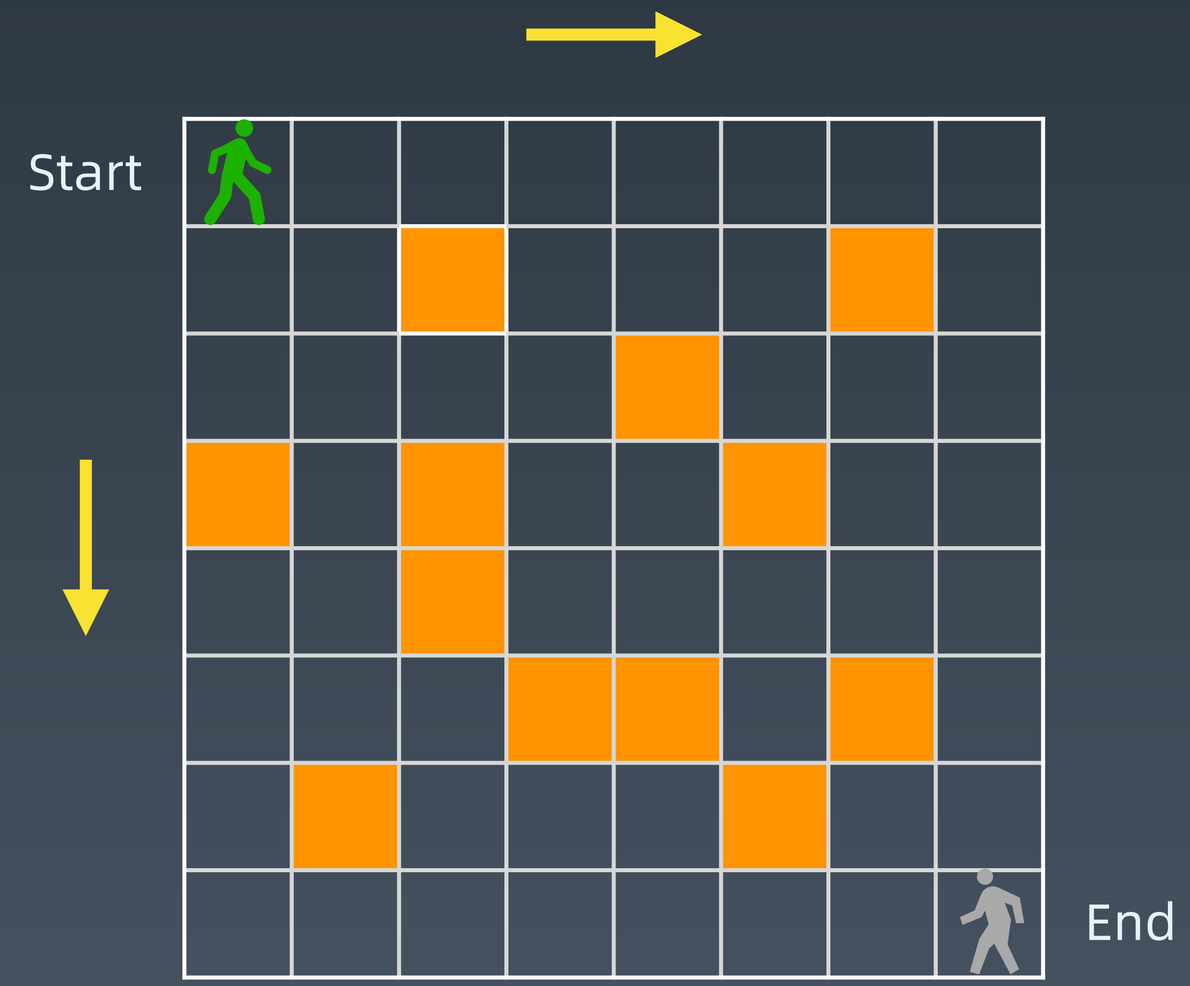
↳ O(n)

Bottom Up

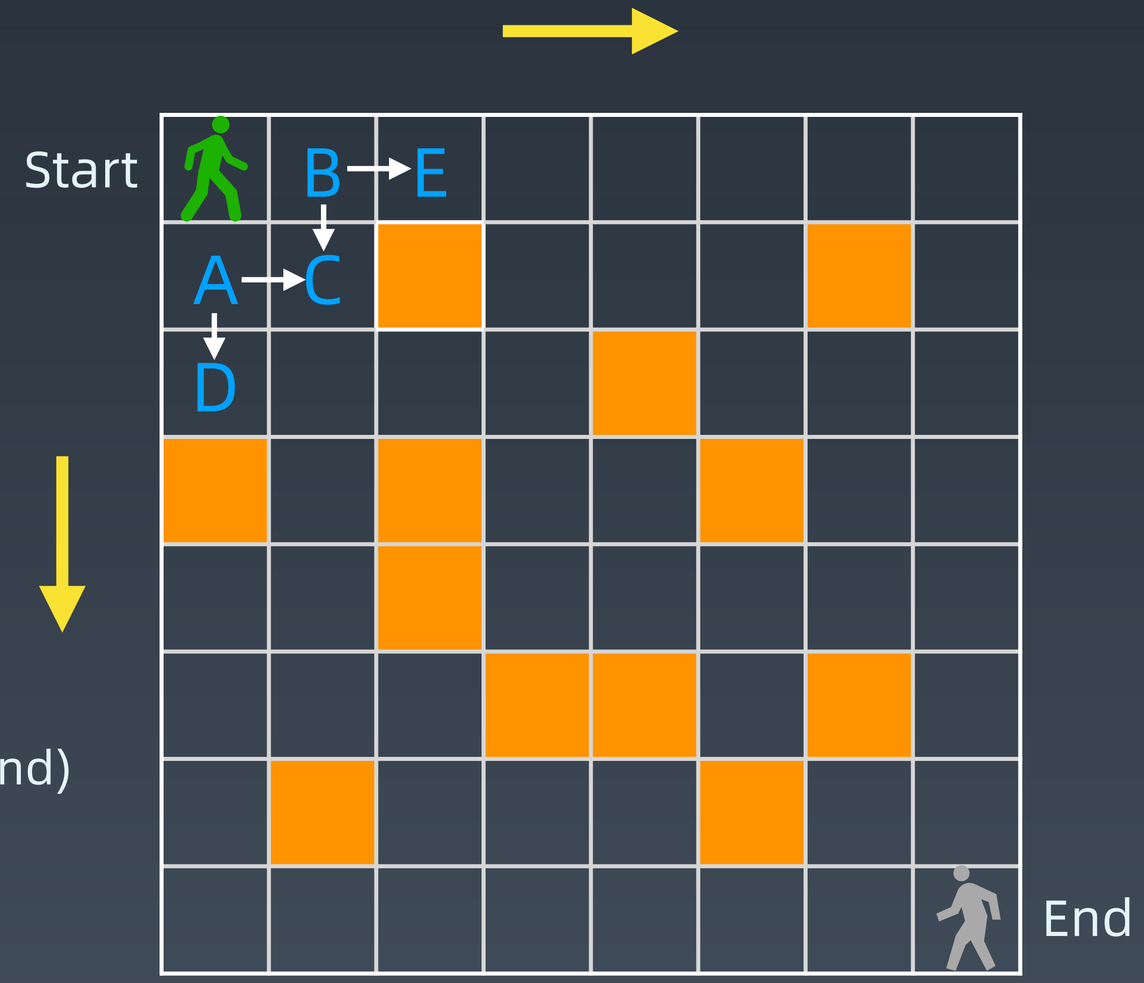
- F[n] = F[n-1] + F[n-2]
- a[0] = 0, a[1] = 1;
  for (int i = 2; i <= n; ++i) {
  a[i] = a[i-1] + a[i-2];
  }
- a[n]
- 0, 1, 1, 2, 3, 5, 8, 13,

实战例题二
路径计数

Count the paths



Count the paths



paths (start, end) =

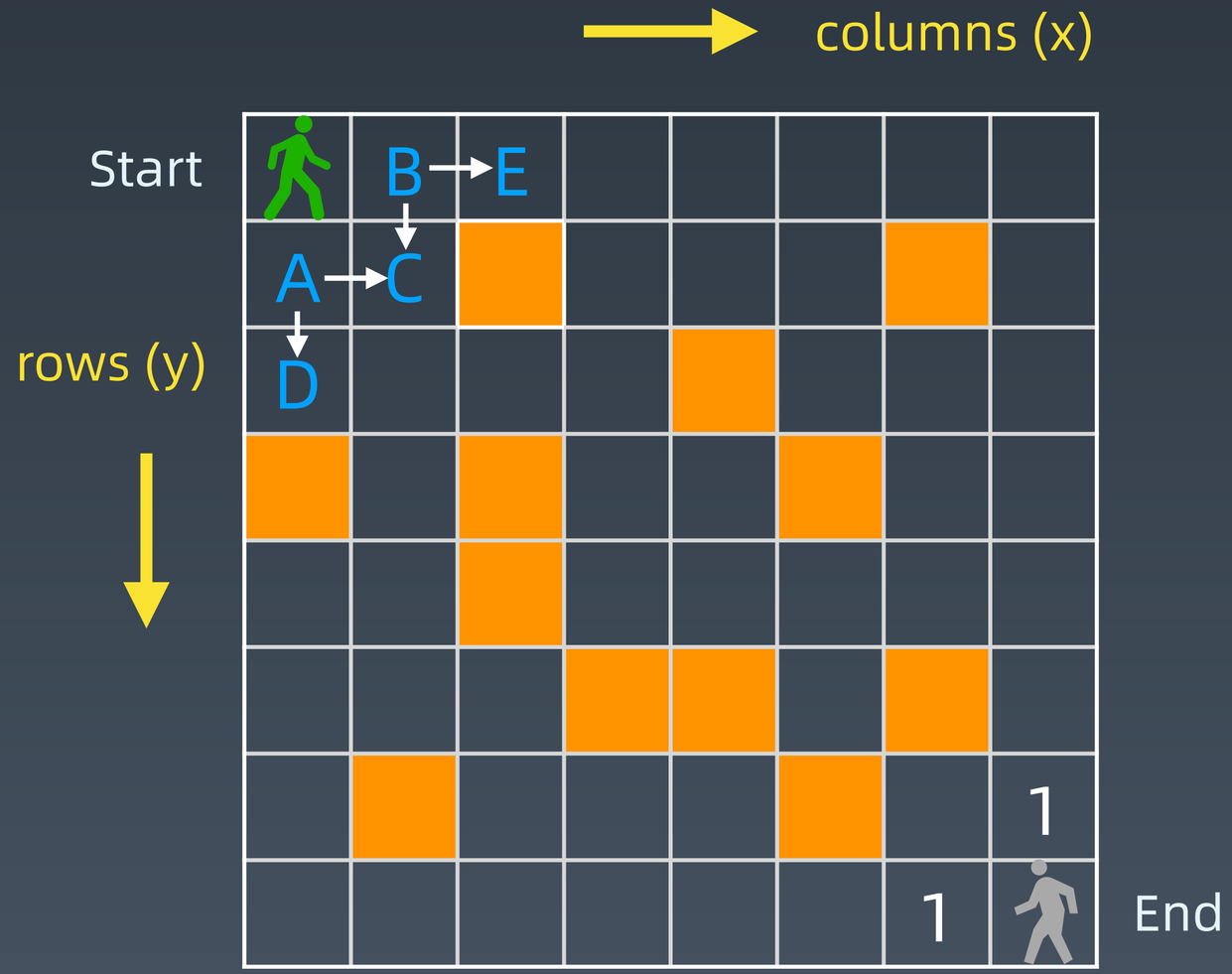
paths (A, end) + paths (B, end)

=

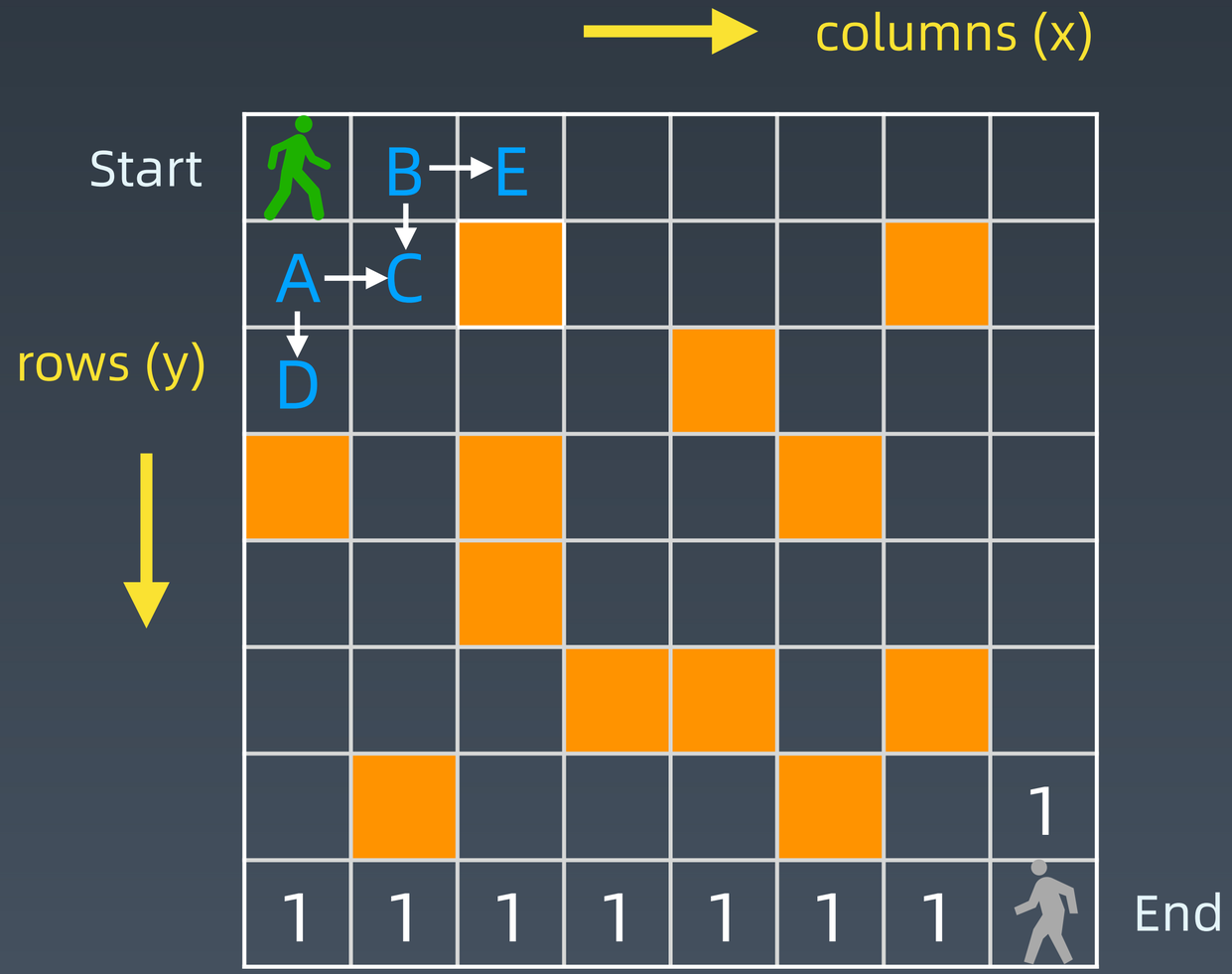
paths (D, end) + paths (C, end) paths (C, end) + paths (E, end)

```
int countPaths (boolean[][]grid, int row, int col) {
  if (!validSquare(grid, row, col)) return 0;
  if (isAtEnd(grid, row, col)) return 1;
  return countPaths (grid, row + 1, col) + countPaths (grid, row, col + 1);
}
```

Count the paths



Count the paths



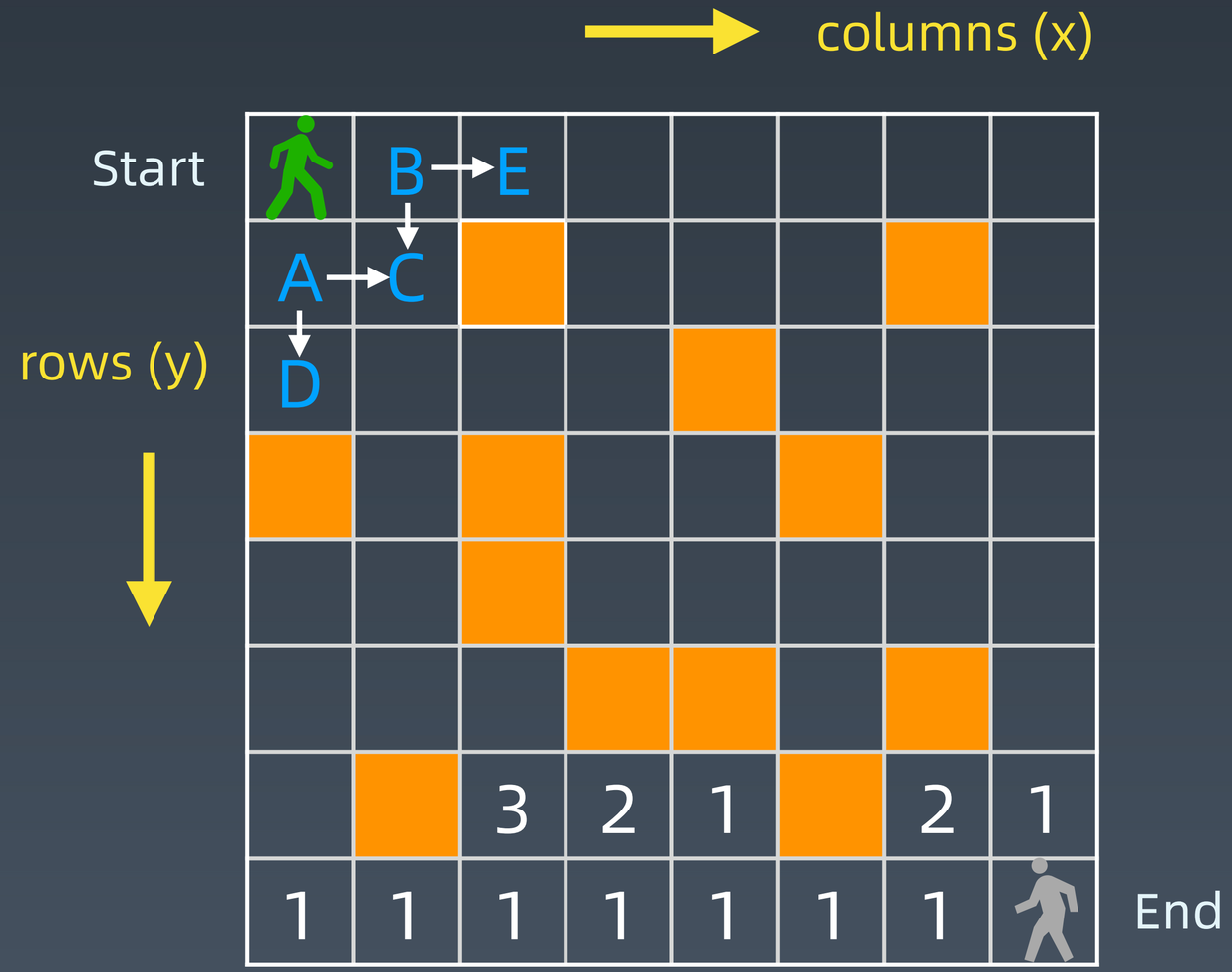
状态转移方程（DP 方程）

opt[i , j] = opt[i + 1, j] + opt[i, j + 1]

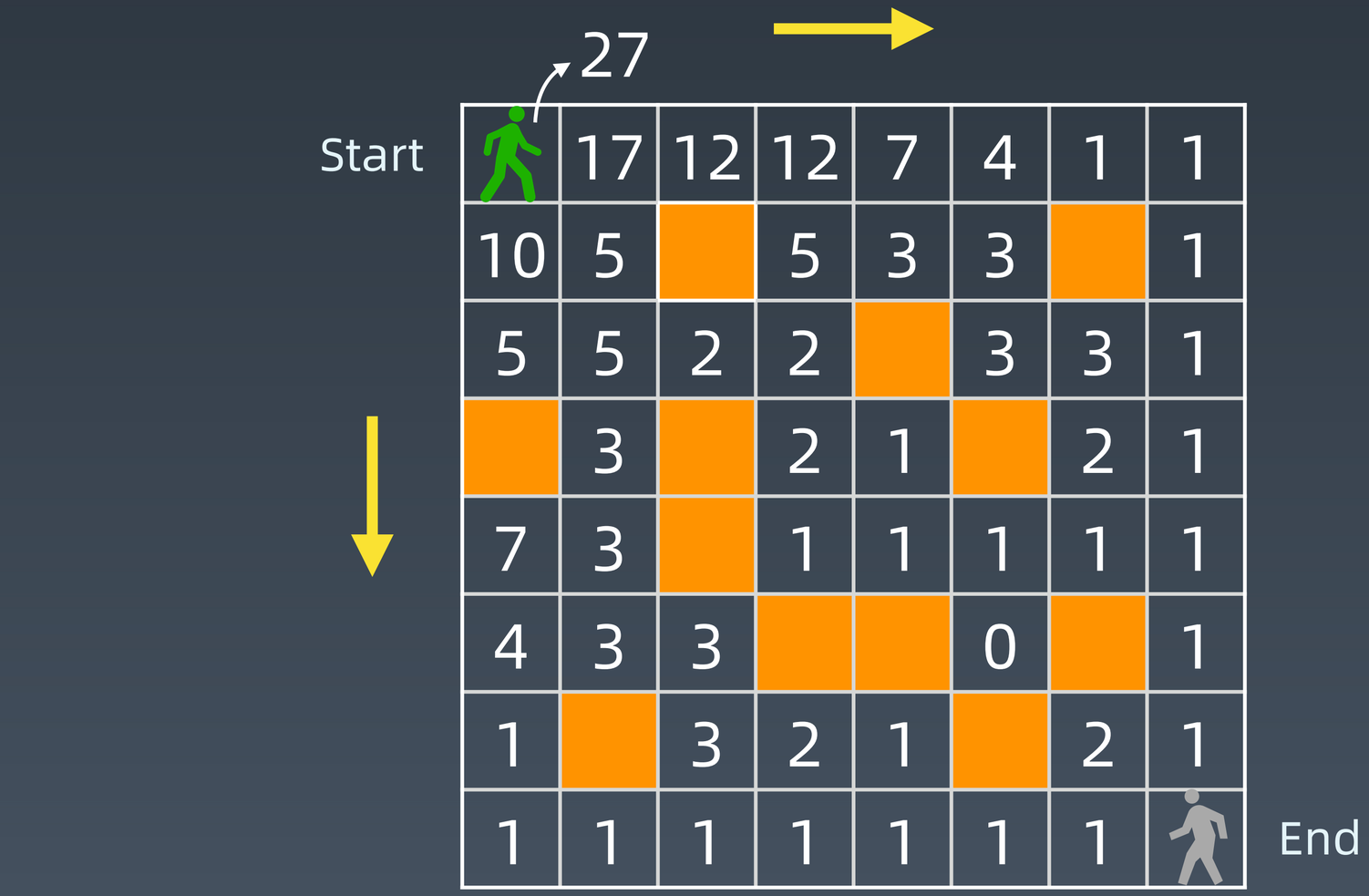
完整逻辑：

```
if a[i, j] = ‘空地’:
    opt[i , j] = opt[i + 1, j] + opt[i, j + 1]
else:
    opt[i , j] = 0
```

Count the paths



Count the paths



动态规划关键点

1. 最优子结构 opt[n] = best_of(opt[n-1], opt[n-2], ...)

2. 储存中间状态：opt[i]

3. 递推公式（美其名曰：状态转移方程或者 DP 方程）

   Fib: opt[i] = opt[n-1] + opt[n-2]

   二维路径：opt[i,j] = opt[i+1][j] + opt[i][j+1] (且判断a[i,j]是否空地）

实战例题三
最长公共子序列

字符串问题

https://leetcode-cn.com/problems/longest-common-subsequence/

给定两个字符串 text1 和 text2，返回这两个字符串的最长公共子序列。

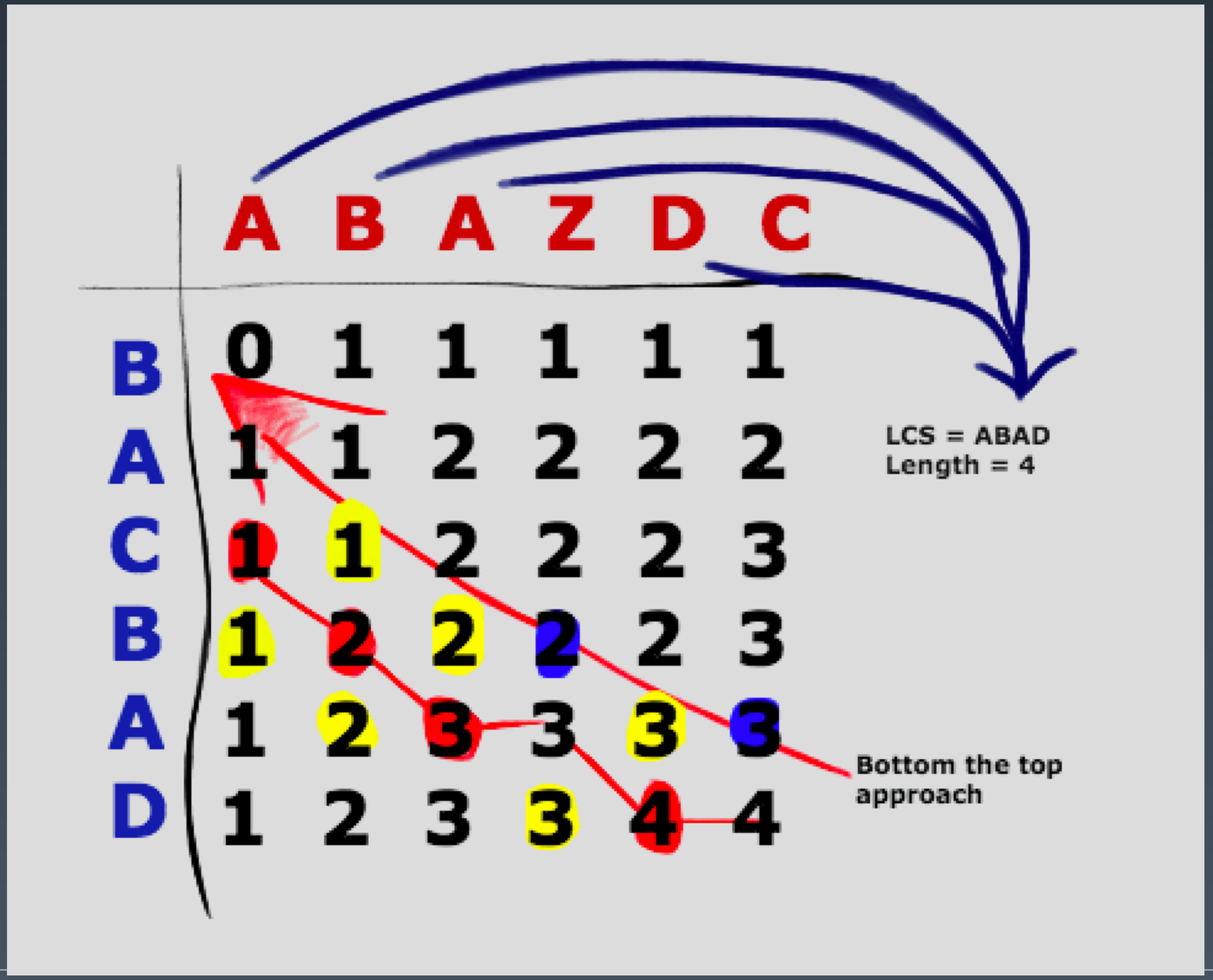
“ABAZDC”，“BACBAD”

字符串问题

1. S1 = “”
   S2 = 任意字符串

2. S1 = “A”
   S2 = 任意

3. S1 = “.....A”
   S2 = “.....A”


A B A Z D C
B 0 1 1 1 1 1
A 1 1 2 2 2 2
C 1 1 2 2 2 3
B 1 2 2 2 2 3
A 1 2 3 3 3 3
D 1 2 3 3 4 4
LCS = ABAD
Length = 4
Bottom the top approach

子问题

- S1 = “ABAZDC”
  S2 = “BACBAD”

- If S1[-1] != S2[-1]: LCS[s1, s2] = Max(LCS[s1-1, s2], LCS[s1, s2-1])

  LCS[s1, s2] = Max(LCS[s1-1, s2], LCS[s1, s2-1], LCS[s1-1, s2-1])

- If S1[-1] == S2[-1]: LCS[s1, s2] = LCS[s1-1, s2-1] + 1

  LCS[s1, s2] = Max(LCS[s1-1, s2], LCS[s1, s2-1], LCS[s1-1, s2-1], LCS[s1-1][s2-1] + 1)

DP 方程

- If S1[-1] != S2[-1]: LCS[s1, s2] = Max(LCS[s1-1, s2], LCS[s1, s2-1])

- If S1[-1] == S2[-1]: LCS[s1, s2] = LCS[s1-1, s2-1] + 1

动态规划小结

1. 打破自己的思维惯性，形成机器思维

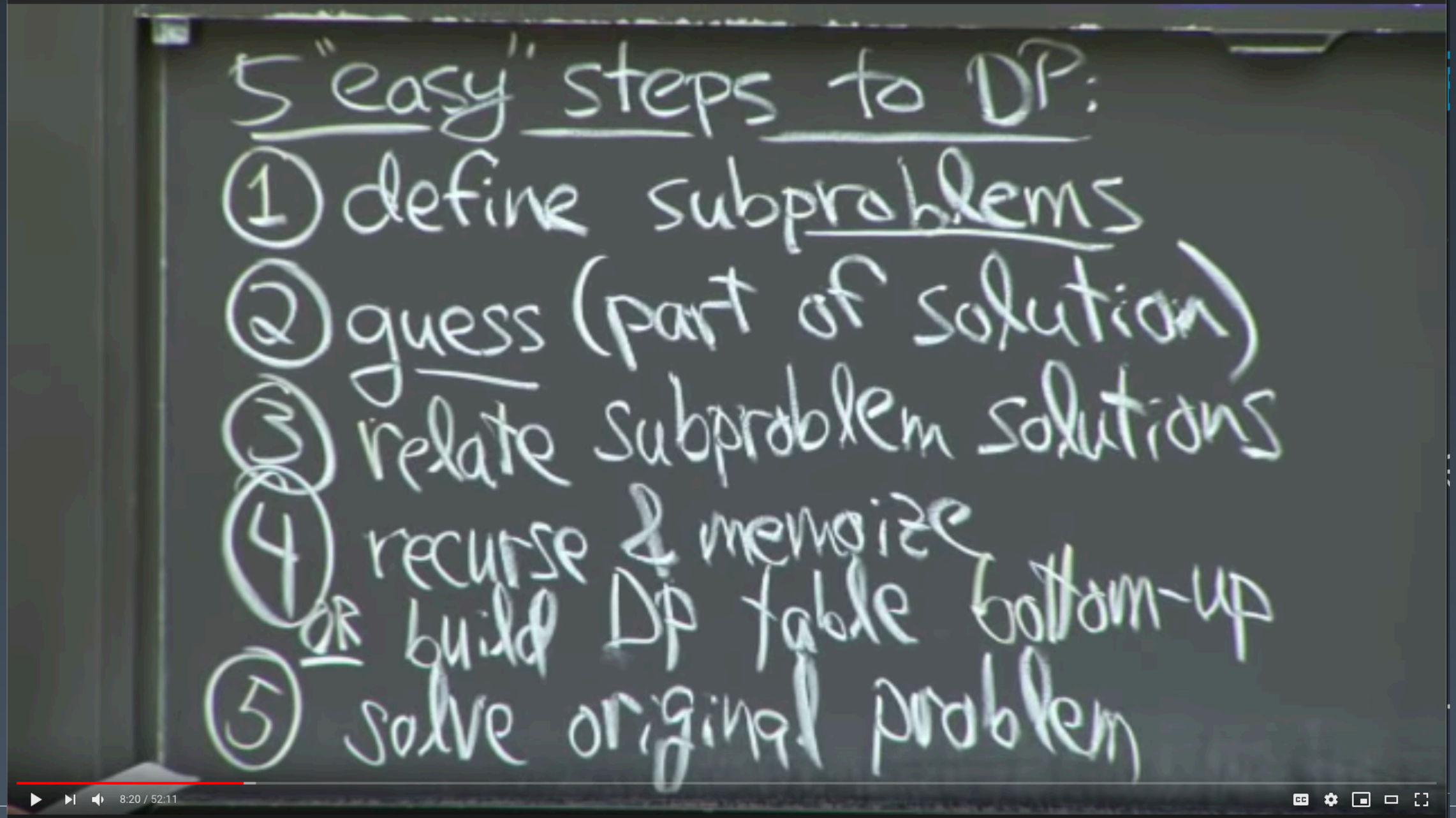
2. 理解复杂逻辑的关键

3. 也是职业进阶的要点要领

MIT algorithm course

B 站搜索： mit 动态规划

https://www.bilibili.com/video/av53233912?from=search&seid=2847395688604491997



20. Dynamic Programming II: Text Justification, Blackjack

295,330 views • Jan 15, 2013

1.6K 42 SHARE SAVE ...

Up next AUTOPLAY

21. DP III: Parenthesization, Edit Distance, Knapsack

实战题目

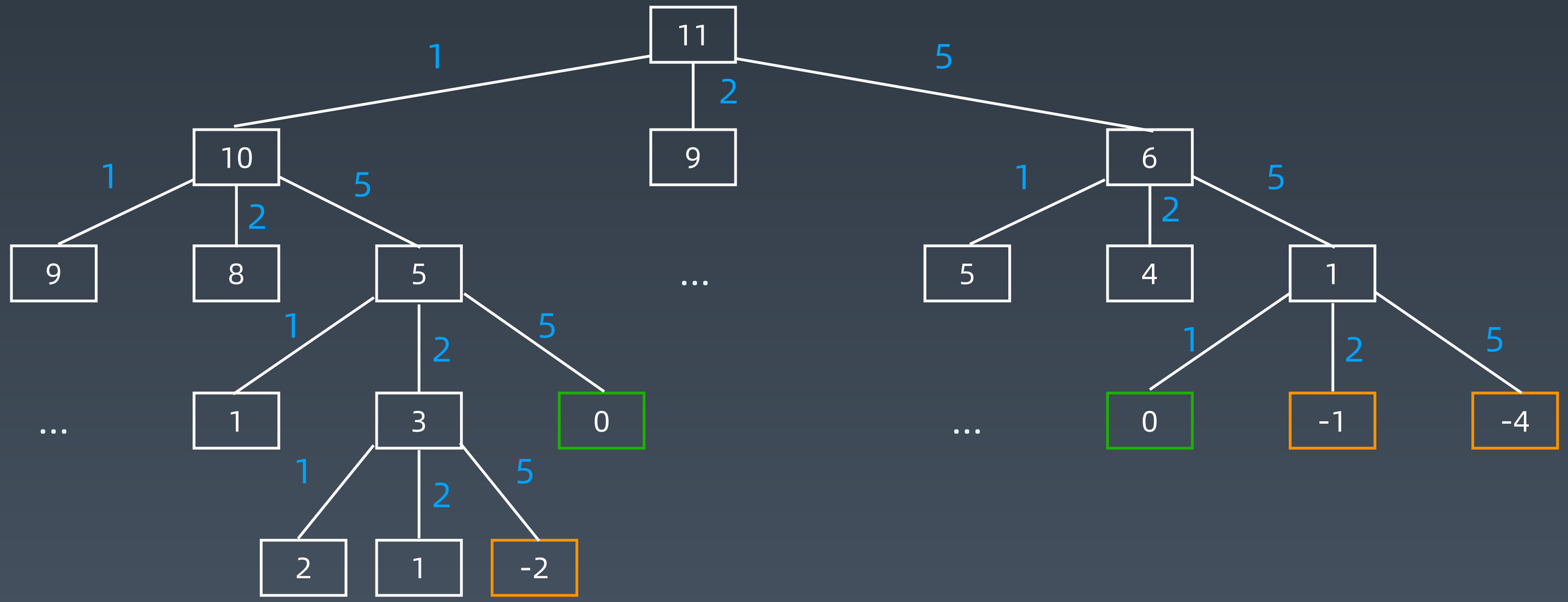
1. https://leetcode-cn.com/problems/climbing-stairs/description/

2. https://leetcode-cn.com/problems/triangle/description/
   (https://leetcode.com/problems/triangle/discuss/38735/Python-easy-to-understand-solutions-(top-down-bottom-up).)

3. https://leetcode-cn.com/problems/maximum-subarray/
   https://leetcode-cn.com/problems/maximum-product-subarray/description/

4. https://leetcode-cn.com/problems/coin-change/description/

Coin change 的状态树



实战题目

1. https://leetcode-cn.com/problems/house-robber/

2. https://leetcode-cn.com/problems/house-robber-ii/description/

3. https://leetcode-cn.com/problems/best-time-to-buy-and-sell-stock/#/description
   https://leetcode-cn.com/problems/best-time-to-buy-and-sell-stock-ii/
   https://leetcode-cn.com/problems/best-time-to-buy-and-sell-stock-iii/
   https://leetcode-cn.com/problems/best-time-to-buy-and-sell-stock-with-cooldown/
   https://leetcode-cn.com/problems/best-time-to-buy-and-sell-stock-iv/
   https://leetcode-cn.com/problems/best-time-to-buy-and-sell-stock-with-transaction-fee/

   https://leetcode-cn.com/problems/best-time-to-buy-and-sell-stock/solution/yi-ge-fang-fa-tuan-mie-6-dao-gu-piao-wen-ti-by-l-3/

实战题目

1. https://leetcode-cn.com/problems/perfect-squares/
2. https://leetcode-cn.com/problems/edit-distance/ （重点）
3. https://leetcode-cn.com/problems/jump-game/
4. https://leetcode-cn.com/problems/jump-game-ii/
5. https://leetcode-cn.com/problems/unique-paths/
6. https://leetcode-cn.com/problems/unique-paths-ii/
7. https://leetcode-cn.com/problems/unique-paths-iii/
8. https://leetcode-cn.com/problems/coin-change/
9. https://leetcode-cn.com/problems/coin-change-2/

Homework

1.https://leetcode-cn.com/problems/longest-valid-parentheses/
2.https://leetcode-cn.com/problems/minimum-path-sum/
3.https://leetcode-cn.com/problems/edit-distance/
4.https://leetcode-cn.com/problems/decode-ways
5.https://leetcode-cn.com/problems/maximal-square/
6.https://leetcode-cn.com/problems/max-sum-of-rectangle-no-larger-than-k/
7.https://leetcode-cn.com/problems/frog-jump/
8.https://leetcode-cn.com/problems/split-array-largest-sum
9.https://leetcode-cn.com/problems/student-attendance-record-ii/
10.https://leetcode-cn.com/problems/task-scheduler/
11.https://leetcode-cn.com/problems/palindromic-substrings/
12.https://leetcode-cn.com/problems/minimum-window-substring/
13.https://leetcode-cn.com/problems/burst-balloons/

THANKS!